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Member American Society of Civil Engineers; Member American Railway Engineering Association; Assistant Professor of Civil Engineering (Railroad Engineering) in the University of Pennsylvania, 1893–1901; Major, Engineer Corps, U. S. A., 1917–1920; etc.

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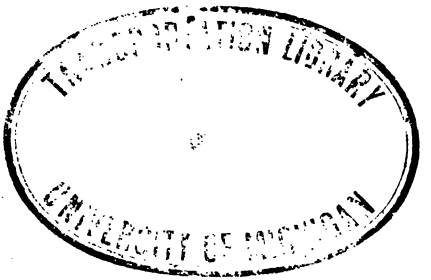
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PREFACE TO SEVENTH EDITION

THE author wishes to reiterate, with even greater emphasis, the statement made in the second paragraph of the preface to the sixth edition. There are few, outside of railroad circles, who realize the great work which is being accomplished by the American Railway Engineering Association. Much of this work has been done during the past five years. One of the notable features is the work of the Special Committee on "Stresses in Track." A very condensed account of the work of this Committee is given in the new added Chapter XXV. Numerous corrections and revisions have also been made throughout this edition to make it conform to the decisions of the recent conventions of the Association.

Some of the more important changes, additions, or developments of subjects, which have been made in this edition, are as follows:

(*a*) The shrinkage of embankments and the subsidence of subsoil under them—Chapter III.

(*b*) Laws governing the life of ties; developments in substitutes for wooden ties—Chapter VIII.

(*c*) Rails; present status of specifications; testing; life of rails; failures; intensity of pressure; rail wear—Chapter IX.

(*d*) Rail joints; causes of failures—Chapter X.

(*e*) Water tanks; principles of construction—Chapter XII.

(*f*) Yards and terminals; hump yards; grades—Chapter XIII (nearly rewritten).

(*g*) Train resistance; resistance of passenger cars, freight cars; resistance through switches—Chapter XVI.

(*h*) Stresses in track, in rails, ties and ballast; static and dynamic stresses—Chapter XXV (new).

WALTER LORING WEBB.

PHILADELPHIA, PA.,
Dec., 1921.

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RAILROAD CONSTRUCTION.

CHAPTER I.

RAILROAD SURVEYS.

THE proper conduct of railroad surveys presupposes an adequate knowledge of almost the whole subject of railroad engineering, and particularly of some of the complicated questions of Railroad Economics, which are not generally studied except at the latter part of a course in railroad engineering, if at all. This chapter will therefore be chiefly devoted to methods of instrumental work, and the problem of choosing a general route will be considered only as it is influenced by the topography or by the application of those elementary principles of Railroad Economics which are self-evident or which may be accepted by the student until he has had an opportunity of studying those principles in detail

The student-engineer should be warned against the hasty and inadequate surveying which has resulted in so much misconstruction in this country. This kind of surveying was especially common forty or fifty years ago, and the methods have more or less continued. The demand for railroad facilities was then so urgent that lax methods were tolerated. A general route would be selected which, at first sight, seemed most obvious and it would be immediately staked out in a manner suitable to a location survey. After correcting some of the most glaring faults, the survey was considered complete and the road was constructed accordingly. The cost of such a survey is comparatively small, but it is almost inevitable that the line is not as good as could have been obtained with a greater amount of

examination and study. The cost of construction and the future cost of operating such a line is always unnecessarily high. The money wasted in construction, plus the capitalized value of the annual waste in future operating expenses, is frequently a hundred times the cost of the extra study and surveying which would have avoided these faults. This has been unquestionably proved by the innumerable cases of reconstruction of portions of old lines which could have been constructed originally on the lines as revised at even less cost. The engineer is not always responsible for ill-advised hasty work. An impatient Board of Directors often insists on commencing to "throw dirt" before a proper survey has been made. The engineer should make, if necessary, the most earnest representations and even strenuous demands, that he be given the requisite time, opportunity and money to conduct his survey in such a manner as to investigate thoroughly every possibility for improving the alinement.

A railroad survey ordinarily consists of three parts: (*a*) the reconnoissance; (*b*) the preliminary survey, and (*c*) the definite location. As explained later, circumstances may modify the relative importance of these divisions, but under ordinary circumstances all three are necessary.

RECONNOISSANCE SURVEYS.

1. Character of a reconnoissance survey. A reconnoissance survey is a very hasty examination of a belt of country to determine which of all possible or suggested routes is the most promising and best worthy of a more detailed survey. It is essentially very rough and rapid. It aims to discover those salient features which instantly stamp one route as distinctly superior to another and so narrow the choice to routes which are so nearly equal in value that a more detailed survey is necessary to decide between them.

A map should be prepared, at a scale not smaller than one mile to the inch, which should show all general routes which are conceivably possible. It is particularly important that the mere lack of data should not exclude consideration of some ral route which might be superior to the one or more obvious which have already been picked out.

2. Selection of a general route. The general question of running a railroad between two towns is frequently a financial rather than an engineering question. Financial considerations usually determine that a road must pass through certain more or less important towns between its termini. It is also possible that there may be certain topographical features in any route between two determined towns on the line, such as a low saddle in crossing a ridge or a difficult crossing of a large river, which, with the towns, may be considered as **control points**, and the problem may be narrowed down to the determination of the best route between these consecutive control points. But care should be taken that control points are not too hastily considered as fixed and unalterable, especially if it results in very unfavorable grades and alinement between consecutive points.

The reconnoissance survey should include the determination of the location and relative elevations of all these control points. These data should be obtained with sufficient accuracy to compute the necessary ruling grade and the general character of the alinement, and the map as thus amplified should be studied by comparing the several possible routes and eliminating all those which are unquestionably less favorable than others.

The engineer should avoid, especially in a rough and wooded country, the influence that an existing highway, or even a path through the woods or of a clearing of the trees, may have in determining the choice of routes. Mere ease of travel, as long as it is not glaringly wrong, has caused many prepossessions in favor of a certain route, when a much better line could be obtained by plunging through the woods or over swampy or rocky ground. As a first trial in selecting the route, the bearing of a line joining two consecutive control points should be determined and then an effort should be made to find a general route which will have the least possible variation from that straight line, without sacrificing the limits of ruling grade, curvature and general type or cost of construction which may have been fixed for the road.

A difficult line between two control points should be studied by beginning at either end for two independent studies. The very obvious route, starting from A toward B, may lead into very difficult construction, which may be avoided by com-

mencing at *B* and finally reaching *A* on a route which, while practicable, would not be considered attractive when starting from *A*.

When a railroad runs through a thickly settled and very flat country, where, from a topographical standpoint, the road may be run by any desired route, the "right-of-way agent" sometimes has a greater influence in locating the road than the engineer. But such modifications of alinement, on account of business considerations, are foreign to the engineer's side of the subject, and it will be hereafter assumed that topography alone determines the location of the line. The consideration of those larger questions combining finance and engineering (such as passing by a town on account of the necessary introduction of heavy grades in order to reach it), will be considered in later chapters.

3. Valley route. This is perhaps the simplest problem. If two control points to be connected lie in the same valley, it is frequently only necessary to run a line which shall have a nearly uniform grade. The reconnoissance problem consists largely in determining the difference of elevation of the two termini of this division and the approximate horizontal distance so that the proper grade may be chosen. If there is a large river running through the valley, the road will probably remain on one side or the other throughout the whole distance, and both banks should be examined by the reconnoissance party to determine which is preferable. If the river may be easily bridged, both banks may be alternately used, especially when better alinement is thereby secured. A river valley has usually a steeper slope in the upper part than in the lower part. A uniform grade throughout the valley will therefore require that the road climbs up the side slopes in the lower part of the valley. In case the "ruling grade"* for the whole road is as great as or greater than the steepest natural valley slope, more freedom may be used in adopting that alinement which has the least cost—regardless of grade. The natural slope of large rivers is almost invariably so low that grade has no influence in determining the choice of location. When bridging is necessary, the river

* The *ruling grade* may here be loosely defined as the maximum grade ch is permissible. This definition is not strictly true, as may be seen later n studying Railroad Economics, but it may here serve the purpose.

banks should be examined for suitable locations for abutments and piers. If the soil is soft and treacherous, much difficulty may be experienced and the choice of route may be largely determined by the difficulty of bridging the river except at certain favorable places.

4. Cross-country route. A cross-country route always has one or more summits to be crossed. The problem becomes more complex on account of the greater number of possible solutions and the difficulty of properly weighing the advantages and disadvantages of each. The general aim should be to choose the lowest summits and the highest stream crossings, provided that by so doing the grades between these determining points shall be as low as possible and shall not be greater than the ruling grade of the road. Nearly all railroads combine cross-country and valley routes to some extent. Usually the steepest natural slopes are to be found on the cross-country routes, and also the greatest difficulty in securing a low through grade. An approximate determination of the ruling grade is usually made during the reconnoissance. If the ruling grade has been previously decided on by other considerations, the leading feature of the reconnoissance survey will be the determination of a general route along which it will be possible to survey a line whose maximum grade shall not exceed the ruling grade.

5. Mountain route. The streams of a mountainous region frequently have a slope exceeding the desired ruling grade. In such cases there is no possibility of securing the desired grade by following the streams. The penetration of such a region may only be accomplished by "development"—accompanied perhaps by tunneling. "Development" consists in deliberately increasing the length of the road between two extremes of elevation so that the rate of grade shall be as low as desired. The usual method of accomplishing this is to take advantage of some convenient formation of the ground to introduce some lateral deviation. The methods may be somewhat classified as follows:

(a) Running the line up a convenient lateral valley, turning a sharp curve and working back up the opposite slope. As shown in Fig. 1, the considerable rise between A and B was surmounted by starting off in a very different direction from the general direction of the road; then, when about one-half of the desired rise had been obtained, the line crossed the valley

and continued the climb along the opposite slope. (b) *Switch back.* On the steep side-hill *BCD* (Fig. 1) a very considerabl gain in elevation was accomplished by the switchback *CD* The gain in elevation from *B* to *D* is very great. On the othe hand, the speed must always be slow; there are two complet stoppages of the train for each run; all trains must run back ward from *C* to *D*. (c) *Bridge spiral.* When a valley is s narrow at some point that a bridge or viaduct of reasonabl length can span the valley at a considerable elevation above th

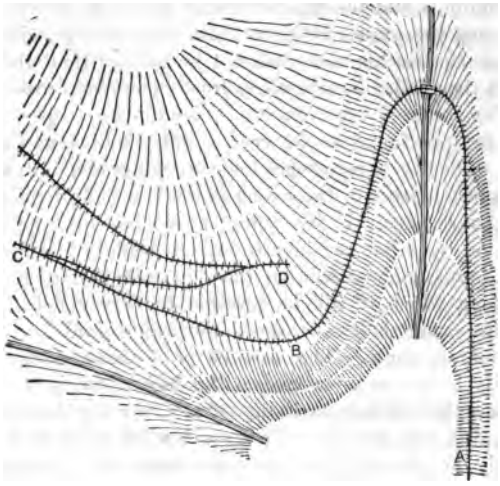
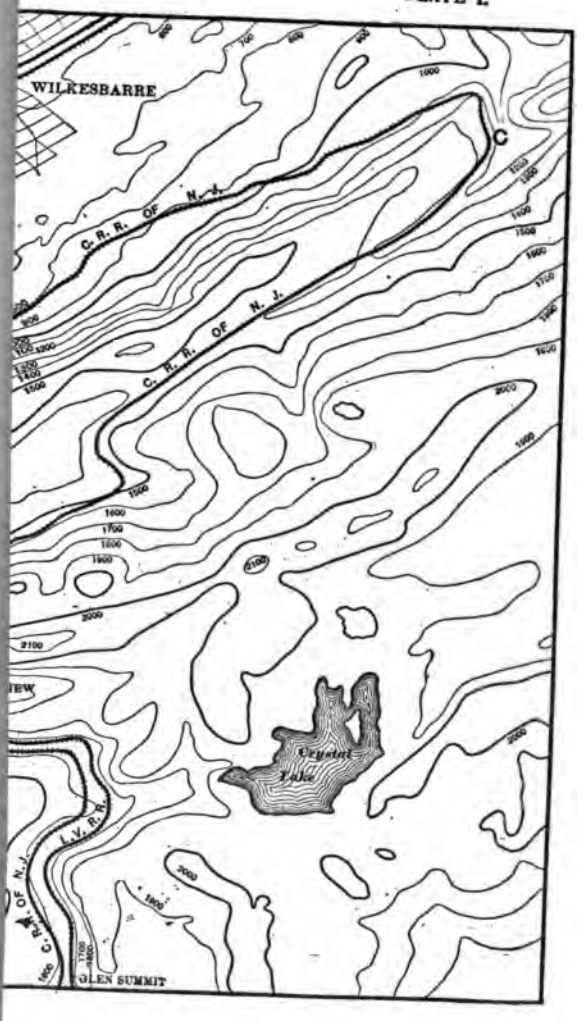


FIG. 1.

bottom of the valley, a bridge spiral may be desirable. In Fig. the line ascends the stream valley past *A*, crosses the stream a *B*, works back to the narrow place at *C*, and there crosses itself having gained perhaps 100 feet in elevation. (d) *Tunne spiral* (Fig. 3). This is the reverse of the previous plan. I implies a thin steep ridge, so thin at some place that a tunne through it will not be excessively long. Switchbacks an spirals are sometimes necessary in mountainous countries, bu they should not be considered as normal types of construction A region must be very difficult if these devices cannot b avoided.

PLATE I.



On Plate I are shown three separate ways (as actually constructed) of running a railroad between two points a little over three miles apart and having a difference of elevation of nearly 1100 feet. At *A* the Central R. R. of New Jersey runs *under* the Lehigh Valley R. R. and soon turns off to the northeast for about six miles, then doubles back, reaching *D*, a fall of about 1050 feet with a track distance of about 12.7 miles. The L. V. R. R. at *A* runs to the westward for six to seven miles,

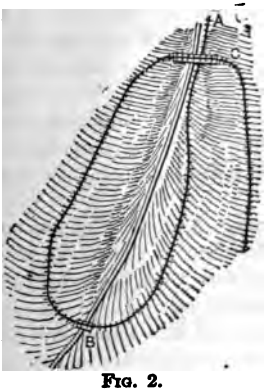


FIG. 2.

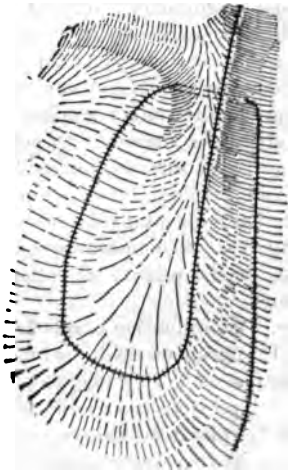


FIG. 3.

then turns back until the roads are again close together at *D*. The track distance is about 14 miles and the drop a little greater, since at *A* the L. V. R. R. crosses *over* the other, while at *D* they are at practically the same level. From *B* to *C* the distance is over eleven miles. From *A* directly down to *D* the C. R. R. of N. J. runs a "gravity" road, used exclusively for freight, on which cars alone are hauled by cable. The main-line routes are remarkable examples of sheer "development." Even as constructed the L. V. R. R. has a grade of about 95 feet per mile, and this grade has proved so excessive for freight work that the company has constructed a cut-off (not shown on the map) which leaves the main line at *A*, nearly parallels the

C. R. R. to *C*, and then running in a northeasterly direction again joins the main line beyond Wilkesbarre. The grade is thereby cut down to 65 feet per mile.

Rack railways and cable roads, although types of mountain railroad construction, will not be here considered.

6. Existing maps. The maps of the U. S. Geological Survey are exceedingly valuable as far as they have been completed. So far as topographical considerations are concerned, they almost dispense with the necessity for the reconnoissance and "first preliminary" surveys. Some of the State Survey maps will give practically the same information. County and township maps can often be used for considerable information as to the relative *horizontal* position of governing points, and even some approximate data regarding elevations may be obtained by a study of the streams. Of course such information will not dispense with surveys, but will assist in so planning them as to obtain the best information with the least work. When the relative horizontal positions of points are reliably indicated on a map, the reconnoissance may be reduced to the determination of the relative elevations of the governing points of the route.

7. Determination of relative elevations. A recent description of European methods includes spirit-leveling in the reconnoissance work. This may be due to the fact that, as indicated above, previous topographical surveys have rendered unnecessary the "exploratory" survey which is required in a new country, and that their reconnoissance really corresponds more nearly to our preliminary.

The perfection to which barometrical methods have been brought has rendered it possible to determine differences of elevation with sufficient accuracy for reconnoissance purposes by the combined use of a mercurial and an aneroid barometer. The mercurial barometer should be kept at "headquarters," and readings should be taken on it at such frequent intervals that any fluctuation is noted, and throughout the period that observations with the aneroid are taken in the field. At each observation there should also be recorded the time, the reading of the attached thermometer, and the temperature of the external air. For uniformity, the mercurial readings should then be "reduced to 32° F." The form of notes for the mercurial barometer readings should be as follows:

Time.	Merc. Barom.	Attached Therm.	Reduction to 32° F.	External Therm.	Corrected reading.
7:00 A. M.	29.872	72°	—.117	73°	29.755
:15	.866	73.5	.121	75	.745
:30	.858	75	.125	76	.733
:45	.850	76	.127	77	.723

The corrections in column 4 are derived from Table XI by interpolation.

Before starting out, a reading of the aneroid should be taken at headquarters coincident with a reading of the mercurial. The difference is one value of the correction to the aneroid. As soon as the aneroid is brought back another comparison of readings should be made. Even though there has been considerable rise or fall of pressure in the interval, the *difference* in readings (the correction) should be substantially the same provided the aneroid is a good instrument. If the difference of elevation is excessive (as when climbing a high mountain) even the best aneroid will "lag" and not recover its normal reading for several hours, but this does not apply to such differences of elevation as are met with in railroad work. The best aneroids read directly to $\frac{1}{100}$ of an inch of mercury and may be estimated to $\frac{1}{1000}$ of an inch—which corresponds to about 0.9 foot difference of elevation. In the field there should be read, at each point whose elevation is desired, the aneroid, the time, and the temperature. These readings, corrected by the mean value of the correction between the aneroid and the mercurial, should then be combined with the reading of the mercurial (interpolated if necessary) for the times of the aneroid observations and the difference of elevation obtained. The field notes for the aneroid should be taken as shown in the first four columns of the tabular form. The "corrected aneroid" readings of column 5 are found by correcting the readings of column 3 by the mean difference between the mercurial and aneroid when compared at morning and night Column 6 is a copy of the "corrected readings" from the office notes, interpolated when necessary for the proper time. Column 7 is similarly obtained. Col. 8 is obtained from cols. 4 and 5, and col. 9 from cols. 6 and 7, with the aid of Table XII. The correction for temperature (col. 11), which is generally small unless the difference of elevation is large, is obtained with the

(Left-hand page of Notes.)

Time.	Place.	Aneroid.	Therm.	Corr. Aner.	Corr. Merc.
7:00	Office	29.628	73°	.....	29.755
7:10	Δ0	29.662	72°	29.789	29.748
7:30	saddle-back	29.374	63°	29.501	29.733
7:50	river cross.	29.548	70°	29.675	29.720

aid of Table XIII. The elevations in Table XII are elevations above an assumed datum plane, where under the given atmospheric conditions the mercurial reading would be 30″. Of course the position of this assumed plane changes with varying atmospheric conditions and so the elevations are to be considered as *relative* and their difference taken. See "Technic of Surveying Instruments and Methods," Prob. 28, by Webb and Fish; John Wiley & Sons. Important points should be observed more than once if possible. Such duplicate observations will be found to give surprisingly concordant results even when a general fluctuation of atmospheric pressure so modifies the tabulated readings that an agreement is not at first apparent. Variations of pressure produced by high winds, thunder-storms, etc., will generally vitiate possible accuracy by this method. By "headquarters" is meant any place whose elevation above any given datum is known and where the mercurial may be placed and observed while observations within a range of several miles are made with the aneroid. If necessary, the elevation of a new headquarters may be determined by the above method, but there should be if possible several independent observations whose accordance will give a fair idea of their accuracy.

The above method should be neither slighted nor used for more than it is worth. When properly used, the errors are compensating rather than cumulative. When used, for example, to determine that a pass *B* is 260 feet higher than a determined bridge crossing at *A* which is six miles distant, and that another pass *C* is 310 feet higher than *A* and is ten miles distant, the figures, even with all necessary allowances for inaccuracy, will give an engineer a good idea as to the choice of route especially as affected by ruling grade. There is no comparison between the time and labor involved in obtaining the above information by barometric and by spirit-leveling methods, and *for recon-*

(Right-hand page of Notes.)

Temp. at headqu.	Approx. field read.	Approx. headq. read.	Diff.	Corr. for temp.	Diff. elev.
—	—	—	—	—	—
75°	192	230	− 38	−(+ 2)	− 40
76	457	244	+213	+(+10)	+223
77	297	256	+ 41	+(+ 2)	+ 43

noissance purposes the added accuracy of the spirit-leveling method is hardly worth its cost.

8. Horizontal measurements, bearings, etc. When reliable maps are unobtainable, rapid exploratory surveys become essential. Since accuracy is sacrificed for rapidity in such surveys, more or less approximate methods are used. "An experienced saddle-horse, whose speeds at his various gaits have been learned accurately by previous timing," is quoted from Beahan * as one means of rapidly measuring distances. The percentage of probable error is evidently large. A **pedometer** (or pace-measurer) is probably more accurate, but its accuracy depends on a knowledge of the average length of the observer's pace. Due allowance must be made for the fact that the length of pace will vary very greatly depending on whether the surface is smooth and level, or is plowed ground, or marshy, or slippery, or consists of rough boulders covered with moss, or is a wilderness of brambles, fallen trees, bogs, etc. It will also depend on whether the observer is fatigued or is in fresh physical condition. Under such a variety of conditions the counting of steps for long distances is sometimes a farce. Even when the surface is fairly smooth and easy, precautions must be taken that paces are not counted during the pauses at important points while bearings are being taken and other data recorded. An **odometer** which records the revolutions of a wheel of known circumference is far more accurate. Such a machine has been made so that it may be trundled like a wheelbarrow and thus go through the woods and over ground that would be impassable to any horse-drawn vehicle. The attachment of an odometer to the wheels of a wagon is very tempting, since it permits the engineer to ride, but it is probably an unreliable method for the reason men-

* "The Field Practice of Railway Location," p. 34.

tioned in Art. 2—permitting the ease of travel over a practicable for a horse and vehicle to deflect the engineer his true course, which is perhaps over rough ground whi impassable for a vehicle.

When the country is quite open and clear of underb very rapid work may be done by the **stadia method**, whi many times more accurate than any of the methods p ously mentioned. Some of the accuracy possible with st may be sacrificed for extreme rapidity and sights may made 1200 and even 2000 feet long. By taking very if any, "side-shots," the progress is very rapid and m miles per day may be covered, with the advantage that three elements of distance, azimuth and relative eleva may be obtained with as great accuracy as is necessary an exploratory survey. The method of using the stadia wi described later.

The **bearings** of the various lines forming the skeleton of survey, and also the bearings of the courses of streams and of lines from the stations on the skeleton line, may be taken r easily with a **prismatic compass.** This instrument has a cular card, or sometimes a metal ring, attached to the ne The edge of the card is graduated into degrees and is usu numbered consecutively (instead of by quadrants), fron up to 360°. This is advantageous since the one number, v out any qualifying letters, *NE* or *NW*, determines the quad definitely without danger of confusion or error. The obse sights through a narrow slit in the desired direction and means of the prismatic reflector, can read directly the nur of degrees, measured *to the right*, and usually from the mag *South*. The makers of prismatic compasses do not al number the graduations in the same manner, and, therefore engineer, who is accustomed to one particular instrument, sh carefully study the markings of any new instrument. In case it should be remembered that the prism reflects the num on that side of the movable card or ring which is *toward th server* rather than on the side toward the object sighted at. prismatic compass has the special advantage that, like a sext it can be used when supported only by hand, while an ordi sight compass of equal accuracy would require a tripod, o least, a Jacob's staff. The declination of the needle in section of the country can be readily determined with suffi

accuracy for the purposes of such a survey. Usually the declination may be ignored. Any errors due to local attraction are never cumulative, but apply only to the point where those individual observations are taken. The angle between two lines radiating from any station may be obtained by subtracting one bearing from the other.

Relative elevations may be obtained systematically, using a barometer, as already explained, but much filling in may be done with the use of a hand-level. Experience soon teaches an engineer that there are many optical illusions about the slopes of ground which have the practical effect of making the apparent slope different from the actual, and, in the case of low grade, may make an actual down grade appear as an up grade. For example, when looking along an actual but slight down grade, especially if there are no obstructions or natural objects which the eye can use as a comparative scale, the eye is apt to foreshorten the distance, which has the effect of lessening the apparent down grade and perhaps of making it appear as a slight up grade. The hand level will immediately detect such errors and its frequent use by a reconnoissance engineer will not only enable him to avoid many errors he might otherwise make, but will also be an effective means of training him to guard against such optical illusions. Such a simple and effective instrument should always be at hand and it should be tested with sufficient frequency to know that it is always as accurate as such an instrument can be. The bubble should be as sensitive as is practicable for an instrument which is held in the hand. A well-made hand level has a bubble of the right sensitiveness, but even a super-sensitive level may be utilized and still better work done by supporting it steadily on the top of a light wooden stick about five feet long.

9. **Importance of a good reconnoissance.** The foregoing instruments and methods should be considered only as aids in exercising an educated common sense, without which a proper location cannot be made. The reconnoissance survey should command the best talent and the greatest experience available. If the general route is properly chosen, a comparatively low order of engineering skill can fill in a location which will prove a paying railroad property; but if the general route is so chosen that the ruling grades are high and the business obtained is small and subject to excessive competition, no amount of perfection in

detailed alinement or roadbed construction can make the road a profitable investment.

PRELIMINARY SURVEYS.

10. Character of survey. A preliminary railroad survey is properly a topographical survey of a belt of country which has been selected during the reconnoissance and within which it is estimated that the located line will lie. The width of this belt will depend on the character of the country. When a railroad is to follow a river having very steep banks the choice of location is sometimes limited at places to a very few feet of width and the belt to be surveyed may be correspondingly narrowed.

But even in such a case, the width surveyed should be sufficient to include not only every possible location of "slope-stakes" but also should indicate the contours and nature of any soil which might give trouble by sliding, after an excavation has been made at the base. It is justifiable and proper to survey a belt considerably wider than it is expected to use, for experience shows that, while there is generally but little or no direct utilization of the extra area surveyed, it frequently becomes essential to know something of the character of the ground considerably to one side of where it was expected to run the line and the inclusion of this area in the original survey has saved an expensive trip to obtain a very small amount of data.

In very flat country the desired width may be only limited by the ability to survey points with sufficient accuracy at a considerable distance from what may be called the "backbone line" of the survey.

11. Cross-section method. This is the only feasible method in a wooded country, and is employed by many for all kinds of country. The backbone line is surveyed either by observing magnetic bearings with a compass or by carrying forward absolute azimuths with a transit. The compass method has the disadvantages of limited accuracy and the possibility of considerable local error owing to local attraction. On the other hand there are the advantages of greater simplicity, no necessity for a back rodman, and the fact that the errors are purely local and not cumulative, and may be so limited, with care, that

they will cause no vital error in the subsequent location survey. The transit method is essentially more accurate, but is liable to be more laborious and troublesome. If a large tree is encountered, either it must be cut down or a troublesome operation of offsetting must be used. If the compass is employed

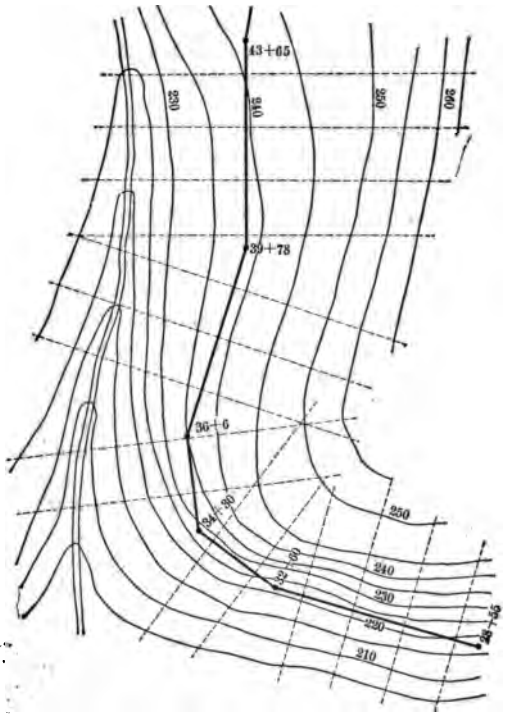


FIG. 4.

under these circumstances, it need only be set up on the far side of the tree and the former bearing produced. An error in reading a transit azimuth will be carried on throughout the survey. An error of only five minutes of arc will cause an offset of nearly eight feet in a mile. Large azimuth errors may, however, be avoided by immediately checking each new azimuth

with a needle reading. It is advisable to obtain true azimuth at the beginning of the survey by an observation on the sun* or Polaris, and to check the azimuths every few miles by azimuth observations. Distances along the backbone line should be measured with a chain or steel tape and stakes set every 100 feet. When a course ends at a substation, as is usually the case, the remaining portion of the 100 feet should be measured along the next course. The level party should immediately obtain the elevations (to the nearest tenth of a foot) of all stations, and also of the lowest points of all streams crossed and even of dry gullies which would require culverts.

12. **Cross-sectioning.** It is usually desirable to obtain contours at five-foot intervals This may readily be done by the use of a Locke level (which should be held on top of a simple five-foot stick), a tape, and a rod ten feet in length graduated to feet and tenths. The method of use may perhaps be best explained by an example. Let Fig. 5 represent a section *perpendicular* to the survey line—such a section as would be made by the dotted lines in Fig. 4. *C* represents the station point. Its elevation as determined by the level is, say, 158.3 above datum. When the Locke level on its five-foot rod is placed at *C*, the level has an elevation of 163.3. Therefore when a point is found (as at *a*) where the level will read 3.3 on the rod, that point has an elevation of 160.0 and its distance from the center gives the position of the 160-foot contour. Leaving the long rod at that point (*a*), carry the level to some point (*b*) such that the level will sight at the *top* of the rod. *b* is then on the 165-foot contour, and the *horizontal* distance *ab* added to the horizontal distance *ac* gives the position of that contour from the center. The contours on the lower side are found similarly. The first rod reading will be 8.3, giving the 155-foot contour. Plot the results in a note-book which is ruled in quarter-inch squares, using a scale of 100 feet per inch in both directions. Plot the work UP the page; then when looking ahead along the line, the work is properly oriented. When a contour crosses the survey line, the place of crossing may be similarly determined. If the ground flattens out so that five-foot contours are very far apart, the absolute elevations of points at even fifty-foot distances from the center should be determined. The

* The method of making such observations is given in the Appendix.

method is exceedingly rapid. Whatever error or inaccuracy occurs is confined in its effect to the one station where it occurs. The work being thus plotted in the field, unusually irregular topography may be plotted with greater certainty and no great error can occur without detection. It would even be possible by this method to detect a gross error that might have been made by the level party.

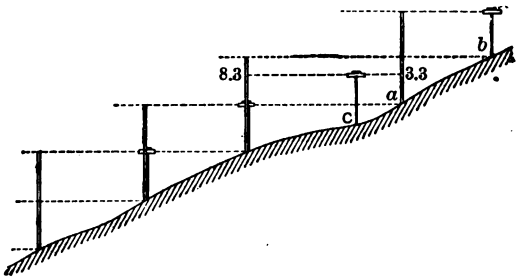


FIG. 5.

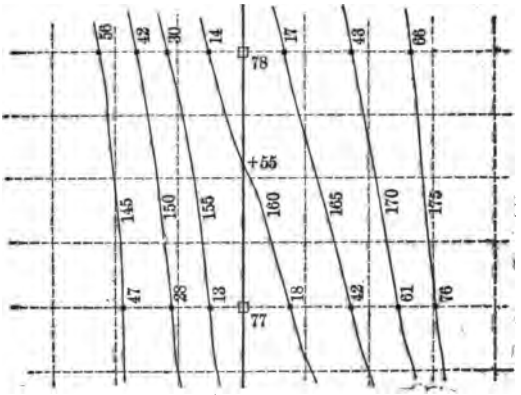


FIG. 6.

13. **Stadia method.** This method is best adapted to fairly open country where a "shot" to any desired point may be taken without clearing. The *backbone* survey line is the same

as in the previous method except that each course is limited to the practicable length of a stadia sight. The distance between stations should be checked by foresight and backsight—also the vertical angle. Azimuths should be checked by the needle. Considering the vital importance of leveling on a railroad survey it might be considered desirable to run a line of levels over the stadia stations in order that the leveling may be as precise as possible; but when it is considered that a preliminary survey is a somewhat hasty survey of a route that *may* be abandoned, and that the errors of leveling by the stadia method (which are compensating) may be so minimized that no proposed route would be abandoned on account of such small error, and that the effect of such an error may be usually neutralized by a slight change in the location, it may be seen that excessive care in the leveling of the preliminary survey is hardly justifiable.

A stadia party should include a locating engineer (or chief of party), and perhaps an assistant, a transitman, a recorder and four rodmen, beside axemen. The transitman should have nothing to do but attend to his instrument. After setting up the transit at an advanced station, a backsight should be taken to the previous station. If the vertical circle is full 360°, the telescope should be plunged and sighted on the backsight with vernier *A* reading the same as the foresight to the station occupied. If the vertical arc is semi-circular (or less), no vertical angle can be taken with the telescope plunged and, therefore, vernier A should read 180° more (or less) than the foresight. The lower plate should be very firmly clamped, and then, after loosening the upper plate, a reference sight and reading on some well-defined natural object should be taken. If there is any reason to suspect that the instrument has been disturbed while occupying that station, the reference point can be sighted at and the instrument can be re-alined, and re-leveled, if necessary, without sending a rodman back to the previous station. When taking a backsight the rod reading for distance should be taken first and immediately compared with the previously recorded foresight. Since the distance between stations will always be taken with especial care so as to avoid "blunders" of an even 10, 20 or perhaps 100 feet, the foresights and backsights should agree to within the proper limits of the stadia method. Similarly the vertical angle should agree with the previous reading, *but with opposite sign.* If especial care is

taken in leveling the instrument immediately before taking both foresights and backsights, these readings should agree to within one minute, or even 30 seconds, with a good transit. The height of the telescope above the ground at the new station must be measured, and the middle wire sighted at that reading on the rod (called the *H. I.*), when taking any vertical angle. Theoretically the rod reading for distance should be taken when the telescope is pointing at the proper vertical angle for that shot, but this will mean, in general, that both the upper and lower cross wires will read odd amounts and that an inconvenient subtraction must be made to get the difference, which is the "rod reading." But it may be demonstrated that no error of distance, amounting to the lowest practicable unit of measurement, can result if the telescope is raised or lowered just enough to set it on the nearest even foot mark. The routine of observing a shot is therefore as follows: (*a*) swing the instrument (the upper plate) horizontally until the telescope sights at the rod and clamp the horizontal motion—but very lightly and perhaps not at all; (*b*) raise or lower the telescope until the middle cross wire is sighting at the H. I., reading on the rod; a target on the rod may be set at the H. I. reading for each set-up and it will facilitate the work; (*c*) read the vertical angle and report it to the recorder, standing at hand; (*d*) raise or lower the telescope just enough so that the lower wire is on the nearest even foot mark and read (calling it out to the recorder) the number of even feet of interval from the lower to the upper wire and the odd amount at the top at the reading of the upper wire; (*e*) dismiss the rodman, who is then directed to another point by the chief of party; (*f*) read the azimuth on the horizontal plate. By that time another rodman has been located at a point where an observation is required, and the routine is repeated. The work of the transitman is thus very strenuous, without any recording work, and the progress of the party depends on him. He, therefore, should not be required to direct the party or even to record his notes, since every moment spent in that way delays the entire party by that amount. The recorder also has all that he can do to record the notes (with perhaps some sketches), as fast as the transitman calls them off. Usually four rodmen can be kept very busy, and they must be on the run between the successive points at which they hold their rods. One of the rodmen or one of the axemen, if axemen are employed, carries and

drives the stakes, which are only required at the instrument points. One or more axemen are generally useful in lopping off branches or cutting down saplings which interfere with desirable sights. The chief of party has plenty to do in directing the rodmen and axemen so that shots may be taken at points which will give the most significant information, and also in picking out the proper location for the advance station at some place from which a maximum of information may be observed with one set-up of the transit. A well-drilled organization and "team work" are necessary. The best work is done when every man is kept busy. Several hundred shots per day can be observed when it is considered advisable to obtain much detailed information and the average number of shots per set-up is large. On the other hand, when the stadia method is used for a rapid exploratory survey, only a few side shots (at some stations perhaps none at all) will be taken at each station. In such a case, the total number of shots taken during a day will be comparatively small, but the progress will be very rapid, and the salient features of several miles of a proposed route can be obtained in a day.

14. Form for stadia notes.

[Left-hand page.]

Inst. at	Azim.	Rod	Vert. angle	Diff. elev.	Elev.	Sighting at
Δ24.....	264° 27′	622	−0° 18′			Δ23
HI = 4.9....	83° 10′	528	+1° 16′			Δ25
Et = 629.2...	184° 23′	264	−2° 18′			bend in creek
.....	5° 47′	218(175)	+26° 20′			top of bluff

The usual six-column note-book can be utilized by ruling an extra line (shown dotted in the Form of Stadia Notes), in the fifth column, since the column is wide enough for both the "difference of elevation" and the "elevation." The "rod reading" (3d column) as recorded should include the $(f+c)$, which in almost all American transits equals 1.0 to 1.3 feet. Since the wire-interval ratio is almost invariably 1 : 100, the rod interval in hundredths of a foot is considered as the number of feet of distance, except that one even foot is added for the $(f+c)$. The sample figures given above are typical of all that needs to be taken in the field. The "difference of elevation" and the "elevation" are computed and entered later.

The "difference of elevation" may be mathematically computed from the formula

$$D = k\,r\,\tfrac{1}{2}\sin 2\alpha + (f+c)\sin\alpha,$$

in which D is the difference of elevation, k is a constant, usually 100, r is the rod intercept and α is the angle of elevation—or depression. The mathematical solution of such an equation for every shot that is taken (except the very few shots which are level) is very laborious and impracticable. But the work of reduction can be shortened by a justifiable approximation. By changing the factor of $(f+c)$ from $\sin\alpha$ to $\tfrac{1}{2}\sin 2\alpha$, the formula may be written

$$D' = [kr + (f+c)]\tfrac{1}{2}\sin 2\alpha.$$

The first term (that within the bracket) is the number recorded under "Rod" in the Form of Notes (622, 528, etc.). The second term ($\tfrac{1}{2}\sin 2\alpha$) may be taken from "Stadia Tables," of which many are published, although the tables usually give these numbers merely as the factors by which the distance is to be multiplied in order to obtain the "Difference of Elevation," and do not mention that the factor is really $\tfrac{1}{2}\sin 2\alpha$. The error of the approximation (when $(f+c)=1$ foot) is less than 0.01 foot for a vertical angle of 15° and less than 0.1 foot for the unusual angle of 30°. Since 0.1 foot is the usual lowest unit of measurement for stadia elevations, probably 99% of all stadia work can use such an approximation without appreciable error. The special cases with high angles can be computed separately if it is considered necessary. The algebraic sign of the vertical angle should *always* be recorded, even if it is plus, or upward; the sign + is a positive statement that it *is* plus and that the sign was not forgotten. The difference of elevation likewise should always have a + or − sign. Adding the difference of elevation to the elevation of the station (or subtracting it), gives the elevation of each point.

Theoretically the true horizontal distance for all inclined sights is always less than the nominal distance, as given by the rod reading. The formula for true distance is

$$L = kr\cos^2\alpha + (f+c)\cos\alpha.$$

As before, we may use the approximation of combining the $(f+c)$ with the kr and say that

$$L' = [kr+(f+c)] \cos^2 \alpha,$$

and that the *correction*, which is subtracted from $[kr+(f+c)]$, and *not* from kr, is

$$\text{Corr.} = [kr+(f+c)] \sin^2 \alpha.$$

The error of this approximation is usually insignificant, as illustrated below. Since $\sin^2 \alpha$ is very much less than $\cos^2 \alpha$ for the usual small values of α, it is easier and more accurate to compute the smaller quantity and mentally subtract it from the nominal reading. When the vertical angle and the distance are both small, the horizontal correction is within the lowest unit of measurement (one foot), and should, therefore, be ignored. The engineer soon learns the approximate limits at which the combination of vertical angle and distance will make a correction necessary. In the above notes no correction is necessary except in the last case, the angle being 26° 20′. The exact mathematical computation is as follows, the rod interval being 2.17 and $(f+c)=1$,

$$L = 217 \cos^2 26° 20' + 1 \cos 26° 20' = 175.20.$$

Using the approximate rule, the correction $= 218 \sin^2 26° 20'$ $= 42.90$.

$$218 - 42.90 = 175.10.$$

The above calculations have been carried to hundredths of a foot for the sole purpose of illustrating that the discrepancy between the approximate and the theoretical value is only 0.10 foot, even for this unusually large angle, and considering that the rod interval is read only to the nearest 0.01 foot, which corresponds to one foot of distance, this discrepancy is utterly inappreciable.

15. **The reduction of stadia observations** is most easily accomplished by using a stadia slide rule, which has one logarithmic scale for distances and for the computed differences of elevation or corrections to distance, and also two other scales one of which gives values for $\frac{1}{2} \sin 2\alpha$, and the other gives values

for $\sin^2 \alpha$. Some scales give values of $\cos^2 \alpha$. To illustrate the difference, in the above case, it is evidently easier to read 43 (two significant figures) than to read 218, which has three figures. When the distance is over 1000 (four figures), the difficulty is even greater. The necessity for subtracting the correction is of no appreciable importance. In this case, the correction would be read from the slide rule as 43, and mentally subtracting 43 from 218, we write at once 175, which is recorded in parenthesis in the Rod column. The draftsman, when plotting the notes, uses this distance (175) instead of 218. Using a slide rule, two men can very quickly compute the differences of elevation for the entire day's work in a very short time. A very little practice will enable them to run down the list, picking out the observations, usually less than 10% of the total number, where the combination of distance and vertical angle is sufficiently great to make it necessary to compute a horizontal correction. The stadia slide rule is so small that it may readily be carried into the field and used there if desired, in which respect it has a great advantage over diagrams, which are sometimes used for the same purpose.

16. Stadia method vs. cross-section method. There is still a difference of opinion among engineers as to the choice of these two methods. When a large part of the route is thickly wooded, the cross-section method is preferable. In open country the stadia method is more rapid and more economical. Although it would be inadvisable to change from one method to the other every mile or so, a very considerable economy is possible by alternating the two methods according to the character of the country. The locating engineer can plan such change of method during his reconnoissance. The real efficiency of the stadia method is due to the fact that the preliminary survey should be considered as the topographical survey of an area or belt, and not the survey of a line, and that in open country the stadia method is the most efficient method of obtaining such topographical data. But the efficiency depends on the handling of the party. When a valley widens out with easy slopes and the possible area in which the location may lie is correspondingly widened, it is far easier and more accurate to widen the belt surveyed by stadia shots of 1000 feet if necessary.

17. "First" and "Second" preliminary surveys. Some engineers advocate two preliminary surveys. When this is done,

the first is a very rapid survey, made perhaps with a compass, and is only a better grade of reconnoissance. Its aim is to rapidly develop the facts which will decide for or against any proposed route, so that if a route is found to be unfavorable another more or less modified route may be adopted without having wasted considerable time in the survey of useless details. By this time the student should have grasped the fundamental idea that both the reconnoissance and preliminary surveys are not surveys of *lines* but of *areas*, that their aim is to survey only those topographical features which would have a determining influence on any railroad line which might be constructed through that particular territory, and that the value of a locating engineer is largely measured by his ability to recognize those determining influences with the least amount of work from his surveying corps. Frequently too little time is spent on the comparative study of preliminary lines. A line will be hastily decided on after very little study; it will then be surveyed with minute detail and estimates carefully worked up, and the claims of any other suggested route will then be handicapped, if not disregarded, owing to an unwillingness to discredit and throw away a large amount of detailed surveying. The cost of two or three extra preliminary surveys (*at critical sections* and not over the whole line) is utterly insignificant compared with the probable improvement in the "operating value" of a line located after such a comparative study of preliminary lines.

LOCATION SURVEYS.

18. "Paper location." When the preliminary survey has been plotted to a proper scale (usually 200 feet per inch), and the contours drawn in, a study may be made for the location survey. Disregarding for the present the effect on location of transition curves, the alinement may be said to consist of straight lines (or "tangents") and circular curves. The "paper location" therefore, consists in plotting on the preliminary map a succession of straight lines which are tangent to the circular curves connecting them. It may be assumed that the general route of the preliminary survey has been so well selected, as the result of the reconnoissance survey, that it is possible to construct a line without excessive earthwork between consecutive control points, and that the grades are within the ruling grade. If the preliminary

survey has been run by locating stations every 100 or 200 feet (see § 11 and Fig. 4), the profile of this line gives the first approximation toward the rate of grade, and from this may be determined whether one uniform grade between the control points is

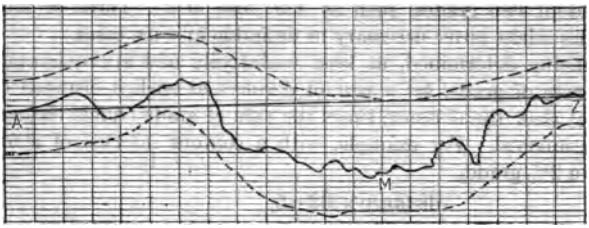


FIG. 7. SINGLE GRADE BETWEEN CONTROL POINTS.

practicable, or whether two or more different grades must be used. If the stadia method was used, the profile of a line running through the station points will serve the same purpose. In Fig. 7 let *AMZ* represent, on a very small scale, the surface profile between two control points, *A* and *Z*, which are, perhaps,

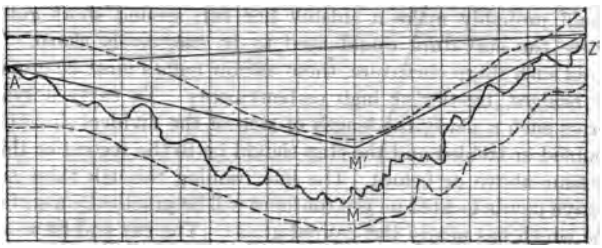


FIG. 8. TWO GRADES BETWEEN CONTROL POINTS,

two miles apart. The upper dotted line shows the elevations of the highest points in the surveyed belt at each of the several stations, and the lower line the corresponding lowest points. If the straight line *AZ* does not go outside of these dotted lines, it indicates that the uniform grade *AZ* will have "supporting ground" for the entire distance and that such a grade is practicable and should be tentatively selected (or at least investi-

gated) for that stretch. If the straight line AZ passes outside the belt of the dotted lines, as in Fig. 8, it implies that there was some definite reason why no higher supporting ground could be found near M', or the preliminary survey, if properly made, would have covered that ground. It then becomes necessary to adopt two grades, such as AM' and $M'Z$. Three or more grades might prove necessary or desirable in some cases.

Having determined, at least tentatively and approximately, the rate of grade, set a pair of dividers at such a distance (to scale) that the distance times the rate of grade equals the contour interval. For example, with a contour interval of 5 feet and a 2% grade,

$$\text{distance} \times .02 = 5,$$

or

$$\text{distance} = 5 \div .02 = 250.$$

Then, with dividers set at 250 feet, put one leg where the line previously located crosses a contour and put the other leg where it reaches the contour next above—or below, if a down grade. Then step to the next contour and so on. If the desired starting point is not on a contour, the distance for the *first* step should be proportionately shortened. A strict application of this method would probably make a sidehill line run around short gullies where the curvature would need to be excessively sharp. To avoid such sharp curvature, these narrow gullies must be crossed by bridges, trestles or high embankments. To carry a grade across such a place, the length of step of the dividers should be doubled or trebled and the step should be to the second or third contour above or below. The line running through these successive points located on the contours will be practically a surface line which has nearly the desired grade. The cut and fill would be almost nothing—except "side-hill work," and the crossing of gullies. No accuracy need be expected on this preliminary trial since the distance is somewhat greater than the air-line distance AZ. It would, in general, be impossible to run a practicable combination of tangents and proper curves through these points, but such a line is very suggestive of a proper alinement which will fulfill the grade and curvature conditions and along which the cut and fill will be reasonably small.

If there are long stretches where, in each case, the line joining a group of consecutive points is nearly straight, the tangents will

predominate and should be located first and then connected by curves. If the line has numerous and long bends, it may be preferable to select the curves first and then connect them with tangents. For such work a series of curves, drawn to proper scale, varying by even degrees from 1° up to 15° or 20°, or whatever is the maximum allowable curvature, and drawn on any transparent material such as tracing cloth, celluloid or glass, is very useful, since different curves may be tried in turn until the curve which best fits the ground is discovered. The contours and other fixed features should have been inked in and then the trial lines and curves may be marked in lightly with a *soft* pencil, so that trial lines may be easily erased until a satisfactory line is obtained. The number of possible combinations is infinite, but certain conditions must be fulfilled which narrows the choice. (1) The connecting tangents must not be too short; 100, 200 and even 300 feet are used as limits. (2) The curvature must be within the adopted limit. If two consecutive curves, which are connected by a very short tangent, bend in the same direction, it is preferable that they should be combined into one simple curve, or into two branches of a compound curve, rather than to make a "broken-backed" curve. If they bend in opposite directions (making a reverse), even 300 feet is none too long for the transition curves which should be used, especially if the curves are sharp. Actual reverse curves (changing the direction of curvature without any separating tangent) should *never* be used, except on switch work and track where the speed is always slow. It would be far preferable to sharpen the curvature enough to introduce a tangent at least 100 feet long. The following considerations should be kept in mind.*

"(1) If the location could follow the grade line [or surface line] precisely, there would be no cuts or fills (practically speaking) on the center line.

"(2) Whenever the location lies on the $\left\{\begin{matrix}\text{down-hill}\\ \text{up-hill}\end{matrix}\right\}$ side of the grade contour [or surface line] there will be $\left\{\begin{matrix}\text{fill}\\ \text{cut.}\end{matrix}\right\}$

"(3) The further the location departs from the grade contour the greater will be the cut or fill, as the case may be."

* Art. 50, Part IV, "Technic of Surveying Instruments and Methods," by Webb and Fish. John Wiley & Sons.

After a location line has been selected which seems satisfactory from the standpoints of easy curvature, not too short tangents, a proper balance of cut and fill, and not too great cuts and fills, as will be approximately indicated by its distance from the surface line, the volume of earthwork may be estimated with sufficient accuracy for comparative purposes by drawing a profile of the surface location line and its roadbed line. Considering the ease with which such lines may be drawn on the preliminary map, it is frequently advisable, after making such a paper location, to begin all over, draw a new line over some specially difficult section and compare results. Profiles of such lines may be readily drawn by noting their intersection with each contour crossed. Drawing on each profile the required grade line will furnish an approximate idea of the *comparative* amount of earthwork required. A comparison of the areas of cut and fill on the profile will show the approximate balance in volume of cut and fill. If it is considered necessary to compute the volume with greater accuracy, it may be done by the use of Table XVII (see also § 126), applying the latter part of the table correctively to allow for side slope. After deciding on the paper location, the length of each tangent, the central angle (see § 51), and the radius of each curve should be measured as accurately as possible. Frequent tie lines and angles should be determined between the plotted location line and the preliminary line. When the preliminary line has been properly run, its "backbone" line will lie very near the location line and will probably cross it at frequent intervals, thus rendering it easy to obtain short and numerous tie lines.

19. Preparation of the notes. This and the actual transfer of the paper location to the ground is a problem in surveying which is so varied in its character that the ingenuity of the engineer is required to use the best method adapted to each particular case, but a few principles may profitably be kept in mind. (1) The scale of the paper location drawing is probably 200 feet per inch, unless the difficulties of the problem demand a larger scale for a particular stretch of the road, so that the paper location may be more accurate. Since a variation of 1/200 inch in the drawing means a variation of one foot on the ground, no close checking of the line on any tie-point need be expected. (2) Since a very small variation in alinement would, if persisted

in, throw the alinement very far from its desired location, it must be expected that there will be more or less adjustment of the paper location alinement (numerically) on nearly every tangent and curve. (3) The intersection of the preliminary line by a paper-location tangent (or the tangent produced) gives a possible tie-point. The position of this tie-point on the preliminary line must be scaled and the angle between the lines determined by measuring the chord of a long arc with its center at the point of intersection or by scaling the sine (or tangent) produced by a perpendicular from one line to the other from a point whose distance from the intersection is a convenient unit length. (4) When there is no intersection at some place where a tie is desired, a perpendicular offset from the preliminary line may be necessary. (5) When the paper location crosses the preliminary line at frequent intervals (say 500 to 1000 feet), it may be more simple to locate the tie-point intersections on the preliminary line and work from one to the other, taking up the inevitable inaccuracies by slight variations in the length of tangents or curves or by some one of the various methods detailed in § 63. When no practicable tie can be obtained for a considerable distance (say one-half mile), it may be desirable to determine the ordinates (latitudes and departures) of all the points on the preliminary and on the paper location between two consecutive intersections. In such a case the precision would depend entirely on the accuracy of scaling the positions of the two intersections and on the accuracy of the preliminary survey. While such a method requires considerable office computation, even that is cheaper than an extensive revision of a located line in the field. For a further development of this method, the student is referred to a course of instruction originally written by Prof. J. C. L. Fish, of Stanford University, and included in "Technic of Surveying Instruments and Methods," by Webb and Fish, published by Wiley & Sons.

As previously stated, the above method has been developed as if the final located line were to be made up only of tangents and circular curves. But transition curves between the tangents and circular curves are essential for the easy operation of trains. Anticipating the more complete demonstration of the subject, § 71, *et seq.*, it may be stated that the effect of the transition curve, or "spiral," is to move the curve inward, or toward its center, or to move the tangent outward. The effect of this is

equivalent to offsetting the tangent outward, or offsetting the curve inward, and then connecting the tangent and circular curve by a transition curve which gradually crosses the offsetted distance. The amount of the offset varies with the degree of the central curve and the desired length of the transition curve, but it is seldom more than three or four feet, and is usually much less. No consideration need be given to these offsets when comparing several trial locations. It is only after the paper location has been settled and it is time to transfer this to the ground that it is necessary to compute these offsets and adjust the lines accordingly. Even then the offsets will seldom be so large that they would appreciably affect the paper location, but when the alinement is actually located on the ground, the proper offsets should be used and the alinement laid out as described in detail in § 80.

20. Surveying methods. A transit should be used for alinement, and only precise work is allowable. The transit stations should be centered with tacks and should be tied to witness-stakes, which should be located outside of the range of the earthwork, so that they will neither be dug up nor covered up. All original property lines lying within the limits of the right of way should be surveyed with reference to the location line, so that the right-of-way agent may have a proper basis for settlement. When the property lines do not extend far outside of the required right of way they are frequently surveyed completely.

The leveler usually reads the target to the nearest thousandth of a foot on turning-points and bench-marks, but reads to the nearest tenth of a foot for the elevation of the ground at stations. Considering that $\frac{1}{1000}$ of a foot has an angular value of about one second at a distance of 200 feet, and that one division of a level-bubble is usually about 30 seconds, it may be seen that it is a useless refinement to read to thousandths unless corresponding care is taken in the use of the level. The leveler should also locate his bench-marks outside of the range of earthwork. A knob of rock protruding from the ground affords an excellent mark. A large nail, driven in the roots of a tree, which is not to be disturbed, is also a good mark. These marks should be clearly described in the note-book. The leveler should obtain the elevation of the ground at all station-points; also at all sudden breaks in the profile line, determining also the distance of these breaks from the previous even station. This will in-

clude the position and elevation of all streams, and even dry gullies, which are crossed

Measurements should preferably be made with a steel tape, care being taken on steep ground to insure horizontal measurements. Stakes are set each 100 feet, and also at the beginning and end of all curves. Transit-points (sometimes called "plugs" or "hubs") should be driven flush with the ground, and a "witness-stake," having the "number" of the station, should be set three feet to the right. For example, the witness-stake might have on one side "137+69.92," and on the other side "P C 4° R," which would signify that the transit hub is 69.92 feet beyond station 137, or 13769.92 feet from the beginning of the line, and also that it is the "point of curve" of a "4° curve" which turns to the *right*.

Alinement. The alinement is evidently a part of the location survey, but, on account of the magnitude and importance of the subject, it will be treated in a separate chapter.

21. Form of Notes. Although the Form of Notes cannot be thoroughly understood until after curves are studied, it is here introduced as being the most convenient place. The right-hand page should have a sketch showing all roads, streams, and property lines crossed with the bearings of those lines. This should be drawn to a scale of 100 feet per inch—the quarter-inch squares which are usually ruled in note-books giving convenient 25-foot spaces. This sketch will always be more or less distorted on curves, since the center line is always shown as *straight* regardless of curves. The station points ("Sta." in first column, left-hand page) should be placed opposite to their sketched positions, which means that even stations will be recorded on every *fourth* line. This allows three intermediate lines for substations, which is ordinarily more than sufficient. The notes should read UP the page, so that the sketch will be properly oriented when looking ahead along the line The other columns on the left-hand page will be self-explanatory when the subject of curves is understood. If the "calculated bearings" are based on azimuthal observations, their agreement (or *constant* difference) with the needle readings will form a valuable check on the curve calculations and the instrumental work.

22. Number of men required in surveying parties. No fixed rules can be given. The general rule of economy and efficiency

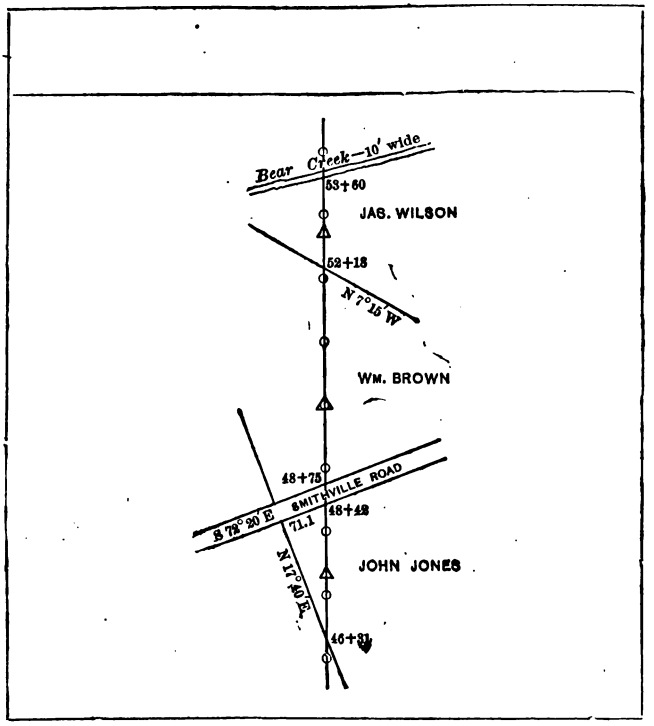
FORM OF NOTES.

[Left-hand page.]

Sta.	Alinement.	Vernier.	Tangential Deflection.	Calculated Bearing.	Needle.
54					
53					
⊙ +72.2	P.T.	9° 11′	18° 22′	N 54° 48′ E	N 62° 15′ E
52		7 57			
51	[3° 24′ curve to right for 18° 22′; tang. dist., 272.5]	6 15			
⊙ 50		4 33			
49		2 51			
48		1 09			
⊙ +32 47	P.C.	0°			
46				N 36° 26′ E	N 44° 0′ E

should govern, and that is, that the organization should be such that all desired data can be obtained at a minimum of cost. This general rule may be subject to the modification that the early completion of the survey is sometimes financially so important as to justify the maximum speed, almost regardless of expense. A common violation of the general rule of economy is the use of too few men, with the mistaken idea that it is economical. This requires the high-priced efficient men to waste their time on work which men at one-half (or even one-third) their salary could do sufficiently well, thus delaying the completion of the work or depreciating its quality by undue haste

[Right-hand page.]



or by neglect to obtain complete data. The work should be so organized that each man is constantly busy at the kind of work for which he is especially qualified, and that no men shall have to wait for others to complete their co-ordinate work. Even if 100% efficiency is unobtainable, it is very uneconomical to have nearly the whole party idle while one or two high-priced men do some work which must be done before the party can proceed but which could have been done by some extra lower-grade men without delaying the party. Reconnoissance. When the territory of the general route has been mapped by the U. S. Geol. Survey, there may be no need of instrumental work on the

reconnoissance, since the approximate ruling grades and general route may perhaps be determined directly from the map, and the purpose of the reconnoissance is the examination of physical features which would affect or modify the general route. In such a case the engineer does his technical work alone and only needs a guide and cook in case camping is necessary. When the reconnoissance partakes more of the nature of a hasty preliminary, distances, elevations and the necessary side topography being determined by rapid approximate methods, more men should be added, keeping in mind that the work should be so organized that each member of the party is kept busy at his own co-ordinate work, and that the chief engineer is not delayed in his own special work by spending his valuable time on a cheaper grade of work which an assistant could do sufficiently well. In other words, it is economical to add to the party an extra assistant whenever the work that he can do will so facilitate the work of the party as a whole that the value of the salaries and expenses saved will more than offset the assistant's salary and expenses. **Preliminary surveys.** No fixed list of members of a party is applicable to all conditions. The following list, with monthly salaries, is given by Mr. Fred Lavis* as having been used on each of five parties in surveying the Choctaw, Oklahoma & Gulf R. R. The list is very full but justifiably so.

Locating engineer	$150	to	$175
Assistant locating engineer	115		125
Transitman	90		100
Levelman	80		90
Draftsman	80		90
Topographers, two	80		90
Level rodman	50		
Head chainman	50		
Rear chainman	40		
Tapemen, two	30		
Back flagman	30		
Stake marker	30		
Axemen, three to five	25	to	30
Cook	50		
Cook's helper	20		
Double teams and driver, furnish their own feed, driver boarded in camp	65	to	90

* Methods of Railroad Location on the Choctaw, Oklahoma & Gulf R. Trans. Am. Soc. C. E., Vol. LIV, page 104.

Other organizations sometimes combine the first two positions on this list and possibly call him "chief of party." For the above work, the locating engineer was relieved altogether from the detailed direction of the party, which was handled by the assistant, and spent nearly all his time in studying the country so as to determine how the line should advance. In nearly all cases, such expense is justified, perhaps many times over, (1) by the saving of uselessly surveying an improper route, (2) by an improvement in the operating value of the route selected, or (3) by an improvement in route which makes a decrease in construction cost. Sometimes those controlling the financial side of the project insist that the chief of party shall also run the transit, as a measure of "economy." Such a policy cannot be too strongly condemned. The work of a transitman requires every instant of his time and every minute that he turns from his transit to direct the party or study the proper route is a minute delay for the entire party. It generally means also a deterioration in the quality of his work as a leader and as a transitman, in his effort to hastily do at one time work which requires the concentrated efforts of two men. In this survey (described by Mr. Lavis), the skeleton or backbone line was a broken line with angles every few hundred feet, and the topography was taken by right-angled offsets every hundred feet or oftener, substantially as described in § 11 and Fig. 4. These offsets were determined by a hand level and pacing by one of the two topographers. The other topographer, using a transit, with the other two tapemen "determined drainage areas, located property lines and section corners, got names of property owners, etc." When, as is usually the case, such essential work cannot be done by the main party without delaying their progress, there is a real economy in adding to the party these comparatively low-priced assistants. It may be noted that the above party includes two chainmen, back flagman and stake-marker, beside three to five axemen. The proper number of axemen manifestly depends on the amount of necessary cutting, but the chainmen or the stake-marker should not be depended on for such work. The steady march of the party should not be halted while a stake-marker or chainman stops his regular work to cut down a tree. One of the duties of the chief of party is to foresee the necessities of tree-cutting and clearing, so far in advance that, by the time the surveying members of the party have reached the spot, the

area is cleared. It is likewise false economy to dispense with the stake-marker and require the head chainman to do such work. A full corps of such men, properly drilled, can add 20 to 50% to the daily progress of the party and much more than save their cost.

MAINTENANCE OF SURVEY PARTIES.

23. Economy and efficiency. When considering the treatment and maintenance of surveying parties, it should be remembered that a false idea of economy is frequently responsible for making the parties too small, overworking the men, depriving them of physical comforts and even necessities, and that the result is a greater net cost and a great deterioration in the quality of the results. A party may cost $40 to $65 per day in salaries and expenses. Any policy which depreciates the net output of their work 20 to 50% (which is easily possible) in order to save a few dollars per day is manifestly poor policy. The men, especially those who must use their brains and who presumably have a finer nervous organism, have only a quite definite sum total of nervous energy. If a considerable part of that energy is spent in needlessly long tramps both morning and evening to and from work, or if that nervous energy is not maintained by plentiful and appetizing food and by sufficient and comfortable rest, there is a reduction in efficiency which is often far greater than any possible saving in expenses. This idea of developing the maximum efficiency of the party is the justification of the recommendations made below regarding outfit, equipment, and other details about managing a party.

24. Country hotels and farm houses. In settled sections of the country, country hotels and even farm houses are sometimes available where men can be provided with living facilities which are unobtainable in camp life and at less total expense. Such accommodations have the advantage that they obviate a considerable capital expenditure to purchase sufficient camp outfit. But if suitable accommodations are unobtainable over a considerable portion of the route and such accommodations as there are on the remaining distance are inconvenient and inadequate, it may be preferable to provide a camping outfit at once. Considering the fact that there is a real economy in making a survey with a large party and that such a party can

seldom if ever be accommodated in a single farmhouse, and that there is a lack of efficiency if the party is separated, the farmhouse plan is frequently impractical. But when villages are so located that there is always one within five miles of any point of the line, the house plan may be preferable, since the party may be taken to and from work in conveyances. The economy of employing conveyances may be judged by comparing the cost of the vehicles and the value of the time and energy saved. A five-mile tramp, carrying an instrument, following a full day's work surveying, will frequently incapacitate a man from doing effective work in the night-work which the higher grade men of the party must generally do. The day's work in the field must be begun later and ended earlier or else the time and strength spent in the morning and evening tramps are uneconomical drains on their total nervous energy.

25. Camping Outfits: Tents. The Choctaw, Oklahoma & Gulf R.R. survey, previously referred to, provided for each party one office tent, with fly, 14×16 feet, three tents, evidently without flies, 14×16 feet, and one cook tent 16×20 feet. The office tent had 5-foot walls; the others 4-foot. H. M. Wilson ("Topographical Surveying," p. 817) recommends 9×9 foot tents, with 4-foot walls. These are easier to erect but have only 36% of the floor area of the 14×16-foot tents and it would require 15 such tents to equal the floor area of the 5 tents described above. For a small party the smaller tents would be preferable. The canvas should be mildew-proof and free from sizing. A "sod-flap" about 8 inches wide, should be attached to the bottom of the wall. When this flap is weighted down with stones or heavy sticks the wind and weather is kept out. Dirt or sod should *not* be used for weights, since they rot the canvas. It pays to use tents which conform to the U. S. Army specifications. Some of the specifications as to material and workmanship are here quoted:

"*Materials.*—Body of tent to be made of Army standard 12$\frac{4}{10}$ ounce cotton duck, 29½ inches wide and the sod cloth of Army standard 8-ounce cotton duck, 28½ inches wide.

"*Workmanship.*—To be made by machine in a workmanlike manner, all seams to be stitched with two rows of stitching, not less than six stitches to the inch, with three-cord twelve-thread Sea Island cotton, white.

"In making tents by hand, to have not less than two and one-

half stitches of equal length to the inch, made with a double thread of five-fold cotton twine, drab, well waxed.

"The seams should be not less than 1 inch in width, flat stitched, and no slack taken in them.

"*Grommet holes.*—Made with malleable iron rings, galvanized, to be worked with four-thread five-fold cotton twine, well waxed.

"*Sod cloth.*—To be 8 inches in width in the clear from the tabling, into which it is inserted 1 inch and extending from door seam to door seam around the tent.

"*Tabling.*—On foot of tent when finished to be 2½ inches in width." (Adopted July 14, 1911.)

A ditch should be dug outside the tent, at least on the up-hill side, if the ground is at all inclined. This will prevent rain-water from draining through the tent. Of course, the bottom of the ditch should have a uniform slope draining to an outfall amply clear of the tent.

26. Tent floors. Dry floors are almost essential to health. Sectional floors, about 3×9 feet per section, made by fastening boards to cross cleats, provide a perfectly dry floor and often repay their transportation. A mere layer of canvas, cut to proper shape and bound on the edges, is worth providing if the ground is dry when the tent is erected and can be kept from getting rainsoaked by proper outside drainage.

27. Tent stoves. For winter work, tents may be made quite comfortable with stoves. Oil stoves are convenient when the oil can be purchased without excessive cost for transportation. "Sibley" stoves, burning wood, are commonly used but they require smoke pipes which must pass through the canvas and this means that the holes must be properly protected with metal or asbestos. If a pipe elbow is provided, the pipe may be taken out through one end of the tent. This obviates a hole in the roof of the tent (and also the fly); it avoids a direct pour of rain on the fire or leakage into the tent around the pipe, and also the danger of sparks dropping on the canvas. A "Sibley" stove for mere heating is a sheet-iron frustum of a cone, about 3 feet high; diameter at bottom 18 to 30 inches; diameter at top 4½ to 6 inches, or so as to fit the stovepipe which is to be used. It has no bottom, or in other words, the bare earth forms the base. A door, large enough for the insertion of such fuel as it is designed to use, is placed in the side. Three or four lengths of pipe, one of which should have a damper, and an elbow,

should be provided. Draft at the bottom is obtained, and may be easily controlled, by packing earth around the base, leaving a small opening which may be easily enlarged or diminished to control the draft. **Cook stove.** A regular 6-hole cooking range, perhaps made of wrought-iron or sheet-steel, is essential to cook meals for twenty or more hearty men. Sporting outfitters supply all sizes of stoves, which must always be selected with due regard for the facilities for transportation. Oil stoves are commonly used. For still smaller parties, or when no cook stove can be permitted in the baggage, a primitive grid may be made from four sticks of *green* timber about 6 inches in diameter and 2 to 4 feet long. Notch two of them, each with a pair of notches about 10 inches apart. Place the other two sticks across the notches and they will steadily support a kettle or a frying pan. If the sticks are sufficiently green and the fuel quite dry the grid will last some time. A folding grid of iron bars may be obtained, which is but a small addition to the weight of the baggage. Another method is to suspend a kettle by a chain or long hook either from a tripod of sticks or from a horizontal stick lying in two forked sticks on each side of the fire.

28. Dining tables. These are justifiable for a large party when the baggage is necessarily great and camp wagons are a part of the equipment. Mr. Lavis, in the article previously referred to, describes a very good table from the standpoint of transportation. The table top consists of three loose planks $1\frac{3}{8}'' \times 12'' \times 18'\ 0''$. Two similar boards are used for seats. During transportation these boards are placed on the bottom of the wagon and, of course, project from the back where they form a support for stoves, etc., which can be roped on. These boards are supported on three trestles or horses, made as shown. For a much smaller party, a table may be improvised by utilizing two "mess-boxes," which carry the cooking utensils and table-ware. These mess-boxes are about 20 inches wide and high and from 24 to 30 inches long. The covers are made to open 180° and may be fastened horizontally. An "inside cover," which can be utilized as a bread board, covers the entire inside area of the box. Two such boxes, set together and with the tops opened out, provide a fairly even surface four times the area of one box.

29. Cooking utensils, table-ware, tools, etc. The size of the party, the individual preferences of the person designing the

outfit and the facilities for transportation, vary such lists almost indefinitely. Agate ware has replaced china for plates and cups. Aluminum ware, although expensive, is preferable from a cooking standpoint and has the advantage of a very material reduction in weight. Out of the very great number of lists which have been published, the following list of articles is quoted as suggestive: Plates, cups, saucers, steel knives and forks, German-silver spoons, large and small, carving knives and forks, large cooking forks and spoons, pepper and salt boxes, tin pans about

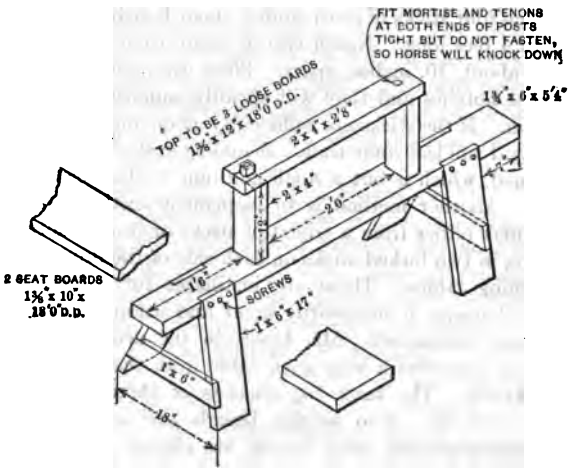

FIT MORTISE AND TENONS AT BOTH ENDS OF POSTS TIGHT BUT DO NOT FASTEN, SO HORSE WILL KNOCK DOWN
TOP TO BE 3 LOOSE BOARDS 1⅛" x 12" x 18'0" D.D.
1¾" x 6" x 5'4"
2" x 4" x 2'8"
2" x 4"
2'0"
2 SEAT BOARDS 1⅛" x 10" x 18'0" D.D.
1'6"
SCREWS
1" x 6" x 17"
1" x 6"
18"

FIG. 9.—CAMP DINING TABLE.

6 inches diameter by 1½ inches deep, utilized for serving soup, cereal, etc., pans and kettles of varying sizes which will "nest" and thus facilitate packing, tea kettle, coffee pot, frying pan, griddle, cake turner, pie plates, dripping pan, chopping bowl and chopper, colander, flour sieve, coffee mill, broiler, corkscrew and can opener, rolling pin, folding table (similar to the drawing table described below), wash basins, kerosene oil can, alarm clock, spring balance. The last two articles are important. The cook is the first man up in the morning—usually before daylight—and may need the alarm clock. A single delay, of even ten minutes of such a party, would cost more than a very valuable clock. A spring balance is very essential to the proper

and economical use of provisions without waste. It pays to have a cook who is able to compute, weigh out and use an amount

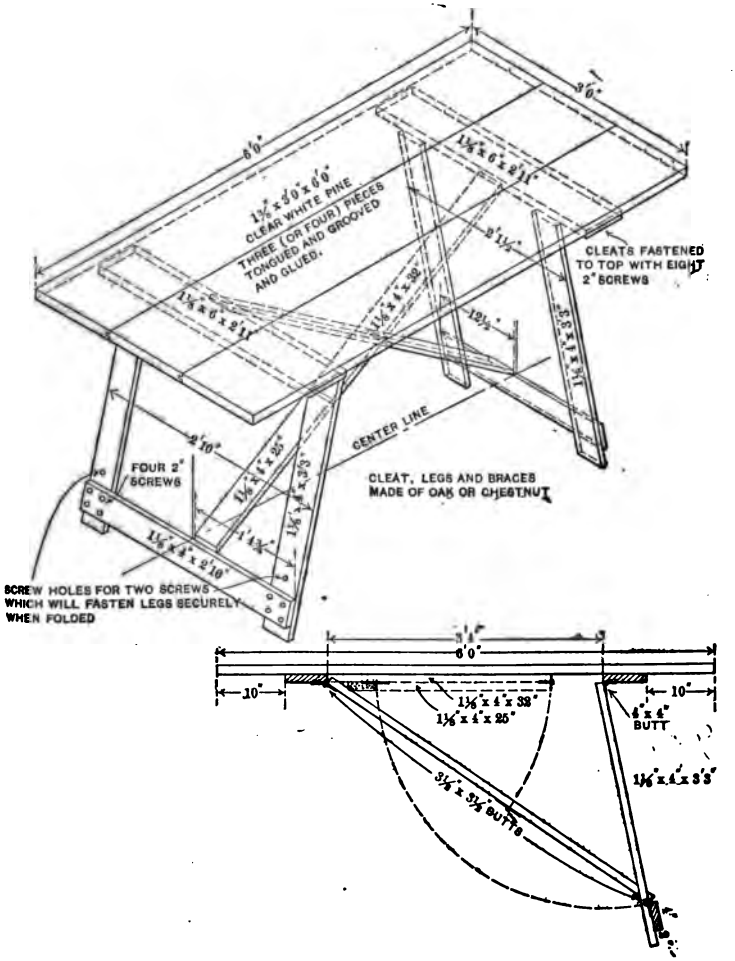


FIG. 10.—FOLDING DRAFTING TABLE.

of each kind of provisions so that there will be sufficient, but no waste. Besides the above, dish towels are practically essen-

tial and tablecloths and napkins are easily carried. A table oilcloth may replace the ordinary tablecloth. Wash tubs and wash board facilitate the washing of table linen and also underwear, so essential to clean, healthy living. Illumination for night work must be provided. Reflecting lanterns will answer for all tents except the office tent, where good lamps, with cylindrical wick and center draft, or similar, should be provided. The farther the party travels from "civilization" the greater the necessity for providing for emergencies, breakages, etc. Axes are essential, apart from their use in the surveying work. Extra handles should be provided. A saw, brace and several sizes of bits, screw drivers, monkey wrench, files, pliers, hatchet, assorted screws and nails, pick, shovel, crowbar, whetstone, rope in various sizes, sailor's needles, palm and sewing twine, will all be useful and even invaluable in times of emergency. Canvas-covered canteens, for each member of the party, when passing through arid regions, may be essential.

30. **Drawing tables.** Complete topographic drawings, made in the field, are absolutely essential. Suitable drawing boards are, therefore, required. The design shown in Fig. 10 fulfills all the working requirements; it also is easily handled when packed up and is not readily broken. By packing them together in pairs, face to face, the surfaces are protected during transportation. The table consists essentially of a drawing board with stiffening cleats. The legs are hinged to the cleats, the braces for each pair of legs being of just such a length that when opened the legs stand at the desired angle. The braces are hinged and fold up, jackknife fashion, so that they nowhere project beyond the legs.

31. **Stationery and map chest.** Considering that the maps, drawings and notebooks may represent thousands of dollars, and that they are likely to be injured, if not irreparably ruined, by rain, when moving camp or during a cyclonic storm, a strong, water-tight chest, of ample capacity for all drawings and notebooks, should be provided. It should be required that *all* drawings and notebooks should be kept in the chest over night and at all other times, except such drawings and notebooks as are in actual use. The net inside length should be a little in excess of the longest roll or drawing, which is perhaps 36 inches. There should be a tray in the top with numerous compartments or boxes for the multitudinous small articles required by a drafts-

man. Handles should be provided for convenience and it should have a lock. A good "steamer" trunk of requisite size will answer the purpose, provided it is waterproof, and it would perhaps be cheaper than a chest of similar size, made to order.

32. Provisions. A "ration" is the estimated amount of food required per man per day. For men engaged in strenuous outdoor work, the food required is far more than that eaten ordinarily. Ration lists should average about 5 to 6 pounds of food per day per man. The amount that must be *transported* may be considerably less than this, in view of the fact that e.g., dried vegetables may be substituted for fresh vegetables in the ratio of 1 lb. of dried for 3 lbs. of fresh, the water used in cooking providing the other two pounds. For explorers, who carry their own provisions, and who must cut down every possible ounce of baggage, still further concentrations are possible.

Article	100 rations
Fresh meat, including fish and poultry, (*a*)	100 lbs.
Cured meat, canned meat, or cheese (*b*)	50 "
Lard	15 "
Flour, bread or crackers	80 "
Corn meal, cereals, macaroni, sago, or cornstarch	15 "
Baking powder or yeast cakes	5 "
Sugar	40 "
Molasses	1 gal.
Coffee	12 lbs.
Tea, chocolate or cocoa	2 "
Milk, condensed (*c*)	10 cans
Butter	10 lbs.
Dried fruits (*d*)	20 "
Rice or beans	20 "
Potatoes, or other fresh vegetables (*e*)	100 "
Canned vegetables or fruit	30 "
Spices	½ "
Flavoring extracts	½ "
Pepper or mustard	½ "
Salt	4 "
Pickles	3 qts.
Vinegar	1 "

"(*a*) Eggs may be substituted for fresh meat in the ratio of 8 eggs for 1 lb. of meat.

"(*b*) Fresh meat and cured meat may be interchanged on the basis of 5 lbs. of fresh for 2 lbs. of cured. [This ratio 5:2 is far higher than is usually allowed, 5:3 or even less is usually stated as the equivalent ratio.]

"(*c*) Fresh milk may be substituted for condensed milk in the ratio of 5 quarts of fresh for 1 can of condensed.

"(*d*) Fresh fruit may be substituted for dried fruit in the ratio of 5 lbs. of fresh for 1 of dried.

"(*e*) Dried vegetables may be substituted for fresh vegetables in the ratio of 3 lbs. of fresh for 1 lb. of dried."

The list at bottom of p. 43 is given by H. M. Wilson ("Topographic Surveying") as the ration list of the U. S. Geol. Survey. The quantities are those required to make up 100 rations, or the food for 5 men for 20 days, or for 100 men for one day. They are considered maximum. The sum total is about 525 lbs. or 5¼ lbs. per day per man.

Wilson states that the cost of the above list of rations should not average more than 45 to 55 cents per day for average conditions and with a maximum of 75 cents, but considering that this statement was written in 1900, some allowance may need to be made for higher prices since then.

The list given below represents the provisions actually supplied to a mining camp in British Columbia. The list has been reduced to the average quantity actually consumed per man per day. The food supply averaged nearly 6 lbs. per day per man.

Meat, etc.:		
Fresh beef	1.89	lbs.
Bacon	.076	"
Ham	.060	"
Codfish	.007	"
Canned salmon	.014	can
Breads, etc.:		
Pilot bread	.007	lb.
Flour	.894	"
Baking powder	.016	"
Corn meal	.037	"
Vegetables:		
Potatoes	1.421	lbs.
Turnips	.010	"
Carrots	.047	"
Beets	.016	"
Parsnips	.023	"
Rice	.043	"
Cabbage	.101	"
Dehydrated onions	.0014	"
" rhubarb	.0029	"
White beans	.0014	"
Bayo "	.027	"
Lima "	.013	"
Split peas	.006	"
Rowan "	.0014	"
Canned tomatoes	.016	can
" beans	.0043	"
" peas	.0014	"
Cereals:		
Pearl barley	.0004	lb.
Rolled oats	.117	"
Beverages:		
Tea	.021	lb.
Coffee	.036	"
Milk, condensed	.137	can

Fruit:		
Dried apples	.040	lb.
" pears	.033	"
" peaches	.029	"
" prunes	.020	"
" apricots	.007	"
" figs	.030	"
Dehydrated cranberries	.004	"
Currants	.021	"
Jam	.001	pint
Condiments, etc:		
Mustard	.001	lb.
Salt	.036	"
Pepper	.001	"
Vinegar, Klondyke	.0003	pint
Worcestershire sauce	.0043	"
Catsup	.0029	gal.
Miscellaneous:		
Sugar	.594	lb.
Lard	.030	"
Cheese	.016	"
Cornstarch	.007	"
Extract	.049	"
Curry powder	.0007	"
Cinnamon	.0009	"
Hops	.0001	"
Nutmeg	.0009	"
Ginger	.0014	"
Mapleine	.0011	oz.
Candied peel	.004	lb.
Butter	.014	"
Macaroni	.003	"
Sago	.011	"
Tapioca	.003	"
Baker's chocolate	.0014	"
Cocoanut	.0003	"
Pickles	.003	gal.

Supplies: candles, .03 lb.; gold dust, .003 lb.; soap, .024 bar.

The following list of provisions was bought to start a camp of 20 to 25 men on the Choctaw, Oklahoma & Gulf R. R. Survey. (F. Lavis, Trans. Am. Soc. C. E., Vol. LIV, p. 104.)

6 hams	100 cakes soap
6 pieces of bacon	1 gal. molasses
50 lbs. fresh beef	1 case condensed milk
1 case eggs	1 doz. tomato catsup
25 lbs. butter	½ " Worcestershire sauce
25 " lard	1 gal. pickles
100 " flour, hard wheat	½ doz. lemon extract
100 " flour, soft wheat	½ " vanilla extract
100 " sugar	1 box dried prunes
5 " baking powder	5 lbs. raisins
2 " tea	4 doz. assorted canned fruits
50 " coffee	1 case tomatoes
50 " navy beans	1 bushel potatoes
25 " lima beans	1 kit salt mackerel
12 " buckwheat flour	20 lbs. salt
5 " macaroni	½ " mustard
35 " cornmeal	1 " pepper
1 cheese, about 15 lbs.	1 qt. vinegar
12 packages oatmeal	½ doz. yeast cakes
10 lbs. rice	

In addition to the above, there must be provided plenty of matches, kerosene oil and perhaps candles. As a matter of health conservation, and the prevention of piles, it is wise to provide toilet paper and to insist, if necessary, on its use. There is economy, when it is practicable, in making wholesale contracts for all provisions, rather than to buy haphazard from small local sources.

35. Beds. When baggage wagons accompany the party, as is virtually necessary to transport other essential equipment, it is desirable that they also transport army cots. These fold up so as to be easily transportable. It is a wise economy to obtain the regular army blankets, since they are what long experience has approved. Bedding rolls should be provided for the bedding. This is essential to keep the bedding in even reasonably cleanly condition, especially while moving. The policy of requiring each member of the party to provide himself with cot, bedding and cover, and to care for them, is debatable. As a matter of business economy, the company should buy all cots and bedding wholesale. Requiring each one to purchase his own is virtually a reduction of salary, for, if a man leaves the party, he usually does not care to take his bedding with him, except in the hope of realizing something on it. But as all this is considered when accepting employment, the company virtually pays for the bedding by an increase of salary over what

they would have to pay if bedding were provided. There is the same reason for owning bedding as for owning dishes, etc. Sterilizing bedding by means of a formaldehyde candle, especially after a man has left the party, is a wise sanitary precaution and nullifies one of the strongest reasons for individual ownership.

34. **Transportation.** The route of travel of a mining engineer, a topographical engineer or an explorer, may be over country with every variety of surface and slope. But, since a practicable railroad route is necessarily on a low grade, except as it may pass over a ridge or mountain to be pierced by a tunnel, the question of grade does not ordinarily influence the method of transportation and wagons can ordinarily be used, provided the nature of the surface will permit. Strong and heavy wagons can usually pick their way between the camping places, even though long detours must be made to avoid swamps or other obstructions. The parties surveying the Choctaw, Oklahoma & Gulf R. R., previously referred to, used two teams regularly, one of which stayed with the topographical party. They used a third team for hauling supplies. Two teams of horses can help each other over a particularly bad place in the trail or in the case of accident. The wagons should have canvas tops, as a protection against rain, especially while moving. Transportation by dogs and sledges is only applicable under very limited and unusual conditions. It implies winter work, which is always uneconomical and inefficient compared with summer work, but in a very swampy country, where the transportation of any considerable amount of baggage is very difficult, and where it freezes during the winter to a comparatively smooth surface, such a method may be preferable in spite of short daylight hours and other disadvantages. "The Duluth, South Shore & Atlantic Railway employed toboggans during the construction of its road throughout the season of 1887." The description of this work, and much other useful information is given in a paper by Chas. H. Snow, Vol. XXIX, p. 164, in the Trans. Am. Inst. Mining Eng'rs. A reconnoissance through a comparatively unexplored country, made with the object of discovering a practicable low-grade route through a mountainous section, might require that all baggage shall be reduced to what may be handled in packs carried by horses, mules, Indian ponies or even by men. The question of the necessary method of transportation must always be studied before beginning a survey, since the entire question

of subsistence, and even many features of the method of work, must depend on what can be included in the baggage.

35. Clothing. While it may seem an unwarranted interference with personal liberty to control the clothing worn by members of the party, it becomes justifiable when the efficiency and progress of the party is impaired by bad health or disability, which is plainly due to neglect of proper precautions in the way of clothing or personal sanitation. Sore feet are responsible for a large part of the disablement of men. Washing the feet *every night*, especially when they have become wet, will often obviate blisters. Stockings should be heavy, made of "natural wool" and should fit tightly enough so that wrinkles will not form. Shoes should have heavy soles and should be made of such tough leather that they will not easily tear. Rubber boots should *not* be worn; they make the feet tender. Although a surveying trip is usually considered as the opportunity to use up discarded clothing, ordinary clothing is usually very unsuitable and quickly becomes unwearable. When camping conditions are rough and the work must last for several months, and possibly years, clothing made of specially suitable material is economical. The material should be tough, so that it will not easily be torn by brambles, etc. It should be waterproof so as to shed rain and yet should be porous. It should be so thoroughy shrunken that moisture cannot appreciably shrink it further. "Mackinaw" is a soft, rough cloth, all wool, thoroughly shrunken, light, warm and waterproof. It is especially suitable for cold weather. "Pontiac" is similar. "Khaki" is a twilled cotton and is especially suitable for warm weather clothing. "Jungle cloth" is somewhat similar, but is particularly noted for its toughness and durability.

Especial care should be taken in the choice of underclothing, so as to avoid sudden chills after becoming overheated. Woolen underclothing is almost essential. "Cholera bands," made of wool, should *always* be worn about the abdomen in tropical countries.

MEDICAL AND SURGICAL TREATMENT.

36. Responsibility of engineer-in-charge. Throughout any surveying trip, where camping is necessary, professional medical aid is usually unobtainable. There rests upon the engineer-in-

charge, as the head of the expedition, some measure of responsibility for the health and care of the party. When some member of the party is seriously injured by accident, bitten by a poisonous snake or insect, or stricken with a sudden and violent attack of disease, and competent medical assistance is absolutely unobtainable for several days or even weeks, the head of the party must choose between seeing the victim die or boldly performing some simple surgical operation or giving medical treatment which he would not dream of doing otherwise. It is the lesser of two evils and the engineer must not shirk his duty. Even though a doctor is *perhaps* obtainable after many days delay by despatching a messenger 50 miles for him, common sense first-aid work and the intelligent use of a few simple methods and remedies may save life or prevent or mitigate permanent disablement. The outfit should include a sufficient supply of the medicines and medical appliances which would most probably be required. All bottles should be carried in cases to prevent breakage and the corks or stoppers secured tightly. When practicable, the drugs should be in tablet form, rather than liquid, and a normal dose should be marked on each bottle or package. They should be doubly labeled and the labels varnished to prevent their coming off in a damp climate. All adhesive plasters, antiseptic gauze, and such appliances, should be kept carefully wrapped up and protected from air and moisture.

37. **Appliances.** The very simplest medical outfit should include a pair of good scissors, which can be made antiseptically clean by wiping off with alcohol; a knife with two razor-sharp blades; a probe; a small saw; dentist's forceps; a pair of mouse-tooth forceps; a hypodermic syringe and two needles, or the more modern individual hypodermic syringe packages; also individual needles and cat-gut "No. 2 chromic" in curved vacuum tubes; a two-quart fountain syringe; supplies of sterilized gauze, adhesive plaster, needles, safety pins. The engineer should thoroughly familiarize himself with the working and manner of use of these. Any engineer who is preparing to head an expedition into a region where medical attention is unobtainable should consider that he can very wisely spend time with some doctor friend in learning the elements of the use of all these appliances.

38. **Antiseptics.** The engineer should warn his men of the danger from the infection of even slight wounds and scratches, especially in hot climates. The best emergency treatment for

any scratch, nail gouge, or nail in the foot, is to apply pure tincture of iodine at the *base* of the wound by cotton on the end of a small stick or probe. A more modern safe-guard against tetanus, or "lock-jaw," is "tetanus antitoxin," put up in individual syringe packages. A few of the many effective antiseptics are here mentioned: **Boric ointment;** one part of powdered boric acid added to nine parts of vaseline. **Carbolic ointment;** one part of carbolic acid to nineteen parts of vaseline. **Iodoform** powder promotes rapid healing of sores and wounds; one part in eight parts of vaseline is a good healing ointment. **Permanganate of potash;** one grain gives a purple color to a gallon of water; if the water is impure, the purple color changes rapidly to brown and this is a rough test of organic impurity; the crystals are soluble in 20 parts of water; it is especially useful in the treatment of snake bites. In a snake-infested country, it is wise for each man to carry permanganate of potash crystals with him, for use in emergency. See "Snake bites," § 44.

39. **Drinking water.** Every chief of party should see to it that his party has a pure supply of drinking water and especially that this supply is not contaminated by excrement from the camp draining into it. If there is any doubt about the purity of the supply (especially if so indicated by the permanganate-of-potash test) it should be part of the duty of the camp cook to maintain a liberal supply of boiled and cooled water. A neglect of such a precaution might easily result in an epidemic of typhoid. In a region where all streams are contaminated, perhaps by decaying vegetation or other natural cause, it may be wise to provide canteens, which the cook should furnish each morning filled with sterilized water.

40. **Bleeding** from an artery or vein can sometimes be stopped by pressing the vessel with sufficient pressure to stop the flow and continuing the pressure until the blood coagulates. If the vein or artery is actually severed but is not too large, the bleeding may be stopped by the use of a pair of forceps; grasp and pinch the vessel and twist it around three or four times. In about ten minutes the forceps may be removed. If the vein or artery is larger, and especially when it is an artery, which may be recognized by spurts of bright red blood, it may be necessary to tie the vessel. This may be done with catgut ligature, which should previously be boiled to prevent infection. While preparing for this, bleeding should be stopped by temporary pres-

sure. This is most easily done when the bleeding vessel may be pressed against a bone. A tourniquet can be improvised for pressing a pad (or even a stone) against the vein or artery of a limb by using a stick and a piece of cloth, or, perhaps, a rope and a small block of wood. Fasten the cloth or rope into a loose loop around the limb and run the stick through the loop; then twist it so that the pad is pressed down as desired. The rope can be so disposed as to press the block, which in turn presses the pad against the vein or artery.

41. Ailments and diseases; medicines; treatment.

Colic or cramp. Essence of ginger, 5 to 20 drops, in a small amount of very hot water.

Diarrhœa. Remove the bowel irritant by a castor-oil purge; then, if diarrhœa continues, give one-half teaspoonful of bismuth sub-carbonate every two or three hours until relieved.

Purgatives. Epsom salts; dose, two teaspoonsful in a small glass of hot water. Calomel; dose, two to five grains; should be followed by Epsom salts. Cascara sagrada; dose, two to six grains. Castor oil; dose, one to three tablespoonsful, which may be made more palatable by mixing with an equal amount of glycerine, and then putting the mixture into a glass of lemonade. Any tendency to constipation, which leads to intestinal poisoning and appendicitis, may be avoided by using a laxative, made as effective as necessary, about once a week.

Emetics. Common salt (two tablespoonsful), or mustard (one tablespoonful) or Ipecacuanha (30 grains) or Zinc Sulphate (30 grains), dissolved in a glass of water. Tickling the throat with a feather may sometimes be effective. Strong "Ivory" soap suds is excellent.

Malaria. Five grains of Quinine as a preventive; ten grains, three times a day, as an ordinary maximum dose. Larger doses are often given but it is dangerous unless under the care of a physician.

Cold-in-head. Rhinitis tablets, given as directed on bottle, are effective to break up an incipient cold. "Dover's powder"; dose, five to ten grains. Keep patient warm, with hot-water bottles and hot drinks.

42. Drowning; electric shock, asphyxiation. The trouble and the remedy is essentially the same in all three cases; respiration has been temporarily suspended and *must* be promptly restored

by artificial means. Loosen the patient's clothing, especially about the neck. In a drowning case, lay the patient on the ground, face down, straddle him and raise him at the hips so that the water in the air passages will drain out. Remove from the mouth any tobacco, false teeth or anything else that might obstruct breathing. Draw the tongue forward with forceps or a handkerchief. Then lay him face down, but with the face turned to one side so as to facilitate breathing, and with the arms extended forward. Then the operator, kneeling astride the patient, facing his head, and with the hands pressing on the lower ribs, *gradually* presses down so as to expel the air from the lungs. Then he suddenly removes the pressure by swinging back, and thus allows air to enter the lungs. Repeat the movements every four or five seconds, until natural breathing commences. Considering the fact that this method has successfully restored breathing after some *hours* of unsuccessful effort, and also that, in those cases, the patient would have died except for the persistency of the effort, the operator must not be discouraged because his efforts are not immediately successful. Promptness in beginning such treatment is so important that it is better to commence at once (even outdoors) rather than allow any material delay in order to get the patient to a house. The patient should be allowed plenty of air; crowding around him should be avoided. A blanket, extra clothing, hot bricks or stones, or hot-water bags, to restore heat to the body, will be of assistance, provided they do not interfere with the respiration operations. Do not attempt to make the patient swallow anything (e. g., a stimulant), until he is fully conscious; otherwise he will choke.

43. **Fractures.** Obtain medical aid if possible, but if this is unobtainable, except after a delay of many days or weeks, and it is uncertain even then, it may be preferable to take the chances of common-sense treatment, even if unskilled, rather than the certain permanent injury due to neglect of all treatment. Fractures are (a) **simple,** when the skin is not broken; (b) **compound,** when the skin is so broken that the fractured bone is more or less exposed to the air; and (c) **comminuted,** when there are two or more breaks of the same bone; a comminuted fracture may be simple or compound. Great care should be used in handling the patient immediately after the accident so that a simple fracture does not become compound. A broken limb should be

carefully straightened out and bound temporarily with the best improvised splints which are available until the patient can be removed to a bed. Even if amateur bone setting is decided to be advisable, setting should not be attempted if there is excessive swelling or tenderness. Apply ice or evaporating lotions to reduce any swelling. Splints should be made which are of proper length and are so rounded and padded with cloth that they cannot produce any concentrated pressure. Usually the dislocated bones are forced past each other, especially if the fracture is oblique rather than perpendicular, and it is always necessary to use considerable force, especially if it is a broken leg, to pull the bones back into position. The amateur must use his best common sense and knowledge of skeleton anatomy to restore the fragments to the same relative position they had previously; and then to secure them rigidly stiff with splints. Comparison with an unbroken arm or leg will be made even by a skilled surgeon, and such a comparison should be carefully studied by the amateur. While the binding should be as firm as it is safe to make it, it may be so tight as to produce swelling and even ulceration, and then the binding must be loosened. Compound fractures require the care of the flesh and skin wound in addition to the bone setting. The wound should be treated as described for wounds, but the splints and binding should be designed so that the wound can be properly dressed without loosening the splints. If the broken bone protrudes through the wound, it *must* be drawn back so that the wound can heal externally; even though the bone setting is beyond the skill of the amateur surgeon. Setting usually requires about six weeks, but, in the case of a limb, the joints above and below the break should be very carefully moved after about three weeks, so as to avoid stiff joints, special care being taken that there is no strain on the healing bone.

44. **Snake or insect bites.** The majority of snake bites occur on the limbs. In such a case (1) **tie a cord or bandage** about the limb just above the wound as promptly as possible, so as to prevent the poisoned blood from getting into the system; (2) **cut into the wound** so as to induce free bleeding; (3) **suck the wound** to aid in drawing out the poisoned blood; there is little or no danger in this, provided the mouth is free from sores, and provided the mouth is immediately rinsed out, preferably with an antiseptic solution, such as a light purple solution of per-

manganate of potash; (4) **inject into the wound a strong** solution of **permanganate of potash**, which may be done hypodermically or, perhaps, even by rubbing into the wound crystals of the drug. When the case is very serious, on account of the known deadly character of the poison, and when no permanganate of potash is obtainable, heroic measures are sometimes necessary. Pure **carbolic acid**, or **caustic**, may be used, if available. Cauterizing the wound with white-hot iron, exploding a pinch of gunpowder over the wound, shooting away the infected part with a gun, or even summary amputation with a hatchet, may sometimes be considered the lesser of two evils. If the limb has been tightly tied, it will, of course, produce great pain, discoloration and swelling, which must not be continued too long. A second ligature should be tied a few inches above the first. When the limb becomes very swelled and painful, loosen the first ligature for about ten seconds and again tighten, and then loosen the second ligature for ten seconds and again tighten. After fifteen minutes, repeat the loosening and tightening. After about eight repetitions, the ligatures may be removed altogether. If the poison is partly sucked out, the remainder partly neutralized with chemicals, and does not get fully into the system for two hours, the danger is greatly diminished. Of course bites on the face or body cannot be tied up and can only be treated by sucking out the poison and by chemicals. Stimulation of the heart is usually essential, which may be done with one teaspoonful of **aromatic spirits of ammonia** in two tablespoonsful of water, or with alcoholic liquor, preferably whiskey. One 1-30th grain **strychnine** tablets, dissolved in two tablespoonsful of water, is also a stimulant. If a hypodermic is available, one tablet may be dissolved in thirty drops of sterile water and inserted in the back or arm, well under the skin.

45. **Wounds.** First, last and all the time, **prevent infection.** The marvelous success of modern surgery is due largely to antiseptic methods. Neglect of cleanliness almost inevitably induces blood poisoning. A perfectly clean cut, after being washed and sterilized with iodine, may be closed with adhesive plaster, taking stitches, if necessary, with sterilized catgut or silk or linen thread. The stitches may be removed in a week. But when the flesh is torn and, especially, when dirt and other matter, which is possibly poisonous or infectious, has been forced into the wound, there is great danger of blood poisoning, and

the wound must be cleansed. First, cover the wound itself with a pad which has been soaked in an antiseptic solution and then wash the skin (shaving off all hair), all around the wound, using first soap and then an antiseptic solution. Then cleanse out all foreign matter from the wound, using antiseptics and pack the wound with strip gauze, soaked in the antiseptic, so as to extend from the deepest part of it to the outside. This will drain the discharges. The dressing should be renewed every day, or even three times per day, according to the severity of the wound, until the wound shows a tendency to heal. A gaping torn wound should not be sewed up, except to bring the edges together temporarily.

45a. Medical outfit to be carried. The quantity of medicine which should be carried is necessarily guess-work. If the party has great good luck, it might bring back the entire supply untouched. On the other hand severe sickness might exhaust some of the medicines long before the survey is complete. But the following list has been estimated as a reasonably proper supply for a party of 25 men which may be out of reach of an adequate source of medical supplies for a period of six months. The list should be varied somewhat according to the climate. The probabilities of disease, snake-bites, etc., in a cold climate are not the same as those of the tropics. Some of the following articles, those commonly required for "first-aid" work, should always be provided, even when the party will never be more than a few hours distance from medical assistance.

Boric acid, powdered, 5 lbs.
Carbolic acid, pure, 1 oz.
Iodoform powder, 2 oz.
Permanganate of potash, 8 oz.
Essence of ginger, 2 oz.
Epsom salts, 50 lbs.
Calomel, 1000 ½-gr. tablets.
Cascara sagrada, 1000 5-gr. tablets.
Castor oil, 4 quarts.
Glycerine, 4 quarts.
Ipecacuanha, 6 oz.
Individual hypoder. syr. packages; Tetanus antitoxin, 12 units; Morphine (¼ gr.), Atropine (1/150 gr.); (for agonizing pain); 24 units; Strychnia (1/20 gr.); 24 units; Camphorated oil; 24 units (for profound shock); Digalen (20 drops); 24 units (for acute heart trouble).
Bismuth sub-carbonate, 2 lbs.
Zinc sulphate, 4 oz.
Quinine 1000 5-gr. tablets.
Rhinitis, 2000 5-gr. tablets.
Dover's powder, 1000 5-gr. tablets.
Caustic, $AgNO_3$, 24 sticks.
Aromatic sp'ts of ammonia, 1 pint.
Strychnine tablets, 1000 1/40 gr.
Carbolized vaseline, 12 1-oz. jars.
Sterilized gauze, 5 dos. individual 1-yard rolls.
Adhesive plaster, 3 5-yard rolls, 12 inches wide.
Needles and catgut, No. 2 chromic, in curved vacuum tubes, 12 packages.
Needles, safety pins, etc.
Instruments, etc., as listed in § 37.

CHAPTER II.

ALINEMENT

In this chapter the alinement of the *center line* only of a pair of rails is considered. When a railroad is crossing a summit in the grade line, although the horizontal projection of the alinement may be straight, the vertical projection will consist of two sloping lines joined by a curve. When a curve is on a grade, the center line is really a spiral, a curve of double curvature, although its horizontal projection is a circle. The center line therefore consists of straight lines and curves of single and double curvature. The simplest method of treating them is to consider their horizontal and vertical projections separately. In treating simple, compound, and transition curves, only the horizontal projections of those curves will be considered.

SIMPLE CURVES.

46. Designation of curves. A curve may be designated either by its radius or by the angle subtended by a chord of unit length. Such an angle is known as the "degree of curve" and is indicated by D. Since the curves that are practically used have very long radii, it is generally impracticable to make any use of the actual center, and the curve is located without reference to it. If AB in Fig. 11 represents a unit chord (C) of a curve of radius R, then by the above definition the angle AOB equals D. Then

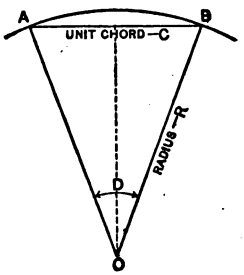


FIG. 11.

$$AO \sin \tfrac{1}{2}D = \tfrac{1}{2}AB = \tfrac{1}{2}C.$$

$$R = \frac{\frac{1}{2}C}{\sin \frac{1}{2}D}, \quad \ldots\ldots (1)$$

or, by inversion,

$$\sin \tfrac{1}{2}D = \frac{C}{2R}. \quad \ldots \ldots \ldots \quad (2)$$

The unit chord is variously taken throughout the world as 100 feet, 66 feet, and 20 meters. In the United States 100 feet is invariably used as the unit chord length, and throughout this work it will be so considered. Table I has been computed on this basis. It gives the radius, with its logarithm, of all curves from a 0° 01′ curve up to a 10° curve, varying by single minutes. The sharper curves, which are seldom used, are given with larger intervals.

An approximate value of R may be readily found from the following simple rule, which should be memorized:

$$R = \frac{5730}{D}.$$

Although such values are not mathematically correct, since R does not strictly vary inversely as D, yet the resulting value is within a tenth of one per cent for all commonly used values of R, and is sufficiently close for many purposes, as will be shown later.

47. Metric Curves. The unit chord for railroad curves on the metric system is 20 meters. If a curve has a 100-foot chord and a central angle of 5°, the radius would, of course, be 1146.3 feet. Since 20 meters = 65.6174 feet, a 20-meter chord between those same radial lines would subtend an arc with a radius of .656174 × 1146.3 feet, or 752.16 feet. But this radius, measured in meters, would be (.656174 × 1146.3) ÷ 3.28087 = 229.26 meters, which is 1146.3 × .20. In other words, the radius of any metric curve, measured in meters, is *numerically* one-fifth of the radius, measured in feet, of the same degree curve, but in actual length is a little less than two-thirds. This practically means that a 10° curve, metric, is actually very much sharper than a 10° curve, using foot-measure, or that the radius is about 66% as much. Therefore, in selecting curves for location, an engineer, who is accustomed to the foot-measure system, should remember that a 10° curve metric, for example, has approximately the same radius as a 15° curve, using foot-measure. While it is more convenient for an engineer, who is constantly using the metric system for curves, to have tables computed directly on

this basis, an engineer need not be dependent on such tables, since it is only necessary to divide the tabular quantities in the foot-table by 5 to obtain the corresponding quantities for the metric system. This applies not only to radii, but also to tangents, external distances and long chords for a 1° curve. A desired logarithm may be obtained by subtracting 0.6989700 from the foot-table logarithm.

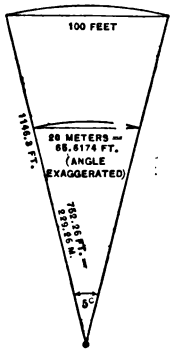


FIG. 12.

For example, anticipating the explanation in Art. 53, what is the tangent distance of a 6° metric curve, when the central angle is 32° 40′. From Table II, we find that by the foot-system the tangent distance for a 1° curve when the central angle is 32° 40′ is 1679.1 feet; then for a 6° curve it is $1679.1 \div 6 = 279.85$ feet; for a 6° metric curve it is $279.85 \div 5 = 55.97$ meters. The radius of the 6° metric curve $= 955.37 \div 5 = 191.074$ meters, which is in actual length about 99% of 955.37 feet.

As another illustration of the transformation from the foot-system to the metric system, or vice versa, the degree of a curve, by the foot system, may be multiplied by .66 and obtain approximately the degree of the equivalent curve by the metric system. For example, a 6° curve, foot system, has about the same actual radius as a $6 \times .66 = 3.96°$ metric curve, or about a 4° curve.

48. **Length of a subchord.** Since it is impracticable to measure along a curved arc, curves are always measured by laying off 100-foot chord lengths. This means that the actual arc is always a little longer than the chord. It also means that a *subchord* (a chord shorter than the unit length), will be a little longer than the ratio of the angles subtended would call for. The truth of this may be seen without calculation by noting that two equal subchords, each subtending the angle $\frac{1}{2}D$, will evidently be slightly longer than 50 feet each. If c be the length of a subchord subtending the angle d, then, as in Eq. 2,

$$\sin \tfrac{1}{2}d = \frac{c}{2R},$$

or, by inversion,

$$c = 2R \sin \tfrac{1}{2}d. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The *nominal length* of a subchord $= 100\frac{d}{D}$. For example, a *nominal* subchord of 40 feet will subtend an angle of $\frac{40}{100}$ of $D°$; its *true length* will be slightly more than 40 feet, and may be computed by Eq. 3. The *difference* between the nominal and true lengths is maximum when the subchord is about 57 feet long, but with the low degrees of curvature ordinarily used the difference may be neglected. With a 10° curve and a nominal chord length of 60 feet, the true length is 60.049 feet. Very sharp curves should be laid off with 50-foot or even 25-foot chords (nominal length). In such cases especially the true lengths of these subchords should be computed and used instead of the nominal lengths.

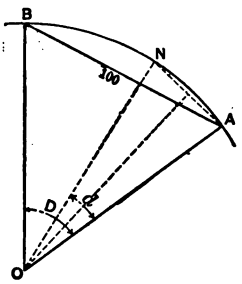


FIG. 13.

For example, assume that a 12° curve begins at Sta. 26+30. The first subchord will be nominally 70 feet and actually 70.066 feet. Assume that the central angle between the tangents is 39° 36′. Then the nominal length of curve is $39.6° \div 12° = 3.30$ stations. $3.30 - .70 = 2.60$, the nominal length of curve beyond the first station point on the curve. The final subchord is nominally 60 feet, but its actual length is 60.070 feet.

The values of these subchords for even degrees between 5° and 30°, and for nominal chord lengths of 10, 20, 30, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90 and 95 feet, are given in Table IIa. The excess values increase approximately as the square of the degree of curvature, but for intervals of 1° simple interpolation will be sufficiently accurate for intermediate values.

49. Length of a curve. The actual mean length of the two rails will be more than the nominal length of the curve, as defined above, and even more than the sum of the full 100-foot lengths and the true lengths of the subchord lengths at the ends. In the above numerical case the mean rail length is

$$39.6° \times \frac{\pi}{180°} \times R = 39.6° \times \frac{\pi}{180°} \times 478.34 = 330.604.$$

The sum of the two full-chord lengths and the two subchords is $70.066+200+60.070=330.136$. A large part of the excess $(330.604-330.136=.468)$ is the excess length (.183) of each arc of a 12° curve over the 100-foot chord. The remainder is the excess of the 70-foot and 60-foot arcs over the true chord lengths. But this excess length is of little practical importance. In the above case (a 12° curve) it adds about 0.2% to the length of rail that must be bought. The excess varies approximately as the square of the degree of curvature. The percentage of excess for the entire length of a road is utterly insignificant and is swallowed up by the 2% excess which is usually allowed for wastage in rail cutting.

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tangents at V, which equals the angle made by the radii to the extremities of the curve, is called the *central angle* (Δ). AV and BV, the two equal tangents from the vertex to the PC and T, are called the *tangent distances* (T). The chord AB is called the *long chord* (LC). The intercept HG from the middle of the long chord to the middle of the arc is called the *middle ordinate* (M). That part of the secant GV from

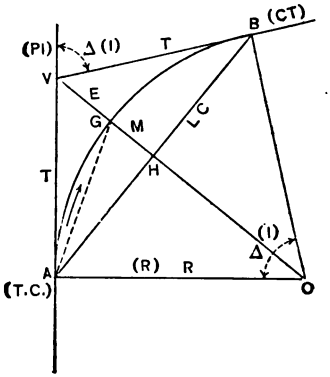

FIG. 14.

the middle of the arc to the vertex is called the *external distance* (E). From the figure it is very easy to derive the following frequently used relations:

$$T = R \tan \tfrac{1}{2}\Delta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$LC = 2R \sin \tfrac{1}{2}\Delta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$M = R \operatorname{vers} \tfrac{1}{2}\Delta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$E = R \operatorname{exsec} \tfrac{1}{2}\Delta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

52. **Relation between T, E, and Δ.** Join A and G in Fig. 14. The angle $VAG = \frac{1}{4}\Delta$, since it is measured by one half of the arc AG between the secant and tangent.

$$AGO = 90° - \tfrac{1}{4}\Delta.$$

$$AV : VG :: \sin AGV : \sin VAG;$$

$$\sin AGV = \sin AGO = \cos \tfrac{1}{2}\Delta;$$

$$T : E :: \cos \tfrac{1}{2}\Delta : \sin \tfrac{1}{2}\Delta;$$

$$T = E \cot \tfrac{1}{2}\Delta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The same relation may be obtained by dividing Eq. 4 by Eq. 7, since $\tan a \div \operatorname{exsec} a = \cot \frac{1}{2}a$.

53. **Elements of a 1° curve.** From Eqs. 1 to 8 it is seen that the elements of a curve vary directly as R. It is also seen to be very nearly true that R varies inversely as D. If the elements of a 1° curve for various central angles are calculated and tabulated, the elements of a curve of $D°$ curvature may be approximately found by dividing by D the corresponding elements of a 1° curve having the same central angle. For small central angles and low degrees of curvature the errors involved by the approximation are insignificant, and even for larger angles the errors are so small that *for many purposes* they may be disregarded

In Table II is given the value of the tangent distances, external distances, and long chords for a 1° curve for various central angles The student should familiarize himself with the degree of approximation involved by solving a large number of cases under various conditions by the exact and by the approximate methods, in order that he may know when the approximate method is sufficiently exact for the intended purpose. The approximate method also gives a ready check on the exact method.

A closer value may be obtained by using the "Corrective Table" found at the end of the main table. The correction is *always* additive and is usually very small and often even too insignificant for attention. A glance at the corrective table will show whether a correction need be made and an easily computed interpolation will show its amount. For example, what is the tangent distance for a 6° curve having a central angle of 42° 15'? Interpolating between 2209.0 and 2218.6, we have 2213.8 as the tangent distance for a 1° curve. Dividing by 6, we have 368.97 as the approximate tangent distance. Interpolating in the corrective table, we have .14 as the correction for a 5° curve and a

central angle of 42° 15′, and .28 as the correction for a 10° curve. Interpolating for 6° between these values of .14 and .28, we have .17, which added to 368.97 equals 369.14. The precise value, computed from Eq. 4, is 369.12. If the approximate value, even after correction, is not considered sufficiently accurate, Eq. 4 should be used. The student should appreciate that the discrepancy of even .02 in the above calculation is not due to any real error in the main table or the corrective table, but is due to the fact that the tangent distances are only computed to the nearest tenth of a foot for values over 1000 feet, and this will produce such discrepancies. The table should not be used where precise values are required.

54. Exercises. (*a*) What is the tangent distance of a 4° 20′ curve having a central angle of 18° 24′?

(*b*) Given a 3° 30′ curve and a central angle of 16° 20′, how far will the curve pass from the vertex? [Use Eq. 7.]

(*c*) An 18° curve is to be laid off using 25-foot (nominal) chord lengths. What is the true length of the subchords?

(*d*) Given two tangents making a central angle of 15° 24′. It is desired to connect these tangents by a curve which shall pass 16.2 feet from their intersection. How far down the tangent will the curve begin and what will be its radius? (Use Eq. 8 and then use Eq. 4 inverted.)

55. Curve location by deflections. The angle between a secant and a tangent (or between two secants intersecting on an arc) is measured by one half of the intercepted arc. Beginning at the *PC* (*A* in Fig. 15), if the first chord is to be a full chord we may deflect an angle *VAa* ($=\frac{1}{2}D$), and the point *a*, which is 100 feet from *A*, is a point on the curve. For the next station, *b*, deflect an *additional* angle *bAa* ($=\frac{1}{2}D$) and, with one end of the tape at *a*, swing the other end until the 100-foot point is on the line *Ab*. The point *b* is then on the curve. If the final chord *cB* is a subchord, its *additional deflection* ($\frac{1}{2}d$) is something less than $\frac{1}{2}D$. The last *deflection* (*BAV*) is

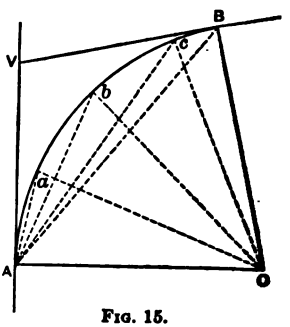

FIG. 15.

of course $\frac{1}{2}\Delta$. It is particularly important, when a curve begins or ends with a subchord and the deflections are odd quantities, that the last additional deflection should be carefully computed and added to the previous deflection, to check the mathematical work by the agreement of this last computed deflection with $\frac{1}{2}\Delta$.

Example. Given a 3° 24′ curve having a central angle of 18° 22′ and beginning at sta. 47+32, to compute the deflections. The nominal length of curve is 18° 22′ ÷ 3° 24′ = 18.367 ÷ 3.40 = 5.402 stations or 540.2 feet. The curve therefore ends at sta. 52+72.2. The deflection for sta. 48 is $\frac{68}{100} \times \frac{1}{2}(3° 24')$ = 0.68 × 1°.7 = 1°.156 = 1° 09′ nearly. For each additional 100 feet it is 1° 42′ additional. The final additional deflection for the final subchord of 72.2 feet is

$$\frac{72.2}{100} \times \tfrac{1}{2}(3° 24') = 1°.2274 = 1° 14' \text{ nearly.}$$

The deflections are

P. C ... Sta.	47+32	0°
	48	0° + 1° 09′ = 1° 09′
	49	1° 09′ + 1° 42′ = 2° 51′
	50	2° 51′ + 1° 42′ = 4° 33′
	51	4° 33′ + 1° 42′ = 6° 15′
	52	6° 15′ + 1° 42′ = 7° 57′
P. T.	52+72.2	7° 57′ + 1° 14′ = 9° 11′

As a check 9° 11′ = ½(18° 22′) = ½Δ. (See the Form of Notes in § 21.)

56. Instrumental work. It is generally impracticable to locate more than 500 to 600 feet of a curve from one station. Obstructions will sometimes require that the transit be moved up every 200 or 300 feet. There are two methods of setting off the angles when the transit has been moved up from the *PC*.

(a) The transit may be sighted at the previous transit station with a reading on the plates equal to the deflection angle from that station to the station occupied, but with the angle set off on the *other* side of 0°, so that when the telescope is turned to 0° it will sight along the tangent at the station occupied. Plunging the telescope, the forward stations may be set off by deflecting the proper deflections from the tangent at the station occupied.

This is a very common method and, when the degree (ture is an even number of degrees and when the transi set at even stations, there is but little objection to it. degree of curvature is sometimes an odd quantity, and gencies of difficult location frequently require that sul be occupied as transit stations. Method (*a*) will ther the recalculation of all deflections for each new station (The mathematical work is largely increased and the pr of error is very greatly increased and not so easily (Method (*b*) is just as simple as method (*a*) even for t simple cases, and for the more difficult cases just referr superiority is very great.

(5) Calculate the deflection for each station and su throughout the curve as though the whole curve were cated from the *PC*. The computations may thus be completed and *checked* (as above) before beginning the instrumental work. If it unexpectedly becomes necessary to introduce a substation at any point, its deflection from the *PC* may be readily interpolated. The stations actually set from the *PC* are located as usual.

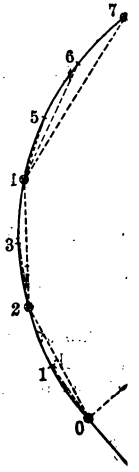

FIG. 16

RULE. **When the transit is set on any forward station, backsight to ANY previous station with the plates set at the deflection angle for the station sighted at. Plunge the telescope and sight at any forward station with the deflection angle originally computed for that station.** When the plates read the deflection angle for the station occupied, the telescope is sighting along the tangent at that station—which is the method of getting the forward tangent when occupying the *PT*. Even though the station occupied is an unexpected substation, when the instrument is properly oriented at that station, the angle reading for *any* station, forward or back, is that originally cc for it from the *PC*. In difficult work, where there structions, a valuable check on the accuracy may be f sighting backward at *any* visible station and noting

deflection agrees with that originally computed. As a ierical illustration, assume a 4° curve, with 28° curvature, ı stations 0, 2, 4, and 7 occupied. After setting stations id 2, set up the transit at sta. 2 and backsight to sta. 0 ı the deflection for sta. 0, which is 0°. The reading on sta. 2°; when the reading is 4° the telescope is tangent to curve, and when sighting at 3 and 4 the deflections will be nd 8°. Occupy 4; sight to 2 with a reading of 4°. When reading is 8° the telescope is tangent to the curve and, by ging the telescope, 5, 6, and 7 may be located with the nally computed deflections of 10°, 12°, and 14°. When oc- ying 7 a backsight may be taken to any visible station with plates reading the deflection for that station; then when

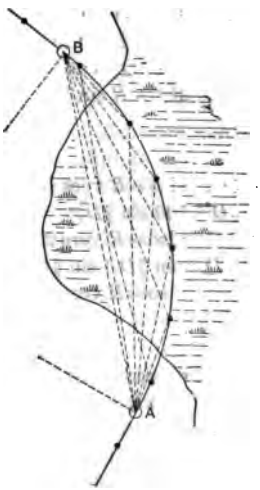

FIG. 17.

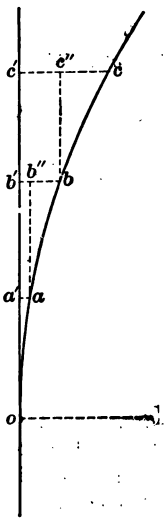

FIG. 18.

plates read 14° the telescope will point along the forward ent.

ıe location of curves by deflection angles is the normal ıod. A few other methods, to be described, should be con- ed as exceptional.

The *nominal length* of a subchord $= 100\frac{d}{D}$. For example, a *nominal* subchord of 40 feet will subtend an angle of $\frac{40}{100}$ of $D°$; its *true length* will be slightly more than 40 feet, and may be computed by Eq. 3. The *difference* between the nominal and true lengths is maximum when the subchord is about 57 feet long, but with the low degrees of curvature ordinarily used the difference may be neglected. With a 10° curve and a nominal chord length of 60 feet, the true length is 60.049 feet. Very sharp curves should be laid off with 50-foot or even 25-foot chords (nominal length). In such cases especially the true lengths of these subchords should be computed and used instead of the nominal lengths.

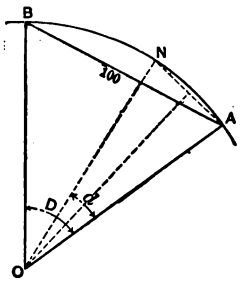

FIG. 13.

For example, assume that a 12° curve begins at Sta. 26+30. The first subchord will be nominally 70 feet and actually 70.066 feet. Assume that the central angle between the tangents is 39° 36′. Then the nominal length of curve is $39.6° \div 12° = 3.30$ stations. $3.30 - .70 = 2.60$, the nominal length of curve beyond the first station point on the curve. The final subchord is nominally 60 feet, but its actual length is 60.070 feet.

The values of these subchords for even degrees between 5° and 30°, and for nominal chord lengths of 10, 20, 30, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90 and 95 feet, are given in Table IIa. The excess values increase approximately as the square of the degree of curvature, but for intervals of 1° simple interpolation will be sufficiently accurate for intermediate values.

49. Length of a curve. The actual mean length of the two rails will be more than the nominal length of the curve, as defined above, and even more than the sum of the full 100-foot lengths and the true lengths of the subchord lengths at the ends. In the above numerical case the mean rail length is

$$39.6° \times \frac{\pi}{180°} \times R = 39.6° \times \frac{\pi}{180°} \times 478.34 = 330.604.$$

The sum of the two full-chord lengths and the two subchords is $70.066+200+60.070=330.136$. A large part of the excess $(330.604-330.136=.468)$ is the excess length (.183) of each arc of a 12° curve over the 100-foot chord. The remainder is the excess of the 70-foot and 60-foot arcs over the true chord lengths. But this excess length is of little practical importance. In the above case (a 12° curve) it adds about 0.2% to the length of rail that must be bought. The excess varies approximately as the square of the degree of curvature. The percentage of excess for the entire length of a road is utterly insignificant and is swallowed up by the 2% excess which is usually allowed for wastage in rail cutting.

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57. Curve location by two transits. A curve might be loc more or less on a swamp where accurate chaining would exceedingly difficult if not impossible. The long chord (Fig. 17) may be determined by triangulation or other and the elements of the curve computed, including (possi subchords at each end. The deflection from A and B to point may be computed. A rodman may then be sent whatever means) to locate long stakes at points determ by the simultaneous sightings of the two transits.

58. Curve location by tangential offsets. When a curv very flat and no transit is at hand the following method may used (see Fig. 18): Produce the back tangent as far forwar necessary. Compute the ordinates Oa', Ob', Oc', etc., and abscissæ $a'a$, $b'b$, $c'c$, etc. If Oa is a full station (100 feet), t

$$\left.\begin{aligned} Oa' &= Oa' && = 100\cos\tfrac{1}{2}D, \text{ also} = R\sin D;\\ Ob' &= Oa' + a'b' && = 100\cos\tfrac{1}{2}D + 100\cos\tfrac{3}{2}D,\\ & && \text{also} = R\sin 2D;\\ Oc' &= Oa' + a'b' + b'c' && = 100(\cos\tfrac{1}{2}D + \cos\tfrac{3}{2}D + \cos\tfrac{5}{2}D),\\ & && \text{also} = R\sin 3D; \end{aligned}\right\}$$

etc.

$$\left.\begin{aligned} a'a &= && 100\sin\tfrac{1}{2}D, \text{ also} = R\,\text{vers}\,D;\\ b'b &= a'a + b''b && = 100\sin\tfrac{1}{2}D + 100\sin\tfrac{3}{2}D,\\ & && \text{also} = R\,\text{vers}\,2D;\\ c'c &= b'b + c''c && = 100(\sin\tfrac{1}{2}D + \sin\tfrac{3}{2}D + \sin\tfrac{5}{2}D),\\ & && \text{also} = R\,\text{vers}\,3D; \end{aligned}\right\}$$

etc.

The functions $\frac{1}{2}D$, $\frac{3}{2}D$, etc., may be more conveniently u *without* logarithms, by adding the several *natural* trigonometr functions and pointing off two decimal places. It may also noted that Ob' (for example) is one half of the long chord four stations; also that $b'b$ is the middle ordinate for f stations. If the engineer is provided with tables giving the l chords and middle ordinates for various degrees of curvat these quantities may be taken (perhaps by interpolation) fr such tables.

If the curve begins or ends at a substation, the angles terms will be correspondingly altered. The modifications n

be readily deduced on the same principles as above, and should be worked out as an exercise by the student.

In Table II are given the long chords for a 1° curve for various values of Δ. Dividing the value as given by the degree of the curve, we have an approximate value which is amply close for low degrees of curvature, especially for laying out curves without a transit. For example, given a 4° 30′ curve, required the ordinate Oc'. This is evidently one half of a chord of six stations, with $\Delta = 27°$. Dividing 2675.1 (which is the long chord of a 1° curve with $\Delta = 27°$) by 4.5 we have 594.47; one half of this is the required ordinate, $Oc' = 297.23$. The exact value is 297.31, an excess of .08, or less than .03 of 1%. The true values are always slightly in excess of the value as computed from Table II.

Exercise. A 3° 40′ curve begins at sta. 18+70 and runs to sta. 23+60. Required the tangential offsets and their corresponding ordinates. The first ordinate = 30 cos $\frac{1}{100} \times 3° 40'$) = 30 × .99995 = 29.9985; the offset = 30 sin 0° 33′ = 30 × .0096 = 0.288. For the second full station (sta. 20) the ordinate = ½ long chord for $\Delta = 2(1° 06' + 3° 40')$ with $D = 3° 40'$. Dividing 476.12, from Table II, by 3½, we have 129.85. Otherwise, by Eq. 9, the ordinate = 30 × cos 0° 33′ + 100 cos (1° 06′ + 1° 50′) = 30.00 + 99.87 = 129.87. The offset for sta. 20 = 30 sin 0° 33′ + 100 sin (1° 06′ + 1° 50′) = 0.288 + 5.12 = 5.41. Work out similarly the ordinates and offsets for sta. 21, 22, 23, and 23+60.

59. Curve location by middle ordinates. Take first the simpler case when the curve begins at an even station. If we consider (in Fig. 19) the curve produced back to z, the chord $za = 2 \times 100 \cos \frac{1}{2}D$, $A'a = 100 \cos \frac{1}{2}D$, and $A'A = am = zn = 100 \sin \frac{1}{2}D$. Set off AA' perpendicular to the tangent and $A'a$ parallel to the tangent. $AA' = aa' = bb' = cc'$, etc. $= 100 \sin \frac{1}{2}D$. Set off aa' perpendicular to $a'A$. Produce Aa' until $a'b = A'a$, thus determining b. Succeeding points of the curve may thus be determined indefinitely.

Suppose the curve begins with a subchord. As before $ra = Am' = c' \cos \frac{1}{2}d'$, and $rA = am' = c' \sin \frac{1}{2}d'$. Also $sz = An' = c'' \cos \frac{1}{2}d''$, and $sA = zn' = c'' \sin \frac{1}{2}d''$, in which $(d' + d'') = D$. The points z and a being determined on the ground, aa' may be computed and set off as before and the curve continued in

full stations. A subchord at the end of the curve may be located by a similar process.

60. Curve location by offsets from the long chord. (Fig 21.) Consider at once the general case in which the curve commences with a subchord (curvature, d'), continues with one or more full

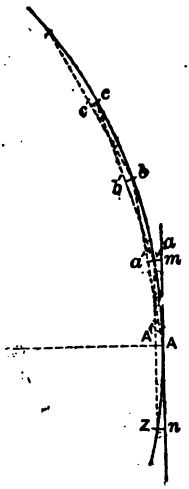


Fig. 19.

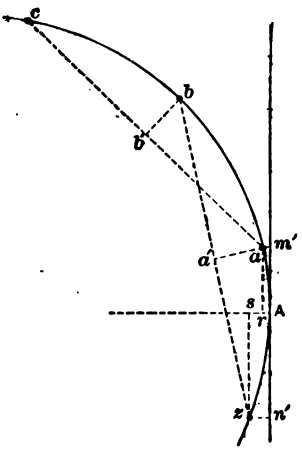


Fig. 20.

Fig. 21.

chords (curvature of each, D), and ends with a subchord with curvature d''. The numerical work consists in computing first AB, then the various abscissæ and ordinates. $AB = 2R \sin \frac{1}{2}\Delta$.

$$\left.\begin{aligned}
Aa' &= Aa' && = a' \cos \tfrac{1}{2}(\Delta - d');\\
Ab' &= Aa' + a'b' && = c' \cos \tfrac{1}{2}(\Delta - d') + 100 \cos \tfrac{1}{2}(\Delta - 2d' - D);\\
Ac' &= Aa' + a'b' + b'c' = c' \cos \tfrac{1}{2}(\Delta - d') + 100 \cos \tfrac{1}{2}(\Delta - 2d' - D)\\
&\qquad + 100 \cos \tfrac{1}{2}(\Delta - 2d'' - D);
\end{aligned}\right\} \quad (11)$$

also

$$= AB - Bc' \qquad = 2R \sin \tfrac{1}{2}\Delta - c'' \cos \tfrac{1}{2}(\Delta - d'').$$

$$\left.\begin{aligned}
a'a &= a'a && = c' \sin \tfrac{1}{2}(\Delta - d');\\
b'b &= a'a + mb = c' \sin \tfrac{1}{2}(\Delta - d') + 100 \sin \tfrac{1}{2}(\Delta - 2d' - D);\\
c'c &= b'b - nb = c' \sin \tfrac{1}{2}(\Delta - d') + 100 \sin \tfrac{1}{2}(\Delta - 2d' - D)\\
&\qquad - 100 \sin \tfrac{1}{2}(\Delta - 2d'' - D);
\end{aligned}\right\} \quad (12)$$

also $\qquad = c'' \sin \tfrac{1}{2}(\Delta - d'')$.

The above formulæ are considerably simplified when the

re begins and ends at even stations. When the curve is r long a regular law becomes very apparent in the formation ll terms between the first and last. There are too few terms ne above equations to show the law.

r. Use and value of the above methods. The chief value he above methods lies in the possibility of doing the work iout a transit. The same principles are sometimes em- ed, even when a transit is used, when obstacles prevent the of the normal method (see § 62, *c*). If the terminal tan- s have already been accurately determined, these methods useful to locate points of the curve when rigid accuracy is essential. Track foremen frequently use such methods to out unimportant sidings, especially when the engineer and ransit are not at hand. Location by tangential offsets (or offsets from the long chord) is to be preferred when the e is flat (i.e., has a small central angle Δ) and there is no ruction along the tangent, or long chord. Location by lle ordinates may be employed regardless of the length of curve, and in cases when both the tangents and the long d are obstructed. The above methods are but samples large number of similar methods which have been devised. choice of the particular method to be adopted must be rmined by the local conditions.

:. Obstacles to location. In this section will be given only a few of the principles involved in this class of problems, with illustrations. The engineer must decide, in each case, which is the best method to use. It is frequently advisable to devise a special solution for some particular case.

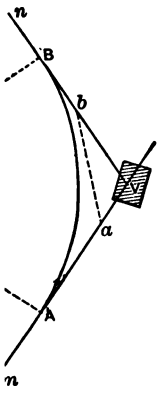


FIG. 22.

a. When the vertex is inaccessible. As shown in § 56, it is not absolutely essential that the vertex of a curve should be located on the ground. But it is very evident that the angle between the terminal tangents is determined with far less probable error if it is measured by a single measurement at the vertex rather than as the result of numerous angle measurements along the curve, involving several positions of the transit and comparatively short sights. Some-

times the location of the tangents is already determined on the ground (as by bn and am, Fig. 22), and it is required to join the tangents by a curve of given radius. *Method.* Measure ab and the angles Vba and baV. Δ is the sum of these angles. The distances bV and aV are computable from the above data. Given Δ and R, the tangent distances are computable, and then Bb and aA are found by subtracting bV and aV from the tangent distances. The curve may then be run from A, and the work may be checked by noting whether the curve as run ends at B—previously located from b.

Example. Assume $ab = 546.82$; angle $a = 15° 18'$; angle $b = 18° 22'$; $D = 3° 40'$; required aA and bB.

$\Delta = 15° 18' + 18° 22' = 33° 40'$

Eq. (4)

R $(3° 40')$	3.19393
$\tan \frac{1}{2}\Delta = \tan 16° 50'$	9.48060
$T = 472.85$	2.67453

$$aV = ab \frac{\sin 18° 22'}{\sin 33° 40'}$$

ab	2.73781
log sin $18° 22'$	9.49941
co-log sin $33° 40'$	0.25629
$aV = 310.81$	2.49351
$AV = 472.85$	
$aA = 162.04$	

$$bV = ab \frac{\sin 15° 18'}{\sin 33° 40'}$$

ab	2.73781
log sin $15° 18'$	9.42198
co-log sin $33° 40'$	0.25629
$bV = 260.29$	2.41608
$BV = 472.85$	
$bB = 212.56$	

b. When the point of curve (or point of tangency) is inaccessible. At some distance (As, Fig. 23) an unobstructed line ps may be run parallel with AV. $nv = py = As = R$ vers a.

$$\therefore \text{vers } a = As \div R.$$

$$ns = ps = R \sin a.$$

At y, which is at a distance ps back from the *computed* position of A, make an offset sA to p. Run pn parallel to the tangent. A tangent to the curve at n makes an angle of a with np. From n the curve is run in as usual

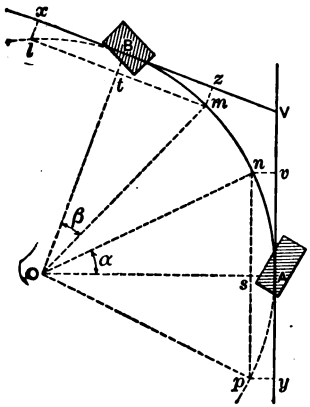


Fig. 23.

If the point of tangency is obstructed, a similar process, somewhat reversed, may be used. β is that portion of Δ still to be laid off when m is reached. $tm = tl = R \sin \beta$. $mz = tB = lx = R$ vers β.

c. When the central part of the curve is obstructed. a is the central angle between two points of the curve between which a chord may be run. a may equal *any* angle, but it is preferable that a should be a multiple of D, the degree of curve, and that the points m and n should be on even stations. $mn = 2R \sin \frac{1}{2}a$. A point s may be located by an offset ks from the chord mn by a similar method to that outlined in § 60.

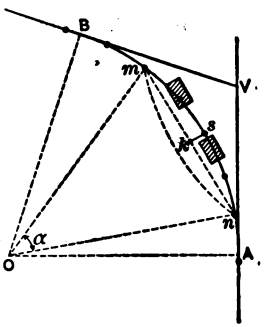


Fig. 24.

The device of introducing the dotted curve mn having the same radius of curvature as the other, although neither necessary nor advisable in the case shown in Fig. 24, is sometimes the best method of surveying around an obstacle. The offset from any point on the dotted curve to the corresponding point on the true curve is twice the "ordinate to the long cord," as computed in § 60.

63. Modifications of location. The following methods may be used in allowing for the discrepancies between the "paper location" based on a more or less rough preliminary survey and the more accurate instrumental location. (See § 18.) They are

also frequently used in locating new parallel tracks and modifying old tracks.

a. To move the forward tangent parallel to itself a distance x, the point of curve (A) remaining fixed. (Fig. 25.)

$$V'h = B'r = x'.$$

$$VV' = \frac{V'h}{\sin hVV'} = \frac{x}{\sin \Delta}. \quad \ldots\ldots \quad (13)$$

$$AV' = AV + VV'.$$

The triangle BmB' is isosceles and $Bm = B'm$.

$$R' - R = O'O = mB = \frac{B'r}{\text{vers } B'mB} = \frac{x'}{\text{vers } \Delta}.$$

$$\therefore R' = R + \frac{x'}{\text{vers } \Delta}. \quad \ldots\ldots \quad (14)$$

The solution is very similar in case the tangent is moved inward to $V''B''$. Note that this method necessarily changes the

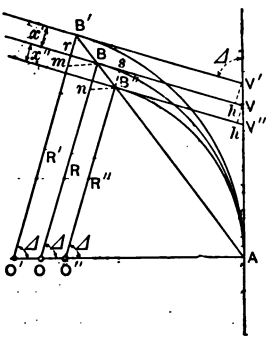


Fig. 25.

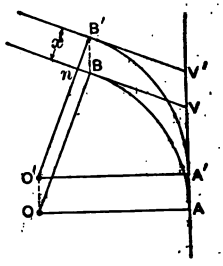


Fig. 26.

radius. If the radius is not to be changed, the point of curve must be altered as follows:

b. To move the forward tangent parallel to itself a distance x, the radius being unchanged. (Fig. 26.) In this case the whole

curve is moved bodily a distance $OO' = AA' = VV' = BB'$, and moved parallel to the first tangent AV

$$BB' = \frac{R'n}{\sin nBB'} = \frac{x}{\sin \Delta} = AA'. \quad . \quad . \quad . \quad (15)$$

c. To change the direction of the forward tangent at the point of tangency. (Fig. 27.) This problem involves a change (a) in the central angle and also requires a new radius. An error in the determination of the central angle furnishes an occasion for its use.

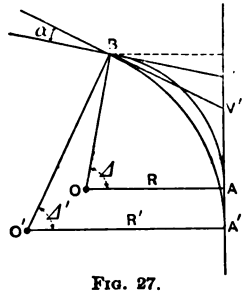


Fig. 27.

R, Δ, a, AV, and BV are known.

$$\Delta' = \Delta - a.$$

$$Bs = R \text{ vers } \Delta. \quad Bs = R' \text{ vers } \Delta'.$$

$$\therefore R' = R\frac{\text{vers } \Delta}{\text{vers } (\Delta - a)}. \quad . \quad (16)$$

$$As = R \sin \Delta. \quad A's = R' \sin \Delta'.$$

$$\therefore AA' = A's - As = R' \sin \Delta' - R \sin \Delta. \quad . \quad . \quad (17)$$

The above solutions are given to illustrate a large class of problems which are constantly arising. All of the ordinary problems can be solved by the application of elementary geometry and trigonometry.

64. Limitations in location. It may be required to run a curve that shall join two given tangents and also pass through a given point The point (P, Fig. 28) is assumed to be determined by its distance (VP) from the vertex and by the angle $AVP = \beta$.

It is required to determine the radius (R) and the tangent distance (AV). Δ is known.

$$PVG = \tfrac{1}{2}(180° - \Delta) - \beta$$
$$= 90° - (\tfrac{1}{2}\Delta + \beta).$$
$$PP' = 2VP \sin PVG$$
$$= 2VP \cos (\tfrac{1}{2}\Delta + \beta).$$
$$PSV = \tfrac{1}{2}\Delta.$$

$$\therefore SP = VP\frac{\sin \beta}{\sin \tfrac{1}{2}\Delta}.$$

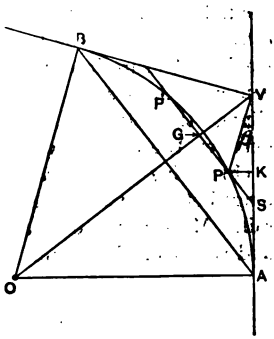


Fig. 28.

$$AS=\sqrt{SP\times SP'}=\sqrt{SP(SP+PP')}$$

$$=\sqrt{VP\frac{\sin\beta}{\sin\frac{1}{2}\Delta}\left[VP\frac{\sin\beta}{\sin\frac{1}{2}\Delta}+2VP\cos(\tfrac{1}{2}\Delta+\beta)\right]}$$

$$=VP\sqrt{\frac{\sin^2\beta}{\sin^2\frac{1}{2}\Delta}+\frac{2\sin\beta\cos(\frac{1}{2}\Delta+\beta)}{\sin\frac{1}{2}\Delta}}.$$

$$SV=VP\frac{\sin(\frac{1}{2}\Delta+\beta)}{\sin\frac{1}{2}\Delta}.$$

$$AV=AS+SV$$

$$=\frac{VP}{\sin\frac{1}{2}\Delta}[\sin(\tfrac{1}{2}\Delta+\beta)+\sqrt{\sin^2\beta+2\sin\beta\sin\tfrac{1}{2}\Delta\cos(\tfrac{1}{2}\Delta+\beta)}]. \quad (18)$$

$$R=AV\cot\tfrac{1}{2}\Delta.$$

In the special case in which P is on the median line OV, $\beta=90°-\frac{1}{2}\Delta$, and $(\frac{1}{2}\Delta+\beta)=90°$. Eq. 18 then reduces to

$$AV=\frac{VP}{\sin\frac{1}{2}\Delta}(1+\cos\tfrac{1}{2}\Delta)=VP\cot\tfrac{1}{2}\Delta,$$

as might have been immediately derived from Eq. 8.

In case the point P is given by the offset PK and by the distance VK, the triangle PKV may be readily solved, giving the distance VP and the angle β, and the remainder of the solution will be as above.

65. Determination of the curvature of existing track. (a) *Using a transit.* Set up the transit at any point in the center of the track. Measure in each direction 100 feet to points also in the center of the track. Sight on one point with the plates at 0°. Plunge the telescope and sight at the other point. The angle between the chords equals the degree of curvature.

(b) *Using a tape and string.* Stretch a string (say 50 feet long) between two points on the inside of the head of the outer rail. Measure the ordinate (x) between the *middle* of the string and the head of the rail. Then

$$R=\frac{\text{chord}^2}{8x}\text{(very nearly)}. \quad . \quad . \quad . \quad . \quad (19)$$

For, in Fig. 29, since the triangles AOE and ADC are similar,

$AO : AE :: AD : DC$ or $R = \frac{1}{2}\overline{AD}^2 \div x$. When, as is usual, the arc is very short compared with the radius, $AD = \frac{1}{2}AB$, very nearly. Making this substitution we have Eq. 19. With a chord of 50 feet and a 10° curve, the resulting difference in x is .0025 of an inch—far within the possible accuracy of such a method. The above method gives the radius of the inner head of the outer rail. It should be diminished by $\frac{1}{2}g$ for the radius of the center of the track. With easy curvature, however, this will not affect the result by more than one or two tenths of one per cent.

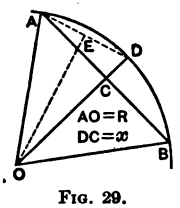


FIG. 29.

The inversion of this formula gives the required middle ordinate for a rail on a given curve. For example, the middle ordinate of a 30-foot rail, bent for a 6° curve, is

$$x = 900 \div (8 \times 955) = .118 \text{ foot} = 1.4 \text{ inches.}$$

Another much used rule is to require the foreman to have a string, knotted at the center, of such length that the middle ordinate, measured in inches, equals the degree of curve. To find that length, substitute (in Eq. 19) $5730 \div D$ for R and $D \div 12$ for x. Solving for *chord*, we obtain *chord* = 61.8 feet. The rule is not theoretically exact, but, considering the uncertain stretching of the string, the error is insignificant. In fact, the distance usually given is 62 feet, which is close enough for all purposes for which such a method should be used.

66. Problems. A systematic method of setting down the solution of a problem simplifies the work. Logarithms should always be used, and *all* the work should be so set down that a revision of the work to find a supposed error may be readily done. The value of such systematic work will become more apparent as the problems become more complicated. The two solutions given below will illustrate such work.

a. Given a 3° curve beginning at Sta. 27 + 60 and running to Sta. 32 + 45. Compute the ordinates and offsets used in locating the curve by tangential offsets.

b. With the same data as above, compute the distances to locate the curve by offsets from the long chord.

c. Assume that in Fig. 22 *ab* is measured as 217.6 feet, the

angle $abV = 17° 42'$, and the angle $baV = 21° 14'$. Join the tangents by a 4° 30′ curve. Determine bB and aA.

d. Assume that in a case similar to Fig. 23 it was noted that a distance (As) equal to 12 feet would clear the building. Assume that $I = 38° 20'$ and that $D = 4° 40'$. Required the value of a and the position of n. *Solution:*

vers $a = As \div R$	$As = 12$	log = 1.07918
	R (for 4° 40′ curve)	log = 3.08923
	$a = 8° 01'$	log vers a = 7.98994
$ns = R \sin a$		log sin a = 9.14445
		log R = 3.08923
	$ns = 171.27$	log = 2.23368

e. Assume that the forward tangent of a 3° 20′ curve having a central angle of 16° 50′ must be moved 3.62 feet *inward*, without altering the *P.C.* Required the change in radius.

f. Given two tangents making an angle of 36° 18′. It is required to pass a curve through a point 93.2 feet from the vertex, the line from the vertex to the point making an angle of 42° 21′ with the tangent. Required the radius and tangent distance. *Solution:* Applying Eq. 18, we have

2	log = 0.30103
$\beta = 42° 21'$	log sin = 9.82844
$\frac{1}{2}I = 18° 09'$	log sin = 9.49346
$(\frac{1}{2}I + \beta) = 60° 30'$	log cos = 9.69234
.20667	9.31527
$\log \sin^2 \beta = 9.65688$ .... .45382	
2 | 9.81987 .... .66049	
9.90993 .... .81271	
nat. sin 60° 30′ .... .8703	
1.6833	log = 0.22610
$VP = 93.2$	log = 1.96941
	2.19551
	log sin $\frac{1}{2}I$ = 9.49346
Tang. dist. $AV = 503.36$	log = 2.70205
	log cot $\frac{1}{2}I$ = 10.48437
$R = 1530.1$	log = 3.18642
$D = 3° 44'$	

COMPOUND CURVES.

67. Nature and use. Compound curves are formed by a succession of two or more simple curves of different curvature. The curves must have a common tangent at the point of compound curvature (*P.C.C.*). In mountainous regions there is frequently a necessity for compound curves having several changes of curvature. Such curves may be located separately as a succession of simple curves, but a combination of two simple curves has special properties which are worth investigating and utilizing. In the following demonstrations R_2 always represents the *longer* radius and R_1 the *shorter*, no matter which succeeds the other. T_1 is the tangent adjacent to the curve of shorter radius (R_1), and is invariably the shorter tangent. Δ_1 is the central angle of the curve of radius R_1, but it may be greater or less than Δ_2

68. Mutual relations of the parts of a compound curve having two branches. In Fig. 30, AC and CB are the two branches of

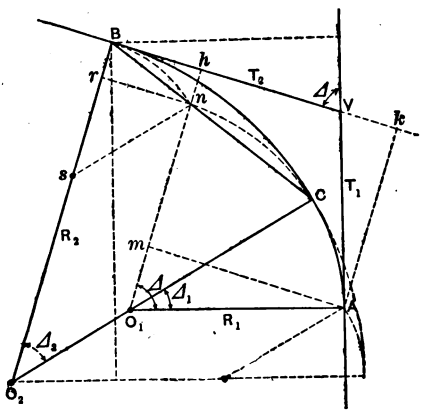


FIG. 30.

the compound curve having radii of R_1 and R_2 and central angles of Δ_1 and Δ_2. Produce the arc AC to n so that $AO_1n = \Delta$. The chord Cn produced *must* intersect B. The line ns, parallel to CO_2, will intersect BO_2 so that $Bs = sn = O_2O_1 = R_2 - R_1$. Draw Am perpendicular to O_1n. It will be parallel to hk.

$$Br = sn \text{ vers } Bsn = (R_2 - R_1) \text{ vers } \Delta_2;$$
$$mn = AO_1 \text{ vers } AO_1n = R_1 \text{ vers } \Delta;$$
$$Ak = AV \sin AVk = T_1 \sin \Delta;$$
$$Ak = hm = mn + nh = mn + Br.$$
$$\therefore T_1 \sin \Delta = R_1 \text{ vers } \Delta + (R_2 - R_1) \text{ vers } \Delta_2. \quad . \quad . \quad (20)$$

Similarly it may be shown that

$$T_2 \sin \Delta = R_2 \text{ vers } \Delta - (R_2 - R_1) \text{ vers } \Delta_1. \quad . \quad . \quad (21)$$

The mutual relations of the elements of compound curves may be solved by these two equations. For example, assume the tangents as fixed (Δ therefore known) and that a curve of given radius R_1 shall start from a given point at a distance T_1 from the vertex, and that the curve shall continue through a given angle Δ_1. Required the other parts of the curve. From Eq. 20 we have

$$R_2 - R_1 = \frac{T_1 \sin \Delta - R_1 \text{ vers } \Delta}{\text{vers } \Delta_2}.$$

$$\therefore R_2 = R_1 + \frac{T_1 \sin \Delta - R_1 \text{ vers } \Delta}{\text{vers } (\Delta - \Delta_1)}. \quad . \quad . \quad . \quad . \quad (22)$$

T_2 may then be obtained from Eq. 21.

As another problem, given the location of the two tangents, with the two tangent distances (thereby locating the PC and PT), and the central angle of each curve; required the two radii. Solving Eq. 20 for R_1, we have

$$R_1 = \frac{T_1 \sin \Delta - R_2 \text{ vers } \Delta_2}{\text{vers } \Delta - \text{vers } \Delta_2}.$$

Similarly from Eq. 21 we may derive

$$R_1 = \frac{T_2 \sin \Delta - R_2(\text{vers } \Delta - \text{vers } \Delta_1)}{\text{vers } \Delta_1}.$$

Equating these, reducing, and solving for R_2, we have

$$R_2 = \frac{T_1 \sin \Delta \text{ vers } \Delta_1 - T_2 \sin \Delta (\text{vers } \Delta - \text{vers } \Delta_2)}{\text{vers } \Delta_2 \text{ vers } \Delta_1 - (\text{vers } \Delta - \text{vers } \Delta_1)(\text{vers } \Delta - \text{vers } \Delta_2)}. \quad (23)$$

Although the various elements may be chosen as above with considerable freedom, there are limitations. For example, in Eq. 22, since R_2 is always greater than R_1, the term to be added to R_1 must be essentially positive—i.e., $T_1 \sin \Delta$ must be greater than R_1 vers Δ. This means that $T_1 > R_1 \frac{\text{vers } \Delta}{\sin \Delta}$, or that

$T_1 > R_1 \tan \frac{1}{2}\Delta$, or that T_1 is greater than the corresponding tangent on a simple curve. Similarly it may be shown that T_2 is *less* than $R_2 \tan \frac{1}{2}\Delta$ or *less* than the corresponding tangent on a simple curve. Nevertheless T_2 is always greater than T_1. In the limiting case when $R_2 = R_1$, $T_2 = T_1$, and $\Delta_2 = \Delta_1$.

69. Modifications of location. Some of these modifications may be solved by the methods used for simple curves. For example:

a. It is desired to move the tangent VB, Fig. 31, parallel to itself to $V'B'$. Run a new curve from the *P.C.C.* which shall reach the new tangent at B', where the chord of the old curve

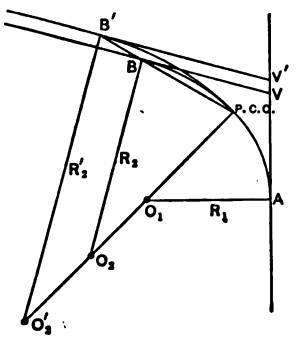


FIG. 31.

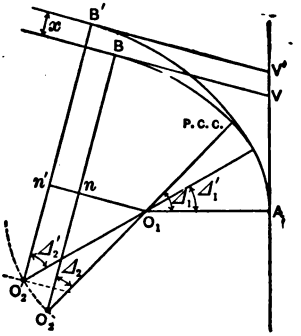


FIG. 32.

intersects the new tangent. The solution is almost identical with that in § 63, *a*.

b. Assume that it is desired to change the forward tangent (as above) but to retain the same radius. In Fig. 32

$$(R_2 - R_1) \cos \Delta_2 = O_2 n;$$
$$(R_2 - R_1) \cos \Delta_2' = O_2' n'.$$
$$x = O_2 n - O_2' n' = (R_2 - R_1)(\cos \Delta_2 - \cos \Delta_2').$$

$$\cos \Delta_2' = \cos \Delta_2 - \frac{x}{R_2 - R_1}. \quad . \quad . \quad . \quad . \quad . \quad (24)$$

The *P.C.C.* is moved *backward* along the sharper curve an angular distance of $\Delta_2' - \Delta_2 = \Delta_1 - \Delta_1'$.

In case the tangent is moved inward rather than outward, the solution will apply by transposing Δ_2 and Δ_2'. Then we shall have

$$\cos \Delta_2' = \cos \Delta_2 + \frac{x}{R_2 - R_1}. \quad . \quad . \quad . \quad . \quad . \quad (25)$$

The *P.C.C.* is then moved *forward*.

c. Assume the same case as (b) except that the larger radius comes first and that the tangent adjacent to the smaller radius is moved. In Fig. 33

$$(R_2-R_1)\cos \Delta_1 = O_1n;$$

$$(R_2-R_1)\cos \Delta_1' = O_1'n'.$$

$$x = O_1'n' - O_1n$$

$$= (R_2-R_1)(\cos \Delta_1' - \cos \Delta_1),$$

$$\cos \Delta_1' = \cos \Delta_1 + \frac{x}{R_2-R_1}. \quad (26)$$

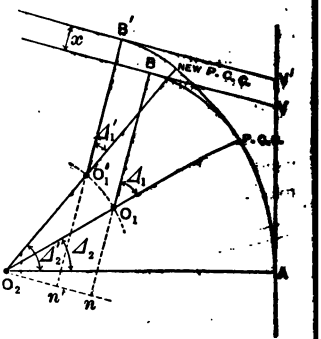


FIG. 33.

The *P.C.C.* is moved *forward* along the easier curve an angular distance of $\Delta_1' - \Delta_1 = \Delta_2 - \Delta_2'$.

In case the tangent is moved *inward*, transpose as before and we have

$$\cos \Delta_1' = \cos \Delta_1 - \frac{x}{R_2-R_1}. \quad (27)$$

The *P.C.C.* is moved *backward*

d. Assume that the radius of one curve is to be altered without changing either tangent. Assume conditions as in Fig. 34. For the diagrammatic solution assume that R_2 is to be increased by O_2S. Then, since R_2' must pass through O_1 and extend beyond O_1 a distance O_1S, the locus of the new center must lie on the arc drawn about O_1 as center and with O_1S as radius. The locus of O_2' is also given by a line $O_2'p$ parallel to *BV* and at a distance of R_2' (equal to $S \ldots P.C.C.$) from it. The new center is therefore at the intersection O_2'. An arc with radius R_2' will therefore be tangent at B' and tangent to the old curve *produced* at NEW *P.C.C.* Draw O_1n' perpendicular to $O_2'B$.

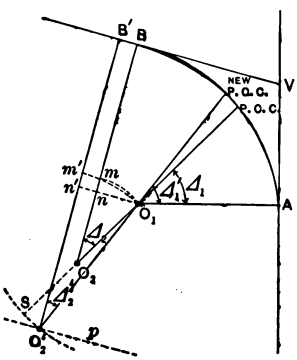


FIG. 34.

With O_2 as center draw the arc O_1m, and with O_2' as center draw the arc O_1m'. $mB=m'B'=R_1$.

$$\therefore\ mn=m'n'=(R_2'-R_1)\ \text{vers}\ \Delta_2'=(R_2-R_1)\ \text{vers}\ \Delta_2.$$

$$\therefore\ \text{vers}\ \Delta_2'=\frac{(R_2-R_1)}{(R_2'-R_1)}\ \text{vers}\ \Delta_2. \quad . \quad . \quad . \quad . \quad (28)$$

$$O_1n=(R_2-R_1)\sin\Delta_2;$$

$$O_1n'=(R_2'-R_1)\sin\Delta_2'.$$

$$BB'=O_1n'-O_1n=(R_2'-R_1)\sin\Delta_2'-(R_2-R_1)\sin\Delta_2. \quad (29)$$

This problem may be further modified by assuming that the radius of the curve is decreased rather than increased, or that the smaller radius follows the larger. The solution is similar and is suggested as a profitable exercise.

It might also be assumed that, instead of making a given change in the radius R_2, a given change BB' is to be made. Δ_2' and R_2' are required. Eliminate R_2' from Eqs. 28 and 29 and solve the resulting equation for Δ_2'. Then determine R_2' by a suitable inversion of either Eq. 28 or 29.

As in §§ 62 and 63, the above problems are but a few, although perhaps the most common, of the problems the engineer may meet with in compound curves. All of the ordinary problems may be solved by these and similar methods.

70. Problems. *a.* Assume that the two tangents of a compound curve are to be 348 feet and 624 feet, and that $\Delta_1=22°\ 16'$ and $\Delta_2=28°\ 20'$. Required the radii.

[*Ans.* $R_1=326.92$; $R_2=1574.85$.]

b. A line crosses a valley by a compound curve which is first a 6° curve for 46° 30′ and then a 9° 30′ curve for 84° 16′. It is afterward decided that the last tangent should be 6 feet farther up the hill. What are the required changes? [*Note.* The second tangent is evidently moved *outward*. The solution corresponds to that in the first part of § 69, *c*. The *P.C.C.* is moved forward 16.39 feet. If it is desired to know how far the *P.T.* is moved in the direction of the tangent (i.e., the *projection* of *BB'*, Fig. 33, on *V'B'*), it may be found by observing that it is equal to $nn'=(R_2-R_1)(\sin\Delta_1-\sin\Delta_1')$. In this case it equals 0.65 foot, which is very small because Δ_1 is nearly 90°. The value of Δ_2 (46° 30′) is not used, since the solution is independent of the value of Δ_2. The student should learn to recognize

which quantities are mutually related and therefore essential to a solution, and which are independent and non-essential.]

TRANSITION CURVES.

71. Superelevation of the outer rail on curves. When a mass is moved in a circular path it requires a centripetal force to keep it moving in that path. By the principles of mechanics we know that this force equals $Gv^2 \div gR$, in which G is the weight, v the velocity in feet per second, g the acceleration of gravity in feet per second in a second, and R the radius of curvature. If the two rails of a curved track were laid on a level (transversely), this centripetal force could only be furnished by the pressure of the wheel-flanges against the rails. As this is very objectionable, the outer rail is elevated so that the reaction of the rails against the wheels shall contain a horizontal component equal to the required centripetal force. In Fig. 35, if ob represents the reaction, oc will represent the weight G, and ao will represent the required centripetal force. From similar triangles we may write $sn : sm :: ao : oc$. Call $g = 32.17$. Call $R = 5730 \div D$, which is sufficiently accurate for this purpose (see § 46). Call $v = 5280V \div 3600$, in which V is the velocity in miles per hour. mn is the distance between rail centers, which, for an 80-lb. rail and standard gauge, is 4.916 feet sm is slightly less than this. As an average value we may call it 4.900, which is its exact value when the superelevation is 4¾ inches. Calling $sn = e$, measured in feet, we have

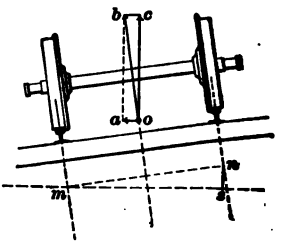


FIG. 35.

$$e = sm\frac{ao}{oc} = 4.9\frac{Gv^2}{gR}\frac{1}{G} = \frac{4.9 \times 5280^2 V^2 D}{32.17 \times 3600^2 \times 5730}.$$

$$e = .0000572V^2D. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

It should be noticed that, according to this formula, the required superelevation varies as the *square* of the velocity, which means that a change of velocity of only 10% would call for a change of superelevation of 21%. Since the velocities of trains over any road are extremely variable, it is impossible to adopt

any superelevation which will fit all velocities even approximately. The above fact also shows why any over-refinement in the calculations is useless and why the above approximations, which are really small, are amply justifiable. For example, the above formula contains the approximation that $R = 5730 \div D$. In the extreme case of a 10° curve the error involved would be about 1%. A change of about ½ of 1% in the velocity, or say from 40 to 40.2 miles per hour, would mean as much. The error in e due to the assumed constant value of sm is never more than a very small fraction of 1%. The rail-laying is not done closer than this. Table XIX is based on Eq. (30):

TABLE XIX. SUPERELEVATION OF THE OUTER RAIL (IN FEET) FOR VARIOUS VELOCITIES AND DEGREES OF CURVATURE.

Velocity in Miles per Hour.	Degree of Curve.									
	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
30	.05	.10	.15	.20	.26	.31	.36	.41	.46	.51
40	.09	.18	.27	.37	.46	.55	.64	.73	.82	.92
50	.14	.29	.43	.57	.71	.86	1.00	1.14	1.29	
60	.20	.41	.62	.82	1.03					

72. **Practical rules for superelevation.** A much used rule for superelevation is to "elevate one half an inch for each degree of curvature." The rule is rational in that e in Eq. 30 varies directly as D. The above rule therefore agrees with Eq. 30 when V is about 27 miles per hour. However applicable the rule may have been in the days of low velocities, the elevation thus computed is too small now. The rule to elevate one inch for each degree of curvature is also used and is precisely similar in its nature to the above rule. It agrees with Eq. 30 when the velocity is about 38 miles per hour, which is more nearly the average speed of trains.

Another (and better) rule is to "elevate for the speed of the fastest trains." This rule is further justified by the fact that a four-wheeled truck, having two parallel axles, will always tend to run to the outer rail and will require considerable flange pressure to guide it along the curve. The effect of an excess of superelevation on the slower trains will only be to relieve this flange pressure somewhat. This rule is coupled with the limitation

hat the elevation should never exceed a limit of six inches—sometimes eight inches. This limitation implies that locomotive engineers must reduce the speed of fast trains around sharp curves until the speed does not exceed that for which the actual superelevation used is suitable. The heavy line in Table XIX shows the six-inch limitation.

Some roads furnish their track foremen with a list of the superelevations to be used on each curve in their sections. This method has the advantage that each location may be separately studied, and the proper velocity, as affected by local conditions (*e.g.*, proximity to a stopping-place for all trains), may be determined and applied.

Another method is to allow the foremen to determine the superelevation for each curve by a simple measurement taken at the curve. The rule is developed as follows: By an inversion of Eq. 19 we have

$$x = chord^2 \div 8R. \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Putting x equal to e in Eq. 30 and solving for "*chord*," we have

$$chord^2 = .0000572V^2D \cdot R$$
$$= 2.621V^2.$$
$$chord = 1.62V. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

To apply the rule, assume that 50 miles per hour is fixed as the velocity from which the superelevation is to be computed. Then $1.62V = 1.62 \times 50 = 81$ feet, which is the distance given to the trackmen. Stretch a tape (or even a string) with a length of 81 feet between two points on the concave side of the head of either the inner or the outer rail. The ordinate at the middle point then equals the superelevation. The values of this chord length for varying velocities are given in the accompanying tabular form.

Velocity in miles per hour...	20	25	30	35	40	45	50	55	60
Chord length in feet.....	32.4	40.5	48.6	56.7	64.8	72.9	81.0	89.1	97.2

The following tabular form shows the standard (at one time) on the N. Y., N. H. & H. R. R. It should be noted that the elevations do not increase proportionately with the radius, and that they are higher for descending grades than for level or

ascending grades. This is on the basis that the velocity on curves and on ascending grades will be less than on descending grades. For example, the superelevation for a 0° 30′ curve on a descending grade corresponds to a velocity of about 54 miles per hour, while for a 4° curve on a level or ascending grade the superelevation corresponds to a velocity of only about 38 miles per hour.

TABLE OF THE SUPERELEVATION OF THE OUTER RAIL ON CURVES.

N. Y., N. H. & H. R. R.

Degree of curve.	Level or ascending grade.	Descending grade.
	inches.	inches.
0° 30′	0¼	1
1 00	1¼	1¼
1 15	1½	2
1 30	2	2¼
1 45	2¼	2½
2 00	2½	2¾
2 15	2¾	3
2 30	2¾	3¼
2 45	3	3½
3 00	3¼	3¾
3 15	3½	3¾
3 30	3¾	4
3 45	3¾	4¼
4 00	4	4½

73. **Transition from level to inclined track.** On curves the track is inclined transversely; on tangents it is level. The transition from one condition to the other must be made gradually. If there is no transition curve, there must be either inclined track on the tangent or insufficiently inclined track on the curve or both. Sometimes the full superelevation is continued through the total length of the curve and the "run-off" (having a length of 100 to 400 feet) is located entirely on the tangents at each end. In other practice it is located partly on the tangent and partly on the curve. Whatever the method, the superelevation is correct at only one point of the run-off. At all other points it is too great or too small. This (and other causes) produces objectionable lurches and resistances when entering and leaving curves. The object of transition curves is to obviate these resistances.

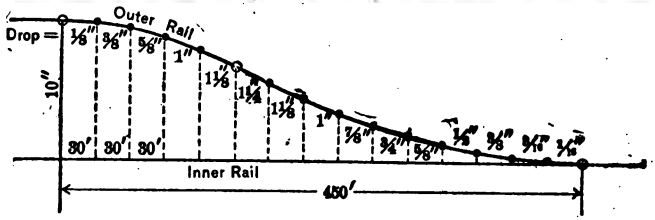
On the Lehigh Valley R. R. the run-off is made in the form of a reversed vertical curve, as shown in the accompanying figure. According to this system the length of run-off varies

from 120 feet, for a superelevation of one inch, to 450 feet, for a superelevation of ten inches. Such a superelevation as ten inches is very unusual practice, but is successfully operated on that road. The curve is concave upward for two-thirds of its length and then reverses so that it is convex upward.

TABLE FOR RUN-OFF OF ELEVATION OF OUTER RAIL OF CURVES.

Drop in inches for each 30-foot rail commencing at theoretical point of curve.

Elevation.	1/8"	1/4"	3/8"	1/2"	5/8"	3/4"	7/8"	1"	1 1/8"	1 1/4"	1 3/8"	1 1/2"	1 5/8"	1 3/4"	1 7/8"	2"	2 1/8"	2 1/4"	2 3/8"	2 1/2"	Total.
1"	..	30	30	..	..	..	..	..	..	..	..	..	..	..	..	..	..	30	..	30	120
2"	..	30	..	..	..	30	..	..	..	..	..	..	..	..	..	30	30	..	..	30	150
3"	..	30	..	..	..	30	..	..	..	..	..	..	..	30	..	30	..	30	..	30	180
4"	..	30	..	30	..	..	30	..	..	..	..	..	..	30	..	30	30	..	30	..	240
5"	..	30	..	30	..	..	..	30	..	..	..	..	30	..	30	30	30	..	30	..	270
6"	..	30	..	30	..	..	30	..	30	..	..	..	30	..	30	30	30	..	30	..	300
7"	..	30	..	30	..	..	30	..	30	..	..	30	30	..	30	30	..	30	30	..	330
8"	..	30	..	30	..	30	..	..	30	30	..	30	30	..	30	30	..	30	..	30	360
9"	30	..	..	30	..	30	..	30	30	..	..	30	30	30	30	30	30	30	..	30	420
10"	30	..	30	..	30	..	..	30	30	30	..	30	30	30	30	30	30	30	..	30	450



The figure (and also the lower line of the tabulated form) shows the drop for each thirty-foot rail length. For shorter lengths of run-off, the drop for each 30 feet is shown by the corresponding lines in the tabular form. Note in each horizontal line that the sum of the drops, under which 30 is found, equals the total superelevation as found in the first column. For example, for 4 inches superelevation, length of curve 240 feet, the successive drops are $\frac{1}{4}''$, $\frac{1}{2}''$, $\frac{7}{8}''$, $\frac{7}{8}''$, $\frac{5}{8}''$, $\frac{1}{2}''$, $\frac{1}{4}''$, and $\frac{1}{8}''$ whose sum is 4 inches. Possibly the more convenient form would be to indicate for each 30-foot point the actual superelevation of the outer rail, which would be for the above case (running from the tangent to the curve) $\frac{1}{8}''$, $\frac{3}{8}''$, $\frac{7}{8}''$, $1\frac{1}{2}''$, $2\frac{3}{8}''$, $3\frac{1}{4}''$, $3\frac{7}{8}''$, $4''$.

74. Fundamental principle of transition curves. If a curve

has variable curvature, beginning at the tangent with a curve of infinite radius, and the curvature gradually sharpens until it equals the curvature of the required simple curve and there becomes tangent to it, the superelevation of such a transition curve may begin at zero at the tangent, gradually increase to the required superelevation for the simple curve, and yet have at every point the superelevation required by the curvature at that point. Since in Eq. (30) e is directly proportional to D, the required curve must be one in which the degree of curve increases directly as the distance along the curve.

75. Varieties of Transition Curves. A theoretically exact transition curve is very complicated and its mathematical solution very difficult. A committee of the Amer. Rwy. Eng. Assoc. investigated the many systems which have been proposed and reported that all of them seemed to be objectionable for one or more of the following reasons: "(1) If simple approximate formulas were used, they were not sufficiently accurate. (2) Accurate formulas were too complex. (3) The curve could not be expressed by formulas. (4) Formulas were of the endless series class. (5) Complex field methods were required to make the field-work agree with formulas with spirals of large angles." The committee then developed a method which gives results whose accuracy is beyond that of the most careful field-work and yet which is sufficiently simple for practical use. The mathematical development is so elaborate that it will not be detailed here, but the working formulas and a condensation of the table together with an explanation of their practical use and application, will be given, with numerical examples.

The general form of these curves, whatever their precise mathematical character, is shown in Fig. 36. AVB are two tangents, joined by the simple circular curve AMB, having the center O. Assume that the entire curve is moved in the direction MO a distance $OO' = MM' = BB' = AA'$. At some point TS on the tangent, the spiral begins and joins the circular curve tangentially at SC. The other spiral runs from CS to ST. The significance of these symbols may be readily remembered from the letters; T, S, and C signify tangent, spiral and circular curve; TS is the point of change from tangent to spiral, SC, the point of change from spiral to curve, etc. At the other end of the circular curve the letters are in reverse order, the station numbers increasing from A to B. The meaning of the various symbols is

indicated in Fig. 36. The student should appreciate the fact of the necessary distortion of the figure in order to make it plain. Based on the figures of the following numerical problem, the distance MM' is about fourteen times its proper amount. Another effect of the distortion is that the dimension U, instead of being

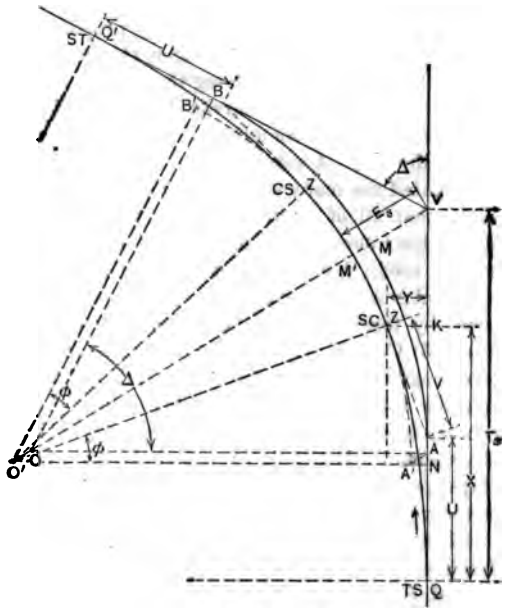


FIG. 36.

nearly twice V, which is usual, as given in Table IV, Part B, is only a little longer than V.

76. Proper length of spiral. This can only be computed on the basis of certain assumptions as to the desired rate of tipping the car, so as to avoid discomfort to passengers, and, of course, this depends on the expected velocity. There is also a maximum limitation, since the sum of the two spiral angles cannot exceed the total central angle of the curve. The *minimum* lengths recommended are as follows:

On curves which limit the speed:

6° and over, 240 feet;

Less than 6°, $5\frac{1}{2}\times$speed in m.p.h. for elevation of 8 inches.

On curves which do not limit the speed:

30 times elevation in inches, or

$\frac{2}{3}\times$ultimate speed in m.p.h.$\times$elevation in inches.

For example. (1) 5° curve which limits speed; speed limit 48 m.p.h. by interpolation in table, § 71; $48\times5\frac{1}{3}=256$ feet minimum length. (2) 3° curve; maximum operating speed 60 m.p.h.; superelevation, .62 feet$=7.44$ inches; $30\times7.44=223.2$ feet; or, $\frac{2}{3}\times60\times7.44=297.6$ feet. Of course the higher value should be used, or say 300 feet as the minimum length.

While it is generally true that the longer transition curves give easier riding, the spiral must not reach the center point of the curve. Since it is approximately true that the spiral extends for equal distances on each side of the original point of curve, it is nearly true that two spirals, each having the same length as the original curve, would just meet at the center. The length of a spiral should in general be very much less than the length of the original curve.

77. Symbols. Beside the symbols whose significance is clearly indicated in Fig. 36, the following are defined:

a The angle between the tangent at the *TS* and the chord from the *TS* to *any* point on the spiral; a_1 is the angle to the *first* chord point.

A The angle between the tangent at the *TS* and the chord from the *TS* to the *SC*.

D The degree of the central circular curve.

Δ The central angle of the original circular curve, or the angle between the tangents.

ϕ The total central angle of the spiral.

k The increase in degree of curve per station on the spiral.

L The length of the spiral in feet from the *TS* to the *SC*.

S The length of the spiral in stations from the *TS* to the *SC*.

s The length of the spiral in stations from the *TS* to any given point.

78. Deflections. The field formulas for deflections are based on the following two equations:

$$a = 10\ ks^2 \text{ minutes},$$

$$A = 10\ kS^2 \text{ minutes}.$$

The first deflection $a_1 = 10\,ks_1^2$ minutes. But k is the increase in degree of curve per station, and since the degree of curve increases as the length, $k = D \div S$, S being expressed in stations. For point 1, since $S = 10s$, $a_1 = 10\left(\frac{D}{10s_1}\right)s_1^2 = Ds_1$, which may be expressed as the degree of the curves times the length of the chord in stations. For example, if the spiral is 400 feet long (which means that $L = 400$ and $S = 4$) and runs on to a 5° curve (then $D = 5$), one chord is 40 feet long and $s = .4$ station. Then $a_1 = 5 \times 0.4 = 2$ minutes of arc for the deflection for the first chord point. And since the deflections are as the square of the number of stations, the deflections from TS to succeeding stations will be 4, 9, 16, 25, 36, 49, 64, 81, and 100 times 2 minutes, these factors being those given in the second vertical column of Part A of Table IV. The last deflection $= A = 100 \times 2' = 200' = 3°\ 20' = \frac{1}{3}\ (10°) = \frac{1}{3}\phi$, ϕ being the total central angle of the spiral. Although it is always nearly true that $A = \frac{1}{2}\phi$, and the error is inappreciable for small angles, the error amounts to 30 seconds of arc when $\phi = 21°\ 30'$, an unusually large angle.

The deflection from any other point of the spiral to any other point, either forward or backward, may be found by multiplying the value of a_1 (in this case 2′), by the coefficients in the proper vertical column of that table.

The spiral angle

$$\phi = \frac{kS^2}{2} = \frac{kL^2}{20000} = \frac{DL}{200} = \frac{5 \times 400}{200} = 10°.$$

Also,

$$\phi = \frac{kS^2}{2} = \frac{DS}{2} = \frac{5 \times 4}{2} = 10°$$

The values of the ratios $U \div L$ and $V \div L$ for even degrees, and for A, $C \div L$, $X \div L$, and $Y \div L$ for half degrees are given in Parts B and C of Table IV. When it is desired to temporarily omit locating the intermediate points of the spiral, the jump from the TS to the SC may be made by measuring the distance U from the TS along the tangent. At that point a deflection ϕ and a measured distance V will give not only the position of SC but also the direction of the tangent at the beginning of the circular curve. Another method of locating the SC without locating the intermediate points is to make the deflection A at the TS

and measure the long chord C. In the above numerical problem this equals $400 \times .998664 = 399.47$, a little over 6 inches short of the full 400 feet. By setting up the transit at the SC, backsighting at the TS, and turning off the angle $(\phi - A)$, which in the above case is $10° - 3°\ 19'\ 57'' = 6°\ 20'\ 03''$, the direction of the tangent at the SC is obtained. In this case, the three seconds variation from the approximate value is utterly negligible. The other dimensions are easily determined from the tables if desired;

$$X = .996975 \times 400 = 398.79,$$
$$Y = .058053 \times 400 = 23.22,$$
$$U = .667742 \times 400 = 267.10$$
$$V = .334313 \times 400 = 133.73.$$

For greater convenience of notation, the points TS, SC, CS, and ST, in Fig. 36 are also indicated by the letters Q, Z, Z' and Q' respectively. The same letters are used for the corresponding points in Figs. 37 and 38.

79. Location of spirals and circular curve with respect to tangents. See Fig. 36. Let AV and BV be the tangents to be connected by a $D°$ curve, having a suitable spiral at each end. If no spirals were to be used, the problem would be solved as in simple curves giving the curve AMB. Introducing the spiral has the effect of throwing the curve away from the vertex a distance MM' and reducing the central angle of the $D°$ curve by 2ϕ. Continuing the curve beyond Z and Z' to A' and B', we will have $AA' = BB' = MM'$. $ZK =$ the Y ordinate and is therefore known. Call $MM' = m$. $A'N = Y - R$ vers ϕ. Then

$$m = MM' = AA' = \frac{A'N}{\cos \frac{1}{2}\Delta} = \frac{Y - R \text{ vers } \phi}{\cos \frac{1}{2}\Delta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

$$NA = AA' \sin \tfrac{1}{2}\Delta = (Y - R \text{ vers } \phi) \tan \tfrac{1}{2}\Delta.$$
$$VQ = QK - KN + NA + AV$$
$$= X - R \sin \phi + (Y - R \text{ vers } \phi) \tan \tfrac{1}{2}\Delta + R \tan \tfrac{1}{2}\Delta$$
$$= X - R \sin \phi + Y \tan \tfrac{1}{2}\Delta + R \cos \phi \tan \tfrac{1}{2}\Delta. \quad . \quad . \quad . \quad (34)$$

When $A'N$ has already been computed, it may be more convenient to write

$$VQ = X + R (\tan \tfrac{1}{2}\Delta - \sin \phi) + A'N \tan \tfrac{1}{2}\Delta. \quad . \quad . \quad . \quad . \quad (35)$$

$$VM' = VM + MM'$$

$$= R \operatorname{exsec} \tfrac{1}{2}\Delta + \frac{Y}{\cos \frac{1}{2}\Delta} - \frac{R \operatorname{vers} \phi}{\cos \frac{1}{2}\Delta}. \quad \ldots\ldots \quad (36)$$

$$AQ = VQ - AV$$

$$= X - R \sin \phi + (Y - R \operatorname{vers} \phi) \tan \tfrac{1}{2}\Delta. \quad \ldots\ldots \quad (37)$$

Example. To join two tangents making an angle of 34° 20′ by a 5° 40′ curve and suitable spirals. Assume that the spiral is 300 feet long. Then

$$\phi = \frac{DS}{2} = \frac{5.67 \times 3}{2} = 8.5° = 8° 30'.$$

Since, from Table IV, Part A, $Y \div L = .049374$ for $\phi = 8° 30'$, $Y = 14.812$; similarly, we find $X = 299.344$ and $C = 299.71$.

[Eq. 33]			R	3.00497
			vers ϕ	8.04076
		11.110		1.04573
		$Y = 14.812$		
		$A'N = 3.702$		0.56843
			cos ½Δ	9.98021
		$m = MM' = AA' = 3.875$		0.58822
[Eq. 36]			R	3.00497
			exsec ½Δ	8.66863
		$VM = 47.164$		1.67360
		$m = 3.875$		
		$VM' = 51.039$		
[Eq. 35]	$X = 299.344$	nat. tan ½Δ =	.30891	
		nat. sin ϕ =	.14781	
			.16110	9.20709
			R	3.00497
	162.954			2.21206
		[See above]	$A'N$	0.56843
			tan ½Δ	9.48984
	1.144		AN	0.05827
	$VQ = 463.442$			
[Eq. 37]			R	3.00497
			tan ½Δ	9.48984
	312.471		AV	2.49481
	$AQ = 150.971$			

It should be noted that AQ is within a foot of equaling one-half the length of the spiral, which illustrates the general fact that a spiral begins at approximately one-half its length from the P.C. of the simple curve. All approximate systems of spirals assume this to be exactly true.

80. Field-work. When the spiral is designed during the original location, the tangent distance VQ should be computed and the point Q located. It is hardly necessary to locate all of the points of the spiral until the track is to be laid. The extremities should be located, and as there will usually be two or more full station points on the spiral, these should also be located. Z may be located by setting off $QK = X$ and $KZ = Y$, or else by the tabular deflection for Z from Q and the distance ZQ, which is the long chord c. Setting up the instrument at Z and sighting back at Q with the proper deflection, the tangent at Z may be found and the circular curve located as usual, its central angle being $\Delta - 2\phi$. A similar operation will locate Q' from Z'.

To locate points on the spiral. Set up at Q, with the plates reading 0° when the telescope sights along VQ. Set off from Q the deflections computed from Table IV for the instrument at Q, using a chord length of $L \div 10$, the process being like the method for simple curves except that the deflections are variable. If a full station-point occurs within the spiral, interpolate between the deflections for the adjacent spiral-points. For example, a 400-foot spiral running on to a 3° 31′ curve begins at Sta. 56+15. The spiral points are 40 feet apart. Sta. 57 comes 5 feet beyond the second spiral point. The first deflection $a_1 = Ds = 3.5 \times .4 = 1.4$ min. The deflection to point 2 is $4 \times 1.4 = 5.6$ min. and that to point 3 is $9 \times 1.4 = 12.6$ min. Then the deflection to Sta. 57 is $\frac{5}{40} \times (12.6 - 5.6) + 5.6 = 6.47$ min.

This method is not theoretically accurate, but the error is small. Arriving at Z, the forward alinement may be obtained by sighting back at Q (or at any other point) with the proper deflection for that point from the station occupied. Then when the plates read 0° the telescope will be tangent to the spiral and to the succeeding curve. All rear points should be checked from Z. If it is necessary to occupy an intermediate station, use the deflections given for that station, orienting as just explained for Z, checking the back points and locating all forward points up to Z if possible.

After the center curve has been located and Z' is reached, the

other spiral must be located but *in reverse order*, i.e., the sharp curvature of the spiral is at Z' and the curvature decreases toward Q'.

81. **To replace a simple curve by a curve with spirals.** This *may* be done by the method of § 79, but it involves shifting the whole track a distance m, which in the given example equals 3.87 feet. Besides this the track is appreciably shortened,

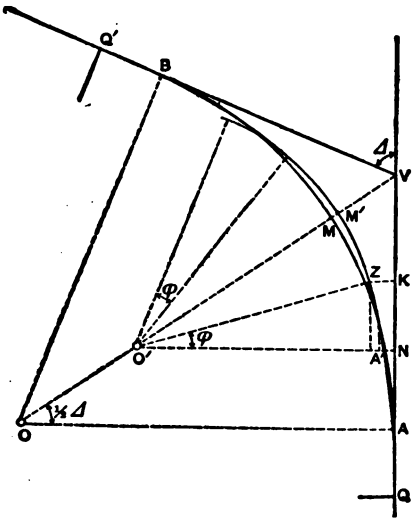


FIG. 37.

which would require rail-cutting. But the track may be kept at practically the same length and the lateral deviation from the old track may be made very small by slightly sharpening the curvature of the old track, moving the new curve so that it is wholly or partially *outside* of the old curve, the remainder of it with the spirals being *inside* of the old curve. It is found by experience that a decrease in radius of from 5% to 10% will answer the purpose. The larger the central angle the less the change. The solution is as indicated in Fig. 37.

$O'N = R' \cos \phi + Y.$

$O'V = O'N \sec \tfrac{1}{2}\Delta$

$\quad = R' \cos \phi \sec \tfrac{1}{2}\Delta + Y \sec \tfrac{1}{2}\Delta.$

$$m = MM' = MV - M'V$$
$$= R \operatorname{exsec} \tfrac{1}{2}\Delta - (O'V - R')$$
$$= R \operatorname{exsec} \tfrac{1}{2}\Delta - R' \cos\phi \sec \tfrac{1}{2}\Delta - Y \sec \tfrac{1}{2}\Delta + R'. \quad . \quad . \quad (38)$$

$$AQ = QK - KN + NV - VA$$
$$= X - R' \sin\phi + (R' \cos\phi + Y) \tan \tfrac{1}{2}\Delta - R \tan \tfrac{1}{2}\Delta$$
$$= X - R' \sin\phi + R' \cos\phi \tan \tfrac{1}{2}\Delta - (R - Y) \tan \tfrac{1}{2}\Delta. \quad . \quad . \quad (39)$$

The length of the old curve from Q to $Q' = 2AQ + 100\frac{\Delta}{D}$.

The length of the new curve from Q to $Q' = 2L + 100\frac{\Delta - 2\phi}{D'}$,

in which L is the length of each spiral.

Example. Suppose the old curve is a 7° 30′ curve with a central angle of 38° 40′. As a trial, compute the relative length of a new 8° 20′ curve with spirals 240 feet long. $\frac{1}{2}\Delta = 19° 20'$; R (for the 7° 30′ curve) = 764.49; R' (for the 8° 20′ curve) = 688.16; $\phi = 10° 0'$; $Y = 13.933$; $X = 239.274$.

[Eq. 38]			R	2.88337
			exsec ½Δ	8.77642
	45.687	. . . . .		1.65979
	$R' = 688.16$			
	733.847		R'	2.83768
			cos ϕ	9.99335
			sec ½Δ	0.02521
		718.200		2.85624
			Y	1.14405
			sec ½Δ	0.02521
		14.766		1.16926
	732.966	732.966		
	$m =$ 0.881			
[Eq. 39]	$X = 239.274$		R'	2.83768
			sin ϕ	9.23967
		119.497		2.07735
			R'	2.83768
			cos ϕ	9.99335
			tan ½Δ	9.54512
	237.770			2.37615
			$R = 764.49$	
			$Y = 13.93$	
			750.56	2.87538
			tan ½Δ	9.54512
	477.044	263.333		2.42050
	382.830	382.830		
	$AQ =$ 94.214			

The length of the old curve from Q to Q' is

$$100\frac{\Delta}{D}=100\frac{38.667}{7.5}= \quad . \quad . \quad . \quad . \quad . \quad . \quad 515.556$$

$$2AQ=2\times 94.214= \quad . \quad . \quad . \quad . \quad . \quad . \quad 188.428$$

$$703.984$$

New curve: $100\frac{\Delta-2\phi}{D'}=100\frac{38.667-20.000}{8.33}=224.000$

$$2L=2\times 240 \qquad =480.000$$

$$704.000 \qquad 704.000$$

Difference in length = 0.016

Considering that this difference may be divided among 21 joints (using 33-foot rails) no rail-cutting would be necessary. If the difference is too large, a slight variation in the value of the new radius R' will reduce the difference as much as necessary. A truer comparison of the lengths would be found by comparing the lengths of the arcs.

82. Application of transition curves to compound curves. Since compound curves are only employed when the location is limited by local conditions, the elements of the compound curve should be determined (as in §§ 68 and 69) regardless of the transition curves, depending on the fact that the lateral shifting of the curve when transition curves are introduced is very small. If the limitations are very close, an estimated allowance may be made for them.

Methods have been devised for inserting transition curves between the branches of a compound curve, but the device is complicated and usually needless, since when the train is once on a curve the wheels press against the outer rail steadily and a change in curvature will not produce a serious jar even though the superelevation is temporarily a little more or less than it should be.

If the easier curve of the compound curve is less than 3° or 4°, there may be no need for a transition curve off from that branch. This problem then has two cases according as transition curves are used at both ends or at one end only.

a. *With transition curves at both ends.* Adopting the method of § 79, calling $\Delta_1=\frac{1}{2}\Delta$, we may compute $m_1=MM_1'$. Similiarly, calling $\Delta_2=\frac{1}{2}\Delta$, we may compute $m_2=MM_2'$. But M_1' and M_2' must be made to coincide. This may be done by moving the curve $Z'M_1'$ and its transition curve parallel to $Q'V$ a distance $M_1'M_3$, and the other curve parallel to QV a distance $M_2'M_3$.

riangle $M_1'M_3M_3'$, the angle at $M_1' = 90° - \Delta_1$, the angle $90° - \Delta_2$, and the angle at $M_3 = \Delta$.

$$\left.\begin{aligned} M_1'M_3 &= M_1'M_2' \frac{\sin(90° - \Delta_2)}{\sin \Delta} = (m_1 - m_2)\frac{\cos \Delta_2}{\sin \Delta}. \\ M_2'M_3 &= M_1'M_2' \frac{\sin(90° - \Delta_1)}{\sin \Delta} = (m_1 - m_2)\frac{\cos \Delta_1}{\sin \Delta}. \end{aligned}\right\} \quad (40)$$

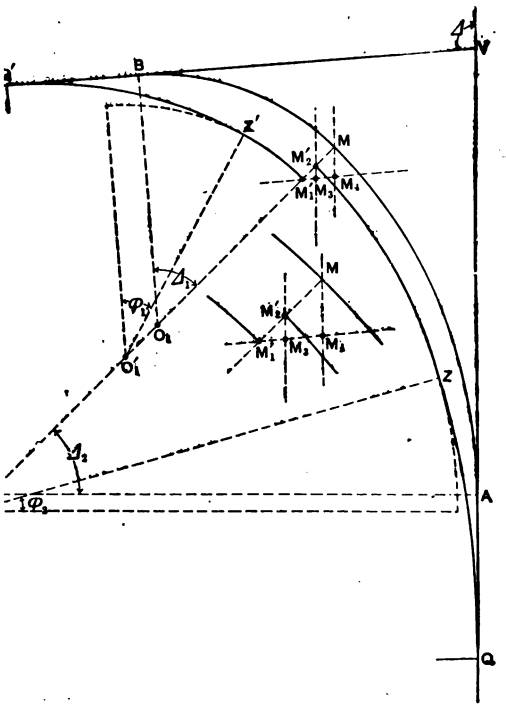


FIG. 38.

th a transition curve on the sharper curve only. Com- $= MM_1'$ as before; then move the curve Z_1M_1' parallel a distance of

$$M_1'M_4 = m_1 \frac{\cos \Delta_2}{\sin \Delta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

The simple curve MA is moved parallel to VA a distance of

$$MM_4 = m_1 \frac{\cos \Delta_1}{\sin \Delta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4$$

If Δ_1 and Δ_2 are both small, $M_1'M_4$ and MM_4 may be m than m_1, but the lateral deviation of the new curve from the will always be less than m_1.

83. To replace a compound curve by a curve with spir The numerical illustration given below employs another meth We first solve for m_1 for the sharper branch of the curve, pl ing $\Delta_1 = \frac{1}{2}\Delta$ in Eq. 38. A value for R_2' may be found wh corresponding value of m_2 will equal m_1. Solving Eq. 38 for we obtain

$$R' = \frac{R \text{ vers } \frac{1}{2}\Delta - m \cos \frac{1}{2}\Delta - Y}{\cos \phi - \cos \frac{1}{2}\Delta} \quad . \quad . \quad . \quad (4$$

Substituting in this equation the known value of m_1 ($=$ and calling $R' = R_2'$, $R = R_2$, and $\Delta_2 = \frac{1}{2}\Delta$, solve for R_2'. Obt the value of AQ for each branch of the curve separately by E 39, and compare the lengths of the old and new lines.

Example. Assume a compound curve with $D_1 = 8°$, $D_2 =$ $\Delta_1 = 36°$, and $\Delta_2 = 32°$. Use 240-foot spirals at each end. Assu that the sharper curve is sharpened from 8° 0′ to 8° 15′.

Eq. 38

		R_1	2.38
		exsec 36°	9.37
169.21	. . . . .	. . . . .	2.22
695.09			
864.30	$\phi_1 = \frac{8.25 \times 240}{2}$	R_1' (8° 15′)	2.84
		cos ϕ_1	9.99
	$= 9.°9 = 9°54'$	sec Δ_1	0.08
		846.39	2.92
	$Y_1 = 240 \times .05747$	Y_1	1.12
	$= 13.79$	sec Δ_1	0.09
		17.05	1.23
863.44		863.44	
$m_1 =$ 0.86			

[Eq. 43] $\phi_2 = \frac{4.05 \times 2.4}{2.}$ R_2 3.15615

$= 4°.86 = 4°51'.6$ vers 32° 9.18170

217.700 . . . . . 2.33785

$Y_2 = .02826 \times 240.$ $m_1 = 0.86$ 9.93450

$= 6.782$ cos 32° 9.92842

0.729 . . . . . 9.86292

$Y_2 = 6.782$

7.511 7.511

210.189 . . . . . 2.32261

nat. cos $\phi_2 = .99640$

nat. cos $\Delta_2 = .84805$

.14835 9.17129

$R_2' = 1416.84$ [4° 2' 41''] 3.15132

Eq. 39] $X_1 = 239.286$ $X_1 = .997024 \times 240$

$= 239.286$ R_1' 2.84204

sin ϕ_1 9.23535

119.505 . . . . . 2.07739

R_1' 2.84204

cos ϕ_1 9.99348

tan $\frac{1}{2}\Delta[\Delta_1 = 36°]$ 9.86126

497.489 . . . . . 2.69678

$R_1 = 716.78$

$Y_1 = 13.70$

703.08 2.84700

tan $\frac{1}{2}\Delta$ 9.86126

736.775

630.325 510.820 . . . . . 2.70826

$\Delta Q_1 = 106.450$ 630.325

[Eq. 39] R_2' 3.15132

$X_2 = .999284 \times 240$ sin ϕ_2 8.92799

$= 239.828$ 120.035 . . . . . 2.07931

R_2' 3.15132

cos ϕ_2 9.99843

tan $\frac{1}{2}\Delta(\Delta_2 = 32°)$ 9.79579

882.145 . . . . . 2.94554

$R_2 = 1432.7$

$Y_2 = 6.8$

1425.9 3.15400

tan $\frac{1}{2}\Delta$ 9.79579

891.00 . . . . . 2.94988

1121.973 1011.03

1011.03

$\Delta Q_2 = 110.94$

For the length of the old track we have:

$$100\,\frac{\Delta_1}{D_1} = 100\,\frac{36°}{8°} = 450.$$

$$100\,\frac{\Delta_2}{D_2} = 100\,\frac{32°}{4°} = 800.$$

$$AQ_1 = 106.45$$

$$AQ_2 = 110.94$$

$$= 1467.39$$

For the length of the new track we have:

$$100\,\frac{\Delta_1 - \phi_1}{D'_1} = 100\,\frac{26°.1}{8°.25} = 316.36$$

$$100\,\frac{\Delta_2 - \phi_2}{D'_2} = 100\,\frac{27.14}{4°.044} = 671.11$$

Spiral on 8° 15′ curve = 240.00

Spiral on 4° 02′ 41″ curve = 240.00

Length of new track = 1467.47

Length of old track = 1467.39

Excess in length of new track = 0.08 feet.

Since the new track is slightly longer than the old, it shows that the new track runs too far *outside* the old track at the *P.C.C.* On the other hand the offset m is only 0.86. The maximum amount by which the new track comes *inside* of the old track at two points, presumably not far from Z' and Z, is very difficult to determine exactly. Since it is desirable that the maximum offsets (inside and outside) should be made as nearly equal as possible, this feature should not be sacrificed to an effort to make the two lines of precisely equal length so that the rails need not be cut. Therefore, if it is found that the offsets inside the old track are nearly equal to m (0.86), the above figures should stand. Otherwise m may be diminished (and the above excess in length of track diminished) by *increasing* R_1' very slightly and making the necessary consequent changes.

VERTICAL CURVES

84. Necessity for their use. Whenever there is a change in the rate of grade, it is necessary to eliminate the angle that would be formed at the point of change and to connect the two grades by a curve. This is especially necessary at a sag between two grades, since the shock caused by abruptly forcing an upward motion to a rapidly moving heavy train is very severe both to the track and to the rolling stock. The necessity for vertical curves was even greater in the days when link couplers were in universal use and the "slack" in a long train was very great.

Under such circumstances, when a train was moving down a heavy grade the cars would crowd ahead against the engine. Reaching the sag, the engine would begin to pull out, rapidly taking out the slack. Six inches of slack on each car would amount to several feet on a long train, and the resulting jerk on the couplers, especially those near the rear of the train, has frequently resulted in broken couplers or even derailments. A vertical curve will practically eliminate this danger if the curve is made long enough.

85. Required length. Theoretically the length should depend on the change in the rate of grade and on the length of the longest train on the road. A sharp change in the rate of grade requires a long curve; a long train requires a long curve; but since the longest trains are found on roads with light grades and small changes of grade, the required length is thus somewhat equalized. The A.R.E.A. rule is: "On class A roads (see § 234) rates of change of 0.1 per cent per station on summits and 0.05 per cent per station in sags should not be exceeded. On minor roads 0.2 per cent per station on summits and 0.1 per cent per station in sags may be used." When changing from a down grade to an up grade (or vice versa) the change of grade equals the numerical sum of the two rates of grade. For example, if a 0.5 per cent down grade is followed by a 0.7 per cent up grade, the road being a "minor" road, then, by the above rule the length of the curve should be at least $[0.5-(-0.7)] \div 0.1 = 12$ stations or 1200 feet. Added length increases the amount of earthwork required both in cuts and fills, but the resulting saving in operating expenses will always justify a considerable increase.

86. Form of curve. In Fig. 39 assume that A and C, equi-

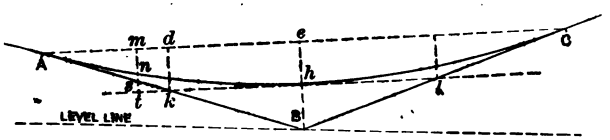


FIG. 39.

distant from B, are the extremities of the vertical curve. Bisect BC at e; draw Be and bisect it at h. Bisect AB and BC at k and l. The line kl will pass through h. A parabola may be drawn with its vertex at h which will be tangent to AB and BC at A and C. It may readily be shown * from the properties of

* See note at end of this chapter.

a parabola that if an ordinate be drawn at *any* point (as at n) we will have

$$sn : eh \text{ (or } hB) : : \overline{Am}^2 : \overline{Ae}^2$$

or

$$sn = eh\frac{\overline{Am}^2}{\overline{Ae}^2} \quad \ldots\ldots\ldots \quad (44)$$

In Fig. 39 the grades are necessarily exaggerated enormously. With the proportions found in practice we may assume that ordinates (such as mt, eB, etc.) are perpendicular to either grade, as may suit our convenience, without any appreciable error. In the numerical case given below, the variation of these ordinates from the vertical is 0° 07′, while the effect of this variation on the calculations in this case (as in the most extreme cases) is absolutely inappreciable. It may easily be shown that the angle CAB = half the *algebraic difference* of the rates of grade. Call the difference, expressed in per cent of grade, r; then $CAB = \frac{1}{2}r$. Let l = length (in "stations" of 100 feet) of the line AC, which is practically equal to the horizontal measurement. Since the angle CAB is one-half the total change of grade at B, it follows that $Be = \frac{1}{2}l \times \frac{1}{2}r$ Therefore

$$Bh = \tfrac{1}{8}lr. \quad \ldots\ldots\ldots \quad (45)$$

Since Bh (or eh) and Ae are constant for any one curve, the correction sn at any point (see Eq. 44) equals a constant times Am^2.

87. Numerical example. Assume that B is located at Sta. 16+20; that the grade of AB is −0.5%, and of BC +0.7%; also that the elevation of B above the datum plane is 162.6. Then the algebraic difference of the grades, r, $= 0.7 - (-0.5) = 1.2$; $l = 12$. $Bh = \frac{1}{8}lr = \frac{1}{8} \times 12 \times 1.2 = 1.8$. A is at Sta. 10+20 and its elevation is $162.6 + (6 \times 0.5) = 165.6$; C is at Sta. 22+20 and its elevation is $162.6 + (6 \times 0.7) = 166.8$. The elevation of Sta. 11 is found by adding sn to the elevation of s on the straight grade line. The constant $(eh \div \overline{Ae}^2)$ equals in this case $1.8 \div 600^2 = \frac{1}{200000}$. Therefore the curve elevations are

A, Sta.	10+20,	$162.6 + (6.00 \times 0.5)$	$= 165.60$
	11	$165.6 - (0.80 \times 0.5) + \frac{1}{200000}\ 80^2$	$= 165.23$
	12	$165.6 - (1.80 \times 0.5) + \frac{1}{200000}\ 180^2$	$= 164.86$
	13	$165.6 - (2.80 \times 0.5) + \frac{1}{200000}\ 280^2$	$= 164.59$
	14	$165.6 - (3.80 \times 0.5) + \frac{1}{200000}\ 380^2$	$= 164.42$
	15	$165.6 - (4.80 \times 0.5) + \frac{1}{200000}\ 480^2$	$= 164.35$
	16	$165.6 - (5.80 \times 0.5) + \frac{1}{200000}\ 580^2$	$= 164.38$

B,	16+20, $162.6+1.80$	$=164.40$
	17 $166.8-(5.20\times0.7)+\frac{1}{2000000}520^2$	$=164.51$
	18 $166.8-(4.20\times0.7)+\frac{1}{2000000}420^2$	$=164.74$
	19 $166.8-(3.20\times0.7)+\frac{1}{2000000}320^2$	$=165.07$
	20 $166.8-(2.20\times0.7)+\frac{1}{2000000}220^2$	$=165.50$
	21 $166.8-(1.20\times0.7)+\frac{1}{2000000}120^2$	$=166.03$
	22 $166.8-(0.20\times0.7)+\frac{1}{2000000}20^2$	$=166.66$
C,	22+20, $162.6+(6.00\times0.7)$	$=166.80$

DEMONSTRATION OF EQ. 44.

he general equation of a parabola passing through the point n (Fig. 36) be written

$$y^2 + y_n^2 = 2p(x + x_n),$$

which

$$x_n = \frac{y^2}{2p} + \frac{y_n^2}{2p} - x.$$

$x = x_A, y = y_A$, and we have

$$x_n = \frac{y_A^2}{2p} + \frac{y_n^2}{2p} - x_A.$$

he general equation of a tangent passing through the point A may be en

$$yy_A = p(x + x_A),$$

which

$$x = \frac{yy_A}{p} - x_A.$$

$x = x_s, y = y_s[= y_n]$, and we have

$$x_s = \frac{y_n y_A}{p} - x_A.$$

$$\overline{sn} = x_n - x_s = \frac{y_A^2 + y_n^2 - 2y_n y_A}{2p}$$

$$= \frac{(y_A - y_n)^2}{2p} = \frac{\overline{Am}^2}{2p},$$

$$2p = \frac{y_A^2}{x_A} = \frac{\overline{Ae}^2}{\overline{eh}}.$$

$$\therefore \quad \overline{sn} = \overline{eh}\frac{\overline{Am}^2}{\overline{Ae}^2}.$$

his proves the general proposition that if secants are drawn parallel to xis of x, intersecting a parabola and a tangent to it, the intercepts be- 1 the tangent and the parabola are proportional to the square of the nces (measured parallel to y) from the tangent point.

CHAPTER III.

EARTHWORK.

FORM OF EXCAVATIONS AND EMBANKMENTS.

88. Usual form of cross-section in cut or fill. The normal form of cross-section in cut is as shown in Fig. 40, in which $e \ldots g$ represents the natural surface of the ground, no matter

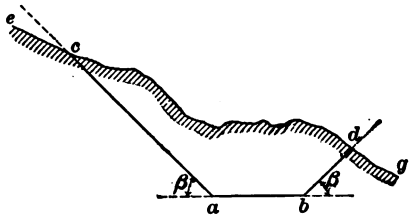


FIG. 40.

how irregular; ab represents the position and width of the required roadbed; ac and bd represent the "side slopes" which begin at a and b and which intersect the natural surface at such

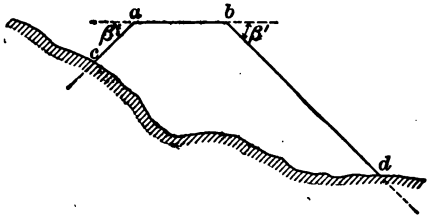


FIG. 41.

points (c and d) as will be determined by the required slope angle (β).

The normal section in fill is as shown in Fig. 41. The points c and d are likewise determined by the intersection of the re-

quired side slopes with the natural surface. In case the required roadbed (*ab* in Fig. 42) intersects the natural surface, both cut

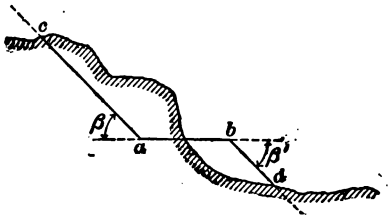


FIG. 42.

and fill are required, and the points *c* and *d* are determined as before. Note that β and β' are not necessarily equal. Their proper values will be discussed later.

89. **Terminal pyramids and wedges.** Fig. 43 illustrates the general form of cross-sections when there is a transition from cut to fill. *a* . . . *g* represents the grade line of the road which

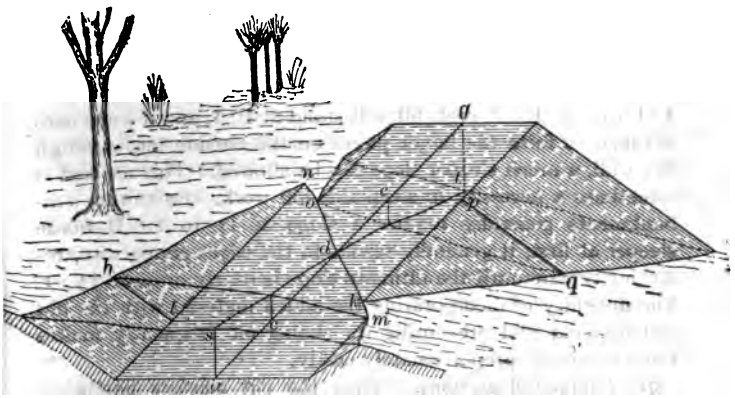


FIG. 43.

passes from cut to fill at *d*. *sdt* represents the surface profile. A cross-section taken at the point where either side of the roadbed *first cuts* the surface (the point *m* in this case) will usually be triangular if the ground is regular. A similar cross-section should be taken at *e*, where the other side of the roadbed cuts the surface. In general the earthwork of cut and fill terminates

in two pyramids. In Fig. 43 the pyramid vertices are at *n* and *k*, and the bases are *lhm* and *opq*. The roadbed is generally wider in cut than in fill, and therefore the section *lhm* and the altitude *ln* are generally greater than the section *opq* and the altitude *pk*. When the line of intersection of the roadbed and natural surface (*nodkm*) becomes perpendicular to the axis of the roadbed (*ag*) the pyramids become wedges whose bases are the nearest convenient cross-sections.

90. Slopes. a. Cuttings. The required slopes for cuttings vary from perpendicular cuts, which may be used in hard rock which will not disintegrate by exposure, to a slope of perhaps 4 horizontal to 1 vertical in a soft material like quicksand or in a clayey soil which flows easily when saturated. For earthy materials a slope of 1 : 1 is the maximum allowable, and even this should only be used for firm material not easily affected by saturation. A slope of 1½ horizontal to 1 vertical is a safer slope for average earthwork. It is a frequent blunder that slopes in cuts are made too steep, and it results in excessive work in clearing out from the ditches the material that slides down, at a much higher cost per yard than it would have cost to take it out at first, to say nothing of the danger of accidents from possible landslides.

b. Embankments. The slopes of an embankment vary from 1 : 1 to 1.5 : 1. A rock fill will stand at 1 : 1, and if some care is taken to form the larger pieces on the outside into a rough dry wall, a much steeper slope can be allowed. This method is sometimes a necessity in steep side-hill work. Earthwork embankments generally require a slope of 1½ to 1. If made steeper at first, it generally results in the edges giving way, requiring repairs until the ultimate slope is nearly or quite 1½ : 1. The difficulty of incorporating the added material with the old embankment and preventing its sliding off frequently makes these repairs disproportionately costly.

91. Compound sections. When the cut consists partly of earth and partly of rock, a compound cross-section must be made. If borings have been made so that the contour of the rock surface is accurately known, then the true cross-section may be determined. The rock and earth should be calculated separately, and this will require an accurate knowledge of where the rock "runs out"—a difficult matter when it must be deter-

mined by boring. During construction the center part of the earth cut would be taken out first and the cut widened until a sufficient width of rock surface had been exposed so that the rock cut would have its proper width and side slopes. Then the earth slopes could be cut down at the proper angle. A "berm" of about three feet should be left on the edges of the rock cut as

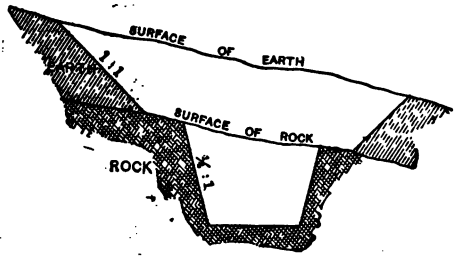


FIG. 44.

a margin of safety against a possible sliding of the earth slopes. After the work is done, the amount of excavation that has been made is readily computable, but accurate preliminary estimates are difficult. The area of the cross-section of earth in the figure must be determined by a method similar to that developed for borrow-pits (see § 120).

92. Width of roadbed. Owing to the large and often disproportionate addition to volume of cut or fill caused by the addition of even one foot to the width of roadbed, there is a natural tendency to reduce the width until embankments become unsafe and cuts are too narrow for proper drainage. The cost of maintenance of roadbed is so largely dependent on the drainage of the roadbed that there is true economy in making an ample allowance for it. The practice of some of the leading railroads of the country in this respect is given in the following table, in which are also given some data belonging more properly to the subject of superstructure.

It may be noted from the table that the average width for an *earthwork* cut, single track, is about 24.7 feet, with a minimum of 19 feet 2 inches. The widths of fills, single track, average over 18 feet, with numerous minimums of 16 feet. The widths for double track may be found by adding the distance between track centers, which is usually 13 feet.

WIDTH OF ROADBED FOR SINGLE AND DOUBLE TRACK—SLOPE RATIOS—DISTANCES BETWEEN TRACK CENTERS.

Road.	Single Track.		Double Track.		Slope Ratios.		Distance Between Track Centers.
	Cut.	Fill.	Cut.	Fill.	Cut.	Fill.	
A., T. & Santa Fé	{ 28′ earth { 22′ rock	20			1 : 1 ¼ : 1	1.5 : 1	
Chicago, Burlington & Quincy	14+(2×5)*	16	28+(2×5)	30	1.5 : 1	1.5 : 1	14′
Chicago, Milwaukee & St. Paul	18+(2×6)	20 to 24	31+(2×6)	33 to 37	1.5 : 1	1.5 : 1	13′
C., C., C. & St. Louis	20+(2×4)	20	33+(2×4)	33	1.5 : 1	1.5 : 1	13′
Illinois Central	32.5	16			1.5 : 1	1.5 : 1	
Erie	20′ 8½″	20′ 8½″	33′ 8½″	33′ 8½″	1.5 : 1	1.5 : 1	13′
Lehigh Valley	14+(2×3.5)	16	27+(2×3.5)	30	1 : 1	1.5 : 1	13′
Lake Shore & Michigan Southern			33+(2×7.25)	32	1.5 : 1	1.5 : 1	13′
Louisville & Nashville	13+(2×4.5)	16			1 : 1	1.5 : 1	
Michigan Central			(33+2×2.5)	33	1.5 : 1	1.5 : 1	13′
N. Y., N. H. & H.			30	30	1.5 : 1	1.5 : 1	12′
Norfolk & Western	{ 21′ 2″ earth { 16′ rock	17′ 2″	34′ 2″ earth 29′ rock	30′ 2′	1.5 : 1 { ¼ : 1 { ½ : 1	1.5 : 1	13′
Pennsylvania	{ 19′ 2″ light traffic { 27′ 2″ heavy "	19′ 2″ 19′ 2″	31′ 4″+(2×4)	31′ 4″	1.5 : 1	1.5 : 1	12′ 2″
Union Pacific	14+(2×3.5)	16			1 : 1	1.5 : 1	

* (2×5) signifies two ditches each 5 feet wide; the following cases should be interpreted similarly.

Am. Rwy. Eng. Assoc. standard for Class A roads, 20 feet for single track fill, 20+width of ditches for cut; 16 feet for Class B roads and 14 feet for Class C roads. See § 334 for classification. In theory, the track, in excavation, rests on a low embankment. By adding the width of side ditches, in excavations, there are uniform conditions, immediately under the track, throughout the line.

93. Form of subgrade. Specifications (or the cross-section drawings) formerly required that the subgrade should have a curved form, convex upward, or that it should slope outward from a slight ridge in the center, with the evident purpose of draining to the sides all water which might percolate through the ballast. If the subsoil were hard and impenetrable by the ballast, the method might answer, but experience has shown that, with ordinary subsoils, the ballast immediately under each rail is forced a little deeper into the subsoil by the passage of each train. Periodical retamping of ballast under the ends of the ties, and little or no tamping under the center, only adds to the accumulation under each rail. A cross-section of a very old roadbed will frequently show twice as much depth of ballast under the rails as there is under the center. This method of tamping quickly obliterates the original line of demarcation between ballast and subsoil and any expected improvement in drainage due to sloping subsoil is not realized. Therefore the A.R.E.A. specifications call for *flat* subgrades.

94. Ditches. "The stability of the track depends upon the strength and permanence of the roadbed and structures upon which it rests; whatever will protect them from damage or prevent premature decay should be carefully observed. The worst enemy is WATER, and the further it can be kept away from the track, or the sooner it can be diverted from it, the better the track will be protected. Cold is damaging only by reason of the water which it freezes; therefore the first and most important provision for good track is drainage." (Rules of the Road Department, Illinois Central R. R.)

The form of ditch generally prescribed has a flat bottom 12″ to 24″ wide and with sides having a minimum slope, except in rock-work, of 1 : 1, more generally 1.5 : 1 and sometimes 2 : 1. Sometimes the ditches are made V-shaped, which is objectionable unless the slopes are low The best form is evidently that which will cause the greatest flow for a given slope, and this will evidently be the form in which the ratio of area to wetted perimeter is the largest. The semicircle fulfills this condition better than any other form, but the nearly vertical sides would be difficult to maintain. (See Fig. 45.) A ditch, with a flat bottom and such

FIG. 45.

slopes as the soil requires, which approximates to the circular form will therefore be the best.

When the flow will probably be large and at times rapid it will be advisable to pave the ditches with stone, especially if the soil is easily washed away. Six-inch tile drains, placed 2′ under the ditches, are prescribed on some roads. (See Fig. 46.) No better method could be devised to insure a dry subsoil. The ditches through cuts should be led off at the end of the cut so that the adjacent embankment will not be injured.

Wherever there is danger that the drainage from the land above a cut will drain down into the cut, a ditch should be made near the edge of the cut to intercept this drainage, and this ditch should be continued, and paved if necessary, to a point where the outflow will be harmless Neglect of these simple and inexpensive precautions frequently causes the soil to be loosened on the shoulders of the slopes during the progress of a heavy rain, and results in a landslide which will cost more to repair than the ditches which would have prevented it for all time.

Ditches should be formed along the bases of embankments; they facilitate the drainage of water from the embankment, and may prevent a costly slip and disintegration of the embankment.

95. Effect of sodding the slopes, etc. Engineers are unanimously in favor of rounding off the shoulders and toes of embankments and slopes, sodding the slopes, paving the ditches, and providing tile drains for subsurface drainage, all to be put in during original construction. (See Fig. 46.) Some of the highest grade specifications call for the removal of the top layer of vegetable soil from cuts and from under proposed fills to some convenient place, from which it may be afterwards spread on the slopes, thus facilitating the formation of sod from grass-seed. But while engineers favor these measures and their economic value may be readily demonstrated, it is generally impossible to obtain the authorization of such specifications from railroad directors and promoters. The addition to the original cost of the roadbed is considerable, but is by no means as great as the capitalized value of the extra cost of maintenance resulting from the usual practice. Fig. 46 is a copy of

designs * presented at a convention of the American Society of Civil Engineers by Mr. D. J. Whittemore, Past President of the Society and Chief Engineer of the Chi., Mil. & St. Paul

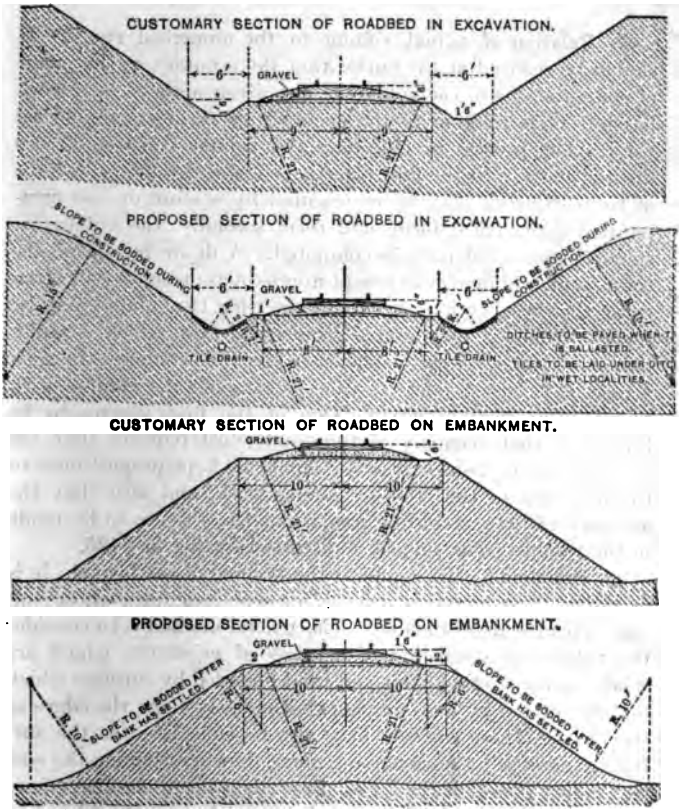

CUSTOMARY SECTION OF ROADBED IN EXCAVATION.
GRAVEL
PROPOSED SECTION OF ROADBED IN EXCAVATION.
SLOPE TO BE SODDED DURING CONSTRUCTION
TILE DRAIN
DITCHES TO BE PAVED WHEN T IS BALLASTED. TILES TO BE LAID UNDER DITC IN WET LOCALITIES.
CUSTOMARY SECTION OF ROADBED ON EMBANKMENT.
PROPOSED SECTION OF ROADBED ON EMBANKMENT.
SLOPE TO BE SODDED AFTER BANK HAS SETTLED

FIG. 46.—"WHITTEMORE ON RAILWAY EXCAVATION AND EMBANKMENTS" Trans. Am. Soc. C. E., Sept. 1894.

R. R. The "customary sections" represent what is, with some variations of detail, the practice of many railroads. The "pro-

* Trans. Am. Soc. Civil Eng., Sept. 1894.

posed sections" elicited unanimous approval. They should be adopted when not prohibited by financial considerations.

EARTHWORK SURVEYS.

96. Relation of actual volume to the numerical result. It should be realized at the outset that the accuracy of the result of computations of the volume of any given mass of earthwork has but little relation to the accuracy of the mere numerical work. The process of obtaining the volume consists of two distinct parts. In the first place it is assumed that the volume of the earthwork may be represented by a more or less complicated geometrical form, and then, secondly, the volume of such a geometrical form is computed. A desire for simplicity (or a frank willingness to accept approximate results) will often cause the cross-section men to assume that the volume may be represented by a very simple geometrical form which is really only a very rough approximation to the true volume. In such a case, it is only a waste of time to compute the volume with minute numerical accuracy. One of the first lessons to be learned is that economy of time and effort requires that the accuracy of the numerical work should be kept proportional to the accuracy of the cross-sectioning work, and also that the accuracy of both should be proportional to the use to be made of the results. The subject is discussed further in § 125.

97. Prismoids. To compute the volume of earthwork, it is necessary to assume that it has some geometric form whose volume is readily determinable. The general method is to consider the volume as consisting of a series of *prismoids*, which are solids having parallel plane ends and bounded by surfaces which may be formed by lines moving continuously along the edges of the bases These surfaces may also be considered as the surfaces generated by lines moving along the edges joining the corresponding points of the bases, these edges being the directrices, and the lines being always parallel to either base, which is a plane director. The surfaces thus developed may or may not be planes. The volume of such a prismoid is readily determinable (as explained in § 108 *et seq.*), while its definition is so very general that it may be applied to very rough ground. The "two plane ends" are sections perpendicular to the axis of the road. The roadbed and side slopes (also plane) form three of

the side surfaces. The only approximation lies in the degree of accuracy with which the plane (or warped) surfaces coincide with the actual surface of the ground between these two sections. This accuracy will depend (*a*) on the number of points which are taken in each cross-section and the accuracy with which the lines joining these points coincide with the actual cross-sections; (*b*) on the skill shown in selecting places for the cross-sections so that the warped surfaces shall coincide as nearly as possible with the surface of the ground. In fairly smooth country, cross-sections every 100 feet, placed at the even stations, are sufficiently accurate, and such a method simplifies the computations greatly; but in rough country cross-sections must be interpolated as the surface demands. As will be explained later, carelessness or lack of judgment in cross-sectioning will introduce errors of such magnitude that all refinements in the computations are utterly wasted.

98. **Cross-sectioning.** The process of cross-sectioning consists in determining at any place the intersection by a vertical plane of the prism of earth lying between the roadbed, the side slopes, and the natural surface. The intersection with the road-

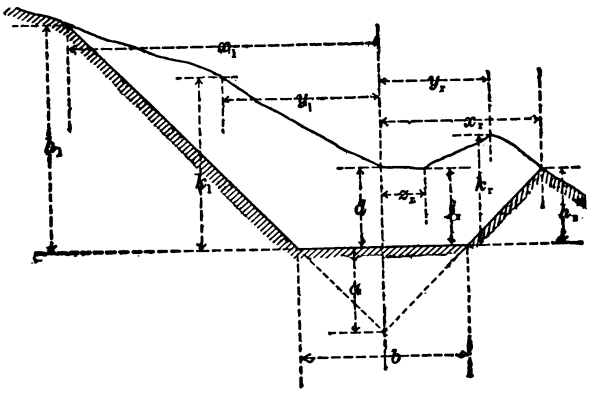


FIG. 47.

bed and side slopes gives three straight lines. The intersection with the natural surface is in general an irregular line. On smooth regular ground or when approximate results are acceptable this line is assumed to be straight. According to the irreg-

ularity of the ground and the accuracy desired more and more "intermediate points" are taken.

The distance (d in Fig. 47) of the roadbed below (or above) the natural surface at the center is known or determined from the profile or by the computed establishment of the grade line. The distances out from the center of all "breaks" are determined with a tape. To determine the elevations for a cut, set up a level at any convenient point so that the line of sight is higher than any point of the cross-section, and take a rod reading on the center point. This rod reading added to d gives the height of the instrument (H. I.) above the roadbed. Subtracting from H. I. the rod reading at any "break" gives the height of that point above the roadbed (h_l, k_l, h_r, etc.). This is true for all cases in excavation. For fill, the rod reading at center minus d equals the H. I., which may be positive or negative. When negative, add to the "H. I." the rod readings of the intermediate points to get their depths below "grade"; when positive, subtract the "H. I." from the rod readings.

The heights or depths of these intermediate points above or below grade need only be taken to the nearest tenth of a foot, and the distances out from the center will frequently be sufficiently exact when taken to the nearest foot. The roughness of the surface of farming land or woodland generally renders useless any attempt to compute the volume with any greater accuracy than these figures would imply unless the form of the ridges and hollows is especially well defined. The position of the slope-stake points is considered in the next section. Additional discussion regarding cross-sectioning is found in § 118.

99. Position of slope-stakes. The slope-stakes are set at the intersection of the required side slopes with the natural surface, which depends on the center cut or fill (d). The distance of the slope-stake from the center for the lower side is $x = \frac{1}{2}b + s(d+y)$; for the up-hill side it is $x' = \frac{1}{2}b + s(d-y')$. s is the "slope ratio" for the side slopes, the ratio of horizontal to vertical. In the above equation both x and y are unknown. Therefore some position must be found by trial which will satisfy the equation. As a preliminary, the value of x for the point $a = \frac{1}{2}b + sd$, which is the value of x for *level* cross-sections. In the case of fills on sloping ground the value of x on the *down-hill* side is *greater* than this; on the *up-hill* side it is *less*. The difference in distance is s times the difference of elevation. Take

numerical case corresponding with Fig. 48. The rod reading on c is 2.9; $d=4.2$; therefore the telescope is $4.2-2.9=1.3$ *below* grade. $s=1.5:1$, $b=16$. Hence for the point a (or for level ground) $x=\frac{1}{2}\times 16+1.5\times 4.2=14.3$. At a distance out of 14.3 the ground is seen to be about 3 feet lower, which will not only require $1.5\times 3=4.5$ more, but enough additional distance so that the added distance shall be 1.5 times the additional drop. As a first trial the rod may be held at 24 feet out and a reading of, say, 8.3 is obtained. $8.3+1.3=9.6$, the depth of the point below grade. The point on the slope line (n) which has this depth below grade is at a distance from the center

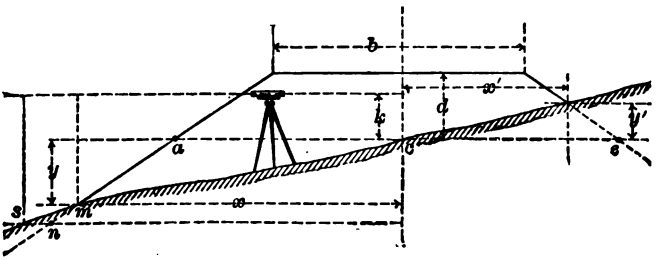


FIG. 48.

$x=8+1.5\times 9.6=22.4$. The point on the surface (s) having that depth is 24 feet out. Therefore the true point (m) is nearer the center. A second trial at 20.5 feet out gives a rod reading of, say, 7.1 or a depth of 8.4 below grade. This corresponds to a distance out of 20.6. Since the natural soil (especially in farming lands or woods) is generally so rough that a difference of elevation of a tenth or so may be readily found by slightly varying the location of the rod (even though the distance from the center is the same), it is useless to attempt too much refinement, and so in a case like the above the combination of 8.4 below grade and 20.6 out from center may be taken to indicate the proper position of the slope-stake. This is usually indicated in the form of a fraction, the distance out being the denominator and the height above (or below) grade being the numerator; the fact of *cut* or *fill* may be indicated by C or F. Ordinarily a second trial will be sufficient to determine with sufficient accuracy the true position of the slope-stake. Experienced men will frequently estimate the required distance

out to within a few tenths at the first trial. The left-hand pages of the note-book should have the station number, surface elevation, grade elevation, center cut or fill, and rate of grade. The right-hand pages should be divided in the center and show the distances out and heights above grade of all points, as is illustrated in §109. The notes should read UP the page, so that when looking ahead along the line the figures are in their proper relative position. The "fractions" farthest from the center line represent the slope-stake points.

100. Setting slope-stakes by means of "automatic" slope-stake rods. The equipment consists of a specially graduated tape and a specially constructed rod. The tape may readily be prepared by marking on the *back* side of an ordinary 50-foot tape which is graduated to feet and tenths. Mark "0" at "$\frac{1}{2}b$" from the tape-ring. Then graduate from the zero backward, at true scale, to the ring. Mark off "feet" and "tenths" on a scale proportionate to the slope ratio. For example, with the usual slope ratio of 1.5:1 each "foot" would measure 18 inches and each "tenth" in proportion.

The rod, 10 feet long, is shod at each end and has an endless tape passing within the shoes at each end and over pulleys—to reduce friction. The tape should be graduated in feet and tenths, from 0 to 20 feet—the 0 and 20 coinciding. By moving the tape so that 0 is at the bottom of the rod—or (practically) so that the 1-foot mark on the tape is one foot above the bottom of the shoe, an index mark may be placed on the back of the rod (say at 15—on the tape) and this readily indicates when the tape is "set at zero."

The method of use may best be explained from the figure and from the explicit rules as stated. The proof is given for two assumed positions of the level.

(1) Set up the level so that it is higher than the "center" and (if possible) higher than both slope-stakes, but not more than a rod-length higher. On very steep ground this may be impossible and each slope-stake must be set by separate positions of the level.

(2) Set the rod-tape at zero (i.e., so that the 15-foot mark on the *back* is at the index mark).

(3) Hold the rod at the center-stake (B) and note the reading (n_1 or n_2). Consider n to be always plus; consider d to be plus for cut and minus for fill.

(4) *Raise* the tape on the *face* side of the rod $(n+d)$. Applied literally (and algebraically), when the level is *below* the roadbed (only possible for fill), $(n+d)=(n_2+(-d_f))=n_2-d_f$. This being numerically negative, the tape is *lowered* (d_f-n_2). With level at (1), for fill, $(n+d)=(n_1+(-d_f))=(n_1-d_f)$; this being positive, the tape is raised. With level at (1), for cut, the tape is raised (n_1+d_c). In every case the effect is the same as if the telescope were set at the elevation of the roadbed.

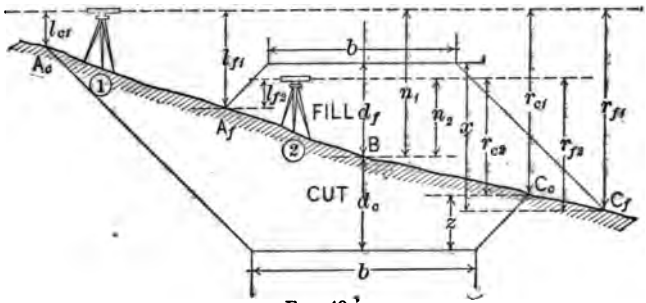


Fig. 49.

(5) With the special distance-tape, so held that its zero is $\frac{1}{2}b$ from the center, carry the rod out until the rod reading equals the reading indicated by the tape. Since in cut the tape is raised $(n+d)$, the zero of the rod-tape is always higher than the level (unless the rod is held at or below the elevation of the roadbed—which is only possible on side-hill work), and the reading at either slope-stake is necessarily *negative*. The reading for slope-stakes in fill is always positive.

(6) Record the rod-tape reading as the numerator of a fraction and the *actual distance* out (read directly from the *other* side of the distance-tape) as the denominator of the fraction.

Proof. Fill. Level at (1). Tape is raised (n_1-d_f). When rod is held at C_f, the rod reading is $+x$, which $=r_{f1}-(n_1-d_f)$. But the reading on the back side of the distance-tape is also x.

Fill. Level at (2). Tape is raised (n_2-d_f), i.e., it is *lowered* (d_f-n_2). When rod is held at C_f, the rod reading is $+x$, which similarly $=r_{f2}-(n_2-d_f)=r_{f2}+(d_f-n_2)$. Distance-tape as before.

Cut Level at (1). Tape is raised (n_1+d_c). When rod is held at C_c the rod reading is $-z$, which $= r_{c_1}-(n_1+d_c)$, i.e., $z=(n_1+d_c)-r_{c_1}$. The distance-tape will read z.

Side-hill work. It is easily demonstrated that the method, when followed literally, may be applied to side-hill work, although there is considerable chance for confusion and error, when, as is usual, $\frac{1}{2}b$ and the slope ratio are different for cut and for fill.

The method appears complicated at first, but it becomes mechanical and a time-saver when thoroughly learned. The advantages are especially great when the ground is fairly level transversely, but decrease when the difference of elevation of the center and the slope-stake is more than the rod length. By setting the rod-tape "at zero," the rod may always be used as an ordinary level rod and the regular method adopted, as in § 99. Many engineers who have thoroughly tested these rods are enthusiastic in their praise as a time-saver.

COMPUTATION OF VOLUME

§ 101. Simple approximations. The principles developed in §§ 96 and 97 show that, except where the ground is abnormally smooth and level, the earthwork to be excavated has a geometrical form whose volume cannot be *accurately* computed by any simple rule. The usual method is to consider that the volume is approximately measured by the product of the mean of the areas of two consecutive sections and the distance between those sections. When the ground is so regular that the error of such an approximation may be tolerated, or when only a rough approximation is necessary, such a computation may be accepted without correction. In any case, the "volume by averaging end areas" is computed as a first approximation and then correction is computed if desired. It should, therefore, be remembered that this approximate method, which is so common that it is often accepted without correction as the true volume, is never mathematically correct except under conditions which practically never exist. Whether a correction should be computed depends on the percentage of accuracy required, on the irregularity of the ground, and on the differences in the depth of adjacent center cuts—or fills. Experience gives the engineer such an idea of the probable amount of this correction under

any given conditions that he may judge when it is necessary to compute the correction in order to obtain the true volume with any desired degree of accuracy. The methods of computing this correction will be given later.

102. Approximate volume, level sections. When the country is very level or when only approximate preliminary results

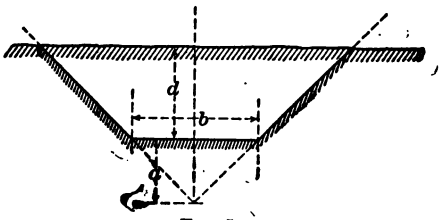


FIG. 50.

are required, it is sometimes assumed that the cross-sections are level. The area of the cross-section may be written

$$(a+d)^2 s - \frac{ab}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

in which a, b and d are dimensions as indicated by the figure and s is the "slope ratio" or the ratio of the horizontal projection of the slope to the vertical. A table is very readily formed giving the area in square feet of a section of given depth and for any given width of roadbed and ratio of side slopes. Usually these tables give a number which equals that area times 100 and divided by 27, which is the volume in cubic yards of a prism 100 feet long and with that cross-sectional area. Table XVII is such a table.

The volume may also be readily determined (as illustrated in the following example), without the use of such a table; a table of squares will facilitate the work. Assuming the cross-sections at equal distances ($=l$) apart, the total approximate volume for any distance will be

$$\frac{l}{2}[A_0+2(A_1+A_2+\ldots A_{n-1})+A_n]. \quad . \quad . \quad . \quad . \quad (47)$$

103. Numerical example: level sections. Given the following center heights for the same number of consecutive stations 100 feet apart; width of roadbed 18 feet; slope 1½ to 1.

The products in the fifth column may be obtained very readily and with sufficient accuracy by the use of the slide-rule described in § 106. The products should be considered as $(a+d)(a+d) \div \frac{1}{s}$. In this problem $s = 1\frac{1}{2}$, $\frac{1}{s} = .6667$. To apply the rule to the first case above, place 6667 on scale B over 89 on scale A, then opposite 89 on scale B will be found 118.8 on scale A. The position of the decimal point will be evident from an approximate mental solution of the problem.

Sta.	Center Height.	$a+d$	$(a+d)^2$	$(a+d)^2 s$	Areas.
17	2.9	8.9	79.21	118.81	118.81
18	4.7	10.7	114.49	171.74	343.48
19	6.8	12.8	163.84	245.76	491.52
20	11.7	17.7	313.29	469.93	939.86
21	4.2	10.2	104.04	156.06	312.12
22	1.6	7.6	57.76	86.64	86.64
					2292.43

$\frac{ab}{2} = \frac{6 \times 18}{2} = 54$

$10 \times 54 = 540$

1752.43

$\frac{1752.43 \times 100}{2 \times 27} = 3245$ cub. yards = approx. vol.

104. Equivalent sections. When sections are very irregular the following method may be used, especially if great accuracy

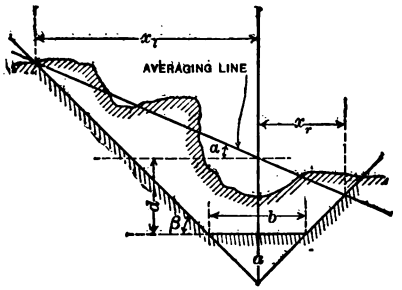


FIG. 51.—EQUIVALENT SECTION.

is not required. The sections are plotted to scale and then a uniform slope line is obtained by stretching a thread so that the undulations are averaged and an *equivalent section* is obtained. Measure the distances (x_l and x_r) from the center. The area

may then be obtained independent of the center depth as follows: Let s = the slope ratio of the side slopes = $\cot \beta = \frac{b}{2a}$. (See Fig. 51.) Then the

$$\text{Area} = \frac{1}{2}\left(\frac{x_l + x_r}{s}\right)(x_l + x_r) - \frac{x_r}{s}\frac{x_r}{2} - \frac{x_l}{s}\frac{x_l}{2} - \frac{ab}{2}$$

$$= \frac{x_l x_r}{s} - \frac{ab}{2}. \quad \ldots\ldots\ldots\ldots \quad (48)$$

These approximate methods are particularly useful for rapidly making up monthly estimates, realizing that the inaccuracies, plus and minus, will be wiped out when the final computation is made by a more accurate method.

105. Three-level sections. The next method of cross-sectioning in the order of complexity, and therefore in the order of

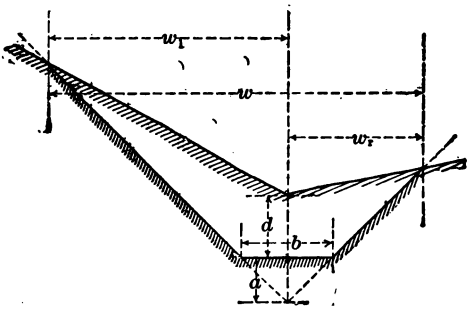


FIG. 52.

accuracy, is the method of three-level sections. The area of the section is $\frac{1}{2}(a+d)(w_r+w_l) - \frac{ab}{2}$, which may be written $\frac{1}{2}(a+d)w - \frac{ab}{2}$, in which $w = w_r + w_l$. If the volume is computed by averaging end areas, it will equal

$$\frac{l}{4}[(a+d')w' - ab + (a+d'')w'' - ab]. \quad \ldots\ldots \quad (49)$$

Notes.				Approx. Volume.				* Prismoidal correction.			Curvature Correction.†		
Station.	Center.	Left.	Right.	$a+d$	w	Yards.		$d'-d''$	$w''-w'$	Pris. Corr.	$x_l \sim x_r$	$\frac{V(x_l \sim x_r)}{3R}$	Curv. Corr.*
17	$2.6F$	$\frac{10.6F}{22.9}$	$\frac{0.8F}{8.2}$	7.3	31.1	210	·				14.7	+1	
18	$8.1F$	$\frac{15.8F}{30.7}$	$\frac{3.4F}{12.1}$	12.8	42.8	507	595	−5.5	+11.7	−20	18.6	+3	+4
+ 40	$10.7F$	$\frac{20.2F}{37.3}$	$\frac{4.8F}{14.2}$	15.4	51.5	734	448	−2.6	+ 8.7	− 3	23.1	+6	+4
19	$6.4F$	$\frac{14.0F}{28.0}$	$\frac{2.1F}{10.1}$	11.1	38.1	392	602	+4.3	−13.4	−11	17.9	+2	+5
20	$3.7F$	$\frac{5.8F}{15.7}$	$\frac{0.2F}{7.3}$	8.4	23.0	179	449	+2.7	−15.1	−13	8.4	+1	+3

Roadbed, 14′ wide in fill.
Slope 1½ to 1.

$a = \frac{b}{2s} = \frac{14}{3} = 4.7$.

$\frac{25}{27}ab = 61$.

Approx. Vol. = 2094 −47 +16
* Pris. corr. = 47
True Vol. = 2047 (disregarding curv. corr.)†

* For the method of computing the prismoidal correction see § 114.

† For the derivation of the curvature correction, see § 124.

If we divide by 27 to reduce to cubic yards, we have, when $l=100$

$$\text{Vol } (\prime \ldots \prime\prime) = \tfrac{25}{27}(a+d')w' - \tfrac{25}{27}ab + \tfrac{25}{27}(a+d'')w'' - \tfrac{25}{27}ab.$$

For the next section

$$\text{Vol } (\prime\prime \ldots \prime\prime\prime) = \tfrac{25}{27}(a+d'')w'' - \tfrac{25}{27}ab + \tfrac{25}{27}(a+d''')w''' - \tfrac{25}{27}ab.$$

For a partial station length compute as usual and multiply result by $\frac{\text{length in feet}}{100}$.

The following example is given to illustrate the method of three-level sections.

In the first column of yards

$$210 = \tfrac{25}{27}(a+d)w = \tfrac{25}{27}\times 7.3\times 31.1;$$

507, 734, etc., are found similarly;

$$595 = 210-61+507-61;$$

$$448 = \tfrac{40}{100}(507-61+734-61);$$

$$602 = \tfrac{60}{100}(734-61+392-61);$$

$$449 = 392-61+179-61.$$

The "*F*" in the columns of center heights, as well as the columns of "right" and "left" are inserted to indicate *fill* for all those points. Cut would be indicated by "*C*."

106. Computation of products. The quantities $\frac{25}{27}(a+d)w$ and $\frac{25}{27}ab$ represent in each case the product of two variable terms and a constant. These products are sometimes obtained from tables which are calculated for all ordinary ranges of the variable terms as arguments. A similar table computed for $\frac{25}{81}(d'-d'')(w''-w')$ will assist similarly in computing the prismoidal correction, see § 114. Prof. Charles L. Crandall, of Cornell University, is believed to be the first to prepare such a set of tables, which were first published in 1886 "Tables for the Computation of Railway and Other Earthwork." Another easy method of obtaining these products is by the use of a slide-rule. Any slide-rule, from which may be read directly three significant figures and from which the fourth may be read by estimation, can be utilized for this purpose. The Thacher or

the Stanley cylindrical rules are still more accurate. To illustrate its use, suppose $(a+d)=28.2$, and $w=62.4$; then

$$\frac{25}{27}(a+d)w=\frac{28.2\times62.4}{1.08}.$$

Set 108 (which, being a constant of frequent use, may be specially marked) on the sliding scale (B) opposite 282 on the other scale (A), and then opposite 624 on scale B will be found 1629 on scale A, the 162 being read directly and the 9 read by estimation. Although strict rules may be followed for pointing off the final result, it only requires a very simple mental calculation to know that the result must be 1629 rather than 162.9 or 16290. For products less than 1000 cubic yards the result may be read directly from the scale; for products between 1000 and 5000 the result may be read directly to the nearest 10 yards, and the tenths of a division estimated. Between 5000 and 10,000 yards the result may be read directly to the nearest 20 yards, and the fraction estimated; but prisms of such volume will never be found as simple triangular prisms—at least, an assumption that any mass of ground was as regular as this would probably involve more error than would occur from faulty estimation of fractional parts. Facilities for reading as high as 10,000 cubic yards would not have been put on the scale except for the necessity of finding such products as $\frac{2}{27}(9.1\times9.5)$, for example. This product would be read off from the same part of the rule as $\frac{2}{27}(91\times95)$. In the first case the product (80.0) could be read directly to the nearest .2 of a cubic yard, which is unnecessarily accurate. In the other case, the product (8004) could only be obtained by estimating $\frac{4}{20}$ of a division.

The computation for the prismoidal correction (see § 114), may be made similarly except that the divisor is 3.24 instead of 1.08. For example, $\frac{2}{81}(5.5\times11.7)=\frac{5.5\times11.7}{3.24}$. Set the 324 on scale B (also specially marked like 108) opposite 55 on scale A, and proceed as before.

107. **Approximate volume. Irregular sections.** In cross-sectioning irregular sections, the distance from the center and the elevation above "grade" of every "break" in the cross-section must be observed. The area of the irregular section may be obtained by computing the area of the trapezoids (as, in Fig. 53) and subtracting the two external triangles. For Fig. 53 the area would be

$$\frac{h_l+k_l}{2}(x_l-y_l)+\frac{k_l+d}{2}y_l+\frac{d+j_r}{2}z_r+\frac{j_r+k_r}{2}(y_r-z_r)$$
$$+\frac{k_r+h_r}{2}(x_r-y_r)-\frac{h_l}{2}\left(x_l-\frac{b}{2}\right)-\frac{h_r}{2}\left(x_r-\frac{b}{2}\right).$$

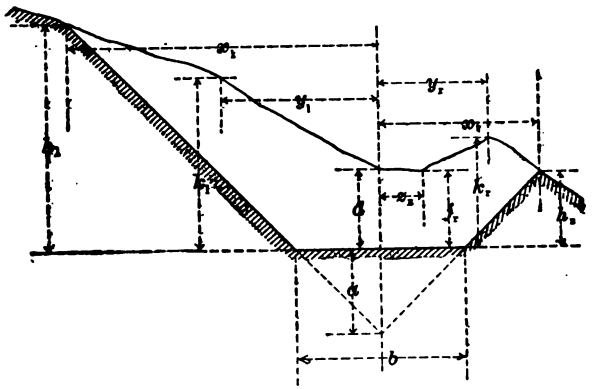


FIG. 53.

Expanding this and collecting terms, of which many will cancel, we obtain

$$\text{AREA}=\frac{1}{2}\Big[x_lk_l+y_l(d-h_l)+x_rk_r+y_r(j_r-h_r)$$
$$+z_r(d-k_r)+\frac{b}{2}(h_l+h_r)\Big]. \quad . \quad . \quad . \quad (50)$$

An examination of this formula will show a perfect regularity in its formation which will enable one to write out a similar formula for any section, no matter how irregular or how many points there are, without any of the preliminary work. The formula may be expressed in words as follows:

AREA equals one-half the sum of products obtained as follows:

the distance to each slope-stake times the height above grade of the point next inside the slope-stake;

the distance to each intermediate point in turn times the height of the point just inside minus the height of the point just outside;

finally, one-half the width of the roadbed times the sum of the slope-stake heights.

If one of the sides is perfectly regular from center to slope-stake, it is easy to show that the rule holds literally good. The "point next inside the slope-stake" in this case is the center; the intermediate terms for that side vanish. The *last term* must always be used. The rule holds good for three-level sections, in which case there are three terms, which may be reduced to two. Since these two terms are both variable quantities for each cross-section, the special method, given in § 105, in which one term ($\frac{1}{2}ab$) is a constant for all sections, is preferable for three-level sections. In the general method, each intermediate "break" adds another term.

108. Volume of an irregular prismoid. This is obtained by computing first the approximate volume by "averaging end areas" or by multiplying the length by the half sum of the end areas, as computed from Eq. (50). In other words, the Approx. volume $= \frac{100}{27} \times \frac{1}{2}$ (area′+area″). But since each area equals *one-half* the sum of products of width times height (see Eq. (50)) we may say that

$$\text{Approx. volume} = \frac{25}{27} \text{ (summation of } \textit{width} \text{ times } \textit{height}) \quad . \quad (51)$$

the terms of width times height being like those found within the bracket of Eq. (50).

As before, for partial station lengths, multiply the result by (length in feet ÷ 100). There will be no constant subtractive term, $\frac{25}{27}ab$, as in § 105.

109. Numerical example; approximate volume; irregular sections. Assume the earthwork notes as given below where the roadbed is 18 feet wide in cut and the slope is 1½ to 1. Note that the stations read UP the page and that when the surveyor is looking ahead along the line the several combinations of *heights* and *distances out* have approximately the same relative position on the notebook as they have on the ground. For example, beginning at the bottom line (Sta. 16), the combination $\frac{8.9c}{21.4}$ means that the extreme left-hand point of that section (the "slope-stake") is 22.4 feet horizontally from the center and that it is 8.9 feet above the required roadbed. The cut (*c*) *would be* 8.9 feet to reach the roadbed, but of course the actual cutting is

zero at the slope stake. The next point is 12.0 feet horizontally from the center and 7.6 feet above the roadbed. The cut at the center is 6.8 feet. The combinations of dimensions on the right-hand side are to be interpreted similarly.

Sta.	Center {cut or fill.}	Left.			Right.	
19	0.6c	$\frac{3.6c}{14.4}$			$\frac{0.1c}{4.2}$	$\frac{0.4c}{9.6}$
18	2.3c	$\frac{4.2c}{15.3}$	$\frac{6.8c}{8.4}$	$\frac{3.2c}{5.2}$		$\frac{1.2c}{10.8}$
17	7.6c	$\frac{8.2c}{21.3}$	$\frac{10.2c}{17.4}$	$\frac{8.0c}{6.1}$		$\frac{4.2c}{15.3}$
+42	10.2c	$\frac{12.2c}{27.3}$		$\frac{12\ 6c}{8.2}$	$\frac{6.2c}{7.5}$	$\frac{8.4c}{21.6}$
16	6.8c	$\frac{8.9c}{22.4}$		$\frac{7.6c}{12.0}$	$\frac{3.2c}{4.1}$	$\frac{2.6c}{12.9}$

The numerical computation is greatly facilitated by a systematic form as given below. For Sta. 16, the first term is "the distance to the left slope stake" (22.4) times "the height above grade of the point next inside" (the height being 7.6), and we place this pair of figures in the columns of "width" and "height." The "distance to the point next inside" is 12.0 and the "height of the point just inside (6.8) minus the height of the point just outside" (8.9) equals (−2.1) and these are the next pair of widths and heights. Taking $\frac{25}{27}$ of the product of each pair of numbers we have the numbers in the first column of "yards." The sum of all these numbers in the first and second groups multiplied by $\frac{42}{100}$ (that section being only 42 feet long) equals 378 cubic yards, the volume by averaging end areas. The determination of center heights and total widths and the application of Eq. (54), to obtain the approximate prismoidal correction (see § 114), is self-evident.

110. Prismoidal correction. The foregoing methods of calculation have been called approximate, although under many

VOLUME OF IRREGULAR PRISMOID, WITH APPROXIMATE PRISMOIDAL CORRECTION.

Sta.	Width	Height	Yards.		Cen. Height.	Total width	$d'-d''$	$w''-w'$	Approx. pris. corr.
16	22 4	7 6	168		−6.8	35.3			
	12 1	−12 1	−23						
	12 9	3 2	40						
	4 1	4 2	16						
	9 3	11 5	98						
+42	27 3	12 6	319		−11.2	48.9	−3.4	+13.6	+14
	8 2	−12 0	−15						
	21 3	0 1	124						
	1 5	11 6	13						
	9 1	20 3	172	378					(−6)
17	21 3	11 2	201		− 7.6	38 6	−2.6	−12.3	−10
	1 4	−11 2	− 3						
	8 1	−12 3	−14						
	15 3	7 6	107						
	3 1	12 4	113	584					(−6)
18	15 3	8 6	95		− 2.1	26 1	−5.3	−10.5	−17
	6 4	−11 2	− 7						
	5 2	−4 5	−22						
	11 6	11 3	93						
	3 3	5 4	45	528					(−17)
19	14 4	3 4	56		− 3.6	24.3	−1.7	−2.1	−1
	3 4	0 1	1						
	4 2	6 1	14						
	3 1	4 1	33	104					(−1)
	Approx. volume = 1367								−90
	Approx. pris. corr. = − 30								
	Corrected volume = 1337 cubic yards								

conditions such results are considered to be sufficiently accurate to serve as final. In any case the approximate result is first computed and then the "prismoidal correction" is computed if necessary. The mathematical necessity for a correction may be at once appreciated from the consideration that the volume of a prismoid having dissimilar and unequal ends is NOT equal to the length times the average of the end areas but is usually somewhat less. In an extreme case the correction is one-third of the approximate volume, or one-half of the true volume. The amount of the prismoidal correction for a triangular prism will be first determined and from that the correction for any kind of prism may be deduced.

Let Fig. 54 represent a triangular prismoid. The two triangles forming the ends lie in parallel planes, but since the angles of the triangle are not equal to the corresponding angles of the

ier triangle, at least two of the surfaces must be *warped*. If ection, parallel to the bases, is made at any point at a dis-

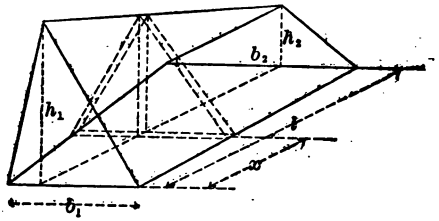


FIG. 54.

ce x from one end, the area of the section will evidently be

$$A_x = \tfrac{1}{2} b_x h_x = \tfrac{1}{2}\left[b_1 + (b_2 - b_1)\frac{x}{l}\right]\left[h_1 + (h_2 - h_1)\frac{x}{l}\right].$$

e volume of a section of infinitesimal length will be $A_x dx$, l the total volume of the prismoid will be *

$$\begin{aligned}
A_x dx &= \tfrac{1}{2}\int_0^l \left[b_1 + (b_2 - b_1)\frac{x}{l}\right]\left[h_1 + (h_2 - h_1)\frac{x}{l}\right] dx \\
&= \tfrac{1}{2}\left[b_1 h_1 x + (b_2 - b_1)h_1 \frac{x^2}{2l} + b_1(h_2 - h_1)\frac{x^2}{2l} \right. \\
&\qquad \left. + (b_2 - b_1)(h_2 - h_1)\frac{x^3}{3l^2}\right]_0^l \\
&= \tfrac{1}{2}\left\{b_1 h_1 l + [(b_2 - b_1)h_1 + b_1(h_2 - h_1)]\frac{l}{2} + (b_2 - b_1)(h_2 - h_1)\frac{l}{3}\right\} \\
&= \frac{l}{2}[\tfrac{1}{3} b_1 h_1 + \tfrac{1}{6} b_1 h_2 + \tfrac{1}{6} b_2 h_1 + \tfrac{1}{3} b_2 h_2] \\
&= \frac{l}{6}[\tfrac{1}{2} b_1 h_1 + \tfrac{1}{2} b_1(h_1 + h_2) + \tfrac{1}{2} b_2(h_1 + h_2) + \tfrac{1}{2} b_2 h_2] \\
&= \frac{l}{6}\left[\tfrac{1}{2} b_1 h_1 + 4\left(\frac{1}{2} \cdot \frac{b_1 + b_2}{2} \cdot \frac{h_1 + h_2}{2}\right) + \tfrac{1}{2} b_2 h_2\right] \\
&= \frac{l}{6}[A_1 + 4A_m + A_2], \qquad \ldots\ldots\ldots\ldots \quad (52)
\end{aligned}$$

Students unfamiliar with the Integral Calculus may take for granted the lamental formulæ that $\int dx = x$, that $\int x dx = \frac{1}{2}x^2$, and that $\int x^2 dx = \frac{1}{3}x^3$; that in integrating between the limits of l and 0 (zero), the value he integral may be found by simply substituting l for x after gration.

in which A_1, A_2, and A_m are the areas respectively of the two bases and of the middle section. Note that A_m is *not* the *mean* of A_1 and A_2, although it does not necessarily differ very greatly from it.

The above proof is absolutely independent of the values, absolute or relative, of b_1, b_2, b_3, or b_4. For example, A_2 may be zero and the second base reduces to a line and the prismoid becomes wedge-shaped; or b_3 and b_4 may both vanish, the second base becoming a point and the prismoid reduces to a pyramid. Since every prismoid as defined in § 37 may be reduced to a combination of triangular prismoids, wedges, and pyramids, and since the formula is true for any one of them individually, it is true for all collectively; therefore it may be stated that *

The volume of a prismoid equals one sixth of the perpendicular distance between the bases multiplied by the sum of the areas of the two bases plus four times the area of the middle section.

While it is always possible to compute the volume of any prismoid by the above method, it becomes an extremely complicated and tedious operation to compute the true value of the middle section, the end sections are complicated in form. It therefore becomes a similar operation to compute volumes by approximate formulæ and sometimes necessary, a correction. The most common methods are as follows:

111. **Correction for triangular prismoid.** The volume of the triangular prismoid Fig. 3e may be computed by averaging end areas, is $\frac{1}{2}(b_1 b_2 + b_3 b_4)$. Subtracting this from the true volume (as given in the formula above Fig. 3e) we obtain the correction

$$\ldots \quad (53)$$

This shows that if the bases are equal, the correction vanishes. It also shows that if the bases are roughly similar and [illegible] occurs, as will be seen at [illegible] which means that the [illegible] gives *too large* results.

[illegible] making a trian-

* [illegible] equation (52) does not [illegible] the following paragraph [illegible]

yramid, then b_1 and h_1 each equal zero and the correction to

$$\frac{l}{12}[(-b_2)(h_2)] = -\frac{lb_2h_2}{12}.$$

e volume of a triangular prismoid is one-third of the altines the area of the base or $\frac{1}{3}l(\frac{1}{2}b_2h_2) = \frac{1}{6}lb_2h_2$. The approx-olume, by averaging end areas, applying the rule strictly, $_2h_2+0) = \frac{1}{4}lb_2h_2$. The correction is therefore one-third of proximate volume, or one-half of the true volume, in this e case. Therefore, when computing the volume of ter-yramids and wedges (see § 89 and Fig. 43), by the method aging end areas, it must be remembered that, although ss volume is comparatively small, the prismoidal correc-relatively very large.

Correction for level sections. Absolutely level sections ctically unknown, and the error involved in assuming any ections as truly level will ordinarily be greater than the ed correction. If greater accuracy is required, more should be obtained in the cross-sectioning, which will ly show that the sections are not truly level. But it easily computed that the correction equals

$$-\frac{l}{12}\frac{b}{a}\Sigma(d' \sim d'')^2.$$

uares of the differences of center depth of consecutive s are always positive, regardless of whether the differences sitive or negative. Therefore the correction is *always* e, showing that the method of averaging end areas, when ions are level, *always* gives too large results.

Prismoidal correction for "equivalent sections." It is le although tedious problem in mathematics to compute ically the true and approximate volumes of a prismoid he areas are determined on the basis of "equivalent s," § 104, and from thence to derive a formula for the dal correction, but it is generally true that the errors such an approximate method of getting the area are so at it is a needless refinement to compute the correction.

Prismoidal correction for three-level sections. The dal correction may be obtained by applying Eq. 53 to le in turn. For the left side we have

$$\frac{l}{12}[(a+d')-(a+d'')](w_l''-w_l'), \text{ which equals}$$

$$\frac{l}{12}(d'-d'')(w_l''-w_l').$$

For the right side we have, similarly,

$$\frac{l}{12}(d'-d'')(w_r''-w_r').$$

The total correction therefore equals

$$\frac{l}{12}(d'-d'')[(w_l''+w_r'')-(w_l'+w_r')]$$

$$=\frac{l}{12}(d'-d'')(w''-w').$$

Reduced to cubic yards, and with $l=100$,

$$\text{Pris. Corr.} = \tfrac{25}{81}(d'-d'')(w''-w'). \quad . \quad . \quad . \quad (54)$$

Applying this formula to the numerical problem worked out in § 105, the several values of $(d'-d'')$ and $w''-w'$ are computed as given in the first two columns under Prismoidal Correction. Then, for example,

$$-20=\tfrac{25}{81}(d'-d'')(w''-w')=\tfrac{25}{81}(2.6-8.1)(42.8-31.1)$$
$$=\tfrac{25}{81}(-5.5)(+11.7).$$

For the next line, $-3=\frac{40}{100}[\frac{25}{81}(-2.6)(+8.7)]$, and similarly for the rest. For this typical case, the correction is over 2% of the volume and is, as usual, negative, or in other words, the approximate method, if used without correction, allows a contractor in this case 2% too much.

115. **Prismoidal correction; irregular sections.** For reasons given in the next article, the correction is computed as if the sections were "three-level" sections. This method was used in the numerical problem worked out in § 109. Instead of considering the heights and widths of the separate triangles, the center height and total width for each section is recorded in two columns and the differences $(d'-d'')$ and $(w''-w')$ are computed. $(-3.4)\times(+13.6)\div 3.24=-14$, which would be the correction for a section 100 feet long. For 42 feet the correction is 42% of -14 or -6. Note that the total prismoidal correction for this stretch of 300 feet is negative, as is usual, and that it is a less than 2%, about the same as the numerical problem of

116. **Magnitude of the probable error of this method.** In previous editions of this work, methods were given for computing the mathematically exact volume of a prismoid whose ends coincide with the "irregular sections" as measured, and whose upper surfaces are assumed to coincide with the actual surface of the ground. As in the previous methods, the "approximate volume" is computed by averaging end areas and then a correction is applied. If the end sections have the same number of intermediate points on each side, and if it can be assumed that the corresponding lines in each section are connected by plane or warped surfaces, which coincide with the surface of the ground, then the mathematically exact or "true" correction may be obtained by dividing the volume into elementary triangular prismoids, finding the correction for each and adding the results. Although such a method appears very complicated, it is readily possible to develop a law by means of which the true prismoidal correction may be written out (similarly to writing out the formula for the area, Eq. (50)) without any preliminary calculation. Such a law has a mathematical fascination, but it should be remembered that when the ground surface is so broken up that the cross-sections are "irregular" it is in general correspondingly rough and irregular between the cross-sections, especially when those sections are 100 feet apart. It is also true that the cross-sections do *not* usually have the same number of intermediate points on corresponding sides of the center. In such a case, unless the actual form of the ground between the cross-sections is observed and measured, the exact method cannot be used. An extra point in one cross-section implies an extra ridge (or hollow) which "runs out" or disappears by the time the adjoining section is reached. Theoretically a cross-section should be taken at the point where such a ridge or hollow runs out. In general this point will not be at an even 100-foot station. The attempt to compute the exact prismoidal correction usually gives merely a false appearance of extreme accuracy to the work which is not justified by the results. It should not be forgotten that it is readily possible to spend an amount of time on the surveying and computing which is worth more than the few cubic yards of earth which represents the additional accuracy of the more precise method. The accuracy of the office computation should be kept proportionate to the accuracy of the cross-sectioning

in the field. The discussion of the magnitude of the prismoid correction in §§ 110–115 shows that it is small except when t two ends of the prismoid are very dissimilar. The dissimilari between the two ends of a prismoid would be substantially t same whether the ends were actually "irregular" or had "thr level" sections, which for each end had the same slope stak and center heights as the irregular sections. Experience prov that the approximate prismoidal correction, computed considering the ground as three-level, is so nearly equal to t true prismoidal correction that the difference is perhaps greater than the *probable* difference between the true volu of earth and the volume of the geometrical prismoid which assumed to represent that volume. The experienced survey will take his cross-sections at such places and so close togeth that the warped surfaces joining the sections will lie very nea in the surface or at least will so average the errors that th will substantially neutralize each other.

117. Numerical illustration of the accuracy of the appro mate rule. The "true" prismoidal correction for the numeri case given in § 109 was computed by the method outlined abo and on the basis of certain figures as to the vanishing of t ridges and valleys found in one section and not found in t adjacent sections. The various quantities for the volum between the cross-sections have been tabulated as show

	1	2	3	4	5	6	7
Sections.	Approx. vol. by averaging end areas.	True prismoidal correction.	True volume.	Approx. pris. corr. on basis of three-level ground.	Error; Col. 4 − col. 2.	Approx. vol. computed from center and side heights *only*.	Error;
16.....16+42	378	− 5	373	− 6	−1	396	−
16+42..17	584	− 3	581	− 6	−3	577	+
17.....18	528	−16	512	−17	−1	463	+
18.....19	177	− 3	174	− 1	+2	147	+
	1667	−27	1640	−30	−3	1583	+

There has also been shown in the last two columns the er involved if the "intermediate points" had been ignored the cross-sectioning. From the tabular form we may learn t

1. The *differences* between the "true" and approxim

rrections is so small that it is *probably* swallowed up by errors sulting from inaccurate cross-sectioning.

2. The error which would have been involved in ignoring e intermediate points is so very large in comparison with e other corresponding errors that (although it proves nothing solutely definite, being an individual case) the *probabilities* the relative error from these sources are clearly indicated.

118. **Cross-sectioning irregular sections.** The slope stake ould preferably be determined first, and then the "breaks" tween the slope stake and the center. When, as is usual, e ground is not even between the cross-section just taken d the section at the next 100-foot station, a point should be lected for a cross-section such that the lines to the previous ction should coincide with the actual surface of the ground as osely as the accuracy of the work demands. § 125 gives numerical illustration of the magnitude of some of these rors. Although it is possible for a skillful surveyor to so oose his cross-sections in rough and irregular ground that e positive and negative errors will nearly balance, it requires ceptional skill. Frequently the work may be simplified by mputing separately the volume of a mound or pit, the istence of which has been ignored in the regular cross-ctioning.

119. **Side-hill work.** When the natural slope cuts the roadbed ere is a necessity for both cut and fill at the same cross-section.

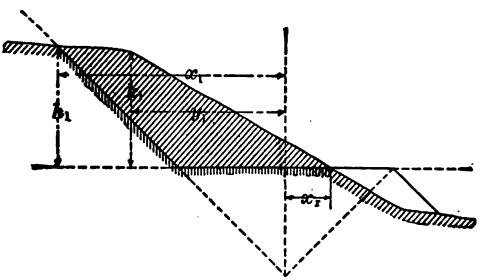


FIG. 55.

hen this occurs the cross-sections of both cut and fill are often nearly triangular that they may be considered as such without eat error, and the volumes may be computed separately as iangular prismoids without adopting the more elaborate form

of computation so necessary for complicated irregular sections. When the ground is too irregular for this the best plan is to follow the uniform system. In computing the cut, as in Fig. 55, the left side would be as usual; there would be a small center cut and an ordinate of zero at a short distance to the right of the center. Then, *ignoring the fill*, and applying Eq. 56 strictly, we have two terms for the left side, one for the right, and the term involving $\frac{1}{2}b$, which will be $\frac{1}{2}bh_l$ in this case, since $h_r = 0$, and the equation becomes

$$\text{Area}_{(\text{Cut})} = \tfrac{1}{2}[x_l k_l + y_l(d - h_l) + x_r d + \tfrac{1}{2}bh_l].$$

The area for fill may also be computed by a strict application of Eq. 50, but for Fig. 56 all distances for the left side are zero and the elevation for the first point out is zero. d also must be

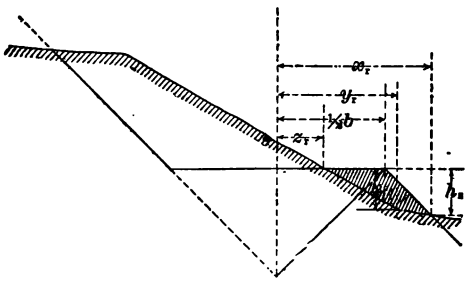


FIG. 56.

considered as zero. Following the rule, § 107, literally, the equation becomes

$$\text{Area}_{(\text{Fill})} = \tfrac{1}{2}[x_r k_r + y_r(o - h_r) + z_r(o - k_r) + \tfrac{1}{2}b(o + h_r)],$$

which reduces to

$$\tfrac{1}{2}[x_r k_r - y_r h_r - z_r k_r + \tfrac{1}{2}bh_r].$$

(*Note* that x_r, h_r, etc., have different significations and values in this and in the preceding paragraphs.) The "terminal pyramids" illustrated in Fig. 43 are instances of side-hill work for very short distances. Since side-hill work always implies *both* cut and fill at the same cross-section, whenever either the cut or fill disappears and the earthwork becomes wholly cut or wholly fill, that point marks the end of the "side-hill work," and a cross-section should be taken at this point.

120. **Borrow-pits.** The cross-sections of borrow-pits will vary not only on account of the undulations of the surface of the

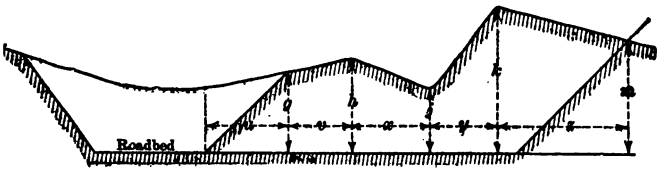


FIG. 57.

ground, but also on the sides, according to whether they are made by widening a convenient cut (as illustrated in Fig. 57) or simply by digging a pit. The sides should always be properly sloped and the cutting made cleanly, so as to avoid unsightly roughness. If the slope ratio on the right-hand side (Fig. 57) is s, the area of the triangle is $\frac{1}{2}sm^2$. The area of the section is $\frac{1}{2}[ug+(g+h)v+(h+j)x+(j+k)y+(k+m)z-sm^2]$. If all the horizontal measurements were referred to one side as an origin, a formula similar to Eq. 50 could readily be developed, but little or no advantage would be gained on account of any simplicity of computation. Since the *exact* volume of the earth borrowed is frequently necessary, the prismoidal correction should be computed; and since such a section as Fig. 57 does not even approximate to a three-level section, the method suggested in § 115 cannot be employed. It will then be necessary to employ the more exact method of dividing the volume into triangular prismoids and taking the summation of their corrections, found according to the general method of § 111.

121. **Correction for curvature.** The volume of a solid, generated by revolving a plane area about an axis lying in the plane but outside of the area, equals the product of the given area times the length of the path of the center of gravity of the area. If the centers of gravity of all cross-sections lie in the center of the road, where the length of the road is measured, there is absolutely no necessary correction for curvature. If all the cross-sections in any given length were exactly the same and therefore had the same eccentricity, the correction for curvature would be very readily computed according to the above principle. But when both the areas and the eccentricities vary from point to point, as is generally the case, a theoretically exact

solution is quite complex, both in its derivation and application. Suppose, for simplicity, a curved section of the road, of uniform cross-sections and with the center of gravity of every cross-section at the same distance e from the center line of the road. The length of the path of the center of gravity will be to the length of the center line as $R \pm e : R$. Therefore we have *True vol.*: *nominal vol.* $:: R \pm e : R$. $\therefore$ *True vol.* $= lA\frac{R \pm e}{R}$ for a volume of uniform area and eccentricity. For any other area and eccentricity we have, similarly, *True vol.'* $= lA'\frac{R \pm e'}{R}$. This shows that the effect of curvature is the same as increasing (or diminishing) the area by a quantity depending on the area and eccentricity, the increased (or diminished) area being found by multiplying the actual area by the ratio $\frac{R \pm e}{R}$. This being independent of the value of l, it is true for infinitesimal lengths. If the eccentricity is assumed to vary uniformly between two sections, the *equivalent area* of a cross-section located midway between the two end cross-sections would be $A_m\frac{\left(R \pm \frac{e' + e''}{2}\right)}{R}$. Therefore the volume of a solid which, when straight, would be $\frac{l}{6}(A' + 4A_m + A'')$, would then become

$$\textit{True vol.} = \frac{l}{6R}\left[A'(R \pm e') + 4\,A_m\left(R \pm \frac{e' + e''}{2}\right) + A''(R \pm e'')\right].$$

Subtracting the nominal volume (the true volume when the prismoid is straight), the

$$\textit{Correction} = \pm\frac{l}{6R}\left[(A' + 2A_m)e' + (2A_m + A'')e''\right]. \quad . \quad (55)$$

Another demonstration of the same result is given by Prof. C. L. Crandall in his "Tables for the Computation of Railway and other Earthwork," in which is obtained by calculus methods the summation of elementary volumes having variable areas with variable eccentricities. The exact application of Eq. 55 requires that A_m be known, which requires laborious computa-

is, but no error worth considering is involved if the equation written approximately

$$Curv.\ corr. = \frac{l}{2R}(A'e' + A''e''), \quad . \quad . \quad . \quad . \quad (56)$$

ch is the equation generally used. The approximation consis in assuming that the difference between A' and A_m equals difference between A_m and A'' but with opposite sign. The r due to the approximation is always utterly insignificant.

1. Eccentricity of the center of gravity. The determination he true positions of the centers of gravity of a long series of gular cross-sections would be a very laborious operation, fortunately it is generally sufficiently accurate to consider cross-sections as three-level ground, or, for side-hill work, to

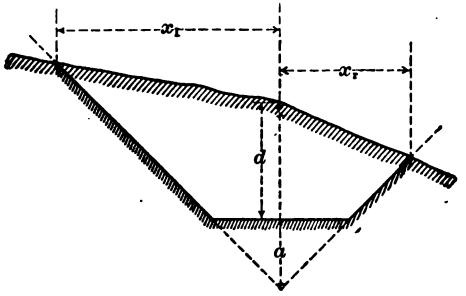


FIG. 58.

riangular, *for the purpose of this correction.* The eccentricity he cross-section of Fig. 58 (including the grade triangle) may written

$$e = \frac{\frac{(a+d)x_l}{2}\frac{x_l}{3} - \frac{(a+d)x_r}{2}\frac{x_r}{3}}{\frac{(a+d)x_l}{2} + \frac{(a+d)x_r}{2}} = \frac{1}{3}\frac{x_l^2 - x_r^2}{x_l + x_r} = \frac{1}{3}(x_l - x_r). \quad . \quad (57)$$

he side toward x_l being considered positive in the above onstration, if $x_r > x_l$, e would be negative, i.e., the center ravity would be on the right side. Therefore, for three-level

ground, the correction for curvature (see Eq. 56) may be written

$$Correction = \frac{l}{6R}[A'(x_l' - x_r') + A''(x_l'' - x_r'')].$$

Since the approximate volume of the prismoid is

$$\frac{l}{2}(A + A') = \frac{l}{2}A' + \frac{l}{2}A'' = V' + V'',$$

in which V' and V'' represent the number of cubic yards corresponding to the area at each station, we may write

$$Corr.\ in\ cub.\ yds. = \frac{1}{3R}[V'(x_l' - x_r') + V''(x_l'' - x_r'')]. \quad (58)$$

It should be noted that the value of e, derived in Eq. 57, is the eccentricity of the whole area including the triangle under the roadbed. The eccentricity of the true area is greater than this and equals

$$e \times \frac{\text{true area} + \frac{1}{2}ab}{\text{true area}} = e_1.$$

The required quantity ($A'e'$ of Eq. 56) equals *true area* $\times e_1$ which equals (*true area* $+ \frac{1}{2}ab) \times e$. Since the value of e is very simple, while the value of e_1 would, in general, be a complex quantity, it is easier to use the simple value of Eq. 57 and add $\frac{1}{2}ab$ to the area. Therefore, in the case of three-level ground the subtractive term $\frac{2}{7}\frac{5}{7}ab$ (§ 105) should *not* be subtracted in computing this correction. For irregular ground, when computed by the method given in §§ 107 and 108, which does not involve the grade triangle, a term $\frac{2}{7}\frac{5}{7}ab$ must be *added* at every station when computing the quantities V' and V'' for Eq. 58.

It should be noted that the factor $1 \div 3R$, which is constant for the length of the curve, may be computed with all necessary accuracy and without resorting to tables by remembering that

$$R = \frac{5730}{\text{degree of curve}}.$$

Since it is useless to attempt the computation of railroad earthwork closer than the nearest cubic yard, it will frequently

ssible to write out all curvature corrections by a simple
l process upon a mere inspection of the computation sheet.
3 shows that the correction for each station is of the form
$\frac{(x_l - x_r)}{}$. $3R$ is generally a large quantity—for a 6° curve
865. $(x_l - x_r)$ is generally small. It may frequently be
by inspection that the product $V(x_l - x_r)$ is roughly twice
ee times $3R$, or perhaps less than half of $3R$, so that the
tive term for that station may be written 2, 3, or 0 cubic
the fraction being disregarded. For much larger absolute
its the correction must be computed with a correspondingly
percentage of accuracy.

algebraic sign of the curvature correction is best deter-
by noting that the center of gravity of the cross-section is
right or left side of the center according as x_r is greater
than x_l, and that the correction is *positive* if the center of
y is on the *outside* of the curve, and *negative* if on the

s frequently found that x_l is uniformly greater (or uni-
less) than x_r throughout the length of the curve. Then
rvature correction for each station is uniformly positive or
ve. But in irregular ground the center of gravity is apt

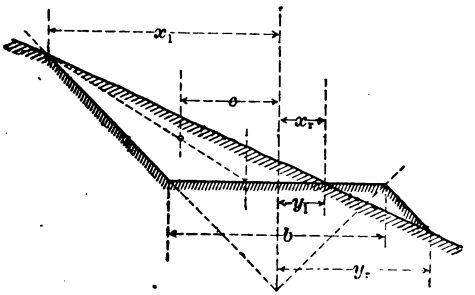


FIG. 59.

irregularly on the outside or on the inside of the curve,
he curvature correction will be correspondingly positive or
ive. If the curve is to the *right*, the correction will be
ve or negative according as $(x_l - x_r)$ is positive or negative;
curve is to the *left*, the correction will be positive or nega-

tive according as $(x_r - x_l)$ is positive or negative. Therefore when computing curves to the *right* use the form $(x_l - x_r)$ in Eqs. 58 and 60; when computing curves to the *left* use the form $(x_r - x_l)$ in these equations; the algebraic sign of the correction will then be strictly in accordance with the results thus obtained.

123. Center of gravity of side-hill sections. In computing the correction for side-hill work the cross-section would be treated as triangular unless the error involved would evidently be too great to be disregarded. The center of gravity of the triangle lies on the line joining the vertex with the middle of the base and at $\frac{1}{3}$ of the length of this line from the base. It is therefore equal to the distance from the center to the foot of this line plus $\frac{1}{3}$ of its horizontal projection. Therefore

$$e = \left[\frac{b}{2} - \frac{1}{2}\left(\frac{b}{2} + x_r\right)\right] + \frac{1}{3}\left[x_l - \left(\frac{b}{2} - \frac{1}{2}\left(\frac{b}{2} + x_r\right)\right)\right]$$

$$= \frac{b}{4} - \frac{x_r}{2} + \frac{x_l}{3} - \frac{b}{12} + \frac{x_r}{6}$$

$$= \frac{b}{6} + \frac{x_l}{3} - \frac{x_r}{3}$$

$$= \frac{1}{3}\left[\frac{b}{2} + (x_l - x_r)\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

By the same process as that used in § 122 the correction equation may be written

$$\text{Corr. in cub. yds.} = \frac{1}{3R}\left[V'\left(\frac{b}{2} + (x_l' - x_r'\right) + V''\left(\frac{b}{2} + (x_l'' - x_r'')\right)\right]. \quad (60)$$

It should be noted that since ade triangle is not used in this computation the volume of grade prism in computing the quantities V'

The eccentricities of cr zero, and are frequently erally quite small. is generally a v in ordinary th

If the tri still be triangle

stakes by y_r and $-y_l$ (note the minus sign). Applying Eq. 59 literally (noting that $\frac{b}{2}$ must here be considered as negative in order to make the notation consistent) we obtain

$$e=\frac{1}{3}\left[-\frac{b}{2}+(-y_l-y_r)\right],$$

which reduces to

$$e=-\frac{1}{3}\left[\frac{b}{2}+y_l+y_r\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

As the algebraic signs tend to create confusion in these formulæ, it is more simple to remember that for a triangle lying on *both* sides of the center e is always numerically equal to $\frac{1}{3}\left[\frac{b}{2}+(x_l\sim x_r)\right]$, and for a triangle entirely on one side, e is numerically equal to $\frac{1}{3}\left[\frac{b}{2}+\text{the numerical } sum \text{ of the two distances out}\right]$. The algebraic sign of e is readily determinable as in § 122.

124. Example of curvature correction. Assume that the fill in § 105 occurred on a 6° curve to the *right*. $\frac{1}{3R}=\frac{1}{2865}$. The quantities 210, 507, etc., represent the quantities V', V'', etc., since they include in each case the 61 cubic yards due to the grade prism. Then

$$\frac{V(x_l\sim x_r)}{3R}=\frac{210(22.9-8.2)}{2865}=\frac{3101.7}{2865}=+1.$$

The sign is plus, since the center of gravity of the cross-section is on the left side of the center and the road curves to the right, thus making the true volume larger. For Sta. 18 the correction, computed similarly, is +3, and the correction for the whole section is 1+3=4. For Sta. 18+40 the correction is computed as 6 yards. Therefore, for the 40 feet, the correction is $\frac{40}{100}(3+6)=3.6$, which is called 4. Computing the others similarly we obtain a total correction of +16 cubic yards.

125. Accuracy of earthwork computations. The preceding methods give the *precise volume* (except where approximations are distinctly admitted) of the prismoids which are *supposed* to represent the volume of the earthwork. To appreciate the accuracy necessary in cross-sectioning to obtain a given accuracy in volume, consider that a fifteen-foot length of the cross-section, which is assumed to be straight, really sags 0.1 foot, so that the cross-section is in error by a triangle 15 feet wide and 0.1 foot high. This sag 0.1 foot high would hardly be detected by the eye, but in a length of 100 feet in each direction it would make an error of volume of 1.4 cubic yards in *each* of the two prismoids, assuming that the sections at the other ends were perfect. If the cross-sections at both ends of a prismoid were in error by this same amount, the volume of that prismoid would be in error by 2.8 cubic yards if the errors of area were both plus or both minus. If one were plus and one minus, the errors would neutralize each other, and it is the compensating character of these errors which permits any confidence in the results as obtained by the usual methods of cross-sectioning. It demonstrates the utter futility of attempting any closer accuracy than the nearest cubic yard. It will thus be seen that if an error really exists at *any* cross-section it involves the prismoids on *both* sides of the section, even though all the other cross-sections are perfect. As a further illustration, suppose that cross-sections were taken by the three-level method (§ 105), and that a cross-section, assumed as uniform from center to side, sags 0.4 foot in a width of 20 feet. Assume an equal error (of same sign) at the other end of a 100-foot section. The error of volume for that one prismoid is 38 cubic yards.

The computations further assume that the warped surface, passing through the end sections, coincides with the surface of the ground.. Suppose that the cross-sectioning had been done with mathematical perfection; and, to assume a simple case, suppose a sag of 0.5 foot between the sections, which causes an error equal to the volume of a pyramid having a base of 20 feet (in each cross-section) times 100 feet (between the cross-sections) and a height of 0.5 foot. The volume of this pyramid is $\frac{1}{3}(20\times100)\times0.5=333$ cub. ft. $=12$ cub. yds. And yet this sag or hump of 6 inches would generally be utterly unnoticed, or at least disregarded.

When the ground is very rough and broken it is sometimes

practically impossible, even with frequent cross-sections, to locate warped surfaces which will closely coincide with all the sudden irregularities of the ground. In such cases the computations are necessarily more or less approximate and dependence must be placed on the compensating character of the errors.

126. Approximate computations from profiles. When a "paper location" has been laid out on a topographical map having contours, it is possible to compute approximately the amount of earthwork required by some very simple and rapid calculations. A profile may be readily drawn by noting the intersections of the proposed center line with the various contours and plotting the surface line on profile paper. Drawing the grade-line on the profile, the depth of cut or fill may be scaled off at any point. When it is only desired to obtain

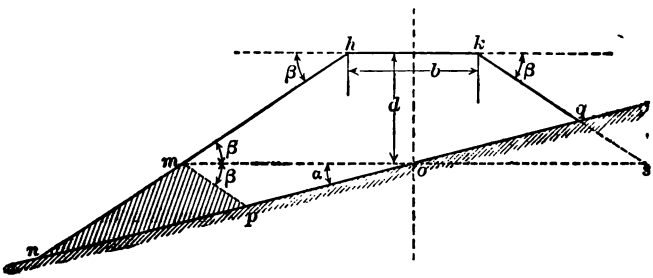


FIG. 60.

very quickly an approximate estimate of the amount of earthwork required on a suggested line, it may be done by the method described in § 103, or by the use of Table XVII. But the assumption that the surface of the ground at each cross-section is level invariably has the effect that the estimated volumes are not as large as those actually required. The difference between the "level section" *hkms* and the actual slope section *hknq* equals the difference between the triangles *mon* and *oqs*, and this difference equals the shaded area *mpn*. The excess volume is proportional to the area of the triangle *mpn*. This area may be expressed by the formula,

$$\text{Area } mpn = 2(\tfrac{1}{2}b + d\cot\beta)^2 \frac{\sin^2\alpha \sin\beta\cos\beta}{\cos 2\alpha - \cos 2\beta}.$$

The percentage of this excess area to the nominal area *hkms* therefore depends on the dimensions b and d and the angles α and β. A solution of this equation for ninety different combinations of various numerical values for these four variables is included in Table XVII for the purpose of making corrections. A study of this correction table points conclusively to the following laws, a thorough understanding of which will enable an engineer to appreciate the degree of accuracy which is attainable by this approximate method:

(a) Increasing the *width* of the roadbed (b), the other three factors remaining constant, *increases* the percentage of error, but the increase is comparatively small.

(b) Increasing the *depth* of cut or fill (d), *decreases* the percentage of error, but the decrease is almost insignificant.

(c) Increasing the angle of the side slopes (β) *decreases* the percentage of error, the decrease being very considerable.

(d) Increasing the angle of the slope of the ground (α), *increases* the percentage of error, the percentage rapidly increasing to infinity as the value of α approaches that of β. This is another method of stating the fact that α must always be less than β and, practically, must be considerably less, so that the slope stake shall be within a reasonable distance from the center.

Since the above value for the corrective area is a function of the angle α, which is usually variable and whose value is frequently known only approximately, it is useless to attempt to apply the correction with great precision, and the following rules will usually be found amply accurate, considering the probable lack of precision in the data used.

1. For embankments or cuts, having a slope of 1.5:1, and with a surface slope of 5° (nearly 9%) the excess of true area over nominal area is *about* 2%. There is only a slight variation from this value for all ordinary depths (d) and widths (b) of roadbed. Therefore the nominal volume would be about 2% too *small*. On the other hand, the effect of the prismoidal correction is such that, even with truly level sections, the nominal volume is too *large*. See §§ 103 and 112. The amount
[illegible]e prismoidal correction depends on the differences between
[illegible]ssive center depths. In the very ordinary numerical case
[illegible] in § 103, the correction was nearly 3%, which more
[illegible]eutralizes the error due to surface slope. Therefore in

many cases on slightly sloping ground the error due to the surface slope will so nearly neutralize the prismoidal correction that the quantities taken directly from the tables (without correction for either cause) will equal the true volume with as close an approach to accuracy as the precision of the surveying will permit.

2. For a cut with a slope of 1:1, and with a surface slope of 5° the error is about 1%. This will be neutralized by still smaller prismoidal corrections. Therefore, for surface slopes of 5° or less, no allowance should be made for this error unless the prismoidal correction is also considered.

3. When the surface slope is 10° (nearly 18%) the error for a 1.5:1 slope is from 7% to 10% and for a 1:1 slope from 3% to 5%.

4. For a 30° surface slope and 1.5:1 side slopes the excess volume is three or four times the nominal volume. Such a steep surface slope implies the probability of "side-hill work" to which the above corrective rules are not applicable. When the surface slopes are very steep careful work must be done to avoid excessive error. For a 1:1 side slope, the errors are from 50% to 80%.

A still closer approximation, especially for the steeper surface slopes, may be obtained by using, directly or by interpolation, figures from the corrective tabular form which forms part of Table XVII. Unless the surface slope angle is known accurately (especially when large) no great accuracy in the final result is possible. Close accuracy would also require the determination of the prismoidal correction. But if such close accuracy is deemed essential, it can be most easily obtained by accurate cross-sectioning at each station and the adoption of other methods of computation—such as are given in §§ 108 and 109.

When the contours have been drawn in for a sufficient distance on either side to include the position of both slope stakes at every station, as will usually be the case, cross-sections may be obtained by drawing lines on the map at each station perpendicular to the center line—see Fig. 4. The intersection of these lines with the contours will furnish the distances for drawing on cross-section paper the transverse profile at each station. Drawing on the same cross-section the lines representing the roadbed and the side slopes, the cross-section of

cut (or fill) is complete and its area may be obtained by scaling from the cross-section paper. If the contours have been located on the map with sufficient accuracy, such a method will determine the cross-sectional area very closely. When cross-sections have been taken with a wye- or hand-level, as described in § 12, the cross-sections as plotted will probably be more accurate than when the contours are run in from points determined by the stadia method. In fact this semi-graphical method is frequently used, in place of the purely numerical methods described in previous sections, to make final estimates of the volume of earthwork.

As a numerical example, an assumed location line was laid out on the contours given in Fig. 4. The volume of cut, as determined by Table XVII for a roadbed 20 feet wide, with side slopes of 1:1, was 5746 cubic yards. The surface slope varied from 3° to 11°. Computing the corrections by a careful interpolation from the corrective table, the total correction was found to be 128 cubic yards, or an average of a little over 2%. On the other hand the negative prismoidal correction amounts to 72 cubic yards, which leaves a net correction of 56 cubic yards—about 1%. It so happens that in this case a correction for curvature would tend further to wipe out this correction. These figures merely verify numerically the general conclusions stated above, although it should not be forgotten that in individual cases the figures taken from Table XVII require ample correction.

The following approximate rule, for which the author is indebted to Mr. W. H. Edinger, is exceedingly useful when it is desired to rapidly determine the approximate volume of earthwork between two points along the road. Its great merit lies in the fact that it only means the memorizing of a comparatively simple rule which will make it possible to make such computations in the field, without the use of tables. The rule is based on the fact that the area of any level section equals $bd+sd^2$; and therefore,

$$\Sigma(\text{vol.}) = (b\Sigma d + s\Sigma d^2)\frac{L}{27},$$

in which L is usually 100 feet. For strict accuracy this would only be the volume provided the total length was an even number of hundred feet, and the various values of d represented

the depths which were uniform for hundred foot sections. It makes no allowance for the comparatively large prismoidal error of the pyramidal and wedge-shaped sections usually found at each end of a cut or fill, but where an approximate estimate is desired, in which this inaccuracy may be neglected, the method is very useful. The method of applying this rule without tables may best be illustrated by a simple numerical example. Assume that the levels on a stretch of fairly level ground, which is about 500 feet long, have been taken, the depths being taken at points 100 feet apart, the first and last points being about 40 or 50 feet from the ends of the cut, or fill. The depths are as given in the first column in the tabular form below; the slope is 1.5:1, and the breadth (b) is 14 feet.

d	d^2
1.6	2.56
2.8	7.84
4.5	20.25
3.1	9.61
0.9	.81
$\Sigma d = 12.9$	$\Sigma d^2 = 41.07$
14	adding one half $= 20.53$
$b\Sigma d = 180.6$	$s\Sigma d^2 = 61.60$
61.60	
242.2	

$24220 \div 27 = 897$ cubic yards.

The 180.6 is the $b\Sigma d$ and the 61.6 is $s\Sigma d^2$; adding these and moving the decimal point two places to multiply by 100, we only have to divide by 27 to obtain the value in cubic yards. Although the above rule requires more work than the employment of earthwork tables, yet it is a very convenient method of estimating the approximate volume of a short section of earthwork when no tables are at hand.

FORMATION OF EMBANKMENTS.

127. Shrinkage of earthwork. The statistical data indicating the amount of shrinkage is very conflicting, a fact which is probably due to the following causes:

1. The various kinds of earthy material act very differently as respects shrinkage. There is a great lack of uniformity in

the *classification of earths* in the tests and experiments which have been made.

2. Very much depends on the *method* of forming an embankment (as will be shown later). Different reports have been based on different methods—often without mention of the method.

3. An embankment requires considerable *time* to shrink to its final volume, and therefore much depends on the time elapsed between construction and the measurement of what is supposed to be the settled volume.

4. A soft subsoil will frequently settle under the weight of a high embankment and apparently indicate a far greater shrinkage than the actual reduction in volume.

5. An embankment of very soft material will sometimes "mush" or widen at the sides, with a consequent settling of the top, due to this cause alone, but such settlement would indicate that unsuitable material had been used to form the embankment.

As a summary of the extensive discussion and wide range of shrinkage factors which might be quoted, the following facts may be stated:

1. The *density* of natural soil increases with its depth below the surface. Some careful and accurate tests of some "clay, loam and gumbo," made on the C. B. & Q. R. R., showed an increase from 70 lbs. per cubic foot at the surface to 121 lbs. per cubic foot (an increase of over 70%) at a depth of 25 feet.

2. Freshly excavated material of whatever character occupies a greater volume in a cart or other conveyor, or when loosely deposited, than it did in the original excavation.

3. After being deposited it usually shrinks more or less from its volume as loose material. This shrinkage increases with age and with the amount of traffic over it. When the material is deposited in small increments from wagons or carts and each layer is subjected to compression from horses' hoofs and from wheels, the contraction during construction is very great and the subsequent shrinkage is comparatively small.

4. Light vegetable mould or "top soil," and, in general, all naturally deposited soil to a depth of 3 to 5 feet, will shrink until its final volume is less than its volume in its original state.

5. On the other hand, compact earth, taken from the bottom of a deep excavation, and also rock, although it may shrink

somewhat from its volume as measured in carts, cars, or other conveyors, will never shrink to its volume in the original excavation, and will always occupy a larger volume in an embankment.

6. An embankment continues to shrink with age, due to the pressure of superincumbent material and also due to the pressure and vibration caused by traffic. This law was clearly indicated by the following figures from the C. B. & Q. R. R. tests, where three embankments were:

(*a*)	17 years old;	no traffic.....	shrinkage,	6.7%
(*b*)	49 "	" abandoned after 32 years of light traffic.	"	12.9%
(*c*)	17 "	" constantly under heavy traffic.....	"	13.6%

7. If an embankment is formed by dropping earth from a trestle, there is no compression during formation and the shrinkage will be long-drawn-out, especially if the material is light and the track continues for some time to be supported by the floor system of the trestle.

8. The mere weight of an embankment, augmented by the vibrating action of heavy traffic, will compress the natural soil on which an embankment is placed. The depth of this compression will vary from zero for a rocky surface to an indefinite and unceasing settlement into a " bottomless bog." This effect, distinct from the shrinkage of the volume of embankment material, is called **subsidence.** It always occurs to some extent on ordinary grazing or agricultural land, which means under the majority of embankments. The *percentage* of subsidence will be greater for a low than for a high embankment, since the area of the base is less and the tamping action of traffic is more direct and effective. Investigation, by borings and the digging of test pits, has shown that there is sometimes as much (or more) deposited material below the original surface line as that in the visible embankment above. This means that when an embankment is to be formed on soft, or even ordinary agricultural ground, considerably more material must be deposited than is called for by the nominal cross-sections above the original surface lines. The extent of such subsidence cannot be accurately predicted. It is even more difficult than to predict the ultimate shrinkage of a volume of excavated material after being formed into an embankment. When subsidence is altogether ignored, the almost inevitable result is a future sag of the grade line on embankments, which can only be restored by comparatively expensive raising of the track under traffic

conditions. Instances are not uncommon where a company has been compelled to change a location after having deposited on a seemingly bottomless bog a volume of material several times the volume of the desired visible embankment. Of course such cases are exceptional, but the engineer must use judgment, aided perhaps by boring into a soft soil, to estimate how much the subsidence will prove to be.

9. A sharp and clear distinction should be made between the *coefficient of extra height* of an embankment and the *coefficient of shrinkage* which determines how many cubic yards of settled embankment may be made from a definite volume of earth or rock measured in the excavation. Even if the coefficient of volume shrinkage were accurately known, the effect of subsidence must still be allowed for, and the coefficient of extra height must be a composite of these two effects. The effect of the method of formation of the embankment must also be considered. If the material is compacted during construction, some of the shrinkage will have been accomplished and some of the subsidence will have taken place by the time the embankment is up to grade line and only the *future* shrinkage and subsidence must be allowed for, although more material has been used than the profile seemed to call for. A rock embankment will not shrink appreciably after formation and in such case only the future subsidence need be allowed for.

10. The very serious expense of raising the grade of a track under traffic may be reduced if not altogether avoided by modifying the normal grade line over an embankment, substantially as shown in Fig. 62. Whatever may be the coefficients of shrinkage and subsidence, the lowering of the grade line by these combined effects will be greater for a high than for a low embankment, and any allowance must be in principle as shown in Fig. 62. From 8% to 15% is sometimes quoted as the required extra height of embankments, although it is strenuously claimed by many that 3% or 2% is sufficient, or even that *no* allowance should be made.

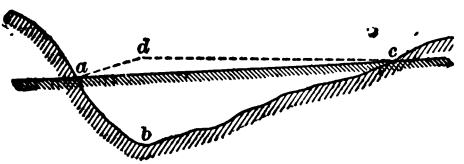


FIG. 62.

128. Proper allowance for shrinkage or subsidence. It follows from the above considerations that no simple and set rules may be prescribed, either for the coefficient of shrinkage (or expansion) or for the coefficient of extra height, since the coefficients will depend on the kind of material, its depth in the cutting, the method of formation of the embankment, the time during which complete settlement is assumed to take place, and even on the intensity of the traffic which will run over the embankment. And also, since an embankment will be formed from materials taken from various depths of excavation, and also from various cuttings containing perhaps several kinds of material, it follows that the real coefficient will be a composite figure whose exact value will be impossible to determine, even if some of the elements could be determined with substantial accuracy. Therefore the allowance to be made when forming any embankment must be estimated according to judgment, after allowing for all of the factors involved. The following figures have the weight of considerable authority and may be used as a guide in making up a composite figure which will best suit any particular case.

Gravel or sand	will shrink about		8%
Clay	"	"	10%
Loam	"	"	12%
Loose vegetable surface soil	"	"	15%
Rock, large pieces	" expand	"	40%
" small pieces	"	"	60%

To utilize such figures we might say, for example, that, if some material will shrink 8%, 1000 cubic yards of it, measured in place, will make $1000-80=920$ cubic yards of *settled* embankment. If the material is a mixture of earth and rock, for which a composite figure of 20% expansion is estimated to be correct, 1000 cubic yards of such excavation will make 1200 cubic yards of settled embankment. Even this calculation ignores subsidence, which must be estimated separately.

129. Methods of forming embankments. Embankments of moderate height are sometimes formed by scraping material with drag scrapers from ditches at the sides, especially if there is little or no cutting to be done in the immediate vicinity. Over a low level swampy stretch this method has the double advantage of building an embankment which is well above the general level and also provides generous drainage ditches which keep the embankment dry. Wheeled scrapers may be used economically up to a distance of 400 feet to excavate

cuts and deposit the material on low embankments. Such methods have the advantage of compacting the embankments during construction and reducing future shrinkage.

When carts are used, an embankment of any height may be formed by "dumping over the end" and building to the full height (or even higher to allow for shrinkage) as the embankment proceeds. The method is especially applicable when the material comes from a place as high as or higher than the grade-line, so that no up-hill hauling is necessary. Only a small contractor's plant is required for all of these methods.

Trestles capable of carrying carts, or even cars and locomotives, from which excavated material may be dropped, are found to be economical in spite of the fact that their cost is a construction expense. There is the disadvantage that such embankments require a long time to settle, but there are the advantages that the earth may be hauled by the train load from a distance of perhaps several miles, dumped from the

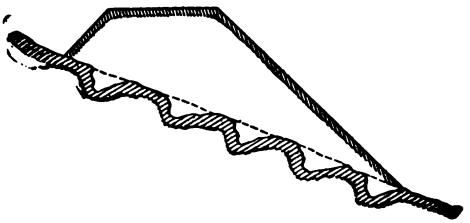


FIG. 63.

cars by train ploughs, or automatically dumped when the material is carried in patent dumping-cars, and all at a comparatively small cost per cubic yard. The disadvantages of slow settlement may be obviated, although at some additional cost, by making the trestle sufficiently strong to support regular traffic until the settlement is complete.

During recent years cableways have been utilized to fill comparatively narrow but deep ravines from material obtainable on either side of the ravine. This method obviates the construction of an excessively high trestle which might otherwise be considered necessary.

When an embankment is to be placed on a steep side hill which has a slippery clay surface, the embankment will some-

times slide down the hill, unless means are taken to prevent it. Some sort of bond between the old surface and the new material becomes necessary. This has sometimes been provided by cutting out steps somewhat as is illustrated in Fig. 63. It is possible that a deep ploughing of the surface would accomplish the result just as effectively and much cheaper. The tendency to slip is generally due not only to the nature of the soil but also to the usual accompanying characteristic that the soil is wet and springy. The sub-surface drainage of such a place with tile drains will still further prevent such slipping, which often proves very troublesome and costly.

COMPUTATION OF HAUL.

130. Nature of subject. As will be shown later when analyzing the cost of earthwork, the most variable item of cost is that depending on the distance hauled. As it is manifestly impracticable to calculate the exact distance to which every individual cartload of earth has been moved, it becomes necessary to devise a means which will give at least an equivalent of the haulage of all the earth moved. Evidently the *average* haul for any mass of earth moved is equal to the distance from the center of gravity of the excavation to the center of gravity of the embankment formed by the excavated material. As a rough approximation the center of gravity of a cut (or fill) may sometimes be considered to coincide with the center of gravity of that part of the profile representing it, but the error is frequently very large. The center of gravity may be determined by various methods, but the method of the "mass diagram" accomplishes the same ultimate purpose (the determination of the haul) with all-sufficient accuracy and also furnishes other valuable information.

131. Mass diagram. In Fig. 64 let $A'B' \ldots G'$ represent a profile and grade line drawn to the usual scales. Assume A' to be a point past which no earthwork will be hauled. Such a point is determined by natural conditions, as, for example, a river crossing, or one end of a long level stretch along which no grading is to be done except the formation of a low embankment from the material excavated from ample drainage ditches on each side. Above the profile draw an indefinite horizontal line ($AC'n$ in Fig. 64) which may be called the "zero line." Above every station point in the profile draw an ordinate (above or be-

low the zero line) which will represent the algebraic sum of the cubic yards of cut and fill (calling cut + and fill −) from the point A' to the point considered. The computations of these ordinates should first be made in tabular form as shown below. In doing this shrinkage must be allowed for by considering how much embankment would actually be made by so many cubic yards of excavation of such material. For example, we will assume that 1000 cubic yards of sand or gravel, measured in place (see § 128) will make about 920 cubic yards of embankment; therefore all cuttings in sand or gravel should be discounted in about this proportion. Excavations in rock should be increased in the proper ratio. In short, all excavations should be valued according to the amount of *settled* embankment that could be made from them. Place in the first column a list of the stations; in the second column, the number of cubic yards of cut or fill between each station and the preceding station; in the third and fourth columns, the kind of material and the proper shrinkage factor; in the fifth column, a repetition of the quantities in cubic yards, except that the excavations are diminished (or increased, in the case of rock) to the number of cubic yards of settled embankment which may be made from them. In the sixth column place the *algebraic sum* of the quantities in the fifth column (calling cuts + and fills −) from the starting-point to the station considered. These algebraic sums at each station will be the ordinates, drawn to some scale, of the mass curve. The scale to be used will depend somewhat on whether

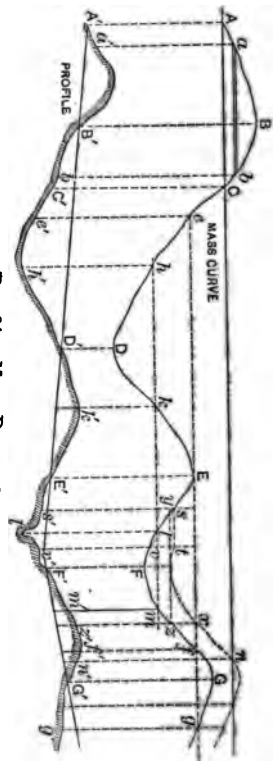


Fig. 64.—Mass Diagram.

the work is heavy or light, but for ordinary cases a scale of 5000 cubic yards per inch may be used. Drawing these ordinates to scale, a curve A, B, ... G may be obtained by joining the extremities of the ordinates.

Sta.	Yards { cut + / fill −	Material.	Shrinkage factor.	Yards, reduced for shrinkage.	Ordinate in mass curve.
46 + 70	.....	.....	.....	.....	0
47	+ 195	Clayey soil	− 10 per cent	+ 175	+ 175
48	+ 1792	" "	− 10 "	+ 1613	+ 1788
+ 60	+ 614	" "	− 10 "	+ 553	+ 2341
49	− 143	.....	.....	− 143	+ 2198
50	− 906	.....	.....	− 906	+ 1292
51	− 1985	.....	.....	− 1985	− 693
52	− 1721	.....	.....	− 1721	− 2414
+ 30	− 112	.....	.....	− 112	− 2526
53	+ 177	Hard rock	+ 60 per cent	+ 283	− 2243
+ 70	+ 180	" "	+ 60 "	+ 289	− 1954
54	− 52	.....	.....	− 52	− 2006
+ 42	− 71	.....	.....	− 71	− 2077
55	+ 276	Clayey soil	− 10 per cent	+ 249	− 1828
56	+ 1242	" "	− 10 "	+ 1118	− 710
57	+ 1302	" "	− 10 "	+ 1172	+ 462

132. Properties of the mass curve.

1. The curve will be rising while over cuts and falling while over fills.

2. A tangent to the curve will be horizontal (as at B, D, E, F, and G) when passing from cut to fill or from fill to cut.

3 When the curve is *below* the "zero line" it shows that material must be drawn *backward* (to the left); and *vice versa*, when the curve is *above* the zero line it shows that material must be drawn *forward* (to the right).

4. When the curve crosses the zero line (as at A and C) it shows (in this instance) that the cut between A' and B' will just provide the material required for the fill between B' and C', and that no material should be hauled past C', or, in general, past any intersection of the mass curve and the zero line.

5. If any horizontal line be drawn (as ab), it indicates that the cut and fill between a' and b' will just balance.

6. When the center of gravity of a given volume of material is to be moved a given distance, it makes no difference (at least theoretically) how far each individual load may be hauled or how any individual load may be disposed of. The summation

of the products of each load times the distance hauled will be a constant, whatever the method, and will equal the total volume times the movement of the center of gravity. The *average haul*, which is the movement of the center of gravity, will therefore equal the summation of these products divided by the total volume. If we draw two horizontal parallel lines at an infinitesimal distance dx apart, as at ab, the small increment of cut dx at a' will fill the corresponding increment of fill at b', and this material must be hauled the distance ab. Therefore the product of ab and dx, which is the product of distance times volume, is represented by the area of the infinitesimal rectangle at ab, and the total area ABC represents the summation of volume times distance for all the earth movement between A' and C'. This summation of products divided by the total volume gives the average haul.

7. The horizontal line, tangent at E and cutting the curve at e, f, and g, shows that the cut and fill between e' and E' will just balance, and that a *possible* method of hauling (whether desirable or not) would be to "borrow" earth for the fill between C' and e', use the material between D' and E' for the fill between e' and D', and similarly balance cut and fill between E' and f' and also between f' and g'.

8. Similarly the horizontal line $hklm$ may be drawn cutting the curve, which will show another *possible* method of hauling. According to this plan, the fill between C' and h' would be made by borrowing; the cut and fill between h' and k' would balance; also that between k' and l' and between l' and m'. Since the area $ehDkE$ represents the measure of haul for the earth between e' and E', and the other areas measure the corresponding hauls similarly, it is evident that the sum of the areas $ehDkE$ and $ElFmf$, which is the measure of haul of all the material between e' and f', is largely in excess of the sum of the areas hDk, kEl, and lFm, plus the somewhat uncertain measures of haul due to borrowing material for $c'h'$ and wasting the material between m' and f'. Therefore to make the measure of haul a minimum a line should be drawn which will make the sum of the areas between it and the mass curve a minimum. Of course this is not necessarily the cheapest plan, as it implies more or less borrowing and wasting of material, which *may* cost more than the amount saved in haul. The comparison of the two methods is quite simple, however. Since the amount

of fill between e' and h' is represented by the *difference* of the ordinates at e and h, and similarly for m' and f', it follows that the amount to be borrowed between e' and h' will exactly equal the amount wasted between m' and f'. By the first of the above methods the haul is excessive, but is definitely known from the mass diagram, and all of the material is utilized; by the second method the haul is reduced to about one-half, but there is a known quantity in cubic yards wasted at one place and the same quantity borrowed at another. The length of haul necessary for the borrowed material would need to be ascertained; also the haul necessary to waste the other material at a place where it would be unobjectionable. Frequently this is best done by widening an embankment beyond its necessary width. The computation of the relative cost of the above methods will be discussed later (§ 148).

9. Suppose that it were deemed best, after drawing the mass curve, to introduce a trestle between s' and v', thus saving an amount in fill equal to tv. If such had been the original design, the mass curve would have been a straight horizontal line between s and t and would continue as a curve which would be at all points a distance tv above the curve $vFmzfGg$. If the line Ef is to be used as a zero line, its intersection with the new curve at x will show that the material between E' and z' will just balance if the trestle is used, and that the amount of haul will be measured by the area between the line Ex and the broken line $Estx$. The same computed result may be obtained without drawing the auxiliary curve $txn \ldots$ by drawing the horizontal line zy at a distance $xz(=tv)$ below Ex. The amount of the haul can then be obtained by adding the triangular area between Es and the horizontal line Ex, the rectangle between st and Ex, and the irregular area between vFz and $y \ldots z$ (which last is evidently equal to the area between tx and $E \ldots x$). The disposal of the material at the right of z' would then be governed by the indications of the profile and mass diagram which would be found at the right of y'. In fact it is difficult to decide with the best of judgment as to the proper disposal of material without having a mass diagram extending to a considerable distance each side of that part of the road under immediate consideration.

133. **Area of the mass curve.** The area may be computed most readily by means of a planimeter, which is capable with reasonable care of measuring such areas with as great accuracy

as is necessary for this work. If no such instrument is obtainable, the area may be obtained by an application of "Simpson's rule." The ordinates will usually be spaced 100 feet apart. Select an *even* number of such spaces, leaving, if necessary, one or more triangles or trapezoids at the ends for separate and independent computation. Let $y_0 \ldots y_n$ be the ordinates, i.e., the number of cubic yards at each station of the mass curve, or the figures of "column six" referred to in § 131. Let the uniform distance between ordinates (=100 feet) be called 1, i.e., one *station*. Then the units of the resulting area will be cubic yards hauled one station. Then the

$$\text{Area} = \tfrac{1}{3}[y_0 + 4(y_1 + y_3 + \ldots y_{(n-1)}) + 2(y_2 + y_4 + \ldots y_{(n-2)}) + y_n]. \quad (62)$$

When an ordinate occurs at a substation, the best plan is to ignore it at first and calculate the area as above. Then, if the difference involved is too great to be neglected, calculate the area of the triangle having the extremity of the ordinate at the substation as an apex, and the extremities of the ordinates at the adjacent stations as the ends of the base. This may be done by finding the ordinate at the substation that would be a proportional between the ordinates at the adjacent full stations. Subtract this from the real ordinate (or *vice versa*) and multiply the difference by $\frac{1}{2} \times 1$. An inspection will often show that the correction thus obtained would be too small to be worthy of consideration. If there is more than one substation between two full stations, the corrective area will consist of two triangles and one or more trapezoids which may be similarly computed, if necessary.

When the zero line (Fig. 64) is shifted to eE, the drop from AC (produced) to E is known in the same units, cubic yards. This constant may be subtracted from the numbers ("column 6," § 131) representing the ordinates, and will thus give, without any scaling from the diagram, the exact value of the modified ordinates.

134. Value of the mass diagram. The great value of the mass diagram lies in the readiness with which different plans for the disposal of material may be examined and compared. When the mass curve is once drawn, it will generally require only a shifting of the horizontal line to show the disposal of the material by any proposed method. The mass diagram also shows the

extreme length of haul that will be required by any proposed method of disposal of material. This brings into consideration the "limit of profitable haul," which will be fully discussed in § 148. For the present it may be said that with each method of carrying material there is some limit beyond which the expense of hauling will exceed the loss resulting from borrowing and wasting. With wheelbarrows and scrapers the limit of profitable haul is comparatively short, with carts and tram-cars it is much longer, while with locomotives and cars it may be several miles. If, in Fig. 64, *eE* or *Ef* exceeds the limit of profitable haul, it shows at once that some such line as *hklm* should be drawn and the material disposed of accordingly.

135. **Changing the grade line.** The formation of the mass curve and the resulting plans as to the disposal of material are based on the mutual relations of the grade line and the surface profile and the amounts of cut and fill which are thereby implied. If the grade line is altered, every cross-section is altered, the amount of cut and fill is altered, and the mass curve is also changed. At the farther limit of the actual change of the grade line the revised mass curve will have (in general) a different ordinate from the previous ordinate at that point. From that point on, the revised mass curve will be parallel to its former position, and the revised curve may be treated similarly to the case previously mentioned in which a trestle was introduced. Since it involves tedious calculations to determine accurately how much the volume of earthwork is altered by a change in grade line, especially through irregular country, the effect on the mass curve of a change in the grade line cannot therefore be readily determined except in an approximate way. Raising the grade line will evidently increase the fills and diminish the cuts, and *vice versa*. Therefore if the mass curve indicated, for example, either an excessively long haul or the necessity for borrowing material (implying a fill) and wasting material farther on (implying a cut), it would be *possible* to diminish the fill (and hence the amount of material to be borrowed) by lowering the grade line near that place, and diminish the cut (and hence the amount of material to be wasted) by raising the grade line at or near the place farther on. Whether the advantage thus gained would compensate for the possibly injurious effect of these changes on the grade line would require patient investigation. But the method outlined shows how the mass

curve might be used to indicate a possible change in grade line which might be demonstrated to be profitable.

136. Limit of free haul. It is sometimes specified in contracts for earthwork that *all* material shall be entitled to free haul up to some specified limit, say 500 or 1000 feet, and that all material drawn farther than that shall be entitled to an allowance on the *excess* of distance. It is manifestly impracticable to measure the excess for each load, as much so as to measure the actual haul of each load. The mass diagram also solves this problem very readily. Let Fig. 65 represent a pro-

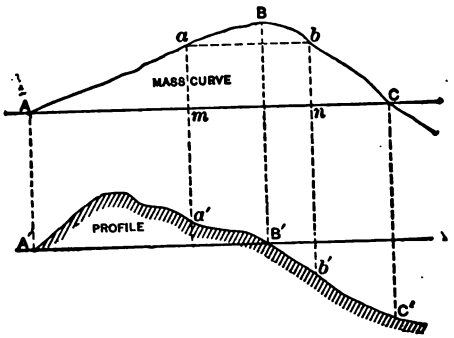


FIG. 65.

file and mass diagram of about 2000 feet of road, and suppose that 800 feet is taken as the limit of free haul. Find two points, a and b, in the mass curve *which are on the same horizontal line* and which are 800 feet apart. Project these points down to a' and b'. Then the cut and fill between a' and b' will just balance, and the cut between A' and a' will be needed for the fill between b' and C'. In the mass curve, the area between the horizontal line ab and the curve aBb represents the haulage of the material between a' and b', which is all free. The rectangle $abmn$ represents the haulage of the material in the cut $A'a'$ across the 800 feet from a' to b'. This is also free. The sum of the two areas Aam and bnC represents the haulage entitled to an allowance, since it is the summation of the products of cubic yards times the *excess* of distance hauled.

If the amount of cut and fill was symmetrical about the point

B', the mass curve would be a symmetrical curve about the vertical line through B, and the two limiting lines of free haul would be placed symmetrically about B and B'. In general there is no such symmetry, and frequently the difference is considerable The area $aBbnm$ will be materially changed according as the two vertical lines am and bn, always 800 feet apart, are shifted to the right or left. It is easy to show that the area $aBbnm$ is a *maximum* when ab is horizontal. The minimum value would be obtained either when m reached A or n reached C, depending on the exact form of the curve. Since the position for the minimum value is manifestly unfair, the best definite value obtainable is the maximum, which must be obtained as above described. Since $aBbnm$ is made maximum, the remainder of the area, which is the allowance for overhaul, becomes a minimum. The areas Aam and bCn may be obtained as in § 120. If the whole area $AaBbCA$ has been previously computed, it may be more convenient to compute the area $aBbnm$ and subtract it from the total area.

Since the intersections of the mass curve and the "zero line" mark limits past which no material is drawn, it follows that there will be no allowance for overhaul except where the distance between consecutive intersections of the zero line and mass curve exceeds the limit of free haul.

Frequently all allowances for overhaul are disregarded; the profiles, estimates of quantities, and the required disposal of material are shown to bidding contractors, and they must then make their own allowances and bid accordingly. This method has the advantage of avoiding possible disputes as to the amount of the overhaul allowance, and is popular with railroad companies on this account. On the other hand the facility with which different plans for the disposal of material may be studied and compared by the mass-curve method facilitates the adoption of the most economical plan, and the elimination of uncertainty will frequently lead to a safe reduction of the bid, and so the method is valuable to both the railroad company and the contractor.

ELEMENTS OF THE COST OF EARTHWORK.

137. Analysis of the total cost into items. The variation in the total cost of excavating earthwork, hauling it a greater or less distance, and forming with it an embankment of definite

form or wasting it on a spoil bank, is so great that the only possible method of estimating the cost under certain assumed conditions is to separate the total cost into elementary items. Ellwood Morris was perhaps the first to develop such a method —see *Journal of the Franklin Institute*, September and October, 1841. Trautwine used the same general method with some modifications. The following analysis will follow the same general plan, will quote some of the figures given by Morris and by Trautwine, but will also include facts and figures better adapted to modern conditions. Since every item of cost (except interest on cost of plant and its depreciation) is a direct function of the current price of common labor, all calculations will be based on the simple unit of $1 per day. Then the actual cost may be obtained by multiplying the calculated cost under the given conditions by the current price of day labor. When possible, figures will be quoted giving the cost of all items of work on a loose sandy soil which is the easiest to work and also for the cost of the heaviest soils, such as stiff clay and hard pan. These represent the extremes, excluding rock, which will be treated separately. The cost of intermediate grades may be interpolated between the extreme values according to the judgment of the engineer as to the character of the soil.

The possible division into items varies greatly according to the method adopted, but the differentiation into items given below (which is strictly applicable to the old fashioned simpler methods of work) can usually be applied to any other method by merely combining or eliminating some of the items. The items are

1. Loosening the natural soil.
2. Loading the soil into whatever carrier may be used.
3. Hauling excavated material from excavation to embankment or spoil bank.
4. Spreading or distributing the soil on the embankment.
5. Keeping roadways or tracks in good running order.
6. Trimming cuts to their proper cross-section (sometimes called "sandpapering").
7. Repairs, wear, depreciation, and interest on cost of plant.
8. Superintendence and incidentals.

138. Item 1. Loosening. (a) Ploughs. Very light sandy soils can frequently be shovelled without any previous loosening, but it is generally economical, even with very light material,

to use a plough. Morris quotes, as the results of experiments, that a three-horse plough would loosen from 250 to 800 cubic yards of earth per day, which at a valuation of $5 per day would make the cost per yard vary from 2 cents to 0.6 cent. Trautwine estimates the cost on the basis of two men handling a two-horse plough at a total cost of $3.87 per day, being $1 each for the men, 75 c. for each horse, and an allowance of 37 c. for the plough, harness, etc. From 200 to 600 cubic yards is estimated as a fair day's work, which makes a cost of 1.9 c. to 0.65 c. per yard, which is substantially the same estimate as above. Extremely heavy soils have sometimes been loosened by means of special ploughs operated by traction-engines.

Gillette estimates that "a two-horse team with a driver and a man holding the plough will loosen 25 cubic yards of fairly tough clay, or 35 cubic yards of gravel and loam per hour." For ten hours per day this would be 250 to 350 cubic yards per day. These values are neither as high nor as low as the extremes above noted. It is probably very seldom that a soil will be so light that a two-horse (or three-horse) plough can loosen as much as 600 (or 800) cubic yards per day.

It is sometimes necessary to plough up a macadamized street. This may be done by using as a plough a pointed steel bar which is fastened to a very strong plough frame. A preliminary hole must be made which will start the bar under the macadam shell. Then, as the plough is drawn ahead, the shell is ripped up. Four or six horses, or even a traction-engine, are used for such work. Gillette quotes two such cases where the cost of such loosening was 2 c. and 6 c. per cubic yard, with common labor at 15 c. per hour. Two-thirds of such figures will reduce them to the $1 per day basis. The cost for ploughing *on the* $1 *per* 10-*hour-day basis* may therefore be summarized as follows:

For very loose sandy soils.....	0.6 c.	per cubic yard
" " heavy clay "	2.0 c.	" " "
" hard pan and macadam, up to ...	4.0 c.	" " "

(b) **Picks.** When picks are used for loosening the earth, as is frequently necessary and as is often done when ploughing would perhaps be really cheaper, an estimate * for a fair day's

* Trautwine.

work is from 14 to 60 cubic yards, the 14 yards being the estimate for stiff clay or cemented gravel, and the 60 yards the estimate for the lightest soil that would require loosening. At $1 per day this means about 7 c. to 1.7 c. per cubic yard, which is about three times the cost of ploughing. Five feet of the face is estimated * as the least width along the face of a bank that should be allowed to enable each laborer to work with freedom and hence economically.

(c) **Blasting.** Although some of the softer shaly rocks may be loosened with a pick for about 15 to 20 c. per yard, yet rock in general, frozen earth, and sometimes even compact clay are most economically loosened by blasting. The subject of blasting will be taken up later, §§ 149–155.

(d) **Steam-shovels.** The items of loosening and loading merge together with this method, which will therefore be treated in the next section.

139. **Item 2. Loading.** (a) **Hand-shovelling.** Much depends on proper management, so that the shovellers need not wait unduly either for material or carts. With the best of management considerable time is thus lost, and yet the intervals of rest need not be considered as entirely lost, as it enables the men to work, while actually loading, at a rate which it would be physically impossible for them to maintain for ten hours. Seven shovellers are sometimes allowed for each cart; otherwise there should be five, two on each side and one in the rear. Economy requires that the number of loads per cart per day should be made as large as possible, and it is therefore wise to employ as many shovellers as can work without mutual interference and without wasting time in waiting for material or carts. The figures obtainable for the cost of this item are unsatisfactory on account of their large disagreements. The following are quoted as the number of cubic yards that can be loaded into a cart by an average laborer in a working day of ten hours, the lower estimate referring to heavy soils, and the higher to light sandy soils: 10 to 14 cubic yards (Morris), 12 to 17 cubic yards (Haskoll), 18 to 22 cubic yards (Hurst), 17 to 24 cubic yards (Trautwine), 16 to 48 cubic yards (Ancelin). As these estimates are generally claimed to be based on actual experience, the discrepancies are probably due to differences of management. If the

* Hurst.

average of 15 to 25 cubic yards be accepted, it means, on the basis of $1 per day, 6.7 c. to 4 c. per cubic yard. These estimates apply only to earth. *Rockwork* costs more, not only because it is harder to handle, but because a cubic yard of solid rock, measured in place, occupies about 1.8 cubic yards when broken up, while a cubic yard of earth will occupy about 1.2 cubic yards. Rockwork will therefore require about 50% more loads to haul a given volume, *measured in place*, than will the same nominal volume of earthwork. The above authorities give estimates for loading rock varying from 6.9 c. to 10 c. per cubic yard. The above estimates apply only to the loading of carts or cars with shovels or by hand (loading masses of rock). The cost of loading wheelbarrows and the cost of scraper work will be treated under the item of hauling.

(b) Steam-shovels.* Whenever the magnitude of the work will warrant it there is great economy in the use of steam-shovels. These have a "bucket" or "dipper" on the end of a long beam, the bucket having a capacity varying from ½ to 2½ cubic yards. Steam-shovels handle all kinds of material from the softest earth to shale rock, earthy material containing large boulders, tree-stumps, etc. The record of work done varies from 200 to 1000 cubic yards in 10 hours. They perform all the work of loosening and loading. Their economical working requires that the material shall be hauled away as fast as it can be loaded, which usually means that cars on a track, hauled by horses or mules, or still better by a locomotive, shall be used. The expenses for a steam-shovel, costing about $5000, will average about $1600 per month. Of this the engineer may get $100; the fireman $50; the cranesman $90; repairs perhaps $250 to $300; coal, from 15 to 25 tons, cost very variable on account of expensive hauling; water, a very uncertain amount, sometimes costing $100 per month; about five laborers and a foreman, the laborers getting $1.25 per day and the foreman $2.50 per day, which will amount to $227.50 per month. This gang of laborers is employed in shifting the shovel when necessary, taking up and relaying

* For a thorough treatment of the capabilities, cost, and management of steam-shovels the reader is referred to "Steam-shovels and Steam-shovel Work," by E. A. Hermann. D. Van Nostrand Co., New York.
This book is now out of print. "Earthwork and its Cost," by H. P. Gillette, to which the student is referred for a more elaborate exposition of the subject, has used many of Hermann's cuts.

tracks for the cars, shifting loaded and unloaded cars, etc. In shovelling through a deep cut, the shovel is operated so as to undermine the upper parts of the cut, which then fall down within reach of the shovel, thus increasing the amount of material handled for each new position of the shovel. If the material is too tough to fall down by its own weight, it is sometimes found economical to employ a gang of men to loosen it or even blast it rather than shift the shovel so frequently. Non-condensing engines of 50 horse-power use so much water that the cost of water-supply becomes a serious matter if water is not readily obtainable. The lack of water facilities will often justify the construction of a pipe line from some distant source and the installation of a steam-pump. Hence the seemingly large estimate of $100 per month for water-supply, although under favorable circumstances the cost may almost vanish. The larger steam-shovels will consume nearly a ton of coal per day of 10 hours. The expense of hauling this coal from the nearest railroad or canal to the location of the cut is often a very serious item of expense and may easily double the cost per ton. Some steam-shovels have been constructed to be operated by electricity obtained from a plant perhaps several miles away. Such a method is especially advantageous when fuel and water are difficult to obtain.

The following general requirements and specifications were recommended in 1907 by the American Railway Engineering Association:

Three important cardinal points should be given careful attention in the selection of a steam-shovel. These are in their order

(1) Care in the selection, inspection and acceptance of all material that enters into every part of the machine.

(2) Design for strength.

(3) Design for production.

GENERAL SPECIFICATIONS.

Weight of shovel: Seventy (70) tons.

Capacity of dipper: Two and one-half (2½) yards.

Steam pressure: One hundred and twenty (120) pounds.

lear height above rail of shovel track at which dipper should
ld: Sixteen (16) feet.

Depth below rail of shovel track at which dipper should dig Four (4) feet.

Number of movements of dipper per minute from time of entering bank to entering bank: Three (3).

Character of hoist: Cable.

Character of swing: Cable.

Character of housing: Permanent for all employes.

Capacity of tank: Two thousand (2000) gallons.

Capacity of coal-bunker: Four (4) tons.

Spread of jack arm: Eighteen (18) feet. A special short arm should be provided.

Form of steam-shovel track: "T" rails on ties.

Length of rails for ordinary work: Six (6) feet.

Form of rail joint: Strap.

Manufacturers of steam-shovels will sometimes "guarantee" that certain of their shovels will excavate, say 3000 cubic yards of earth per day of ten hours. Even if it were possible for a shovel to fill a car at the rate of 5 cubic yards per minute, it is always impracticable to maintain such a speed, since a shovel must always wait for the shifting of cars and for the frequent shifting of the shovel itself. There are also delays due to adjustments and minor breakdowns. The best shovel records are made when the cars are large—other things being equal. The item of interest and depreciation of the plant is very large in steam-shovel work. This will be discussed further later. The cost of loading alone will usually come to between 3 and 4 c. per cubic yard. The cost of shifting the cars so as to place them successively under the shovel, haul them to the dumping place, dump them and haul them back, will generally be as much more. Gillette quotes five jobs on one railroad where the total cost for loading and hauling varied from 5.9 c. to 11.4 c. per cubic yard. But as these figures are based on *car* measurement, the cost per cubic yard in *place* measurement must be increased about one-fourth, or from 7.4 c. to 14.2 c.

140. Item 3. Hauling. The cost of hauling depends on the number of round trips per day that can be made by each vehicle employed. As the cost of each vehicle is practically the same whether it makes many trips or few, it becomes important that the number of trips should be made a maximum, and to that end there should be as little delay as possible in loading and

unloading. Therefore devices for facilitating the passage of the vehicles have a real money value.

(a) Carts. The average speed of a horse hauling a two-wheeled cart has been found to be 200 feet per minute, a little slower when hauling a load and a little faster when returning empty. This figure has been repeatedly verified. It means an allowance of one minute for each 100 feet (or "station") of "lead—the lead being the distance the earth is hauled." The time lost in loading, dumping, waiting to load, etc., has been found to average 4 minutes per load. Representing the number of stations (100 feet) of lead by s, the number of loads handled in 10 hours (600 minutes) would be $600 \div (s+4)$. The number of loads per cubic yard, measured in the bank, is differentiated by Morris into three classes, viz.:

3 loads per cubic yard in descending hauling;
3½ " " " " " level hauling; and
4 " " " " " ascending hauling.

Attempts have been made to estimate the effect of the grade of the roadway by a theoretical consideration of its rate, and of the comparative strength of a horse on a level and on various grades. While such computations are always practicable on a railway (even on a temporary construction track), the traction on a temporary earth roadway is always very large and so very variable that any refinements are useless. On railroad earthwork the hauling is generally nearly level or it is *descending*—forming embankments on low ground with material from cuts in high ground. The only common exception occurs when an embankment is formed from borrow-pits on low ground. One method of allowing for ascending grade is to add to the horizontal distance 14 times the difference of elevation for work with carts and 24 times the difference of elevation for work with wheelbarrows, and use that as the lead. For example, using carts, if the lead is 300 feet and there is a difference of elevation of 20 feet, the lead would be considered equivalent to $300+(14\times 20)=580$ feet on a level.

Trautwine assumes the average load for all classes of work to be ⅓ cubic yard, which figure is justified by large experience. Using one figure for all classes of work simplifies the calculations and gives the number of cubic yards carried per day of 10 hours equal to $\frac{600}{3(s+4)}$. Dividing the cost of a cart per day by the

number of cubic yards carried gives the cost of hauling per yard. In computing the cost of a cart per day, Trautwine refers to the practice of having one driver manage four carts, thus making a charge of 25 c. per day for each cart for the driver. Although this might be an economical method when the haul is very long, it is not economical for short hauls. A safer estimate is to allow not more than two carts per driver and in many cases a driver for each cart. Some contractors employ a driver for each cart and then require that the drivers shall assist in loading. The policy to be adopted is sometimes dependent on labor union conditions, which may demand that drivers must not assist in loading. The supply of labor and the amount of work on hand have a great influence on the methods of work which a contractor may adopt, for a strike will often disarrange all plans.

The cost of a horse and cart must practically include a charge for the time of the horse on Sundays, rainy days and holidays. The cost of repairs of cart and harness is generally included in this item for simplicity, but, under a strict application of the analysis suggested in § 137, it should properly be included under Item 7, Repairs, etc.

Since the time required for loading loose rock is greater than for earthwork, less loads will be hauled per day. The time allowance for loading, etc., is estimated by Trautwine as 6 minutes instead of 4 as for earth. Considering the great expansion of rock when broken up (see § 128), one cubic yard of solid rock, measured in place, would furnish the equivalent of five loads of earthwork of $\frac{1}{3}$ cubic yard. Therefore, on the basis of five loads per cubic yard, the number of cubic yards handled per ten-hour day per cart would be $\frac{600}{5(s+6)}$.

Let C represent the daily cost of a horse and cart and of the proportional cost of the driver (according to the number of carts handled by one driver), then the cost per cubic yard, *measured in the cut*, for hauling may be given by the formula:

$$\left.\begin{aligned} &\text{Cost per cu. yd. of hauling earth in carts} = \frac{C\times 3(s+4)}{600} \\ &\text{“ “ “ “ “ “ rock “ “ } = \frac{C\times 5(s+6)}{600} \end{aligned}\right\} . \quad (63)$$

(b) **Wagons.** For longer leads (i.e., from ¼ to ¾ of a mile) wagons drawn by two (or three) horses are more economical. The old-style wagons (about 0.8 cu. yd.) have bottoms of loose thick narrow boards. Raising them individually deposits the load underneath. Modern dump wagons contain from 1.0 to 2.0 cu. yds. The daily cost may be estimated on the same principle as the cost of carts.

The number of wagon trips per 10 hours will depend somewhat on the management of the shovellers. Too many shovellers per wagon is not economical, measured in yards shovelled per man, although it may reduce the time consumed in loading any one wagon. At an average figure of 20 cubic yards, measured in place, per shoveller per 10 hours, seven shovellers would load 14 cubic yards per hour or one cubic yard in 4.3 minutes. This would be the allowance for a wagon with a capacity of about 1½ yards of loose earth. Adding time for unloading, waiting to load and other possible "lost time," there is perhaps a total of six minutes. This figure will vary very considerably according to the number of shovellers per wagon, the capacity of the wagon, the type of wagon (whether self-dumping) and other details in the method of management. Adopting six minutes as the time used for loading, unloading, and other "lost time," the formula becomes.

$$\text{Cost per cubic yard of hauling in wagons} = \frac{C(s+6)}{600\,c}, \quad \ldots \quad (64)$$

in which C is the cost of the wagon, team and driver per day of 10 hours; s is the distance hauled in stations of 100 feet, and c is the capacity of the wagon in cubic yards, *place measurement*, which should be about three fourths of the nominal capacity of the wagon for earth and about sixty per cent when handling rock.

(c) **Wheelbarrows.** Gillette has computed from observations that a man will trundle a wheelbarrow at the rate of 250 feet per minute or 1.25 *stations* of lead per minute for the round trip. The time required for loading is estimated at 2½ minutes and for unloading, adjusting wheeling planks, short rests, etc., ¾ minute, or a total of three minutes per trip for all purposes except hauling. Gillette allows for a load only 1/15 cubic yard,

measured in place, or about 1/11 yard, 2.5 cubic feet, on the wheelbarrow. With notation as before, and for a ten-hour day,

$$\left.\begin{array}{l}\text{Cost per cubic yard of loading and}\\ \text{hauling earth in wheelbarrows}\end{array}\right\} = \frac{C \times 15(1.25s+3)}{600}. \quad (65)$$

In this equation C is the cost of both loading and hauling, and usually includes the allowance (Item 7) for the cost, repairs and depreciation of the wheelbarrows, whose service is very short lived. Trautwine estimates this at five cents per day or a total of $1.05 for labor and wheelbarrow.

The number of wheelbarrow loads required for a cubic yard of rock, measured in place, is about twenty-four. The time required for loading should also be increased about one fourth; the time required for all purposes except hauling is therefore about 3.75 minutes, and the corresponding equation becomes

$$\left.\begin{array}{l}\text{Cost per cubic yard of loading and}\\ \text{hauling rock in wheelbarrows}\end{array}\right\} = \frac{C \times 24(1.25s+3.75)}{600}. \quad (66)$$

(d) **Scrapers.** These are made in three general ways, "buck" scrapers, "drag" scrapers and "wheeled" scrapers. The buck scraper in its original form consisted merely of a wide plank, shod with an iron strap on the lower edge and provided with a pole and a small platform on which the driver may stand to weight it down. The earth is not loaded on to any receptacle and carried, but is merely pushed over the ground. Notwithstanding the apparent inefficiency of the method, its extreme implicity has caused its occasional adoption for the construction of canal embankments out of material from the bed of the canal. The occasions are rare when their use for railroad work would be practicable, and even then drag scrapers would probably be preferable.

A drag scraper is an immense "scoop shovel" about three feet long and three feet wide. There are usually two handles and a rail in front by which it is dragged by a team of horses. The nominal capacity varies from 7.5 cubic feet for the largest sizes, down to 3 cubic feet for the "one-horse" size, but these figures must be discounted by perhaps 40 or 50% for the actual average volume (as measured in the cut) loaded on during one scoop. The expansion of the earth during loosening is alone respons-

ible for a discount of 25%. These scrapers cost from $10 to $18.

A wheeled scraper is essentially an extra-large drag scraper which may be raised by a lever and carried on a pair of large wheels. Their nominal capacity ranges from 10 to 17 cubic feet, which should usually be liberally discounted when estimating output. They are loaded by dropping the scoop so that it scrapes up its load. The lever raises the scoop so that the load is carried on wheels instead of being dragged. At the dump the scoop is tipped so as to unload it. The movement of the scraper is practically continuous. They cost from $40 to $75. Their advantages over drag scrapers consist (1) in their greater capacity, (2) in the economy of transporting the load on wheels instead of by dragging, and (3) in the far greater length of haul over which the earth may be economically handled.

Morris estimated the speed of drag scrapers to be 140 feet per minute, or 70 feet of *lead* per minute. The "lead" should be here interpreted as the average distance from the center of the pit to the center of the dump. Gillette declares the speed to be 220 feet per minute. Some of this variation may be due to differences in the method of measuring the distance actually travelled, especially when the lead is very short, since the scraper teams must always travel a considerable extra distance at each end in order to turn around most easily. This extra distance is practically constant whether the lead is long or short. Gillette quotes an instance where the length of lead was actually about 20 feet, but the scraper teams travelled about 150 feet for each load carried. On this account Gillette adopts a minimum of 75 feet of lead no matter how short the lead actually may be. Of course the speed depends considerably on how strictly the men are kept to their work and also on the care which may be taken to obtain a full load for each scraper. As a compromise between Morris's and Gillette's estimates we may adopt the convenient rate of speed of 200 feet per minute, or 100 feet of lead per minute. There should also be allowed for the time lost in loading and unloading and for travelling the extra distance travelled by the teams in making the circuit, $1\frac{1}{3}$ minutes. Allowing the average value of seven loads per cubic yard and letting C represent the cost of scraper team and driver per ten-hour day, we have for the cost as follows:

$$\left.\begin{array}{l}\text{Cost per cubic yard of loading and}\\ \text{hauling earth in drag scrapers}\end{array}\right\} = \frac{C \times 7\,(s+1\frac{1}{4})}{600} \quad . \quad . \quad . \quad (67)$$

In this formula C should include the cost of not only the driver, team, and scraper, but also the proper proportion of the wages of an extra man, who assists each driver in loading his scraper, and whose wages should be divided among the two (or three) scrapers to which he is assigned. Scraper work nearly always implies ploughing, the cost of which should be computed as under Item 1.

When a low embankment is formed from borrow-pits on each side of the road, it may be done with scrapers, which move from one borrow-pit to the other, taking a load alternately from each side to the center and making but one half turn for each load carried. This reduces the time lost in turning by one third of a minute and reduces the constant in the numerator in Eq. (67) from $1\frac{1}{4}$ to 1. In this case the lead will usually be not greater than 75 feet, and therefore, if we consider this as a minimum value, s will ordinarily equal .75 and the quantity in the parenthesis will equal 1.75.

When using wheeled scrapers the catalogue capacity, which varies from 9 or 10 feet for a No. 1 scraper to 16 or 17 feet for a No. 3 scraper, must be reduced to 5 loads per cubic yard (place measurement) for a No. 1 scraper and to $2\frac{1}{2}$ loads per cubic yard for a No. 3, not only on account of the expansion of the earth during loosening, but also on account of the impracticability of loading these scrapers to their maximum nominal capacity. When the haul or lead for wheeled scrapers is 300 feet or over, it will be justifiable to employ shovellers to fill up the bowl of the shovel, especially when the soil is tough and when it is impracticable to fill the shovel even approximately full by the ordinary method. A snatch team to assist in loading the scrapers it also economical, especially with the larger scrapers. The proportionate number of snatch teams to the total number of scrapers of course depends on the length of haul. The cost of these extra shovellers and extra snatch teams must be divided proportionally among the number of scrapers assisted, in determining the value C in the formula given below. The extra time to be allowed on account of turning, loading, and dumping is about $1\frac{1}{4}$ minutes. The speed is considered one station of lead per minute as before. If we call C the average

daily cost of one scraper and n the capacity of the scraper, or the number of loads per cubic yard, we may write the following formula, on the basis of a ten-hour day:

$$\text{Cost per cubic yard of loading and hauling earth in wheeled scrapers} = \frac{C \times n(s+1\frac{1}{2})}{600} \quad . \quad . \quad (68)$$

(e) **Cars and horses.** The items of cost by this method are (*a*) charge for horses employed, (*b*) charge for men employed strictly in hauling, (*c*) charge for shifting rails when necessary, (*d*) repairs, depreciation, and interest on cost of cars and track. Part of this cost should strictly be classified under items 5 and 7, mentioned in § 137, but it is perhaps more convenient to estimate them as follows:

The traction of a car on rails is so very small that grade resistance constitutes a very large part of the total resistance if the grade is 1% or more. For all ordinary grades it is sufficiently accurate to say that the grade resistance is to the gross weight as the rise is to the distance. If the distance is supposed to be measured along the slope, the proportion is strictly true; i.e., on a 1% grade the grade resistance is 1 lb. per 100 of weight or 20 lbs. per ton. If the resistance on a level at the usual velocity is $\frac{1}{120}$, a grade of 1:120 (0.83%) will exactly double it. If the material is hauled *down* a grade of 1:120, the cars will run by gravity after being started. The work of hauling will then consist practically of hauling the empty cars up the grade. The grade resistance depends only on the rate of grade and the weight, but the tractive resistance will be *greater per ton of weight* for the unloaded than for the loaded cars. The tractive power of a horse is less on a grade than on a level, not only because the horse raises his own weight in addition to the load, but is anatomically less capable of pulling on a grade than on a level. In general it will be possible to plan the work so that loaded cars need not be hauled up a grade, unless an embankment is to be formed from a low borrow-pit, in which case another method would probably be advisable. These computations are chiefly utilized in designing the method of work—the proportion of horses to cars. An example may be quoted from English practice (Hurst), in which the cars had a capacity of $3\frac{1}{4}$ cubic yards, weighing 30 cwt. empty. Two horses took five "wagons" $\frac{3}{4}$ of a mile on a level

railroad and made 15 journeys per day of 10 hours, i.e., they handled 250 yards per day. In addition to those on the "straight road," another horse was employed to make up the train of loaded wagons. With a short lead the straight-road horses were employed for this purpose. In the above example the number of men required to handle these cars, shift the tracks, etc., is not given, and so the exact cost of the above work cannot be analyzed. It may be noticed that the two horses travelled 22½ miles per day, drawing in one direction a load, including the weight of the cars, of about 57,300 lbs., or 28.65 net tons. Allowing $\frac{1}{120}$ as the necessary tractive force, it would require a pull of 477.5 lbs., or 239 lbs. for each horse. With a velocity of 220 feet per minute this would amount to 1½ horse-power per horse, exerted for only a short time, however, and allowing considerable time for rest and for drawing only the empty cars. Gillette claims that the rolling resistance for such cars on a contractor's track should be considered as 40 lbs. per ton (the equivalent of a 2% grade) and quotes many figures to support the assertion. Unquestionably the resistance on tracks with very light rails, light ties with wide spacing and no tamping, would be very great and might readily amount to 40 lbs. per ton. In the above case, the resistance could not have been much if any over $\frac{1}{120}$. A resistance of 40 lbs. per ton would have required each horse to pull about 573 lbs. for nearly five hours per day, beside pulling the empty cars the rest of the time. This is far greater exertion than any ordinary horse can maintain. The cars generally used in this country have a capacity of 1½ cubic yards and cost about $65 apiece. Besides the shovellers and dumping-gang, several men and a foreman will be required to keep the track in order and to make the constant shifts that are necessary. Two trains are generally used, one of which is loaded while the other is run to the dump. Some passing-place is necessary, but this is generally provided by having a switch at the cut and running the trains on each track alternately. This insures a train of cars always at the cut to keep the shovellers employed. The cost of hauling per cubic yard can only be computed when the number of laborers, cars, and horses employed are known, and these will depend on the lead, on the character of the excavation, on the grade, if any, etc., and must be so proportioned that the shovellers need not wait for cars to fill, nor the dumping-gang

for material to handle, nor the horses and drivers for cars to haul. Much skill is necessary to keep a large force in smooth running order.

(f) **Cars and locomotives.** 30-lb. rails are the lightest that should be used for this work, and 35- or 40-lb. rails are better. One or two narrow-gauge locomotives (depending on the length of haul), costing about $2500 each, will be necessary to handle two trains of about 15 cars each, the cars having a capacity of about 2 cubic yards and costing about $100 each. Some cars can be obtained as low as $70. A force of about five men and a foreman will be required to shift the tracks. The track-shifters, except the foreman, may be common laborers. The dumping-gang will require about seven men. Even when the material is all taken *down* grade the grades may be too steep for the safe hauling of loaded cars down the grade, or for hauling empty cars up the grade. Under such circumstances temporary trestles are necessary to reduce the grade. When these are used, the uprights and bracing are left in the embankment—only the stringers being removed. This is largely a necessity, but is partially compensated by the fact that the trestle forms a core to the embankment which prevents lateral shifting during settlement. The average speed of the trains may be taken as 10 miles per hour or 5 miles of lead per hour. The time lost in loading and unloading is estimated (Trautwine) as 9 minutes or .15 of an hour. The number of trips per day of 10 hours will equal $\frac{10}{\frac{1}{5}(\text{miles of lead}) + .15}$ or $\frac{50}{(\text{miles of lead}) + .75}$. Of course this quotient *must* be a whole number. Knowing the number of trains and their capacity, the total number of cubic yards handled is known, which, divided into the total daily cost of the trains, will give the cost of hauling per yard. The daily cost of a train will include

(*a*) Wages of engineer, who frequently fires his own engine;

(*b*) Fuel, about ¼ to 1 ton of bituminous coal, depending on work done;

(*c*) Water, a very variable item, frequently costing $3 to $5 per day;

(*d*) Repairs, variable, frequently at rate of 50 to 60% per year;

(*e*) Interest on cost and depreciation, 16 to 40%.

To these must be added, to obtain the total cost of haul,

(*f*) Wages of the gang employed in shifting track.

The above calculation for the number of train loads depends on the assumption that 9 minutes is total time lost by a locomotive for each round trip. If the haul is very short it may readily happen that a steam-shovel cannot fill one train of cars before the locomotive has returned with a load of empties and is ready to haul a loaded train away. The estimation of the number of train loads is chiefly useful in planning the work so as to have every tool working at its highest efficiency. Usually the capacity of the steam-shovel or the ability to promptly "spot" the cars under the shovel is the real limiting agent which determines the daily output.

141. **Choice of method of haul dependent on distance.** In light side-hill work in which material need not be moved more than 12 or 15 feet, i.e., moved *laterally* across the roadbed, the earth may be moved most cheaply by mere shovelling. Beyond 12 feet scrapers are more economical. At about 100 feet drag-scrapers and wheelbarrows are equally economical. Between 100 and 200 feet wheelbarrows are generally cheaper than either carts or drag-scrapers, but wheeled scrapers are always cheaper than wheelbarrows. Beyond 500 feet two-wheeled carts become the most economical up to about 1700 feet; then four-wheeled wagons become more economical up to 3500 feet. Beyond this cars on rails, drawn by horses or by locomotives, become cheaper. The economy of cars on rails becomes evident for distances as small as 300 feet provided the volume of the excavation will justify the outlay. Locomotives will always be cheaper than horses and mules, providing the work to be done is of sufficient magnitude to justify the purchase of the necessary plant and risk the loss in selling the plant ultimately as second-hand equipment, or keeping the plant on hand and idle for an indefinite period waiting for other work. Horses will not be economical for distances much over a mile. For greater distances locomotives are more economical, but the question of "limit of profitable haul" (§ 148) must be closely studied, as the circumstances are certainly not common when it is advisable to haul material much over a mile.

142. **Item 4.** SPREADING. The cost of spreading varies with the method employed in dumping the load. When the earth

is tipped over the edge of an embankment there is little if any necessary work. Trautwine allows about ¼ c. per cubic yard for keeping the dumping-places clear and in order. This would represent the wages of one man at $1 per day attending to the unloading of 1200 two-wheeled carts each carrying ½ cubic yard. 1200 carts in 10 hours would mean an average of two per minute, which implies more rapid and efficient work than may be depended on. The allowance is probably too small. When the material is dumped in layers some levelling is required, for which Trautwine allows 50 to 100 cubic yards as a fair day's work, costing from 1 to 2 cents per cubic yard. The cost of spreading will not ordinarily exceed this and is frequently nothing—all depending on the method of unloading. It should be noted that Mr. Morris's examples and computations (Jour. Franklin Inst., Sept. 1841) disregard altogether any special charge for this item.

143. Item 5. KEEPING ROADWAYS IN ORDER. This feature is important as a measure of true economy, whatever the system of transportation, but it is often neglected. A petty saving in such matters will cost many times as much in increased labor in hauling and loss of time. With some methods of haul the cost is best combined with that of other items.

(a) **Wheelbarrows.** Wheelbarrows should generally be run on planks laid on the ground. The adjusting and shifting of these planks is done by the wheelers, and the time for it is allowed for in the "¾ minute for short rests, adjusting the wheeling plank, etc." The actual cost of the planks must be added, but it would evidently be a very small addition per cubic yard in a large contract. When the wheelbarrows are run on planks placed on "horses" or on trestles the cost is very appreciable; but the method is frequently used with great economy. The variations in the requirements render any general estimate of such cost impracticable.

(b) **Carts and wagons.** The cost of keeping roadways in order for carts and wagons is sometimes estimated merely as so much per cubic yard, but it is evidently a function of the *lead*. The work consists in draining off puddles, filling up ruts, picking up loose stones that may have fallen off the loads, and in general doing everything that will reduce the traction as much as possible. Temporary inclines, built to avoid excessive grade

at some one point, are often measures of true economy. Trautwine suggests $\frac{1}{10}$ c. per cubic yard per 100 feet of lead for earthwork and $\frac{2}{10}$ c. for rockwork, as an estimate for this item when carts are used.

(c) **Cars.** When cars are used a shifting-gang, consisting of a foreman and several men (say five), are constantly employed in shifting the track so that the material may be loaded and unloaded where it is desired. The average cost of this item may be estimated by dividing the total daily cost of this gang by the number of cubic yards handled in one day.

144. **Item 6. TRIMMING CUTS TO THEIR PROPER CROSS-SECTION.** This process, often called "sand-papering," must be treated as an expense, since the payment received for the very few cubic yards of earth excavated is wholly inadequate to pay for the work involved. Gillette quotes bids of 2 cents per *square* yard of surface trimmed, and from this argues that, for *average* excavations, it adds to the cost *four* cents per cubic yard of the total excavation. The shallower the cut the greater is the proportionate cost. Of course the actual cost to the contractor will depend largely on the accuracy of outline demanded by the engineer or inspector.

145. **Item 7. REPAIRS, WEAR, DEPRECIATION, AND INTEREST ON COST OF PLANT.** The amount of this item evidently depends upon the character of the soil—the harder the soil the worse the wear and depreciation. The *interest on cost* depends on the current borrowing value of money. The estimate for this item has already been included in the allowances for horses, carts, ploughs, harness, wheelbarrows, steam-shovels, etc. Trautwine estimates $\frac{1}{4}$ c. per cubic yard for picks and shovels. Depreciation is generally a large percentage of the cost of earth-working tools, the life of all being limited to a few years, and of many tools to a few months or weeks.

146. **Item 8. SUPERINTENDENCE AND INCIDENTALS.** The incidentals include the cost of water-boys, timekeepers, watchmen, blacksmiths, fences, and other precautions to protect the public from possible injury, cost of casualty insurance for workmen, etc. Although the cost of some of these sub-items may be definitely estimated, others are so uncertain that it is only possible to make a lump estimate and add say 5 to 7% of the sum of the previous items for this item.

147. Contractor's profit and contingencies. The word "contingencies" here refers to the abnormal expenses caused by freshets, continued wet weather, and "hard luck," as distinguished from mere incidentals which are really normal expenses. They are the expenses which literally cannot be foreseen, and on which the contractor must "take chances." They are therefore included with the expected profit. The allowance for these two elements combined is variously estimated up to 25% of the previously estimated cost of the work, according to the sharpness of the competition, the contractor's confidence in the accuracy of his estimates, and the possible uncertainty as to true cost owing to unfavorable circumstances. The contractor's real profit may vary considerably from this. He often pays clerks, boards and lodges the laborers in shanties built for the purpose, or keeps a supply-store, and has various other items both of profit and expense. His profit is largely dependent on skill in so handling the men that all can work effectively without interference or delays in waiting for others. An unusual season of bad weather will often affect the cost very seriously. It is a common occurrence to find that two contractors may be working on the same kind of material and under precisely similar conditions and at the same price, and yet one may be making money and the other losing it—all on account of difference of management.

148. Limit of profitable haul. As intimated in §§ 134 and 141, there is with every method of haul a limit of distance beyond which the expense for excessive hauling will exceed the loss resulting from borrowing and wasting. This distance is somewhat dependent on local conditions, thus requiring an independent solution for each particular case, but the general principles involved will be about as follows: Assume that it has been determined, as in Fig. 64, that the cut and fill will exactly balance between two points, as between e and x, assuming that, as indicated in § 132 (9), a trestle has been introduced between s and t, thus altering the mass curve to $Estxn \ldots$ Since there is a balance between A' and C', the material for the fill between C' and e' must be obtained either by "borrowing" in the immediate neighborhood or by transportation from the excavation between z' and n'. If cut and fill have been approximately

balanced in the selection of grade line, as is ordinarily done, borrowing material for the fill $C'e'$ implies a wastage of material at the cut $z'n'$. To compare the two methods, we may place against the plan of borrowing and wasting, (*a*) cost, if any, of extra right of way that may be needed from which to obtain earth for the fill $C'e'$; (*b*) cost of loosening, loading, hauling a distance equal to that between the centers of gravity of the borrow-pit and of the fill, and the other expenses incidental to borrowing M cubic yards for the fill $C'e'$; (*c*) cost of loosening, loading, hauling a distance equal to that between the centers of gravity of the cut $z'n'$ and of the spoil-bank, and the other expenses incidental to wasting M cubic yards at the cut $z'n'$: (*d*) cost, if any, of land needed for the spoil-bank. The cost of the other plan will be the cost of loosening, loading, hauling (the hauling being represented by the trapezoidal figure *Cexn*), and the other expenses incidental to making the fill $C'e'$ with the material from the cut $z'n'$, the amount of material being M cubic yards, which is represented in the figure by the vertical ordinate from *e* to the line *Cn*. The difference between these costs will be the cost, if any, of land for borrow-pit and spoil-bank *plus* the cost of loosening, loading, etc. (except hauling and roadways) of M cubic yards, *minus* the difference in cost of the excessive haul from *Ce* to *xn* and the comparatively short hauls from borrow-pit and to spoil-bank.

As an illustration, taking some of the estimates previously given for operating with average material, the cost of all items, except hauling and roadways, would be about as follows: loosening, with plough, 1.2 c., loading 5.0 c., spreading 1.5 c., wear, depreciation, etc., .25 c., superintendence, etc., 1.5 c.; total 8.95 c. Suppose that the haul for both borrowing and wasting averages 100 feet or 1 station. Then the cost of haul per yard, using carts, would be (§ 140, *a*) $[125\times3(1+4)]\div600 = 3.125$ c. The cost of roadways would be about 0.1 c. per yard, making a total of 3.225 c. per cubic yard. Assume $M = 10000$ cubic yards and the area $Cexn = 180000$ yards-stations or the equivalent of 10000 yards hauled 1800 feet. This haul would cost $[125\times3(18+4)]\div600 = 13.75$ c. per cubic yard. The cost of roadways will be $18\times.1$ or 1.8 c., making a total of 15.55 c. for hauling and roadways. The difference of cost of hauling and roadways will be $15.55-(2\times3.225) = 9.10$ c. per yard or $910

for the 10000 yards. Offsetting this is the cost of loosening, etc., 10000 yards, at 8.95 c., costing $895. These figures may be better compared as follows:

Long Haul.	Loosening, etc.,	10000 yards,	@ 8.95 c.	$ 895.
	Hauling, "	10000 "	@ 15.55 c.	1555.
				$2450.
Borrowing and Wasting.	Loosening, etc.,	10000 yards (borrowed),	@ 8.95 c.	$895.
	" "	10000 " (wasted),	@ 8.95 c.	895.
	Hauling, etc.,	10000 " (borrowed),	@ 3.225 c.	322.50
	" "	10000 " (wasted),	@ 3.225 c.	322.50
				$2435.00

These costs are practically balanced, but no allowance has been made for right of way. If any considerable amount had to be paid for that, it would decide this particular case in favor of the long haul. This shows that *under these conditions* 1800 feet is *about* the limit of profitable haul, the land costing nothing extra.

BLASTING.

149. Explosives. The effect of blasting is due to the extremely rapid expansion of a gas which is developed by the decomposition of a very small amount of solid matter. Blasting compounds may be divided into two general classes, (*a*) slow-burning and (*b*) detonating. Gunpowder is a type of the slow-burning compounds. These are generally ignited by heat; the ignition proceeds from grain to grain; the heat and pressure produced are comparatively low. Nitro-glycerine is a type of the detonating compounds. They are exploded by a shock which *instantaneously* explodes the whole mass. The heat and pressure developed are far in excess of that produced by the explosion of powder. Nitro-glycerine is so easily exploded that it is very dangerous to handle. It was discovered that if the nitro-glycerine was absorbed by a spongy material like infusorial earth, it was much less liable to explode, while its power when actually exploded was practically equal to that of the amount of pure nitro-glycerine contained in the dynamite, which is the name given to the mixture of nitro-glycerine and infusorial earth. Nitro-glycerine is expensive; many other explosive chemical compounds which properly belong to the slow-burning

class are comparatively cheap. It has been conclusively demonstrated that a mixture of nitro-glycerine and some of the cheaper chemicals has a greater explosive force than the sum of the strengths of the component parts when exploded separately. Whatever the reason, the fact seems established. The reason is possibly that the explosion of the nitro-glycerine is sufficiently powerful to produce a *detonation* of the other chemicals, which is impossible to produce by ordinary means, and that this explosion caused by detonation is more powerful than an ordinary explosion. The majority of the explosive compounds and "powders" on the market are of this character—a mixture of 20 to 60 per cent. of nitro-glycerine with variable proportions of one or more of a great variety of explosive chemicals.

The choice of the explosive depends on the character of the rock. A hard brittle rock is most effectively blasted by a detonating compound. The rapidity with which the full force of the explosive is developed has a shattering effect on a brittle substance. On the contrary, some of the softer tougher rocks and indurated clays are but little affected by dynamite. The result is but little more than an enlargement of the blast-hole. Quarrying must generally be done with blasting-powder, as the quicker explosives are too shattering. Although the results obtained by various experimenters are very variable, it may be said that pure nitro-glycerine is eight times as powerful as black powder, dynamite (75% nitro-glycerine) six times, and gun-cotton four to six times as powerful. For open work where time is not particularly valuable, black powder is by far the cheapest, but in tunnel-headings, whose progress determines the progress of the whole work, dynamite is so much more effective and so expedites the work that its use becomes economical.

150. Drilling. Although many very complicated forms of drill-bars have been devised, the best form (with slight modifications to suit circumstances) is as shown in Fig. 66 (*a*), and (*b*). The width should flare at the bottom (*a*) about 15 to 30%. For hard rock the curve of the edge should be somewhat flatter and for soft rock somewhat more curved than shown, Fig. 66, (*a*). Sometimes the angle of the two faces is varied from that given, Fig. 66, (*b*) and occasionally the edge is purposely blunted so as to give a crushing rather than a cutting effect. The drills will require sharpening for each 6 to 18 inches depth of hole, and will require a new edge to be worked every 2 to 4 days.

For drilling vertical holes the *churn-drill* is the most economical. The drill-bar is of iron, about 6 to 8 feet long, $1\frac{1}{4}''$ in diameter, weighs about 25 to 30 lbs., and is shod with a piece of steel welded on. The bar is lifted a few inches between each blow, turned partially around, and allowed to fall, the impact doing the work. From 5 to 15 feet of holes, depending on the character of the rock, is a fair day's work—10 hours. In very soft rocks even more than this may be done. This method is

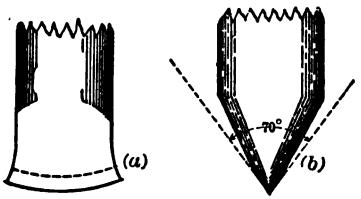


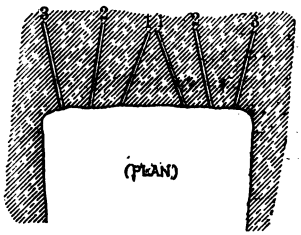
FIG. 66.

inapplicable for inclined holes or even for vertical holes in confined places, such as tunnel-headings. For such places the only practical *hand* method is to use hammers. This may be done by light drills and light hammers (one-man work), or by heavier drills held by one man and struck by one or two men with heavy hammers. The conclusion of an exhaustive investigation as to the relative economy of light or heavy hammers is that the light-hammer method is more economical for the softer rocks, the heavy-hammer method is more economical for the harder rocks, but that the light-hammer method is always more expeditious and hence to be preferred when time is important.

The subject of machine rock-drills is too vast to be treated here. The method is only practicable when the amount of work to be done is large, and especially when time is valuable. The machines are generally operated by compressed air although steam is also used to operate the drills. Gasoline as a motive power is even more economical for a small-scale plant. The cost per foot of hole drilled is quite variable, but is usually somewhat less than that of hand-drilling—sometimes but a small fraction of it.

151. **Position and direction of drill-holes.** As the cost of drilling holes is the largest single item in the total cost of blasting, it is necessary that skill and judgment should be used in so

locating the holes that the blasts will be most effective. The greatest effect of a blast will evidently be in the direction of the "line of least resistance." In a strictly homogeneous material this will be the shortest line from the center of the explosive to the surface. The variations in homogeneity on account of laminations and seams require that each case shall be judged according to experience. In open-pit blasting it is generally easy to obtain two and sometimes three exposed faces to the rock, making it a simple matter to drill holes so that a blast will do effective work. When a solid face of rock must be broken into, as in a tunnel-heading, the work is necessarily ineffectual and expensive. A conical or wedge-shaped mass will first be blown out by simultaneous blasts in the holes marked 1, Fig. 67; blasts in the holes marked 2 and 3 will then complete the cross-section of the heading. A great saving in cost may often be secured by skilfully taking advantage of seams, breaks, and irregularities. When the work is economically done there is but little noise or throwing of rock, a covering of old timbers and branches of trees generally sufficing to confine the smaller pieces which would otherwise fly up.



DRILL HOLES IN TUNNEL HEADING

FIG. 67.

152. Amount of explosive. The amount of explosive required varies as the cube of the line of least resistance. The best results are obtained when the line of least resistance is $\frac{3}{4}$ of the depth of the hole; also when the powder fills about $\frac{1}{3}$ of the hole. For average rock the amount of powder required is as follows:

Line of least resistance.....	2 ft.	4 ft.	6 ft.	8 ft.
Weight of powder.....	¼ lb.	2 lbs.	6¾ lbs.	16 lbs.

Strict compliance with all of the above conditions would require that the diameter of the hole should vary for every case. While this is impracticable, there should evidently be some variation in the size of the hole, depending on the work to be done. For example, a 1″ hole, drilled 2′ 8″ deep, with its line of least resistance 2′. and loaded with ¼ lb. of powder, would

be filled to a depth of 9½", which is nearly ⅓ of the depth. A 3" hole, drilled 8' deep, with its line of least resistance 6', and loaded with 6¾ lbs. of powder, would be filled to a depth of over 28", which is also nearly ⅓ of the depth. One pound of blasting-powder will occupy about 28 cubic inches. Quarrying necessitates the use of numerous and sometimes repeated light charges of powder, as a heavy blast or a powerful explosive like dynamite is apt to shatter the rock. This requires more powder to the cubic yard than blasting for mere excavation, which may usually be done by the use of ¼ to ⅓ lb. of powder per cubic yard of easy open blasting. On account of the great resistance offered by rock when blasted in headings in tunnels, the powder used per cubic yard will run up to 2, 4, and even 6 lbs. per cubic yard. As before stated, nitro-glycerine is about eight times (and dynamite about six times) as powerful as the same *weight* of powder.

153. **Tamping.** Blasting-powder and the slow-burning explosives require thorough tamping. Clay is probably the best, but sand and fine powdered rock are also used. Wooden plugs, inverted expansive cones, etc., are periodically reinvented by enthusiastic inventors, only to be discarded for the simpler methods. Owing to the extreme rapidity of the development of the force of a nitro-glycerine or dynamite explosion, tamping is not so essential with these explosives, although it unquestionably adds to their effectiveness. Blasting under water has been effectively accomplished by merely pouring nitro-glycerine into the drilled holes through a tube and then exploding the charge without any tamping except that furnished by the superincumbent water. It has been found that air-spaces about a charge make a material reduction in the effectiveness of the explosion. It is therefore necessary to carefully ram the explosive into a solid mass. Of course the liquid nitro-glycerine needs no ramming, but dynamite should be rammed with a *wooden* rammer. Iron should be carefully avoided in ramming gunpowder. A copper bar is generally used.

154. **Exploding the charge.** Black powder is generally exploded by means of a fuse which is essentially a cord in which there is a thin vein of gunpowder, the cord being protected by tar, extra linings of hemp, cotton, or even gutta-percha. The fuse is inserted into the middle of the charge, and the tamping carefully packed around it so that it will not be injured. To

produce the detonation required to explode nitro-glycerine and dynamite, there must be an initial explosion of some easily ignited explosive. This is generally accomplished by means of caps containing fulminating-powder which are exploded by electricity. The electricity (in one class of caps) heats a very fine platinum wire to redness, thereby igniting the sensitive powder, or (in another class) a spark is caused to jump through the powder between the ends of two wires suitably separated. Dynamite can also be exploded by using a small cartridge of gunpowder which is itself exploded by an ordinary fuse.

155. Cost. As a rough estimate, the cost of loosening and loading rock work, reduced to the uniform basis of $1.00 per 10-hour day, may be said to vary from 30c. for easy but *brittle* rock and increasing to 80c. per cubic yard when the cutting is shallow, the rock especially tough, and the strata unfavorably placed. For a detailed analysis of cost, which is essential for close estimating, see Gillette's "Rock Excavation, Methods and Cost."

156. Classification of excavated material. The classification of excavated material is a fruitful source of dispute between contractors and railroad companies, owing mainly to the fact that the variation between the softest earth and the hardest rock is so gradual that it is very difficult to describe distinctions between different classifications which are unmistakable and indisputable The classification frequently used is (*a*) earth, (*b*) loose rock, and (*c*) solid rock. As blasting is frequently used to loosen "loose rock" and even "earth" (if it is frozen), the fact that blasting is employed cannot be used as a criterion, especially as this would (if allowed) lead to unnecessary blasting for the sake of classifying material as rock.

Earth. This includes clay, sand, gravel, loam, decomposed rock and slate, boulders or loose stones not greater than 1 cubic foot (3 cubic feet, P. R. R.), and sometimes even "hard-pan." In general it will signify material which *can* be loosened by a plough with two horses, or with which one picker can keep one shoveller busy.

Loose rock. This includes boulders and loose stones of more than one cubic foot and less than one cubic yard; stratified rock, not more than six inches thick, separated by a stratum of clay; also all material (not classified as earth) which may be loosened by pick or bar and which "*can* be quarried without blasting, although blasting may occasionally be resorted to."

Solid rock includes all rock found in masses of over one cubic yard which cannot be removed except by blasting.

It is generally specified that the engineer of the railroad company shall be the judge of the classification of the material, but frequently an appeal is taken from his decisions to the courts.

157. Specifications for earthwork. The following specifications, issued by the Norfolk and Western R. R., represent the average requirements. It should be remembered that very strict specifications invariably increase the cost of the work, and frequently add to the cost more than is gained by improved quality of work.

1. The grading will be estimated and paid for by the cubic yard, and will include clearing and grubbing, and all open excavations, channels, and embankments required for the formation of the roadbed, and for turnouts and sidings; cutting all ditches or drains about or contiguous to the road; digging the foundation-pits of all culverts, bridges, or walls; reconstructing turnpikes or common roads in cases where they are destroyed or interfered with; changing the course or channel of streams; and all other excavations or embankments connected with or incident to the construction of said Railroad.

2. All grading, except where otherwise specified, whether for cuts or fills, will be measured in the excavations and will be classified under the following heads, viz.: Solid Rock, Loose Rock, Hard-pan, and Earth.

Solid Rock shall include all rock occurring in masses which, in the judgment of the said Engineer Maintenance of Way, may be best removed by blasting.

Loose Rock shall include all kinds of shale, soapstone, and other rock which, in the judgment of the said Engineer Maintenance of Way, can be removed by pick and bar, and is soft and loose enough to be removed without blasting, although blasting may be occasionally resorted to; also, detached stone of less than one (1) cubic yard and more than one (1) cubic foot.

Hard-pan shall consist of tough indurated clay or cemented gravel, which requires blasting or other equally expensive means for its removal, or which cannot be ploughed with less than four horses and a railroad plough, or which requires two pickers to a shoveller, the said Engineer Maintenance of Way to be the judge of these conditions.

EARTH shall include all material of an earthy nature, of whatever name or character, not unquestionably loose rock or hardpan as above defined.

POWDER. The use of powder in cuts will not be considered as a reason for any other classification than earth, unless the material in the cut is clearly other than earth under the above specifications.

3. Earth, gravel, and other materials taken from the excavations, except when otherwise directed by the said Engineer Maintenance of Way or his assistant, shall be deposited in the adjacent embankment; the cost of removing and depositing which, when the distance necessary to be hauled is not more than sixteen hundred (1600) feet, shall be included in the price paid for the excavation.

4. EXTRA HAUL will be estimated and paid for as follows: whenever material from excavations is necessarily hauled a greater distance than sixteen hundred (1600) feet, there shall be paid in addition to the price of excavation the price of extra haul per 100 feet, or part thereof, after the first 1600 feet; the necessary haul to be determined in each case by the said Engineer Maintenance of Way or his assistant, from the profile and cross-sections, and the estimates to be in accordance therewith.

5. All embankments shall be made in layers of such thickness and carried on in such manner as the said Engineer Maintenance of Way or his assistant may prescribe, the stone and heavy materials being placed in slopes and top. And in completing the fills to the proper grade such additional heights and fulness of slope shall be given them, to provide for their settlement, as the said Engineer Maintenance of Way, or his assistant, may direct. Embankments about masonry shall be built at such times and in such manner and of such materials as the said Engineer Maintenance of Way or his assistant may direct.

6. In procuring materials for embankments from without the line of the road, and in wasting materials from cuttings, the place and manner of doing it shall in each case be indicated by the Engineer Maintenance of Way or his assistant; and care must be taken to injure or disfigure the land as little as possible. Borrow-pits and spoil-banks must be left by the Contractor in regular and sightly shape.

7. The lands of the said Railroad Company shall be cleared to the extent required by the said Engineer Maintenance of

Way, or his assistant, of all trees, brushes, logs, and other perishable materials, which shall be destroyed by burning or deposited in heaps as the said Engineer Maintenance of Way, or his assistant, may direct. Large trees must be cut not more than two and one-half (2½) feet from the ground, and under embankments less than four (4) feet high they shall be cut close to the ground. All small trees and bushes shall be cut close to the ground.

8. Clearing shall be estimated and paid for by the acre or fraction of an acre.

9. All stumps, roots, logs, and other obstructions shall be grubbed out, and removed from all places where embankments occur less than two (2) feet in height; also, from all places where excavations occur and from such other places as the said Engineer Maintenance of Way or his assistant may direct.

10. Grubbing shall be estimated and paid for by the acre or fraction of an acre.

11. Contractors, when directed by the said Engineer Maintenance of Way or his assistant in charge of the work, will deposit on the side of the road, or at such convenient points as may be designated, any stone, rock, or other materials that they may excavate; and all materials excavated and deposited as above, together with all timber removed from the line of the road, will be considered the property of the Railroad Company, and the Contractors upon the respective sections will be responsible for its safe-keeping until removed by said Railroad Company, or until their work is finished.

12. Contractors will be accountable for the maintenance of safe and convenient places wherever public or private roads are in any way interfered with by them during the progress of the work. They will also be responsible for fences thrown down, and for gates and bars left open, and for all damages occasioned thereby.

13. Temporary bridges and trestles, erected to facilitate the progress of the work, in case of delays at masonry structures from any cause, or for other reasons, will be at the expense of the Contractor.

14. The line of road or the gradients may be changed in any manner, and at any time, if the said Engineer Maintenance of Way or his assistant shall consider such a change necessary or expedient; but no claim for an increase in prices of excavation

or embankment on the part of the Contractor will be allowed or considered unless made in writing before the work on that part of the section where the alteration has been made shall have been commenced. The said Engineer Maintenance of Way or his assistant may also, on the conditions last recited, increase or diminish the length of any section for the purpose of more nearly equalizing or balancing the excavations and embankments, or for any other reason.

15. The roadbed will be graded as directed by the said Engineer Maintenance of Way or his assistant, and in conformity with such breadths, depths, and slopes of cutting and filling as he may prescribe from time to time, and no part of the work will be finally accepted until it is properly completed and dressed off at the required grade.

CHAPTER IV.

TRESTLES.

158. Extent of use. Trestles constitute from 1 to 3% of the length of the average railroad. It was estimated in 1889 that there was then about 2400 miles of single-track railway trestle in the United States, divided among 150,000 structures and estimated to cost about $75,000,000. The annual charge for maintenance, estimated at $\frac{1}{8}$ of the cost, therefore amounted to about $9,500,000 and necessitated the annual use of perhaps 300,000,000 ft. B. M. of timber. The corresponding figures at the present time must be somewhat in excess of this. The magnitude of this use, which is causing the rapid disappearance of forests, has resulted in endeavors to limit the use of timber for this purpose. Trestles may be considered as justifiable under the following conditions:

a. Permanent trestles.

1. Those of *extreme* height — then called viaducts and frequently constructed of steel, as the Kinzua viaduct, 302 feet high.

2. Those across wide shallow waterways—*e.g.*, that across Lake Pontchartrain, near New Orleans, 22 miles long.

3. Those across swamps of soft deep mud, or across a river-bottom, liable to occasional overflow.

b. Temporary trestles.

1. To open the road for traffic as quickly as possible—often a reason of great financial importance.

2. To quickly replace a more elaborate structure, destroyed by accident, on a road already in operation, so that the interruption to traffic shall be a minimum.

3. To form an earth embankment with earth brought from a distant point by the train-load, when such a measure would cost less than to borrow earth in the immediate neighborhood.

4. To bridge an opening temporarily and thus allow time to learn the regimen of a stream in order to better proportion the

size of the waterway and also to facilitate bringing *suitable* stone for masonry from a distance. In a new country there is always the double danger of either building a culvert too small, requiring expensive reconstruction, perhaps after a disastrous washout, or else wasting money by constructing the culvert unnecessarily large. Much masonry has been built of a very poor quality of stone because it could be conveniently obtained and because good stone was unobtainable except at a prohibitive cost for transportation. Opening the road for traffic by the use of temporary trestles obviates both of these difficulties.

159. Trestles vs. embankments. Low embankments are very much cheaper than low trestles both in first cost and maintenance. Very high embankments are very expensive to construct, but cost comparatively little to maintain. A trestle of equal height may cost much less to construct, but will be expensive to maintain—perhaps $\frac{1}{8}$ of its cost per year. To determine the height beyond which it will be cheaper to maintain a trestle rather than build an embankment, it will be necessary to allow for the cost of maintenance. The height will also depend on the relative cost of timber, labor, and earthwork. At the present average values, it will be found that for less heights than 25 feet the *first cost* of an embankment will *generally* be less than that of a trestle; this implies that a permanent trestle should never be constructed with a height less than 25 feet except for the reasons given in § 158. The height at which a permanent trestle is certainly cheaper than earthwork is more uncertain. A high grade line joining two hills will invariably imply at least a culvert if an embankment is used. If the culvert is built of masonry, the cost of the embankment will be so increased that the height at which a trestle becomes economical will be materially reduced. The cost of an embankment increases much more rapidly than the height—with very high embankments more nearly as the square of the height—while the cost of trestles does not increase as rapidly as the height. Although local circumstances may modify the application of any set rules, it is probably seldom that it will be cheaper to build an embankment 40 or 50 feet high than to permanently maintain a wooden trestle of that height. A steel viaduct would probably be the best solution of such a case. These are frequently used for permanent structures, especially when very high. The cost of maintenance is much less than that of wood, which makes the

use of steel preferable for permanent trestles unless wood is abnormally cheap. Neither the cost nor the construction of steel trestles will be considered in this chapter.

160. Two principal types. There are two principal types of wooden trestles—pile trestles and framed trestles. The great objection to pile trestles is the rapid rotting of the portion of the pile which is underground, and the difficulty of renewal. The maximum height of pile trestles is about 30 feet, and even this height is seldom reached. Framed trestles have been constructed to a height of considerably over 100 feet They are frequently built in such a manner that any injured piece may be readily taken out and renewed without interfering with traffic. Trestles consist of two parts—the supports called "bents," and the stringers and floor system. As the stringers and floor system are the same for both pile and framed trestles, the "bents" are all that need be considered separately.

PILE TRESTLES.

161. Pile bents. A pile bent consists generally of four piles driven into the ground deep enough to afford not only sufficient vertical resistance but also lateral resistance. On top of these piles is placed a horizontal "cap." The caps are fastened to the tops of the piles by methods illustrated in Fig. 68. The method of fastening shown in each case should not be considered as applicable only to the particular type of pile bent used to illustrate it. Fig. 68 (*a* and *d*) illustrates a mortise-joint with a hardwood pin about $1\frac{1}{4}''$ in diameter. The hole for the pin should be bored separately through the cap and the mortise, and the hole through the cap should be at a slightly higher level than that through the mortise, so that the cap will be drawn down tight when the pin is driven. Occasionally iron dowels (an iron pin about $1\frac{1}{2}''$ in diameter and about 8″ long) are inserted partly in the cap and partly in the pile. The use of drift-bolts, shown in Fig. 68 (*b*), is cheaper in first cost, but renders repairs and renewals very troublesome and expensive. "Split caps," shown in Fig. 68 (*c*), are formed by bolting two half-size strips on each side of a tenon on top of the pile. Repairs are very easily and cheaply made without interference with the traffic and without injuring other pieces of the bent. The smaller pieces are more easily obtainable in a sound condition; the

decay of one does not affect the other, and the first cost is but little if any greater than the method of using a single piece. For further discussion, see § 170.

For very light traffic and for a height of about 5 feet three vertical piles will suffice, as shown in Fig. 68 (*a*). Up to a height

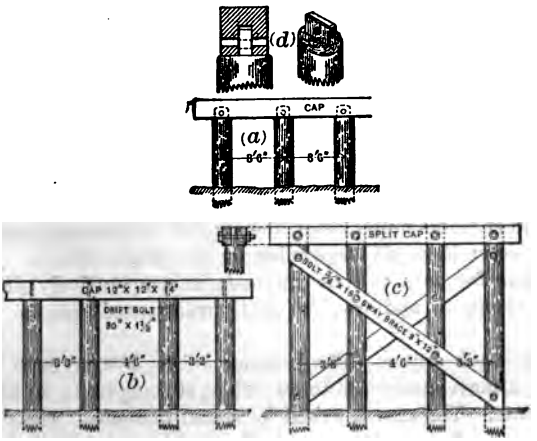


FIG. 68.

of 8 or 10 feet four piles may be used without sway-bracing, as in Fig. 68 (*b*), if the piles have a good bearing. For heights greater than 10 feet sway-bracing is generally necessary. The outside piles are frequently driven with a batter varying from 1 : 12 to 1 : 4.

Piles are made, if possible, from timber obtained in the vicinity of the work. Durability is the great requisite rather than strength, for almost any timber is strong enough (except as noted below) and will be suitable if it will resist rapid decay. The following list is quoted as being in the order of preference on account of durability:

1. Red cedar	5. White pine	9. White oak	12. Black oak
2. Red cypress	6. Redwood	10. Post-oak	13. Hemlock
3. Pitch-pine	7. Elm	11. Red oak	14. Tamarac
4. Yellow pine	8. Spruce		

Red-cedar piles are said to have an average life of 27 years with a possible maximum of 50 years, but the timber is rather

weak, and if exposed in a river to flowing ice or driftwood is apt to be injured. Under these circumstances oak is preferable, although its life may be only 13 to 18 years.

162. Methods of driving piles. The following are the principal methods of driving piles:

a. A hammer weighing 2000 to 3000 lbs. or more, sliding in guides, is drawn up by horse-power or a portable engine, and the "nippers" or "tongs," which hold the hammer, are released by a light trip-rope, which permits the hammer to fall *freely*.

b. The drum of a steam hoisting engine is gripped and released by some form of clutch. When the hammer has been raised and the clutch released, the hammer falls, dragging the rope and turning the drum. The energy of the blow is thus somewhat reduced, falsely increasing the apparent resistance. But the hammer works much faster, the number of blows per minute varying from 12 to 25, depending on the height of fall. The mechanism for both of these methods is comparatively simple and inexpensive, and can be easily transported into a new country.

c. Steam pile-drivers. The hammer weighs 3000 to 5000 lbs., and has a movement of 36 to 40 inches, striking 60 to 80 blows per minute. The ram is raised by steam pressure. The older types are single-acting the ram falling by gravity. Some later types are double-acting, the ram being forced down by steam pressure, which increases both the force and the rapidity of the blows. Very rapid blows, which do not allow time for the soil to settle around the pile between consecutive blows, are more effective and encounter less resistance. The destructive impact of a weight of 5000 lbs. falling only 3 feet is but a small part of that of 3000 lbs. falling 20 feet and there is less danger of overdriving and rupturing the pile.

d. Water jet. Whenever a sufficient supply of water is available, and especially when the soil is sandy, pile driving is facilitated by forcing water through a pipe driven into the ground near the desired location of the pile. Two or even three pipes per pile may be used. The former practice was to attach the pipes to the pile, but the pipes were often broken when withdrawn, and present practice keeps the jet independent of the pile, churning it up and down near the pile point by means of a rope running through a block on the driver leads and leading to a hand-winch or to a nigger-head on the engine. When the soil is very soft,

piles may be sunk, using the jet only, or with the aid of weights loaded on the pile, but a hammer is essential for harder ground, especially for driving the last few blows, the penetration of which will give a measure of the resisting power of the pile—see § 163. Although the jet has been employed using a hand-pump, effective work requires the use of a power pump, with a 2″ pipe for the jet, a pressure up to a maximum of about 200 lbs per square inch. and a flow of 250 to 500 gallons per minute. Many other details regarding pile driving are given in § 167.

Excessive driving frequently fractures the pile below the surface and thereby greatly weakens its bearing power. To prevent excessive "brooming" of the top of the pile, owing to the action of the hammer, the top should be protected by an iron ring fitted to the top of the pile. The "brooming" not only renders the driving ineffective and hence uneconomical, but vitiates the value of any test of the bearing power of the pile by noting the sinking due to a given weight falling a given distance. If the pile is so soft that brooming is unavoidable, the top should be adzed off frequently, and especially should it be done just before the final blows which are to test its bearing-power.

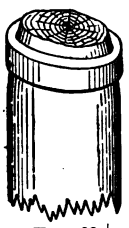


FIG. 69.

In a new country judgment and experience will be required to decide intelligently whether to employ a simple drop-hammer machine, operated by horse-power and easily transported but uneconomical in operation, or a more complicated machine working cheaply and effectively after being transported at greater expense.

163. Pile-driving formulæ. If R=the resistance of a pile, and s the set of the pile during the last blow, w the weight of the pile-hammer, and h the fall during the last blow, then we may state the approximate relation that $Rs=wh$, or $R=\frac{wh}{s}$. This is the basic principle of all rational formulæ, but the maximum weight which a pile will sustain after it has been driven some time is by no means the same as the resistance of the pile during the last blow. There are also many other modifying elements which have been variously allowed for in the many proposed formulæ. The formulæ range from the extreme of empirical simplicity to very complicated attempts to allow

properly for all modifying causes. As the simplest rule, the A. R. E. A. specifications require that the piles shall be driven until the pile will not sink more than $2\frac{1}{2}$ inches under five consecutive blows of a 3000-lb hammer falling 15 feet. The "*Engineering News* formula" * gives the safe load as $\frac{2wh}{s+1}$, in which w = weight of hammer, h = fall in *feet*, s = set of pile in *inches* under the last blow. This formula is derived from the above basic formula by calling the safe load $\frac{1}{6}$ of the final resistance, and by adding (arbitrarily) 1 to the final set (s) as a compensation for the extra resistance caused by the settling of earth around the pile between each blow. This formula is used only for ordinary hammer-driving. When the piles are driven by a steam pile-driver the formula becomes safe load $= \frac{2wh}{s+0.1}$. In the last formula the constant in the denominator is changed from $s+1$ to $s+0.1$. The constant (1.0 or 0.1) is supposed to allow, as before stated, for the effect of the extra resistance caused by the earth settling around the pile between each blow. The more rapid the blows the less the opportunity to settle and the less the proper value of the constant.

The above formulae have been given on account of their simplicity and their practical agreement with experience. Many other formulae have been proposed, the majority of which are more complicated and attempt to take into account the weight of the pile, resistance of the guides, etc. While these elements, as well as many others, have their influence, their effect is so overshadowed by the indeterminable effect of other elements—as, for example, the effect of the settlement of earth around the pile between blows—that it is useless to attempt to employ anything but a purely empirical formula.

For the most careful work, dependence is placed on the actual load which may be carried, without yielding, by test piles, driven on the site of the work. In § 167, par. 16–20, some Am. Rwy. Eng. Assoc. rules are quoted regarding the use of test piles.

Examples. 1. A pile was driven with an ordinary hammer weighing 2500 pounds until the sinking under five consecutive blows was $15\frac{1}{2}$ inches. The fall of the hammer during the last

* *Engineering News*, Nov. 17, 1892.

blows was 24 feet. What was the safe bearing power of the pile?

$$\frac{2wh}{s+1}=\frac{2\times2500\times24}{(\frac{1}{4}\times15.5)+1}=\frac{120000}{4.1}=29300 \text{ pounds.}$$

2. Piles are being driven into a firm soil with a steam pile-driver until they show a *safe* bearing power of 20 tons. The hammer weighs 5500 pounds and its fall is 40 inches. What should be the sinking under the final blow?

$$40000=\frac{2wh}{s+0.1}=\frac{2\times5500\times3.33}{s+0.1},$$

$$s=\frac{36667}{40000}-0.1=.81 \text{ inch.}$$

164. Pile-points and pile-shoes. Piles are generally sharpened to a blunt point. If the pile is liable to strike boulders, sunken logs, or other obstructions which are liable to turn the point, it is necessary to protect the point by some form of shoe. Several forms in cast iron have been used, also a wrought-iron shoe, having four "straps" radiating from the apex, the straps being nailed on to the pile, as shown in Fig. 70 (*b*). The cast-iron form shown in Fig. 70 (*a*) has a base cast around a drift-bolt. The recess on the top of the base receives the bottom of the pile and prevents a tendency to split the bottom of the pile or to force the shoe off laterally. See § 167, par. 23.

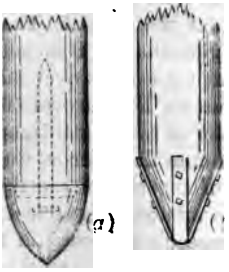


FIG. 70.

165. Details of design. No theoretical calculations of the strength of pile bents need be attempted on account of the extreme complication of the theoretical strains, the uncertainty as to the real strength of the timber used, the variability of that strength with time, and the insignificance of the economy that would be possible even if exact sizes could be computed. The caps are generally 14 feet long (for single track) with a cross-section 12″×12″ or 12″×14″. "Split caps" would consist

of two pieces 6″×12″. The sway-braces, never used for less heights than 6′, are made of 3″×12″ timber, and are spiked on with ⅝″ spikes 8″ long. The floor system will be the same as that described later for framed trestles.

166. **Specifications for timber piles** (Adopted 1909 by Amer. Rwy. Eng. Assoc.). 1. This grade [railroad heart grade] includes white, burr, and post oak; longleaf pine, Douglas fir, tamarack, Eastern white and red cedar, chestnut, Western cedar, redwood and cypress. 2. Piles shall be cut from sound trees; shall be close-grained and solid, free from defects, such as injurious ring shakes, large and unsound or loose knots, decay or other defects, which may materially impair their strength or durability. In Eastern red or white cedar a small amount of heart rot at the butt, which does not materially injure the strength of the pile, will be allowed. 3. Piles must be butt cut above the ground swell and have a uniform taper from butt to tip. Short bends will not be allowed. A line drawn from the center of the butt to the center of the tip shall lie within the body of the pile. 4. Unless otherwise allowed, piles must be cut when sap is down. Piles must be peeled soon after cutting. All knots shall be trimmed close to the body of the pile. 5. The minimum diameter at the tips of round piles shall be 9 inches for lengths not exceeding 30 feet; 8 inches for lengths over 30 feet but not exceeding 50 feet, and 7 inches for lengths over 50 feet The minimum diameter at one-quarter of the length from the butt shall be 12 inches and the maximum diameter at the butt 20 inches. 6. The minimum width of any side of the tip of a square pile shall be 9 inches for lengths not exceeding 30 feet; 8 inches for lengths over 30 feet but not exceeding 50 feet and 7 inches for lengths over 50 feet. The minimum width of any side at one-quarter of the length from the butt shall be 12 inches. 7. Square piles shall show at least 80% heart on each side at any cross-section of the stick, and all round piles shall show at least 10½ inches diameter of heart at the butt.

The second grade ("Railroad falsework grade") includes other woods which "will stand driving" and which cannot pass the specification for proportion of heart; also, they are usually not peeled.

167. **Pile driving—principles of practice.** As adopted by the Amer. Rwy. Eng. Assoc. 1911 and revised 1915.

1. A thorough exploration of the soil by borings, or preliminary

test piles, is the most important prerequisite to the design and construction of pile foundations.

2. Soil consisting wholly or chiefly of sand is most favorable to the use of the water-jet.

3. In harder soils containing gravel the use of the jet may be advantageous, if sufficient volume and pressure be provided.

4. In clay it may be economical to bore several holes in the soil with the aid of the jet before driving the pile, thus securing the accurate location of the pile, and its lubrication while being driven.

5. In general, the water-jet should not be attached to the pile, but handled separately.

6. Two jets will often succeed where one fails. In special cases a third jet extending a part of the depth aids materially in keeping loose the material around the pile.

7. Where the material is of such a porous character that the water from the jets may be dissipated and fail to come up in the immediate vicinity of the pile, the utility of the jet is uncertain, except for a part of the penetration.

8. A steam or drop hammer should be used in connection with the water-jet, and used to test the final rate of penetration.

9. The use of the water jet is one of the most effective means of avoiding injury to piles by overdriving.

10. There is danger from overdriving when the hammer begins to bounce. Overdriving is also indicated by the bending, kicking or staggering of the pile.

11. The brooming of the head of the pile dissipates a part, and in some cases all, of the energy due to the fall of the hammer.

12. The steam hammer is usually more effective than the drop hammer in securing the penetration of a wooden pile without injury, because of the shorter interval between blows.

13. Where shock to surrounding material is apt to prove detrimental to the structure, the steam hammer should always be used instead of the drop hammer. This is especially true in the case of sheet piling which is intended to prevent the passage of water. In some cases also the jet should not be used.

14. In general, the resistance of piles, penetrating soft material, depending solely upon skin friction, is materially increased after a period of rest. This period may be as short as fifteen minutes, and rarely exceeds twelve hours.

15. Where a pile penetrates muck or a soft yielding material and bears upon a hard stratum at its foot, its strength should be determined as a column or beam; omitting the resistance, if any, due to skin friction.

16. Unless the record of previous experience at the same site is available, the approximate bearing power may be obtained by loading test piles. The results of loading test piles should be used with caution, unless their condition is fairly comparable with that of the piles in the proposed foundation.

17. In case the piles in a foundation are expected to act as columns, the results of loading test piles should not be depended upon unless they are sufficient in number to insure their action in a similar manner; and unless they are stayed against lateral motion.

18. Before testing the penetration of a pile in a soft material where its bearing power depends principally, or wholly, upon skin friction, the pile should be allowed to rest for 24 hours after driving.

19. Where the resistance of piles depends mainly upon skin friction it is possible to diminish the combined strength, or bearing capacity, of a group of piles, by driving additional piles within the same area.

20. Where piles will foot in a hard stratum, investigation should be made to determine that this stratum is of sufficient depth and strength to carry the load.

21. Timber piles may be advantageously pointed, in some cases, to a 4-inch or 6-inch square at the end.

22. Piles should not be pointed when driven into soft material.

23. Shoes should be provided for piles when the driving is very hard, especially in riprap or shale. These shoes should be so constructed as to form an integral part of the pile.

24. The use of a cap is advantageous in distributing the impact of the hammer more uniformly over the head of the pile, as well as in holding it in position during driving.

168. Cost of pile trestles. The cost, per linear foot, of piling depends on the method of driving, the scarcity of suitable timber,

the price of labor, the length of the piles, and the amount of shifting of the pile-driver required. The cost of soft-wood piles varies from 8 to 15 cents per lineal foot, and the cost of oak piles varies from 10 to 30 cents per foot, according to the length, the longer piles costing more per foot. The total cost of putting the piles in place is so dependent on other items than the cost of driving, such as the cost of shifting the driver, getting the piles into the leaders, straightening and bracing them, leveling and nailing guide strips for sawing them off, and then the actual sawing, that there is a wide variation in the figures that are obtainable for the cost of such work. Of course the cost per pile of driving is also dependent on the total number of piles in the job. The cost per pile of placing a dozen piles for a single foundation would be far greater than the cost per pile for several hundred piles in one job. Among a large number of obtainable figures the average figure of $1.54 per pile for driving 1267 piles in 46 days is typical. Another quoted figure is $2.88 each, for driving 391 piles in 32 working days. On another job it cost $150 to drive thirty 30-foot piles, or an average of $5 each. In this case the piles cost $1.50 each or only 5 cents per lineal foot. The above cost figures are taken from Gillette's "Handbook of Cost Data" to which the student is referred for numerous examples of the cost of piles and pile-driving, as well as innumerable other cost analyses.

Specifications generally say that the piling will be paid for per lineal foot of piling *left in the work*. The wastage of the tops of piles sawed off is always something, and is frequently very large. Sometimes a small amount per foot of piling sawed off is allowed the contractor as compensation for his loss. This reduces the contractor's risk and possibly reduces his bid by an equal or greater amount than the extra amount actually paid him.

FRAMED TRESTLES

169. Typical design. A typical design for a framed trestle bent is given in Fig. 71. This represents, with slight variations of detail, the plan according to which a large part of the framed trestle bents of the country have been built—i.e., of those less than 20 or 30 feet in height, not requiring multiple story construction.

170. Joints. (a) The mortise-and-tenon joint is illustrated in

Fig. 71 and also in Fig. 68 (*a*). The tenon should be about

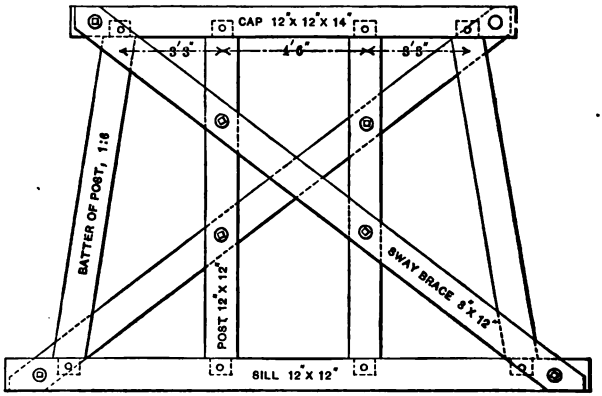


FIG. 71.

3″ thick, 8″ wide, and 5½″ long. The mortise should be cut a little deeper than the tenon. "Drip-holes" from the mortise to the outside will assist in draining off water that may accumulate in the joint and thus prevent the rapid decay that would otherwise ensue. These joints are very troublesome if a single post decays and requires renewal. It is generally required that the mortise and tenon should be thoroughly daubed with paint before putting them together. This will tend to make the joint water-tight and prevent decay from the accumulation and retention of water in the joint.

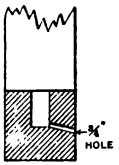


FIG. 72.

(b) **The plaster joint.** This joint is made by bolting and spiking a 3″×12″ plank on both sides of the joint. The cap and sill should be notched to receive the posts. Repairs are greatly facilitated by the use of these joints. This method has been used by the Delaware and Hudson Canal Co. [R. R.].

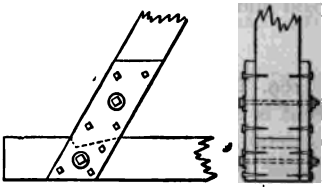


FIG. 73.

(c) **Iron plates.** An iron plate of the form shown in Fig. 74

(*b*) is bent and used as shown in Fig. 74 (*a*). Bolts passing through the bolt-holes shown secure the plates to the timbers and make a strong joint which may be readily loosened for repairs. By slight modifications in the design the method may be used for inclined posts and complicated joints.

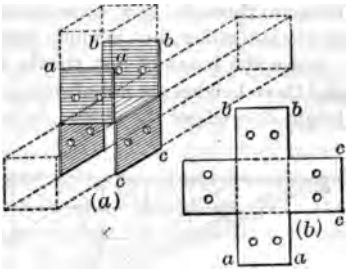


FIG. 74.—JOINT PLATES.

(d) **Split caps and sills.** These are described in § 161. Their advantages apply with even greater force to framed trestles.

(e) **Dowels and drift-bolts.** These joints facilitate cheap and rapid construction, but renewals and repairs are very difficult, it being almost impossible to extract a drift-bolt, which has been driven its full length, without splitting open the pieces containing it. Notwithstanding this objection they are extensively used, especially for temporary work which is not expected to be used long enough to need repairs.

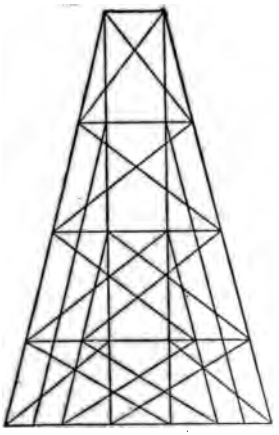


FIG. 75.

171. Multiple-story construction. Single-story framed trestle bents are used for heights up to 18 or 20 feet and exceptionally up to 30 feet. For greater heights some such construction as is illustrated in a skeleton design in Fig. 75 is used. By using split sills between each story and separate vertical and batter posts in each story, any piece may readily be removed and renewed if necessary. The height of these stories varies, in different designs, from 15 to 25 and even 30 feet. In some designs the structure of each story is independent of the stories above and below. This greatly facilitates both the original construction and subsequent repairs.

In other designs the verticals and batter-posts are made c tinuous through two consecutive stories. The structur somewhat stiffer, but is much more difficult to repair.

Since the bents of any trestle are usually of variable he and those heights are not always an even multiple of the unif height desired for the stories, it becomes necessary to make

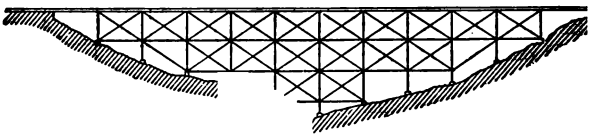


FIG. 76.—SKELETON ELEVATION OF TRESTLE.

upper stories of uniform height and let the odd amount go to lowest story, as shown in Figs. 75 and 76.

172. **Span.** The shorter the span the greater the numbe trestle bents; the longer the span the greater the required strei of the stringers supporting the floor. Economy demands adoption of a span that shall make the sum of these requ

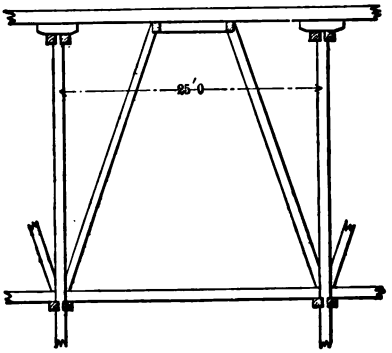


FIG. 77.—KNEE-BRACES FOR LONG-SPAN STRINGERS.

ments a minimum. The higher the trestle the greater the of each bent, and the greater the span that would be justifia Nearly all trestles have bents of variable height, but the adv tage of employing uniform standard sizes is so great that m

roads use the same span and sizes of timber not only for the panels of any given trestle, but also for all trestles regardless of height. The spans generally used vary from 10 to 16 feet. The Norfolk and Western R. R. uses a span of 12′ 6″ for all single-story trestles, and a span of 25′ for all multiple-story trestles. The stringers are the same in both cases, but when the span is 25 feet, knee-braces are run from the sill of the first story below to near the middle of each set of stringers. These knee-braces are connected at the top by a "straining-beam" on which the stringers rest, thus supporting the stringer in the center and virtually reducing the span about one-half.

173. **Foundations.** (a) **Piles.** Piles are frequently used as a foundation, as in Fig. 78, particularly in soft ground, and also for temporary structures. These foundations are cheap, quickly constructed, and are particularly valuable when it is financially necessary to open the road for traffic as soon as possible and with the least expenditure of money; but there is the disadvantage of inevitable decay within a few years unless the piles are chemically treated, as will be discussed later. Chemical treatment, however, increases the cost so that such a foundation would often cost more than a foundation of stone. A pile should be driven under each post as shown in Fig. 78.

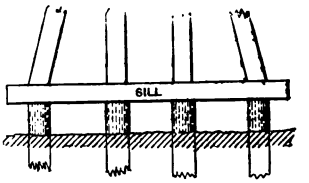


FIG. 78.—PILE FOUNDATION.

(b) **Mud-sills.** Fig. 79 illustrates the use of mud-sills as built by the Louisville and Nashville R. R. Eight blocks 12″×12″×6′ are used under each bent. When the ground is very soft, two additional timbers (12″×12″×length of bent-sill), as shown by the dotted lines, are placed underneath. The number required evidently depends on the nature of the ground.

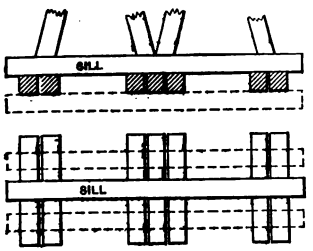


FIG. 79.—MUD-SILL FOUNDATION.

(c) **Stone foundations.** Stone foundations are the best and the most expensive. For very high trestles the Norfolk and

Western R. R. employs foundations as shown in Fig. 80, the walls being 4 feet thick. When the height of the trestle is 72 feet or less (the plans requiring for 72′ in height a foundation-wall 39′ 6″ long) the foundation is made continuous. The sill

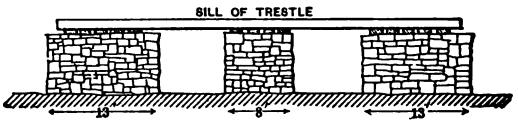


FIG. 80.—MASONRY TRESTLE FOUNDATION.

of the trestle should rest on several short lengths of 3″×12″ plank laid transverse to the sill on top of the wall.

174. **Longitudinal bracing.** This is required to give the structure longitudinal stiffness and also to reduce the columnar length of the posts. This bracing generally consists of horizontal "waling-strips" and diagonal braces. Sometimes the braces are placed wholly on the outside posts unless the trestle is very high. For single-story trestles the P. R. R. employs the "laced" system, i.e., a line of posts joining the cap of one bent with the sill of the next, and the sill of that bent with the cap of the next. Some plans employ braces forming an × in alternate panels. Connecting these braces in the center more than doubles their columnar strength. Diagonal braces, when bolted to posts, should be fastened to them as near the ends of the posts as possible. The sizes employed vary largely, depending on the clear length and on whether they are expected to act by tension or compression. 3″×12″ planks are often used when the design would require tensile strength only, and 8″×8″ posts are often used when compression may be expected.

175. **Lateral bracing.** Several of the more recent designs of trestles employ diagonal lateral bracing between the caps of adjacent bents. It adds greatly to the stiffness of the trestle and better maintains its alignment. 6″×6″ posts, forming an × and connected at the center, will answer the purpose.

176. **Abutments.** When suitable stone for masonry is at hand and a suitable subsoil for a foundation is obtainable without too much excavation, a masonry abutment will be the best. Such an abutment would probably be used when masonry footings for trestle bents were employed (§ 173, *c*).

Another method is to construct a "crib" of 10″×12″ timber,

laid horizontally, drift-bolted together, securely braced and embedded into the ground. Except for temporary construction such a method is generally objectionable on account of rapid decay.

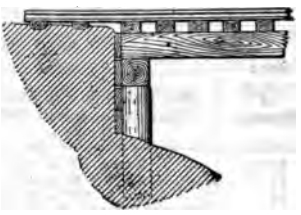


FIG. 81.—ABUTMENT PILE BENT.

Another method, used most commonly for pile trestles, and for framed trestles having pile foundations (§ 173, *a*), is to use a pile bent at such a place that the natural surface on the *up-hill* side is not far below the cap, and the thrust of the material, filled in to bring the surface to grade, is insignificant. 3″×12″ planks are placed behind the piles, cap, and stringers to retain the filled material.

FLOOR SYSTEMS.

177. Stringers. The general practice is to use two, three, and even four stringers under each rail. Sometimes a stringer is placed under each guard-rail. Generally the stringers are made of two panel lengths and laid so that the joints alternate. A few roads use stringers of only one panel length, but this practice is strongly condemned by many engineers. The stringers should be separated to allow a circulation of air around them and prevent the decay which would occur if they were placed close together. This is sometimes done by means of 2″ planks, 6′ to 8′ long, which are placed over each trestle bent. Several bolts, passing through all the stringers forming a group and through the separators, bind them all into one solid construction. Cast-iron "spools" or washers, varying from 4″ to ¾″ in length (or thickness), are sometimes strung on each bolt so as to separate the stringers. Sometimes washers are used between the separating planks and the stringers, the object of the separating planks then being to bind the stringers, especially abutting stringers, and increase their stiffness.

The most common size for stringers is 8″×16″. The Pennsylvania Railroad varies the width, depth, and number of stringers under each rail according to the clear span. It may be noticed that, assuming a uniform load per running foot, both the pressure per square inch at the ends of the stringers (the

caps having a width of 12") and also the stress due to transverse strain are kept *approximately* constant for the variable gross load on these varying spans.

Span c. to c. of bents.	Y. P. stringers under each rail.	
	For H6b and E3d engines [Max. mom. about 200,000 ft.-lbs.].	For heavier than H6b and E3d engines.
10 ft.	2 pcs. 10"×16"	2 pcs. 10"×16"
12	3 " 8"×16"	3 " 10"×16"
14	3 " 10"×16"	Steel stringers

178. Corbels. A corbel (in trestle-work) is a stick of timber (perhaps two placed side by side), about 3' to 6' long, placed underneath and along the stringers and resting on the cap. There are strong prejudices for and against their use, and a corresponding diversity in practice. They are bolted to the stringers and thus stiffen the joint. They certainly reduce the objectionable crushing of the fibers at each end of the stringer, but if the corbel is no wider than the stringers, as is generally the case, the area of pressure between the corbels and the cap is

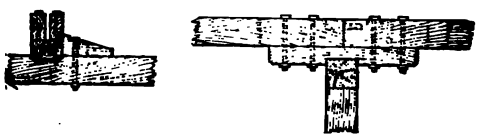


FIG. 82.—CORBEL.

no greater and the pressure per square inch on the cap is no less than the pressure on the cap if no corbels were used. If the corbels and cap are made of hard wood, as is recommended by some, the danger of crushing is lessened, but the extra cost and the frequent scarcity of hard wood, and also the extra cost and labor of using corbels, may often neutralize the advantages obtained by their use.

179. Guard-rails. These are frequently made of 6"×8" stuff, notched 1" for each tie. The sizes vary up to 8"×8", and the depth of notch from ¾" to 1½". They are generally bolted to every third or fourth tie. It is frequently specified that they shall be made of oak, white pine, or yellow pine. The joints are made over a tie, by halving each piece, as illustrated in Fig. 83. The joints on opposite sides of the trestle should be "stag-

gered." Some roads fasten every tie to the guard-rail, using a bolt, a spike, or a lag-screw.

Guard-rails were originally used with the idea of preventing the wheels of a derailed truck from running off the ends of the ties. But it has been found that an outer guard-rail alone (without an inner guard-rail) becomes an actual element of danger, since it has frequently happened that a derailed wheel has caught on the outer guard-rail, thus causing the truck to slew around

FIG. 83.—GUARD TIMBER.

and so produce a dangerous accident. The true function of the *outside* guard-rail is thus changed to that of a tie-spacer, which keeps the ties from spreading when a derailment occurs. The inside guard-rail generally consists of an ordinary steel rail spiked about 10 inches inside of the running rail. These inner guard-rails should be bent inward to a point in the center of the track about 50 feet beyond the end of the bridge or trestle. If the inner guard-rails are placed with a clear space of 10 inches inside the running rail, the outer guard-rails should be *at least* 6′ 10″ apart. They are generally much farther apart than this.

180. Ties on trestles. If a car is derailed on a bridge or trestle, the heavily loaded wheels are apt to force their way between the ties by displacing them unless the ties are closely spaced and fastened. The clear space between ties is generally equal to or less than their width. Occasionally it is a little more than their width. 6″×8″ ties, spaced 14″ to 16″ from center to center, are most frequently used. The length varies from 9′ to 12′ for single track. They are generally notched ½″ deep on the under side where they rest on the stringers. Oak ties are generally required even when cheaper ties are used on the other sections of the road. Usually every third or fourth tie is bolted to the stringers. When stringers are placed underneath the guard-rails, bolts are run from the top of the guard-rail to the under side of the stringer. The guard-rails thus hold down the whole system of ties, and no direct fastening of the ties to the stringers is needed.

181. Superelevation of the outer rail on curves. The location of curves on trestles should be avoided if possible, especially when the trestle is high. Serious additional strains are intro-

duced especially when the curvature is sharp or the speed high. Since such curves are sometimes practically unavoidable, it is necessary to design the trestle accordingly. If a train is stopped on a curved trestle, the action of the train on the trestle is evidently vertical. If the train is moving with a considerable velocity, the resultant of the weight and the centrifugal action is a force somewhat inclined from the vertical. Both of these conditions may be expected to exist at times. If the *axis* of the system of posts is vertical (as illustrated in methods *a*, *b*, *c*, *d*, and *e*), any lateral force, such as would be produced by a moving train, will tend to rack the trestle bent. If the stringers are set vertically, a centrifugal force likewise tends to tip them sidewise. If the axis of the system of posts (or of the stringers) is inclined so as to coincide with the pressure of the train on the trestle when the train is moving at its normal velocity, there is no tendency to rack the trestle when the train is moving at that velocity, but there will be a tendency to rack the trestle or twist the stringers when the train is stationary. Since a moving train is usually the normal condition of affairs, as well as the condition which produces the maximum stress, an inclined axis is evidently preferable from a theoretical standpoint; but whatever design is adopted, the trestle should evidently be sufficiently cross-braced for either a moving or a stationary load, and any proposed design must be studied as to the effect of *both* of these conditions. Some of the various methods of securing the requisite superelevation may be described as follows:

(a) Framing the outer posts longer than the inner posts, so that the cap is inclined at the proper angle; axis of posts vertical. (Fig. 84.) The method requires more work in framing the trestle, but simplifies subsequent track-laying and maintenance, unless it should be found that the superelevation adopted is unsuitable, in which case it could be corrected by one of the other methods given below. The stringers tend to twist when the train is stationary.

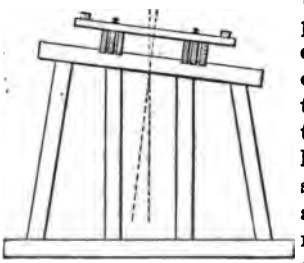


FIG. 84.

(b) Notching the cap so that the stringers are at a different

elevation. (Fig. 85.) This weakens the cap and requires that all ties shall be notched to a bevelled surface to fit the stringers, which also weakens the ties. A centrifugal force will tend to twist the stringers and rack the trestle.

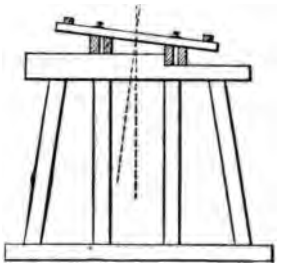


FIG. 85.

(c) **Placing wedges underneath the ties at each stringer.** These wedges are fastened with two bolts. Two or more wedges will be required for each tie. The additional number of pieces required for a long curve will be immense, and the work of inspection and keeping the nuts tight will greatly increase the cost of maintenance.

(d) **Placing a wedge under the outer rail at each tie.** This requires but one extra piece per tie. There is no need of a wedge under the inner tie in order to make the rail normal to the tread. The resulting inward inclination is substantially that produced by some forms of rail-chairs or tie-plates. The spikes (a little longer than usual) are driven through the wedge into the tie. Sometimes "lag-screws" are used instead of spikes. If experience proves that the superelevation is too much or too little, it may be changed by this method with less work than by any other.

(e) **Corbels of different heights.** When corbels are used (see § 178) the required inclination of the floor system may be obtained by varying the depth of the corbels.

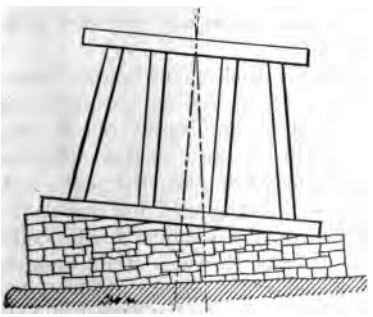


FIG. 86.

(f) **Tipping the whole trestle.** This is done by placing the trestle on an inclined foundation. If very much inclined, the trestle bent must be secured against the possibility of slipping sidewise, for the slope would be considerable with a sharp curve, and the

vibration of a moving train would reduce the coefficient of friction to a comparatively small quantity.

(g) **Framing the outer posts longer.** This case is identical with case (*a*) except that the axis of the system of posts is inclined, as in case (*f*), but the sill is horizontal.

The above-described plans will suggest a great variety of methods which are possible and which differ from the above only in minor details.

182. Protection from fire. Trestles are peculiarly subject to fire, from passing locomotives, which may not only destroy the trestle, but perhaps cause a terrible disaster. This danger is sometimes reduced by placing a strip of galvanized iron along the top of each set of stringers and also along the tops of the caps. Still greater protection was given on a long trestle on the Louisville and Nashville R. R. by making a solid flooring of timber, covered with a layer of ballast on which the ties and rails were laid as usual.

Barrels of water should be provided and kept near all trestles, and on very long trestles barrels of water should be placed every two or three hundred feet along its length. A place for the barrels may be provided by using a few ties which have an extra length of about four feet, thus forming a small platform, which should be surrounded by a railing. The track-walker should be held accountable for the maintenance of a supply of water in these barrels, renewals being frequently necessary on account of evaporation. Such platforms should also be provided as REFUGE-BAYS for track-walkers and trackmen working on the trestle. On very long trestles such a platform is sometimes provided with sufficient capacity for a hand-car.

183. Timber. Any strong durable timber may be used when the choice is limited, but oak, pine, or cypress are preferred when obtainable. When all of these are readily obtainable, the various parts of the trestle will be constructed of different kinds of wood—the stringers of long-leaf pine, the posts and braces of pine or red cypress, and the caps, sills, and corbels (if used) of white oak. The use of oak (or a similar hard wood) for caps, sills, and corbels is desirable because of its greater strength in resisting crushing across the grain, which is the critical test for these parts. There is no physiological basis to the objection, sometimes made, that different species of timber, in contact with each other, will rot quicker than if only one

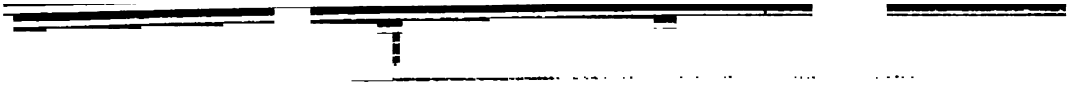
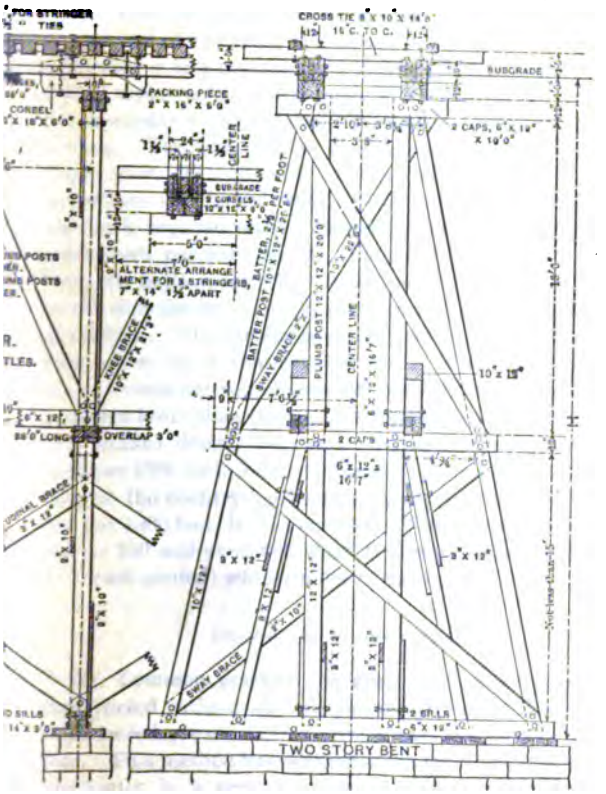
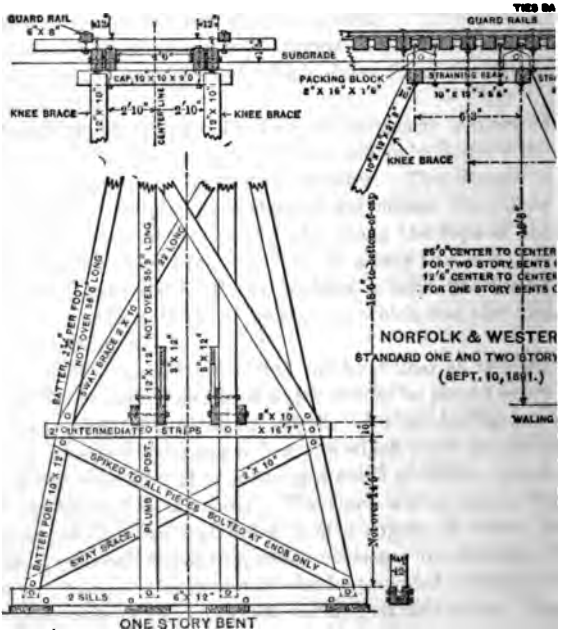


PLATE 11.


CROSS TIE 8" X 10" X 14'-0"
16" C. TO C.
SUBGRADE
PACKING PIECE
2" X 16" X 6'-0"
CENTER LINE
2 CAPS, 6" X 12" X 10'-0"
ALTERNATE ARRANGEMENT FOR 8 STRINGERS, 7" X 14" 1½" APART
BATTER POST 12" X 12" X 20'-0"
PLUMB POST 12" X 12" X 20'-0"
KNEE BRACE
SWAY BRACE
2 CAPS
OVERLAP 3'-0"
2 SILLS
TWO STORY BENT



$(H \div 8.4) + 1'1'' =$ LENGTH OF SILL.

H — 1'11" = LENGTH OF 12" X 12" TIMBER REQUIRED FOR PLUMB POSTS, SINGLE SER

H — 21'11" = " " " " " " " " IN LOWER STORY, DOUBLE "

H — 5'6" = " " PLUMB POSTS BETWEEN SHOULDERS, SINGLE "

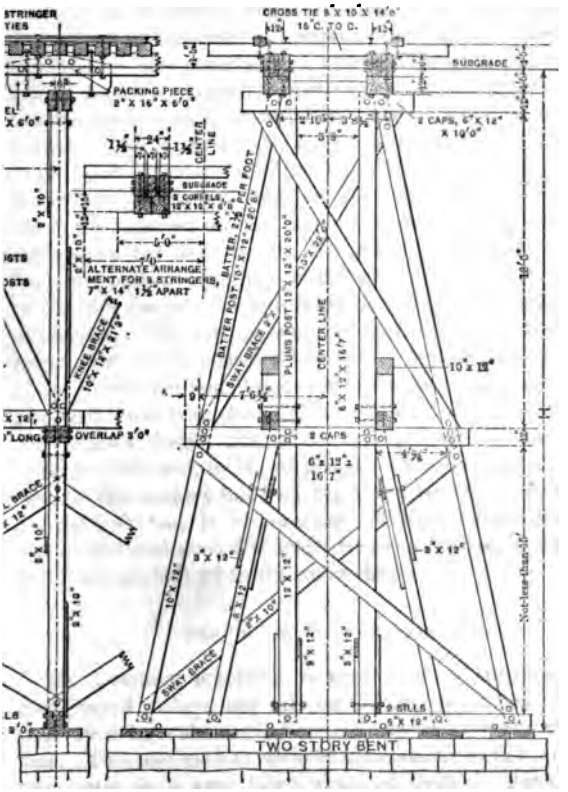
H — 22'11" = " " " " " " " " " " DOUBLE "

(LENGTH OF PLUMB POST X 1.081) + 3" = LENGTH OF BATTER POST, EXCEPT IN 8" X 12" INTERMEDIATE BATTER POSTS WHERE ADD 9" INSTEAD OF 3"

WITH ALTERNATIVE ARRANGEMENT OF STRINGERS, ADD 2" TO LENGTHS GIVEN BY ABOVE FORMULAS.

(To face page 216.)

PLATE II.


CROSS TIE 8"X 10"X 14'0"
16" C. TO C.
SUBGRADE
PACKING PIECE
2" X 16" X 6'0"
2 CAPS, 6"X 12"
X 10'0"
CENTER LINE
SUBGRADE
2 CORBELS
10"X 10"X 6'0"
BATTER 2½" PER FOOT
ALTERNATE ARRANGE-
MENT FOR 3 STRINGERS,
7"X 14" 1'6" APART
KNEE BRACE
10"X 14"X 41'0"
BATTER POST 10"X 12"X 20'0"
SWAY BRACE 2"X
PLUMB POST 12"X 12"X 20'0"
CENTER LINE
8"X 12"X 16'7"
10"X 12"
2 CAPS
OVERLAP 3'0"
3"X 12"
3"X 12"
10"X 12"
8"X 12"
SWAY BRACE
2 SILLS
8"X 10"
TWO STORY BENT

ing approximate percentages of standard stringer size, of 12×12-inch stuff, 10×10-inch stuff, etc., and a liberal proportion of 3- or 4-inch plank, all lengths thrown in. The 12×12-inch stuff, etc., is ordered all lengths, from a certain specified length up. In case of a wreck, washout, burn-out, or sudden call for a trestle to be completed in a stated time, it is much more economical and practical to order a certain number of carloads of 'trestle stuff' to the ground and there to select piece after piece as fast as needed, dependent only upon the length of stick required. When there is time to make the necessary surveys of the ground and calculations of strength, and to wait for a special bill of timber to be cut and delivered, the use of different sizes for posts in a structure would be warranted to a certain extent." * For new construction, when there is generally sufficient time to design and order the proper sizes, such wastefulness is less excusable, and under any conditions it is both safer and more economical to prepare *standard designs* which can be made applicable to varying conditions and which will at the same time utilize as much of the strength of the timber as can be depended on. In the following sections will be given the elements of the preparation of such standard designs, which will utilize uniform sizes with as little waste of timber as possible. It is *not* to be understood that special designs should be made for each individual trestle.

186. **Required elements of strength.** The *stringers* of trestles are subject to transverse strains, to crushing across the grain at the ends, and to shearing along the neutral axis. The strength of the timber must therefore be computed for all these kinds of stress. *Caps* and *sills* will fail, if at all, by crushing across the grain; although subject to other forms of stress, these could hardly cause failure in the sizes usually employed. There is an apparent exception to this: if piles are improperly driven and an uneven settlement subsequently occurs, it may have the effect of transferring practically all of the weight to two or three piles, while the *cap* is subjected to a severe transverse strain which may cause its failure. Since such action is caused generally by avoidable errors of construction it may be considered as abnormal, and since such a failure will generally occur by a *gradual* settlement, all danger may be avoided by reasonable

* From "Economical Designing of Timber Trestle Bridges."

care in inspection. *Posts* must be tested for their columnar strength. These parts form the bulk of the trestle and are the parts which can be definitely designed from known stresses. The stresses in the bracing are more indefinite, depending on indeterminate forces, since the inclined posts take up an unknown proportion of the lateral stresses, and the design of the bracing may be left to what experience has shown to be safe, without involving any large waste of timber.

187. Strength of timber. Until recently tests of the strength of timber have generally been made by testing small, selected, well-seasoned sticks of "clear stuff," free from knots or imperfections. Such tests would give results so much higher than the vaguely known strength of large unseasoned "commercial" timber that very large factors of safety were recommended—factors so large as to detract from any confidence in the whole theoretical design. Recently the U. S. Government has been making a thoroughly scientific test of the strength of full-size timber under various conditions as to seasoning, etc. The work has been so extensive and thorough as to render possible the economical designing of timber structures.

One important result of the investigation is the determination of the great influence of the moisture in the timber and the law of its effect on the strength. It has been also shown that timber soaked with water has substantially the same strength as green timber, even though the timber had once been thoroughly seasoned. Since trestles are exposed to the weather they should be designed on the basis of using green timber. It has been shown that the strength of green timber is very regularly about 55 to 60% of the strength of timber in which the moisture is 12% of the dry weight, 12% being the proportion of moisture usually found in timber that is protected from the weather but not heated, as, *e.g.*, the timber in a barn. Since the moduli of rupture have all been reduced to this standard of moisture (12%), if we take *one-eighth* of the rupture values, it still allows a factor of safety of about five, even on green timber. In Table XX there are quoted the values taken from the U. S. Government reports on the strength of timber, the tests probably being the most thorough and reliable that were ever made.

In Table XXI are given the "working unit stresses for structural timber, expressed in pounds per square inch," as recommended by the committee on "Wooden Bridges and Trestles,"

of the American Railway Engineering Association. The report was presented at their tenth annual convention, held in Chicago, in March, 1909.

TABLE XX. MODULI OF RUPTURE FOR VARIOUS TIMBERS.
[12% moisture.]
(Condensed from U. S. Forestry Circular, No. 15.)

No.	Species.	Weight per cubic foot.	Cross-bending.		Crushing endwise.	Crushing across the grain.	Shearing along the grain.
			Ultimate Strength.	Modulus of Elasticity.			
1	Long-leaf pine....	38	12 600	2 070 000	8000	1180	700
2	Cuban "	39	13 600	2 370 000	8700	1220	700
3	Short-leaf "	32	10 100	1 680 000	6500	960	700
4	Loblolly "	33	11 300	2 050 000	7400	1150	700
5	White "	24	7 900	1 390 000	5400	700	400
6	Red "	31	9 100	1 620 000	6700	1000	500
7	Spruce "	39	10 000	1 640 000	7300	1200	800
8	Bald cypress.....	29	7 900	1 290 000	6000	800	500
9	White cedar.....	23	6 300	910 000	5200	700	400
10	Douglas spruce....	32	7 900	1 680 000	5700	800	500
11	White oak.....	50	13 100	2 090 000	8500	2200	1000
12	Overcup "	46	11 300	1 620 000	7300	1900	1000
13	Post "	50	12 300	2 030 000	7100	3000	1100
14	Cow "	46	11 500	1 610 000	7400	1900	900
15	Red "	45	11 400	1 970 000	7200	2300	1100
16	Texan "	46	13 100	1 860 000	8100	2000	900
19	Willow "	45	10 400	1 750 000	7200	1600	900
20	Spanish "	46	12 000	1 930 000	7700	1800	900
21	Shagbark hickory..	51	16 000	2 390 000	9500	2700	1100
27	Pignut " ..	56	18 700	2 730 000	10900	3200	1200
28	White elm.....	34	10 300	1 540 000	6500	1200	800
29	Cedar "	46	13 500	1 700 000	8000	2100	1300
30	White ash.....	39	10 800	1 640 000	7200	1900	1100

188. **Loading.** As shown in § 172, the span of trestles is always small, is generally 14 feet, and is never greater than 18 feet except when supported by knee-braces. The greatest load that will ever come on any one span will be the concentrated loading of the drivers of a very heavy locomotive. With spans of 14 feet or less it is impossible for even the four pairs of drivers to be on the same span at once. The weight of the rails, ties, and guard-rails should be added to obtain the total load on the stringers, and the weight of these, plus the weight of the stringers, ould be added to obtain the pressure on the caps or corbels

Kind of Timber.	Bending.			Shearing.				Compression.						Ratio of length of stringer to depth.
	Extreme fiber stress.		Modulus of elasticity.	Parallel to grain.		Longitudinal shear in beams		Perpendicular to grain.		Parallel to grain.		For colmns. under 15 diams. safe stress.	Formulas for working stress in long column over 15 diams.	
	Aver. ultimate.	W'rking stress	Average.	Aver. ultimate.	W'rking stress	Aver. ultimate.	Working stress.	Elastic limit.	Working stress.	Aver. ultimate.	Working stress.			
Douglas fir.....	6100	1200	1,510,000	690	170	270	110	630	310	3600	1200	900	$1200\left(1-\frac{L}{60D}\right)$	10
Long-leaf pine...	6500	1300	1,610,000	720	180	300	120	520	260	3800	1300	980	$1300\left(1-\frac{L}{60D}\right)$	10
Short-leaf pine..	5600	1100	1,480,000	710	170	330	130	340	170	3400	1100	830	$1100\left(1-\frac{L}{60D}\right)$	10
White pine.....	4400	900	1,130,000	400	100	180	70	290	150	3000	1000	750	$1000\left(1-\frac{L}{60D}\right)$	10
Spruce.....	4800	1000	1,310,000	600	150	170	70	370	180	3200	1100	830	$1100\left(1-\frac{L}{60D}\right)$	.....
Norway pine....	4200	800	1,190,000	590	130	250	100	.....	150	2600*	800	600	$800\left(1-\frac{L}{60D}\right)$	.....
Tamarack.....	4600	900	1,220,000	670	170	260	100	.....	220	3200*	1000	750	$1000\left(1-\frac{L}{60D}\right)$	.....
Western hemlock	5800	1100	1,480,000	630	160	270*	100	440	220	3500	1200	900	$1200\left(1-\frac{L}{60D}\right)$	.....
Redwood.....	5000	900	800,000	300	80	.....	.....	400	150	3300	900	680	$900\left(1-\frac{L}{60D}\right)$	.....
Bald cypress....	4800	900	1,150,000	500	120	.....	.....	340	170	3900	1100	830	$1100\left(1-\frac{L}{60D}\right)$	.....
Red cedar.....	4200	800	860,000	.....	.....	.....	.....	470	230	2800	900	680	$900\left(1-\frac{L}{60D}\right)$	.....
White oak.....	5700	1100	1,150,000	840	210	270	110	920	450	3500	1300	980	$1300\left(1-\frac{L}{60D}\right)$	12

Note.—These unit stresses are for a green condition of timber and are to be used without increasing the live load stresses for impact. * Partially air-dry.

L = length in inches.
D = least side in ins.

These working stresses are for railroad bridges and trestles. For highway bridges and trestles increase the figures by 25 per cent. For buildings, etc., when protected from weather and free from impact, increase them 50 per cent. To compute deflection, under long-continued loading, use 50 per cent of modulus of elasticity.

This dead load is almost insignificant compared with the live load and may be included with it. The weight of rails, ties, etc., may be estimated at 240 pounds per foot. To obtain the weight on the caps the weight of the stringers must be added, which depends on the design and on the weight per cubic foot of the wood employed. But as the weight of the stringers is comparatively small, a considerable percentage of variation in weight will have but an insignificant effect on the result. Disregarding all refinements as to actual dimensions, the ordinary maximum loading for standard-gauge railroads may be taken as that due to four driving-axles, spaced 5′ 0″ apart and giving a pressure of 40000 pounds per axle. This should be increased to 54000 pounds per axle (same spacing) for the heaviest traffic. On the basis of 40000 pounds per axle or 20000 pounds per wheel the following results have been computed: This loading is assumed to allow for impact.

STRESSES ON VARIOUS SPANS DUE TO MOVING LOADS OF 20000 POUNDS, SPACED 5′ 0″ APART, WITH 120 POUNDS PER FOOT OF DEAD LOAD.

Span in feet.	Max. moment, ft. lbs.	Max. shear.	Max. load on one cap under one rail.
10	51 500	30 600	41 200
12	82 160	35 720	49 440
14	112 940	39 410	57 680
16	123 840	43 460	65 920
18	164 860	47 747	75 160

Although the dead load does not vary in proportion to the live load, yet, considering the very small influence of the dead load, there will be no appreciable error in assuming the corresponding values, for a load of 54000 lbs. per axle, to be $\frac{4}{5}$ of those given in the above tabulation.

189. **Factors of safety.** The most valuable result of the government tests is the knowledge that under given moisture conditions the strength of various species of sound timber is not the variable uncertain quantity it was once supposed to be, but that its strength can be relied on to a comparatively close percentage. This confidence in values permits the employment of lower factors of safety than have heretofore been permissible. Stresses, which when excessive would result in immediate destruction, such as cross-breaking and columnar stresses, should be allowed a higher factor of safety—say 6 or 8 for green timber. Other stresses, such as crushing across the grain and shearing along the

neutral axis, which will be apparent to inspection before it is dangerous, may be allowed lower factors—say 3 to 5.

190. Design of stringers. The strength of rectangular beams of equal width varies as the square of the depth; therefore deep beams are the strongest. On the other hand, when any cross-sectional dimension of timber much exceeds 12″ the cost is much higher per M, B. M., and it is correspondingly difficult to obtain thoroughly sound sticks, free from wind-shakes, etc. Wind-shakes especially affect the shearing strength. Also, if the required transverse strength is obtained by using high narrow stringers, the area of pressure between the stringers and the cap may become so small as to induce crushing across the grain. This is a very common defect in trestle design. As already indicated in § 172, the span should vary roughly with the average height of the trestle, the longer spans being employed when the trestle bents are very high, although it is usual to employ the same span throughout any one trestle.

To illustrate, if we select a span of 14 feet, the load on one cap will be 57680 lbs. If the stringers and cap are made of long-leaf yellow pine, the allowable value, according to Table XXI, for "compression across the grain" is 260 pounds per square inch; this will require 222 square inches of surface. If the cap is 12″ wide, this will require a width of 18.5 inches, or say 2 stringers under each rail, each 9 inches wide. For rectangular beams.

$$\text{Moment} = \tfrac{1}{6} R' b h^2.$$

Using for R' the safe value 1300 lbs. per square inch, we have

$$112940 \times 12 = \tfrac{1}{6} \times 1300 \times 18 \times h^2,$$

from which $h = 18''.7$. If desired, the width may be increased to 10″ and the depth correspondingly reduced, which will give similarly $h = 17''.7$ or say 18″. This shows that two beams, 10″×18″, under each rail will stand the transverse bending and have more than enough area for crushing.

The shear per square inch will equal

$$\frac{3}{2} \frac{\text{total shear}}{\text{cross-section}} = \frac{3}{2} \frac{39410}{2 \times 10 \times 18} = 164 \text{ lbs. per sq. inch.}$$

This is higher than the recommended working value. The combination suggested in § 177, viz., 3 beams 10″×16″ for 14 feet span, gives a far safer value. Considering that wooden beams,

tested to destruction, usually fail by shearing, the three-beam combination is safer.

The deflection should be computed to see if it exceeds the somewhat arbitrary standard of $\frac{1}{200}$ of the span. The deflection for *uniform loading* is

$$\Delta = \frac{5Wl^3}{32bh^3E},$$

in which l = length in inches;

W = total load, assumed as uniform = 57680;
E = modulus of elasticity, given as 1610000 lbs.

per sq. in. for long-leaf pine, according to Table XXI. Then

$$\Delta = \frac{5 \times 57680 \times 168^3}{32 \times 30 \times 16^3 \times 1610000} = 0''.216$$

$$\frac{1}{200} \times 168'' = 0''.84,$$

so that the calculated deflection is well within the limit. Of course the loading is not strictly uniform, but even with a liberal allowance the deflection is still safe.

For the heaviest practice (65000 lbs. per axle) these stringer dimensions must be correspondingly increased.

191. Design of posts. Four posts are generally used for single-track work. The inner posts are usually braced by the cross-braces, so that their columnar strength is largely increased; but as they are apt to get more than their share of work, the advantage is compensated and they should be treated as unsupported columns for the total distance between cap and sill in simple bents, or for the height of stories in multiple-story construction. The caps and sills are assumed to have a width of 12". It facilitates the application of bracing to have the columns of the same width and vary the other dimension as required.

Unfortunately the experimental work of the U. S. Government on timber testing has not yet progressed far enough to establish unquestionably a general relation between the strength of long columns and the crushing strength of short blocks. The

following formula has been suggested, but it cannot be considered as established:

$$f = F \times \frac{700 + 15c}{700 + 15c + c^2}, \text{ in which}$$

f = allowable working stress per sq. in for long columns;
F = " " " " " " " " short blocks;
$c = \frac{l}{d}$;
l = length of column in inches;
d = least cross-sectional dimensions in inches.

The formula recommended by the A. R. E. A. is found in Table XXI. For all columns of which the length is less than 15 times the least diameter, a uniform unit stress is recommended. For longer columns, a unit stress is multiplied by the factor $(1 - l \div 60d)$, which is always less than unity. For the above case, $l = 240$ and $d = 12$, and the factor $= .667$, which, multiplied by 1300, gives a unit stress of 867 lbs. per square inch for a long-leaf yellow pine column of these dimensions.

$867 \times 144 = 124848$ lbs., the *working load* for *each* post. This is more than the total load on one trestle bent and illustrates the usual great waste of timber. Making the post $8'' \times 12''$ and calculating similarly, we have $f = 650$, and the working load per column is $650 \times 96 = 62400$ lbs. As considerable must be allowed for "weathering," which destroys the strength of the outer layers of the wood, and also for the dynamic effect of the live load, $8'' \times 12''$ may not be too great, but it is certainly a safe dimension, considered as a column. One method of allowing for weathering is to disregard the outer half-inch on all sides of the post, i.e., to calculate the strength of a post one inch smaller in each dimension than the post actually employed. On this basis an $8'' \times 12'' \times 20'$ post, computed as a $7'' \times 11''$ post, would have a *safe* columnar strength of 556 lbs. per square inch. With an area of 77 square inches, this gives a working load of 42812 lbs. for *each post*, or 171248 lbs. for the four posts. Considering that 115360 lbs. is the maximum load on one cap (14 feet span), the great excess of strength is apparent.

192. Design of caps and sills. The stresses in caps and sills are very indefinite, except as to crushing across the grain. As

the stringers are placed almost directly over the inner posts, s as the sills are supported just under the posts, the transve stresses are almost insignificant. In the above case four pc have an area of $4\times12''\times8''=384$ sq. in. The total load 115 lbs. will then give a pressure of 300 pounds per square in which is more than the allowable limit. This one feature r require the use of $12''\times12''$ (or at least $10''\times12''$) posts rat than $8''\times12''$ posts, for the smaller posts, although proba strong enough as posts, would produce an objectionably h pressure.

193. **Bracing.** Although some idea of the stresses in bracing could be found from certain assumptions as to wi pressure, etc., yet it would probably not be found wise to crease, for the sake of economy, the dimensions which pract has shown to be sufficient for the work. The economy t would be possible would be too insignificant to justify any ri Therefore the usual dimensions, given in §§ 174 and 175, sho be employed.

CHAPTER V.

TUNNELS.

SURVEYING.

194. Surface surveys. As tunnels are always dug from each end and frequently from one or more intermediate shafts, it is necessary that an accurate surface survey should be made between the two ends. As the natural surface in a locality where a tunnel is necessary is almost invariably very steep and rough, it requires the employment of unusually refined methods of work to avoid inaccuracies. It is usual to run a line on the surface that will be at every point vertically over the center line of the tunnel. Tunnels are generally made straight unless curves are absolutely necessary, as curves add greatly to the cost. Fig. 87 represents roughly a longitudinal section of the

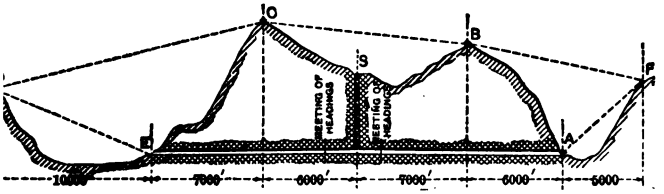


FIG. 87.—SKETCH OF SECTION OF THE HOOSAC TUNNEL.

Hoosac Tunnel. Permanent stations were located at *A*, *B*, *C*, *D*, *E*, and *F*, and stone houses were built at *A*, *B*, *C*, and *D*. These were located with ordinary field transits at first, and then all the points were placed as nearly as possible in one vertical plane by repeated trials and minute corrections, using a very large specially constructed transit. The stations *D* and *F* were necessary because *E* and *A* were invisible from *C* and *B*. The alinement at *A* and *E* having been determined with great accuracy, the true alinement was easily carried into the tunnel.

The relative elevations of *A* and *E* were determined great accuracy. Steep slopes render necessary many se of the level per unit of horizontal distance and require th work be unusually accurate to obtain even fair accurac unit of distance. The levels are usually re-run many until the probable error is a very small quantity

The exact horizontal distance between the two ends tunnel must also be known, especially if the tunnel is grade. The usual steep slopes and rough topography lil render accurate horizontal measurements very difficult. quently when the slope is steep the measurement is be tained by measuring along the slope and allowing for This may be very accurately done by employing two t (level or transit tripods serve the purpose very well), s them up slightly less than one tape-length apart and mea between horizontal needles set in wooden blocks inserted top of each tripod. The elevation of each needle is also obs The true horizontal distance between two successive po of the needles then equals the square root of the differe the squares of the inclined distance and the difference of tion. Such measurements will probably be more accurat those made by attempting to hold the tape horizonts plumbing down with plumb-bobs, because (1) it is prac difficult to hold both ends of the tape truly horizontal; steep slopes it is impossible to hold the down-hill end of foot tape (or even a 25-foot length) on a level with the end, and the great increase in the number of applications unit of measurement very greatly increases the probabl of the whole measurement; (3) the vibrations of a plun introduce a large probability of error in transferring the urement from the elevated end of the tape to the groun the increased number of such applications of the unit of urement still further increases the probable error.

195. Surveying down a shaft. If a shaft is sunk, as Fig. 87, and it is desired to dig out the tunnel in both dire from the foot of the shaft so as to meet the headings fro outside, it is necessary to know, when at the bottom shaft, the elevation, alinement, and horizontal distance each end of the tunnel.

The elevation is generally carried down a shaft by me a steel tape. This method involves the least number of

ıs of the unit of measurement and greatly increases the acy of the final result.

e *horizontal distance from each end* may be easily trans- l down the shaft by means of a plumb-bob, using some of recautions described in the next paragraph.

transfer the *alinement* from the surface to the bottom of ft requires the highest skill because the shaft is always , and to produce a line perhaps several thousand feet long direction given by two points 6 or 8 feet apart requires the two points must be determined with extreme accuracy. ıminently successful method adopted in the Hoosac Tunnel ɔe briefly described: Two beams were securely fastened ɜ the top of the shaft (1030 feet deep), the beams being d transversely to the direction of the tunnel and as far as possible and yet allow plumb-lines, hung from the ıection of each beam with the tunnel center line, to swing at the bottom of the shaft. These intersections of the s with the center line were determined by averaging the s of a large number of careful observations for alinement. fine parallel wires, spaced about $\frac{1}{16}$" apart, were then hed between the beams so that the center line of the l bisected at all points the space between the wires. b-bobs, weighing 15 pounds, were suspended by fine wires ɜ each cross-beam, the wires passing between the two lel alinement wires and bisecting the space. The plumb- were allowed to swing in pails of water at the bottom. s of air up the shaft required the construction of boxes unding the wires. Even these precautions did not suffice ɔsolutely prevent vibration of the wire at the bottom gh a very small arc. The mean point of these vibrations ch case was then located on a rigid cross-beam suitably d at the bottom of the shaft and at about the level of the of the tunnel. Short plumb-lines were then suspended these points whenever desired; a transit was set (by trial) at its line of collimation passed through both plumb-lines he line at the bottom could thus be prolonged.

ne recent experience in the "Tamarack" shaft, 4250 feet shows that the accuracy of the results may be affected by rrents to an unsuspected extent. Two 50-lb. cast-iron ɔ-bobs were suspended with No. 24 piano-wire in this The carefully measured distances between the wires

at top and bottom were 16.32 and 16.43 feet respe
After considerable experimenting to determine the ca
the variation, it was finally concluded that air-current
alone responsible. The variation of the bobs from a tr
tical plane passing through the wires at the top was of
an unknown quantity, but since the variation in *one* di
amounted to 0.11 foot, the accuracy in other directio
very questionable. This shows that a careful comp
measurement between the wires at top and bottom
always be made as a test of their parallelism.

196. Underground surveys. Survey marks are freq
placed on the timbering, but they are apt to prove un
on account of the shifting of the timbering due to sett
of the surrounding material. They should never be pla
the bottom of the tunnel on account of the danger of
disturbed or covered up. Frequently holes are drilled
roof and filled with wooden plugs in which a hook is s
exactly on line Although this is probably the safest m
even these plugs are not always undisturbed, as the m
unless very hard, will often settle slightly as the exca
proceeds. When a tunnel is perfectly straight and not to
alinement-points may be given as frequently as desire
permanent stations located
the tunnel where they are not
to disturbance. This has be
complished by running the
ment through the upper part
cross-section, at one side of th
ter, where it is out of the w
the piles of masonry m
débris, etc., which are so
choke up the lower part
cross-section. The position
line relative to the cross-
being fixed, the alignment
required point of the cross-
is readily found by means of
frame or template with a fix
get located where this line would intersect the frame
properly placed. A level-bubble on the frame will as
setting the frame in its proper position.

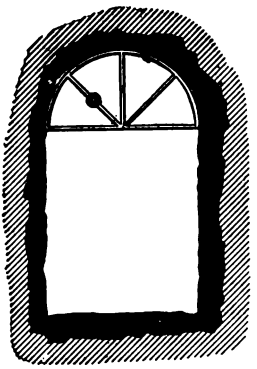


FIG. 88.

In all tunnel surveying the cross-wires must be illuminated by a lantern, and the object sighted at must also be illuminated. A powerful dark-lantern with the opening covered with *ground glass* has been found useful. This may be used to illuminate a plumb-bob string or a very fine rod, or to place behind a brass plate having a narrow slit in it, the axis of the slit and plate being coincident with the plumb-bob string by which it is hung.

On account of the interference to the surveying caused by the work of construction and also by the smoke and dust in the air resulting from the blasting, it is generally necessary to make the surveys at times when construction is temporarily suspended.

197. Accuracy of tunnel surveying. Apart from the very natural desire to do surveying which shall check well, there is an important financial side to accurate tunnel surveying. If the survey lines do not meet as desired when the headings come together, it may be found necessary, if the error is of appreciable size, to introduce a slight curve, perhaps even a reversed curve, into the alinement, and it is even conceivable that the tunnel section would need to be enlarged somewhat to allow for these curves. The cost of these changes and the perpetual annoyance due to an enforced and undesirable alteration of the original design will justify a considerable increase in the expenses of the survey. Considering that the cost of surveys is usually but a small fraction of the total cost of the work, an increase of 10 or even 20% in the cost of the surveys will mean an insignificant addition to the total cost and frequently, if not generally, it will result in a saving of many times the increased cost. The accuracy actually attained in two noted American tunnels is given as follows: The Musconetcong tunnel is about 5000 feet long, bored through a mountain 400 feet high. The error of alinement at the meeting of the headings was 0′.04, error of levels 0′.015, error of distance 0′.52. The Hoosac tunnel is over 25000 feet long. The heading from the east end met the heading from the central shaft at a point 11274 feet from the east end and 1563 feet from the shaft. The error in alinement was $\frac{5}{16}$ of an inch, that of levels "a few hundredths," error of distance "trifling." The alinement, corrected at the shaft, was carried on through and met the heading from the west end at a point 10138 feet from the west end and 2056 feet from the shaft. Here the error of alinement was $\frac{9}{16}$″ and that of levels 0.134 foot.

DESIGN

198. Cross-section. Nearly all tunnels have cross-sections peculiar to themselves—all varying at least in the details. The general form of a great many tunnels is that of a rectangle surmounted by a semi-circle or semi-ellipse. In very soft material an inverted arch is necessary along the bottom. In such cases the sides will generally be arched instead of vertical. The sides are frequently battered. In very long tunnels, several forms of cross-section will often be used in the same tunnel, owing to differences in the material encountered. In solid rock, which will not disintegrate upon exposure, no lining is required, and the cross-section will be the irregular section left by the blasting, the only requirement being that no rock shall be left within the required cross-sectional figure. Farther on, in the same tunnel, when passing through some very soft treacherous material, it may be necessary to put in a full arch lining—top, sides, and bottom—which will be nearly circular in cross-section. For an illustration of this see Figs. 89 and 90.

The cross-section recommended by the A. R. E. A. for single track is a rectangle 16 feet wide by 16 feet 6 inches high, surmounted by a semi-circle with a radius of 8 feet. The top of the tie is to be 2 feet above the bottom which is at sub-grade. If the surrounding material is yielding and exerts great pressure, the sides should be battered inward 1 foot at the bottom. For a double track tunnel the design is similar, except that the width is increased by the standard spacing between double tracks and the top is a compound curve made up of two 8-foot-radius curves at the sides which compound into a curve over the center which will give a clear height of 22 feet 6 inches over the center of each tie. The base of the roof curve is 13 feet 6 inches above the top of the ties. The bottom slopes to a central gutter which is 6 inches below the side corners, which are at sub-grade. Six-inch cast-iron pipes should be spaced as needed and run from each side to the central gutter. The width of both single and double track tunnels should be increased, if the tunnel is on a curve, and the track centers should also be displaced, so that the clearance on each side is as great as on a tangent. Figs. 89, 90 and 91,* show some typical cross-sections.

199. Grade. A grade of at least 0.2% is needed for drainage. If the tunnel is at the summit of two grades, the tunnel grade

* Drinker's "Tunneling."

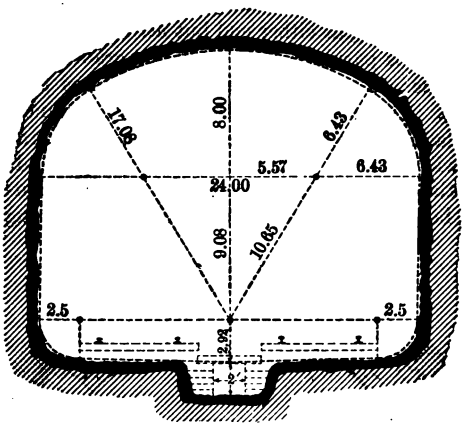


FIG. 89.—HOOSAC TUNNEL. SECTION THROUGH SOLID ROCK.

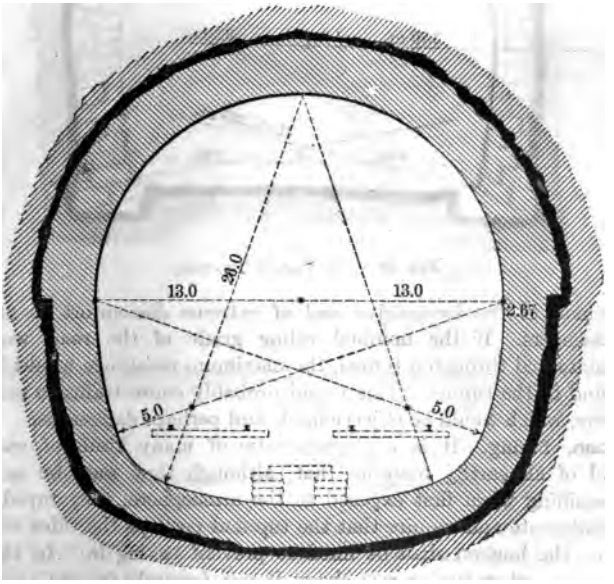


FIG. 90.—HOOSAC TUNNEL SECTION THROUGH SOFT GROUND.

should be practically level, with an allowance for drainage, the actual summit being at either end but *not* in the center. When the tunnel forms part of a long ascending grade, it is advisable to reduce the grade through the tunnel unless the tunnel is very short. The additional atmospheric resistance and the decreased adhesion of the driver wheels on the damp rails in a tunnel will cause an engine to work very hard and still more rapidly vitiate the atmosphere until the accumulation of poisonous gases becomes a source of actual danger to the engineer and

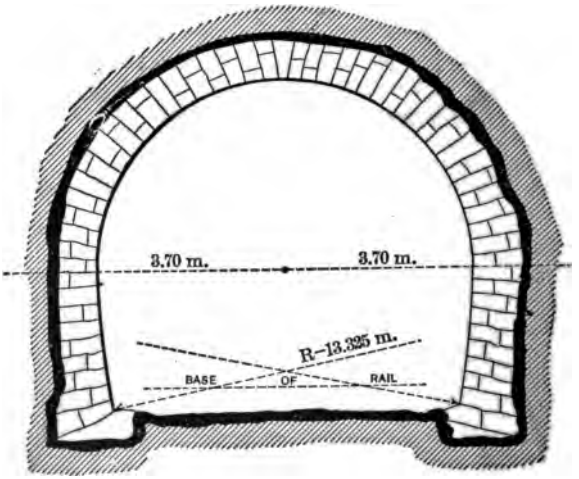


FIG. 91. —ST. CLOUD TUNNEL.

fireman of the locomotive and of extreme discomfort to the passengers. If the nominal ruling grade of the road were maintained through a tunnel, the maximum resistance would be found in the tunnel. This would probably cause trains to stall there, which would be objectionable and perhaps dangerous.

200. Lining. It is a characteristic of many kinds of rock and of all earthy material that, although they may be self-sustaining when first exposed to the atmosphere, they rapidly disintegrate and require that the top and perhaps the sides and even the bottom shall be lined to prevent caving in. In this country, when timber was cheap, it was formerly framed as an arch and used as the *permanent* lining (see Fig. 92), but in

any such case the cross-section should be made extra large so that a masonry lining may subsequently be placed inside the wooden lining and thus postpone a large expense until the road is better able to pay for the work. In very soft unstable material, like quicksand, an arch of cut stone voussoirs may be necessary to withstand the pressure. A good quality of brick is occasionally used for lining, as they are easily handled and make good masonry if the pressure is not excessive. Only the best of cement mortar should be used, economy in this feature being the worst of folly. Of course the excavation must include the outside line of the lining. Any excavation which is made outside of this line (by the fall of earth or loose rock or by excessive blasting) must be refilled with stone well packed in. Occasionally it is necessary to fill these spaces with concrete. Of course it is not necessary that the lining be uniform throughout the tunnel.

201. Shafts. Shafts are variously made with square, rectangular, elliptical, and circular cross-sections. The rectangular

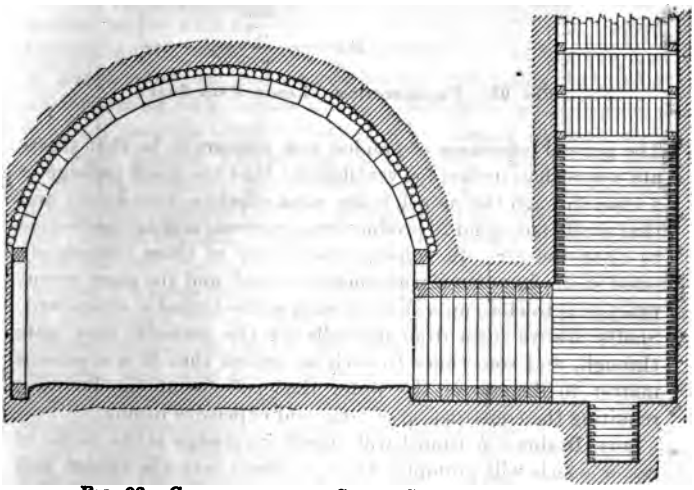


FIG. 92.—CONNECTION WITH SHAFT, CHURCH HILL TUNNEL.

cross-section, with the longer axis parallel with the tunnel, is most usually employed. Generally the shaft is directly over the center of the tunnel, but that always implies a complicated connection between the linings of the tunnel and shaft, provided

such linings are necessary. It is easier to sink a shaft near to one side of the tunnel and make an opening through the nearly vertical side of the tunnel. Such a method was employed in the Church Hill Tunnel, illustrated in Fig. 92.* Fig. 93 † shows a cross-section for a large main shaft. Many shafts have been built with the idea of being left open permanently for ventilation and have therefore been elaborately lined with masonry.

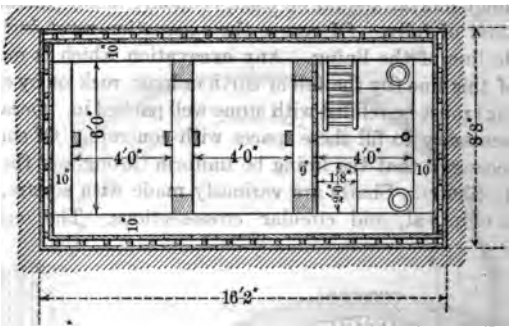


FIG. 93.—CROSS-SECTION. LARGE MAIN SHAFT.

The general consensus of opinion now appears to be that shafts are worse than useless for ventilation; that the quick passage of a train through the tunnel is the most effective ventilator; and that shafts only tend to produce cross-currents and are ineffective to clear the air. In consequence, many of these elaborately lined shafts have been permanently closed, and the more recent practice is to close up a shaft as soon as the tunnel is completed. Shafts always form drainage-wells for the material they pass through, and sometimes to such an extent that it is a serious matter to dispose of the water that collects at the bottom, requiring the construction of large and expensive drains.

202. Drains. A tunnel will almost invariably strike veins of water which will promptly begin to drain into the tunnel and not only cause considerable trouble and expense during construction, but necessitate the provision of permanent drains for its perpetual disposal. These drains must frequently be so large as

* Drinker's "Tunneling."

† Rziha, "Lehrbuch der Gesammten Tunnelbaukunst."

to appreciably increase the required cross-section of the tunnel. Generally a small open gutter on each side will suffice for this purpose, but in double-track tunnels a large covered drain is often built between the tracks. It is sometimes necessary to thoroughly grout the outside of the lining so that water will not force its way through the masonry and perhaps injure it, but may freely drain down the sides and pass through openings in the side walls near their base into the gutters.

CONSTRUCTION.

203. Headings. The methods of all tunnel excavation depend on the general principle that all earthy material, except the softest of liquid mud and quicksand, will be self-sustaining over a greater or less area and for a greater or less time after excavation is made, and the work consists in excavating some material and immediately propping up the exposed surface by timbering and poling-boards. The excavation of the cross-section begins with cutting out a "heading," which is a small horizontal drift whose breast is constantly kept 15 feet or more in advance of the full cross-sectional excavation. In solid self-sustaining rock, which will not decompose upon exposure to air, it becomes simply a matter of excavating the rock with the least possible expenditure of time and energy. In soft ground the heading must be heavily timbered, and as the heading is gradually enlarged the timbering must be gradually extended and perhaps replaced, according to some regular system, so that when the full cross-section has been excavated it is supported by such timbering as is intended for it. The heading is sometimes made on the center line near the top; with other plans, on the center line near the bottom; and sometimes two simultaneous headings are run in the two lower corners. Headings near the bottom serve the purpose of draining the material above it and facilitating the excavation. The simplest case of heading timbering is that shown in Fig. 94, in which cross-timbers are placed at intervals just under the roof, set in notches cut in the side walls and supporting poling-boards which sus-

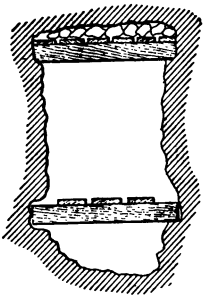


FIG. 94.

tain whatever pressure may come on them. Cross-timbers near the bottom support a flooring on which vehicles for transporting material may be run and under which the drainage may freely escape. As the necessity for timbering becomes greater, side timbers and even bottom timbers must be added, these timbers supporting poling-boards, and even the breast of the heading must be protected by boards suitably braced,

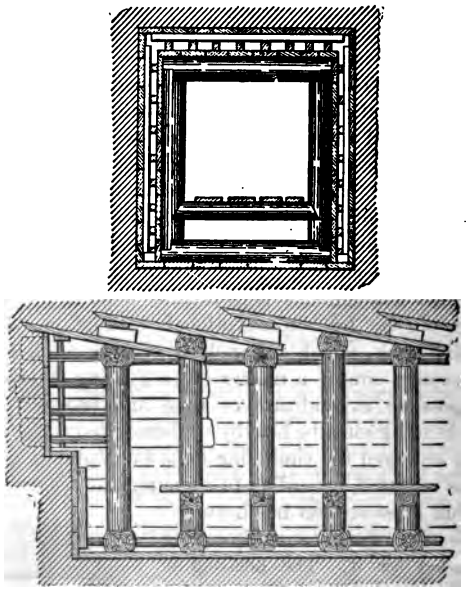


FIG. 95.—TIMBERING FOR TUNNEL HEADING.

as shown in Fig. 95. The supporting timbers are framed into collars in such a manner that added pressure only increases their rigidity.

204. Enlargement. Enlargement is accomplished by removing the poling-boards, one at a time, excavating a greater or less amount of material, and immediately supporting the exposed material with poling-boards suitably braced. (See Figs. 95 and 96.) This work being systematically done, space is thereby obtained in which the framing for the full cross-section may be gradually introduced. The framing is constructed with a cross-

section so large that the masonry lining may be constructed within it.

205. Distinctive features of various methods of construction. There are six general systems, known as the English, German, Belgian, French, Austrian, and American. They are so named

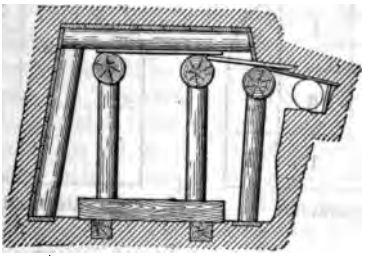


FIG. 96.

from the origin of the methods, although their use is not confined to the countries named. Fig. 97 shows by numbers (1 to 5) the order of the excavation within the cross-sections. The English, Austrian, and American systems are alike in excavating the entire cross-section before beginning the construction of the masonry lining. The German method leaves a solid core (5) until practically the whole of the lining is complete. This has the disadvantage of extremely cramped quarters for work, poor ventilation, etc. The Belgian and French methods agree in excavating the upper part of the section, building the arch at once, and supporting it temporarily until the side walls are built. The Belgian method then takes out the core (3), removes very short sections of the sides (4) immediately underpinning the arch with short sections of the side walls and thus gradually constructing the whole side wall. The French method digs out the sides (3), supporting the arch temporarily with timbers and then replacing the timbers with masonry; the core (4) is taken out last. The French method has the same disadvantage as the German—working in a cramped space. The Belgian and French systems have the disadvantage that the arch, supported temporarily on timber, is very apt to be strained and cracked by the slight settlement that so frequently occurs in soft material. The English, Austrian, and American methods differ mainly in the

design of the timbering. The English support the roof by lines of very heavy *longitudinal* timbers which are supported at comparatively wide intervals by a heavy framework occupying the

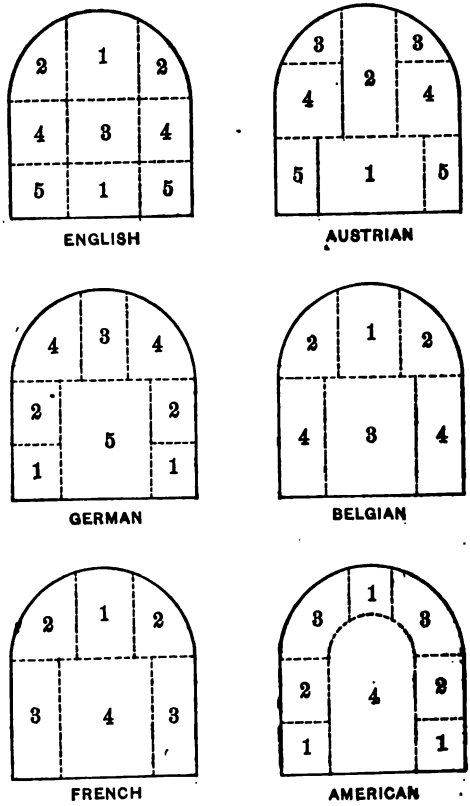


Fig. 97.—Order of Working by the Various Systems.

whole cross-section. The Austrian system uses such frequent cross-frames of timber-work that poling-boards will suffice to support the material between the frames. The American system agrees with the Austrian in using frequent cross-frames

supporting poling-boards, but differs from it in that the "cross frames" consist simply of arches of 3 to 15 wooden voussoirs, the voussoirs being blocks of 12″×12″ timber about 2 to 8 feet long and cut with joints normal to the arch. These arches are put together on a centering which is removed as soon as the arch is keyed up and thus immediately opens up the full cross-section, so that the center core (4) may be immediately dug out and the masonry constructed in a large open space. The American system has been used successfully in very soft ground, but its advantages are greater in loose rock, when it is much cheaper than the other methods which employ more timber. Fig. 92 and Plate III illustrate the use of the American system. Fig. 92 shows the wooden arch in place. The masonry arch may be placed when convenient, since it is *possible* to lay the track and commence traffic as soon as the wooden arch is in place. The student is referred to Drinker's "Tunneling" and to Rziha's "Lehrbuch der Gesammten Tunnelbaukunst" for numerous illustrations of European methods of tunnel timbering.

206. Ventilation during construction. Tunnels of any great length must be artificially ventilated during construction. If the excavated material is rock so that blasting is necessary, the need for ventilation becomes still more imperative. Fresh air is forced *into* the tunnel at or near the heading ("plenum process") and the foul air is thereby crowded out, or the foul air is sucked out ("vacuum process") and fresh air rushes into the tunnel at the entrance. "Compressed air wasted from power drills is so contaminated with oil from the cylinders that it cannot be taken into consideration as ventilation." The draft of air up a shaft will occasionally modify, and perhaps assist, the work of ventilation, but, in general, the work must be done by means of power fans.

207. Excavation for the portals. Under normal conditions there is always a greater or less amount of open cut preceding and following a tunnel. Since all tunnel methods depend (to some slight degree at least) on the capacity of the exposed material to act as an arch, there is implied a considerable thickness of material above the tunnel. This thickness is reduced to nearly zero over the tunnel portals and therefore requires special treatment, particularly when the material is very soft. Fig. 98 *

* **Rziha**, "Lehrbuch der Gesammten Tunnelbaukunst."

illustrates one method of breaking into the ground at a portal. The loose stones are piled on the framing to give stability to the framing by their weight and also to retain the earth on the

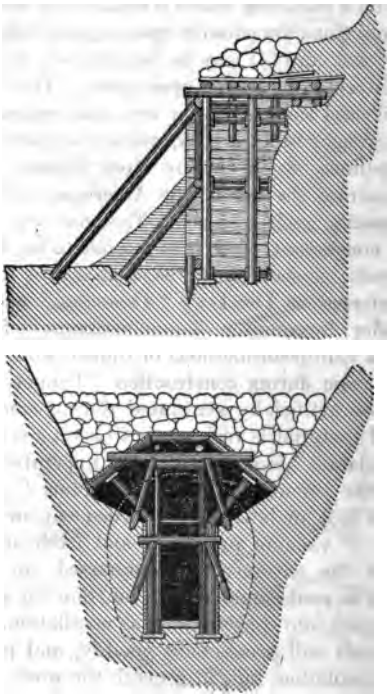
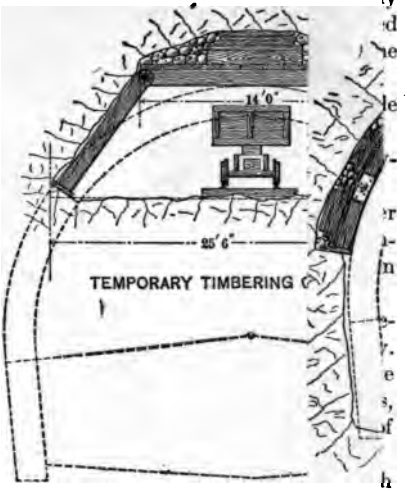
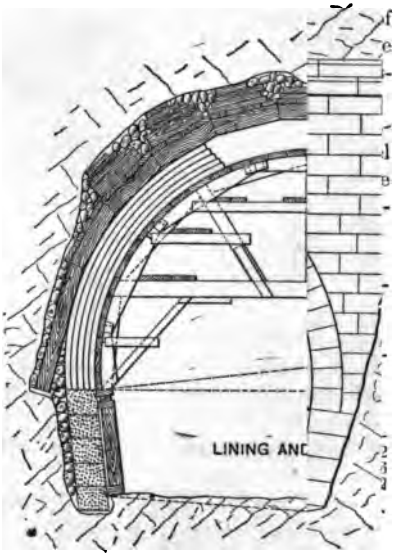


FIG. 98.—TIMBERING FOR TUNNEL PORTAL.

slope above. Another method is to sink a temporary shaft to the tunnel near the portal; immediately enlarge to the full size and build the masonry lining; then work back to the portal. This method is more costly, but is preferable in very treacherous ground, it being less liable to cause landslides of the surface material.

208. Tunnels vs. open cuts. In cases in which an open cut rather than a tunnel is a possibility the ultimate consideration is generally that of first cost combined with other financial con-





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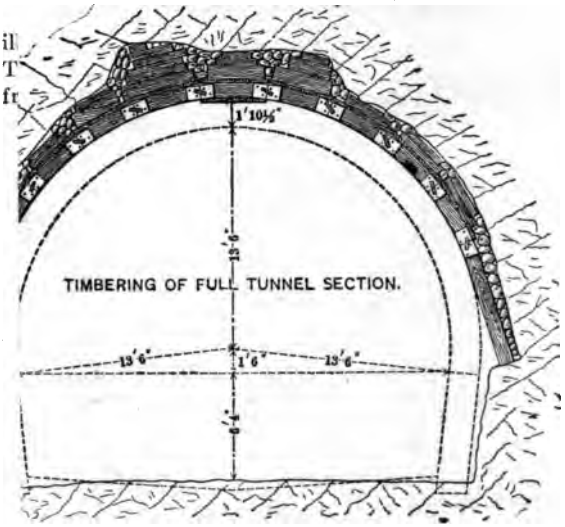
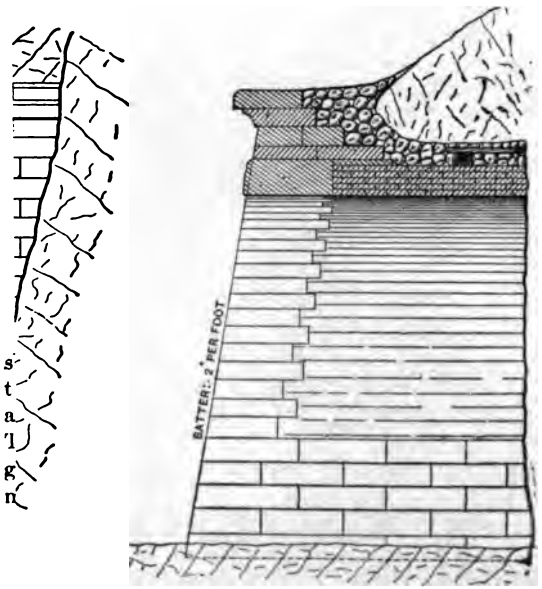




LONGITUDINAL SECTION OF PORTAL.

rations and annual maintenance charges directly or indirectly nected with it. Even when an open cut may be constructed he same cost as a tunnel (or perhaps a little cheaper) the nel may be preferable under the following conditions:

. When the soil indicates that the open cut would be liable andslides.

. When the open cut would be subject to excessive snow-ts or avalanches.

. When land is especially costly or it is desired to run under ting costly or valuable buildings or monuments. When run-; through cities, tunnels are sometimes constructed as open and then arched over.

hese cases apply to tunnels *vs.* open cuts when the aline-t is fixed by other considerations than the mere topography. broader question of excavating tunnels to avoid excessive es or to save distance or curvature, and similar problems, ardly susceptible of general analysis except as questions of ray economics and must be treated individually.

9. Cost of tunneling. The cost of any construction which lves such uncertainties as tunneling is very variable. It nds on the material encountered, the amount and kind of bering required, on the size of the cross-section, on the price abor, and especially on the *reconstruction* that *may* be neces-on account of mishaps.

eadings generally cost $4 to $5 per cubic yard for excava-, while the remainder of the cross-section in the same tunnel cost about half as much. The average cost of a large ber of tunnels in this country may be seen from the follow-table:*

aterial.	Cost per cubic yard.				Cost per lineal foot.	
	Excavation.		Masonry.			
	Single.	Double.	Single.	Double.	Single.	Double.
rock.....	$5.89	$5.45	$12.00	$8.25	$69.76	$142.82
rock.....	3.12	3.48	9.07	10.41	80.61	119.26
round....	3.62	4.64	15.00	10.50	135.31	174.42

* Figures derived from Drinker's "Tunneling."

The above figures are averages for tunnels constructed between 1831 and 1877. The prices paid for labor varied from $1.00 to $2.75 per day for " miners " and 0.75 to $2.00 for unskilled labor. The lower figures were usually paid during the earlier years. As an approximate average, the figures of $2.00 per day for miners and $1.50 per day for unskilled labor may be said to correspond to the average costs given in the tabular form. On the basis that all other expenses (explosives, cost of equipment, etc.) vary proportionately to wages, the tabular figures can even now be utilized by increasing them according to the present scale for labor. The figures are also instructive since they show the relative cost of tunneling through hard rock, loose rock and soft ground.

CHAPTER VI.

CULVERTS AND MINOR BRIDGES.

210. Definition and object. Although a variable percentage of the rain falling on any section of country soaks into the ground and does not immediately reappear, yet a very large percentage flows over the surface, always seeking and following the lowest channels. The roadbed of a railroad is constantly intersecting these channels, which frequently are normally dry. In order to prevent injury to railroad embankments by the impounding of such rainfall, it is necessary to construct waterways through the embankment through which such rainflow may freely pass. Such waterways, called culverts, are also applicable for the bridging of very small although perennial streams, and therefore in this work the term culvert will be applied to all water-channels passing through a railroad embankment which are not of sufficient magnitude to require a special structural design, such as is necessary for a large masonry arch or a truss bridge.

211. Elements of the design. A well-designed culvert must afford such free passage to the water that it will not "back up" over the adjoining land nor cause any injury to the embankment or culvert. The ability of the culvert to discharge freely all the water that comes to it evidently depends chiefly on the area of the waterway, but also on the form, length, slope, and materials of construction of the culvert and the nature of the approach and outfall. When the embankment is very low and the amount of water to be discharged very great, it sometimes becomes necessary to allow the water to discharge "under a head," i. e., with the surface of the water above the top of the culvert. Safety then requires a much stronger construction than would otherwise be necessary to avoid injury to the culvert or embankment by washing. The necessity for such construction should be avoided if possible.

AREA OF THE WATERWAY.

212. **Elements involved.** The determination of the required area of the waterway involves such a multiplicity of indeterminate elements that any close determination of its value from purely theoretical considerations is a practical impossibility. The principal elements involved are:

a. **Rainfall.** The real test of the culvert is its capacity to discharge without injury the flow resulting from the extraordinary rainfalls and "cloud bursts" that may occur once in many years. Therefore, while a knowledge of the average annual rainfall is of very little value, a record of the maximum rainfall during heavy storms for a long term of years may give a relative idea of the maximum demand on the culvert.

b. **Area of watershed.** This signifies the total area of country draining into the channel considered. When the drainage area is very small it is sometimes included within the area surveyed by the preliminary survey. When larger it is frequently possible to obtain its area from other maps with a percentage of accuracy sufficient for the purpose. Sometimes a special survey for the purpose is considered justifiable.

c. **Character of soil and vegetation.** This has a large influence on the rapidity with which the rainflow from a given area will reach the culvert. If the soil is hard and impermeable and the vegetation scant, a heavy rain will run off suddenly, taxing the capacity of the culvert for a short time, while a spongy soil and dense vegetation will retard the flow, making it more nearly uniform and the maximum flow at any one time much less.

d. **Shape and slope of watershed.** If the watershed is very long and narrow (other things being equal), the water from the remoter parts will require so much longer time to reach the culvert that the flow will be comparatively uniform, especially when the slope of the whole watershed is very low. When the slope of the remoter portions is quite steep it may result in the nearly simultaneous arrival of a storm-flow from all parts of the watershed, thus taxing the capacity of the culvert.

e. **Effect of design of culvert.** The principles of hydraulics show that the slope of the culvert, its length, the form of the cross-section, the nature of the surface, and the form of the

approach and discharge all have a considerable influence on the area of cross-section required to discharge a given volume of water in a given time, but unfortunately the combined hydraulic effect of these various details is still a very uncertain quantity.

213. Methods of computation of area. There are three possible methods of computation.

(a) **Theoretical.** As shown above it is a practical impossibility to estimate correctly the combined effect of the great multiplicity of elements which influence the final result. The nearest approach to it is to estimate by the use of empirical formulæ the amount of water which will be presented at the upper end of the culvert in a given time and then to compute, from the principles of hydraulics, the rate of flow through a culvert of given construction, but (as shown in § 212, *e*) such methods are still very unreliable, owing to lack of experimental knowledge. This method has *apparently* greater scientific accuracy than other methods, but a little study will show that the elements of uncertainty are as great and the final result no more reliable. The method is most reliable for streams of uniform flow, but it is under these conditions that method (*c*) is most useful. The theoretical method will not therefore be considered further.

(b) **Empirical.** As illustrated in § 214, some formulæ make the area of waterway a function of the drainage area, the formula being affected by a coefficient the value of which is estimated between limits according to the judgment Assuming that the formulæ are sound, their use only narrows the limits of error, the final determination depending on experience and judgment in the choice of the proper coefficient.

(c) **From observation.** This method, considered by far the best for permanent work, consists in observing the high-water marks on contracted channel-openings which are on the same stream and as near as possible to the proposed culvert. If the country is new and there are no such openings, the wisest plan is to bridge the opening by a temporary structure in wood which has an ample waterway (see § 158, *b*, 4) and carefully observe all high-water marks on that opening during the 6 to 10 years which is ordinarily the minimum life of such a structure. As shown later, such observations may be utilized for a close computation of the required waterway. Method (*b*) may be utilized for an approximate calculation for the required area for the tem-

porary structure, using a value which is intentionally excessive, so that a permanent structure of sufficient capacity may subsequently be constructed *within* the temporary structure.

214. **Empirical formulæ.** Two of the best known empirical formulæ for area of the waterway are the following:

(a) **Myer's formula:**

Area of waterway in square feet $= C \times \sqrt{\text{drainage area in acres}}$, where C is a coefficient varying from 1 for flat country to 4 for mountainous country and rocky ground. As an illustration, if the drainage area is 100 acres, the waterway area should be from 10 to 40 square feet, according to the value of the coefficient chosen. It should be noted that this formula does not regard the great variations in rainfall in various parts of the world nor the design of the culvert, and also that the final result depends largely on the choice of the coefficient.

(b) **Talbot's formula:**

Area of waterway in square feet $= C \times \sqrt[4]{(\text{drainage area in acres})^3}$.

" For steep and rocky ground C varies from $\frac{3}{4}$ to 1. For rolling agricultural country subject to floods at times of melting snow, and with the length of the valley three or four times its width, C is about $\frac{1}{2}$; and if the stream is longer in proportion to the area, decrease C. In districts not affected by accumulated snow, and where the length of the valley is several times the width, $\frac{1}{3}$ or $\frac{1}{4}$, or even less, may be used. C should be increased for steep side slopes, especially if the upper part of the valley has a much greater fall than the channel at the culvert." * As an illustration, if the drainage area is 100 acres the area of waterway should be $C \times 31.6$. The area should then vary from 5 to 31 square feet, according to the character of the country. Like the previous estimate, the result depends on the choice of a coefficient and disregards local variations in rainfall, except as they may be arbitrarily allowed for in choosing the coefficient.

215. **Value of empirical formulæ.** The fact that these formulæ, as well as many others of similar nature that have been suggested, depend so largely upon the choice of the coefficient shows that they are valuable "more as a guide to the judgment" than as a working rule," as Prof. Talbot explicitly declares in

* Prof. A. N. Talbot, "Selected Papers of the Civil Engineers' Club of the Univ. of Illinois."

commenting on his own formula. In short, they are chiefly valuable in indicating a probable maximum and minimum between which the true result probably lies.

216. Results based on observation. As already indicated in § 213, observation of the stream in question gives the most reliable results. If the country is new and no records of the flow of the stream during heavy storms has been taken, even the life of a temporary wooden structure may not be long enough to include one of the unusually severe storms which must be allowed for, but there will usually be some high-water mark which will indicate how much opening will be required. The following quotation illustrates this: "A tidal estuary may generally be safely narrowed considerably from the extreme water lines if stone revetments are used to protect the bank from wash. Above the true estuary, where the stream cuts through the marsh, we generally find nearly vertical banks, and we are safe if the faces of abutments are placed even with the banks. In level sections of the country, where the current is sluggish, it is usually safe to encroach somewhat on the general width of the stream, but in rapid streams among the hills the width that the stream has cut for itself through the soil should not be lessened, and in ravines carrying mountain torrents the openings must be left very much larger than the ordinary appearance of the banks of the stream would seem to make necessary." *

As an illustration of an observation of a storm-flow through a temporary trestle, the following is quoted: "Having the flood height and velocity, it is an easy matter to determine the volume of water to be taken care of. I have one ten-bent pile trestle 135 feet long and 24 feet high over a spring branch that ordinarily runs about six cubic inches per second. Last summer during one of our heavy rainstorms (four inches in less than three hours) I visited this place and found by float observations the surface velocity at the highest stage to be 1.9 feet per second. I made a high-water mark, and after the flood-water receded found the width of stream to be 12 feet and an average depth of $2\frac{3}{4}$ feet. This, with a surface velocity of 1.9 feet per second, would give approximately a discharge of 50

* J. P. Snow, Boston & Maine Railway. From Report to Association of Railway Superintendents of Bridges and Buildings. 1897.

cubic feet, or 375 gallons, per second. Having this information it is easy to determine size of opening required." *

217. Degree of accuracy required. The advantages resulting from the use of standard designs for culverts (as well as other structures) have led to the adoption of a comparatively small number of designs. The practical use made of a computation of required waterway area is to determine which one of several standard designs will most nearly fulfill the requirements. For example, if a 24-inch iron pipe, having an area of 3.14 square feet, is considered to be a little small, the next size (30-inch) would be adopted; but a 30-inch pipe has an area of 4.92 square feet, which is 56% larger. A similar result, except that the percentage of difference might not be quite so marked, will be found by comparing the areas of consecutive standard designs for stone box culverts.

The advisability of designing a culvert to withstand any storm-flow that may *ever* occur is considered doubtful. Several years ago a record-breaking storm in New England carried away a very large number of bridges, etc., hitherto supposed to be safe. It was not afterward considered that the design of those bridges was faulty, because the extra cost of constructing bridges capable of withstanding such a flood, added to interest for a long period of years, would be enormously greater than the cost of repairing the damages of such a storm once or twice in a century. Of course the element of danger has some weight, but not enough to justify a great additional expenditure, for common prudence would prompt unusual precautions during or immediately after such an extraordinary storm.

PIPE CULVERTS.

218. Advantages. Pipe culverts, made of cast iron or earthenware, are very durable, readily constructed, moderately cheap, will pass a larger volume of water in proportion to the area than many other designs on account of the smoothness of the surface, and (when using iron pipe) may be used very close to the track when a low opening of large capacity is required. Another advantage lies in the ease with which they may be inserted through a somewhat larger opening that has been

* A. J. Kelley, Kansas City Belt Railway. From Report to Association of Railway Superintendents of Bridges and Buildings. 1897.

temporarily lined with wood, without disturbing the roadbed or track

219. Construction. Permanency requires that the foundation shall be firm and secure against being washed out. To accomplish this, the soil of the trench should be hollowed out to fit the lower half of the pipe, making suitable recesses for the bells. In very soft treacherous soil a foundation-block of concrete is sometimes placed under each joint, or even throughout the whole length. When pipes are laid through a slightly larger timber culvert great care should be taken that the pipes are properly supported, so that there will be no settling nor development of unusual strains when the timber finally decays and gives way. To prevent the washing away of material around the pipe the ends should be protected by a bulkhead. This is best constructed of masonry (see Fig. 99), although wood is sometimes used for cheap and minor constructions. The joints should be calked, especially when the culvert is liable to run full or when the outflow is impeded and the culvert is liable to be partly or wholly filled during freezing weather. The cost of a calking of clay or even hydraulic cement is insignificant compared with the value of the additional safety afforded. When the grade of the pipe is perfectly uniform, a very low rate of grade will suffice to drain a pipe culvert, but since some unevenness of grade is inevitable through uneven settlement or imperfect construction, a grade of 1 in 20 should preferably be required, although much less is often used. The length of a pipe culvert is approximately determined as follows:

$$Length = 2s\ (depth\ of\ embankment) + (width\ of\ roadbed),$$

in which s is the slope ratio (horizontal to vertical) of the banks. In practice an even number of lengths should be used which will equal or exceed the length given by this formula.

220. Iron-pipe culverts. Simple cast-iron pipes are used in sizes from 12″ to 48″ diameter. These are usually made in lengths of 12 feet with a few lengths of 6 feet, so that any required length may be more nearly obtained. The lightest pipes made are sufficiently strong for the purpose, and even those which would be rejected because of incapacity to withstand considerable internal pressure may be utilized for this work. In Fig. 99 are shown the standard plans used on the C. C. C. & St. L. Ry., which may be considered as typical plans.

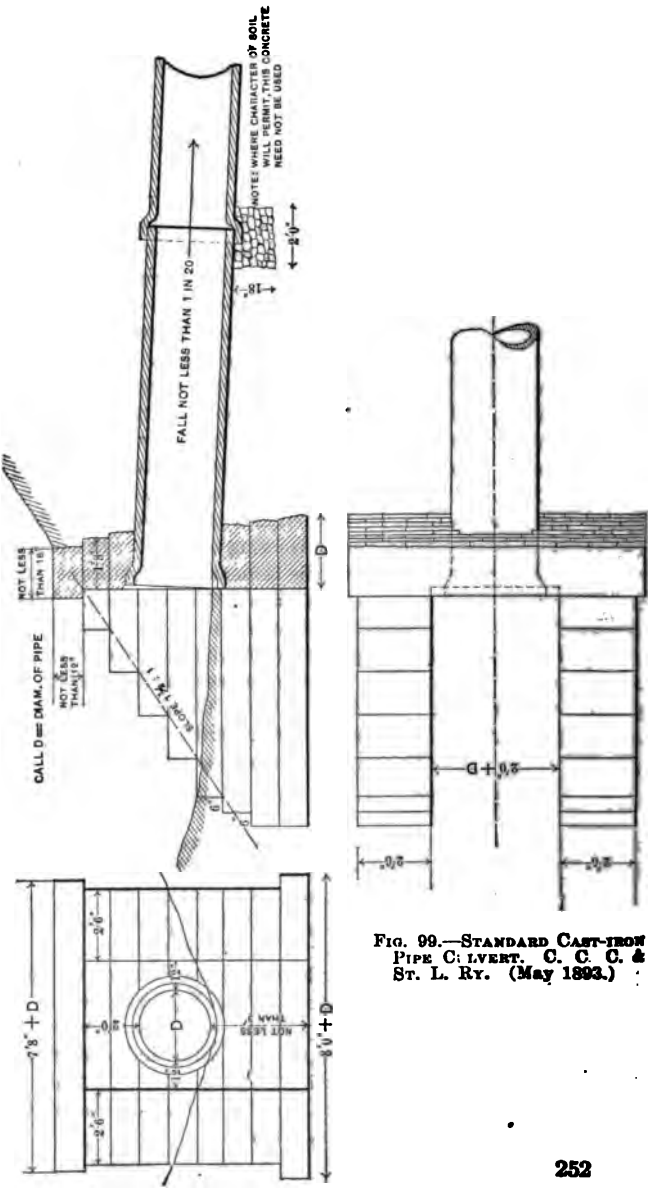

FALL NOT LESS THAN 1 IN 20
NOTE: WHERE CHARACTER OF SOIL WILL PERMIT, THIS CONCRETE NEED NOT BE USED
CALL D = DIAM. OF PIPE
NOT LESS THAN 18"
NOT LESS THAN 12"
NOT LESS THAN 6"
NOT LESS THAN 12"
2'0" + D
2'8" + D
8'0" + D
2'0"

FIG. 99.—STANDARD CAST-IRON PIPE CULVERT. C. C. C. & ST. L. RY. (May 1893.)

Pipes formed of cast-iron segments have been used up to 12 feet diameter. The shell is then made comparatively thin, but is stiffened by ribs and flanges on the outside. The segments break joints and are bolted together through the flanges. The joints are made tight by the use of a tarred rope, together with neat cement.

221. Tile-pipe culverts. The pipes used for this purpose vary from 12″ to 30″ in diameter. When a larger capacity is required two or more pipes may be laid side by side, but in such a case another design might be preferable. It is frequently specified that "double-strength" or "extra-heavy" pipe shall be used.

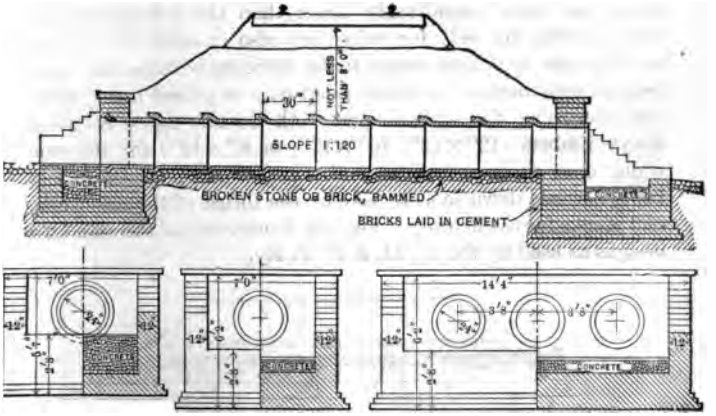


FIG. 100.—STANDARD VITRIFIED-PIPE CULVERT. PLANT SYSTEM. (1891.)

The author's personal experience is that tile pipe are very unreliable as culvert pipe, especially if there is any subsidence of the original soil which supports the embankment. See § 127–8. When a tile pipe is laid in a sewer, the soil on which it is laid is usually compact and there is no subsequent settlement. But when a culvert pipe is laid on soft meadow soil and a high embankment is formed over it, there is almost inevitably a settlement, which is probably *not* uniform and the culvert settles out of line, even if it does not break up and collapse. If the bed of the stream is rocky (precluding future settlement) and the pipes are bedded in concrete, there is less chance of failure. In Fig. 100 are shown the standard plans for vitrified-pipe culverts as used

on the "Plant system." Tile pipe is much cheaper than iron pipe, but is made in much shorter lengths and requires much more work in laying and especially to obtain a uniform grade.

Concrete pipes, factory made, both plain and with metal reinforcement, 12″ to 48″ in diameter, have come into use in recent years. They are stronger and more dependable than tile and there is no deterioration.

BOX CULVERTS.

222. Wooden box culverts. This form serves the purpose of a cheap temporary construction which allows the use of a ballasted roadbed. As in all temporary constructions, the area should be made considerably larger than the calculated area (§§ 213–216), not only for safety but also in order that, if the smaller area is demonstrated to be sufficiently large, the permanent construction (probably pipe) may be placed inside without disturbing the embankment. All designs agree in using heavy timbers (12″×12″, 10″×12″, or 8″×12″) for the side walls, cross-timbers for the roof, every fifth or sixth timber being notched down so as to take up the thrust of the side walls, and planks for the flooring. Fig. 101 shows some of the standard designs as used by the C., M. & St. P. Ry.

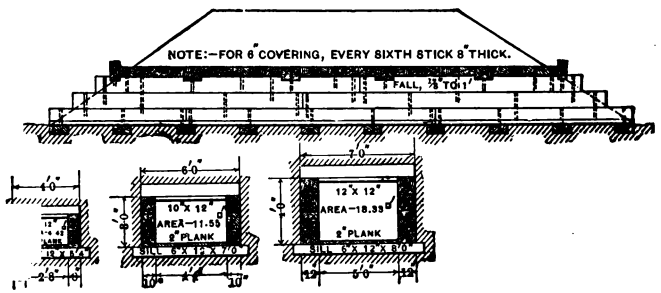
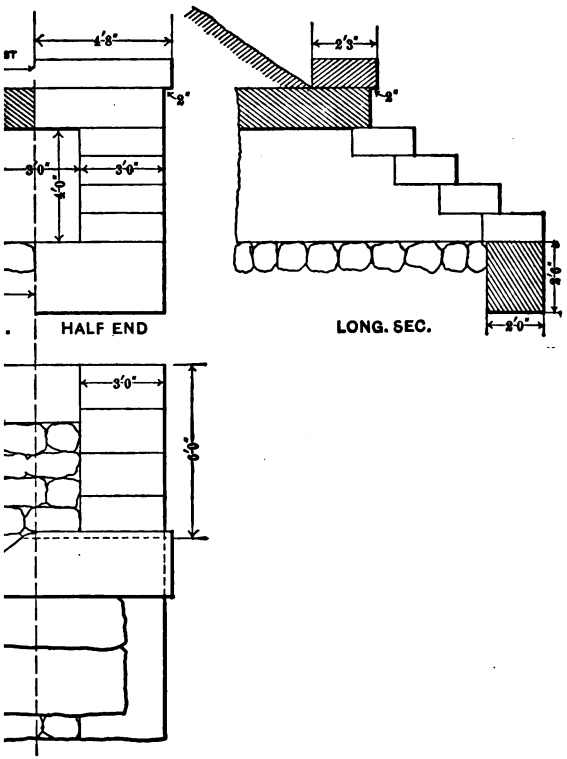


FIG. 101.—STANDARD TIMBER BOX CULVERT. C., M. & ST. P. RY. (Feb. 1889.)

223. Stone box culverts. In localities where a good quality of stone is cheap, stone box culverts are the cheapest form of permanent construction for culverts of medium capacity, but their use is decreasing owing to the frequent difficulty in obtaining really suitable stone within a reasonable distance of the culvert. The clear span of the cover-stones varies from 2 to 4 feet. The required thickness of the cover-stones is sometimes

by the theory of transverse strains on the basis of umptions of loading—as a function of the height of kment and the unit strength of the stone used. Such s simply another illustration of a class of calculations very precise and beautiful, but which are worse than cause misleading) on account of the hopeless uncer-



STANDARD SINGLE STONE CULVERT (3'×4'). N. & W. R.R. (1890.)

to the true value of certain quantities which must be e computations In the first place the true value of nsile strength of stone is such an uncertain and variable

quantity that calculations based on any assumed value for it of small reliability. In the second place the weight of the pri of earth lying directly above the stone, plus an allowance for l load, is by no means a measure of the load on the stone nor the forces that tend to fracture it. All earthwork will tend

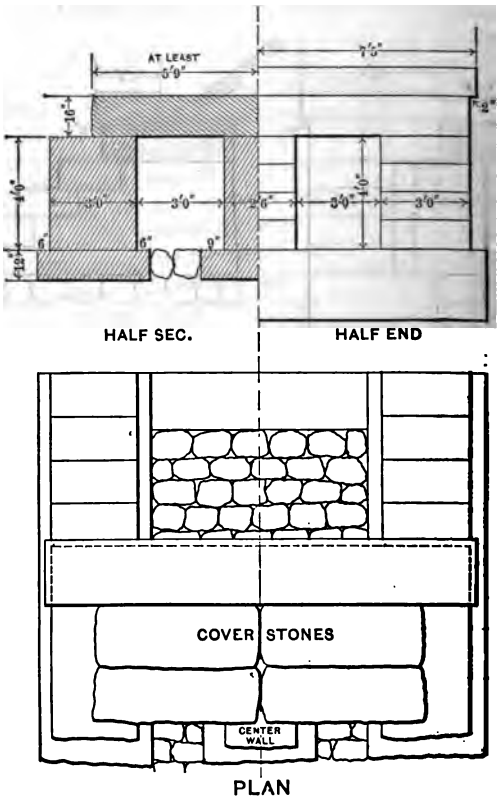


FIG. 103.—STANDARD DOUBLE STONE CULVERT (3′×4′). N. & W. R. (1890.)

form an arch above any cavity and thus relieve an uncert and probably variable proportion of the pressure that m otherwise exist. The higher the embankment the less the

portionate loading, until at some uncertain height an increase in height will not increase the load on the cover-stones. The effect of frost is likewise large, but uncertain and not computable. The usual practice is therefore to make the thickness such as experience has shown to be safe with a good quality of stone, i.e., about 10 or 12 inches for 2 feet span and up to 16 or 18 inches for 4 feet span. The side walls should be carried down deep enough to prevent their being undermined by scour or heaved by frost. The use of cement mortar is also an important feature of first-class work, especially when there is a rapid scouring current or a liability that the culvert will run under a head. In Figs. 102 and 103 are shown standard plans for single and double stone box culverts as used on the Norfolk & Western R.R.

224. Old-rail culverts. It sometimes happens (although very rarely) that it is necessary to bring the grade line within 3 or 4 feet of the bottom of a stream and yet allow an area of 10 or 12 square feet. A single large pipe of sufficient area could not be used in this case. The use of several smaller pipes side by side would be both expensive and inefficient. For similar reasons neither wooden nor stone box culverts could be used. In such cases, as well as in many others where the head-room is not so limited, the plan illustrated in Fig. 104 is a very satisfactory

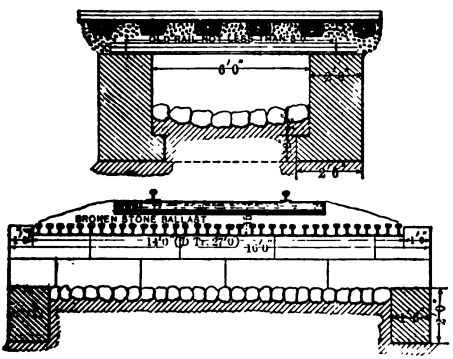


FIG. 104.—STANDARD OLD-RAIL CULVERT. N. & W. R.R. (1895.)

solution of the problem. The old rails, having a length of 8 or 9 feet, are laid close together across a 6-foot opening. Sometimes the rails are held together by long bolts passing through

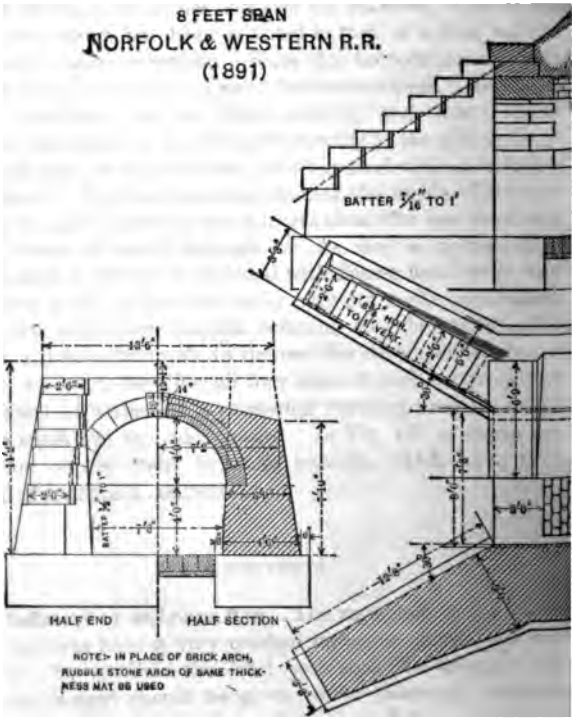
the webs of the rails. In the plan shown the rails are by low end walls on each abutment. This plan requ 15 inches between the base of the rail and the top of th channel. It also gives a continuous ballasted roadbed.

225. **Reinforced Concrete Culverts.** The develop reinforced concrete as a structural material is illustra extensive adoption for arches and also for culverts. C special types which has been adopted is that of a bo which has a concrete bottom. Since this bottom can so that it will withstand an upward transverse stress, it a broad foundation for the whole culvert, and thus eliminates the necessity for extensive footing to the sid the culvert, such as are necessary in soft ground with an stone culvert. Another advantage is that the inside c vert may be made perfectly smooth and thus offer less to the passage of water through it. As may be not Fig. 105, such a culvert is provided with flaring head w sunken end walls, so that the water may not scour u the culvert, and other features common to other ty attempt will here be made to discuss the design of : concrete, except to say that all four sides of such a bo are designed to withstand a computed bursting press tends to crush the flat sides inward. In Fig. 105 is a illustration of the many types of culverts which h designed of reinforced concrete.

ARCH CULVERTS.

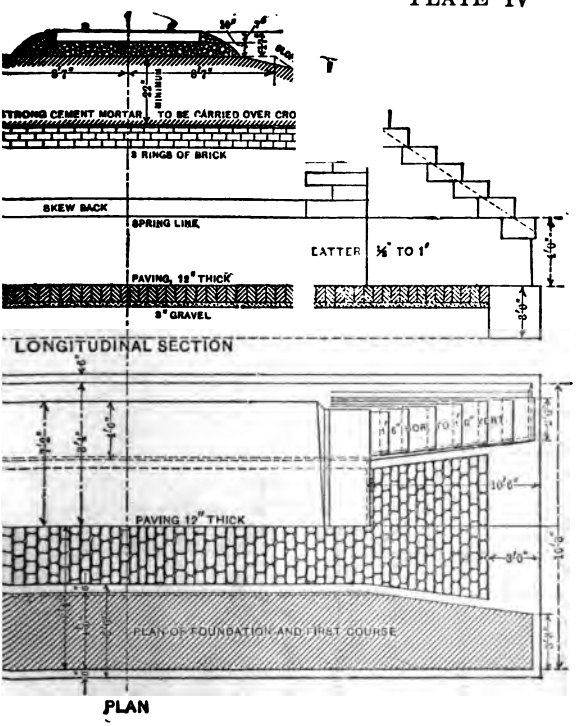
226. **Influence of design on flow.** The variations in of arch culverts have a very marked influence on th efficiency. To combine the least cost with the gre ciency, due weight should be given to the following (*a*) amount of masonry, (*b*) the simplicity of the co work, (*c*) the design of the wing walls, (*d*) the desi junction of the wing walls with the barrel and faces o and (*e*) the safety and permanency of the constructio elements are more or less antagonistic to each othe defects of most designs are due to a lack of proper in the design of these opposing interests. The sim struction (satisfying elements *b* and *e*) is the straight l

STANDARD ARCH CULVERT


8 FEET SPAN
NORFOLK & WESTERN R.R.
(1891)
BATTER 3/16" TO 1'
HALF END
HALF SECTION
NOTE:- IN PLACE OF BRICK ARCH, RUBBLE STONE ARCH OF SAME THICKNESS MAY BE USED

(*To face page* 259.)

PLATE IV


STRONG CEMENT MORTAR TO BE CARRIED OVER CRO
3 RINGS OF BRICK
SKEW BACK
SPRING LINE
BATTER ½" TO 1'
PAVING, 12" THICK
3" GRAVEL
LONGITUDINAL SECTION
PAVING 12" THICK
PLAN OF FOUNDATION AND FIRST COURSE
PLAN

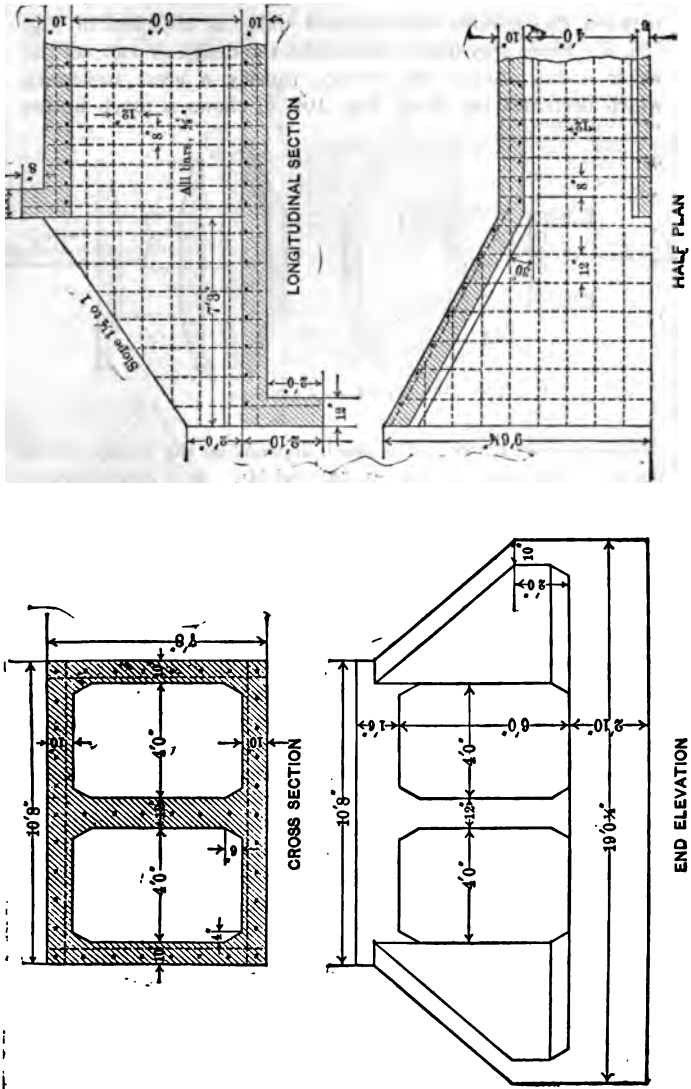


FIG. 105.—REINFORCED CONCRETE BOX CULVERT.

between two parallel vertical head walls, as sketched in Fig. 106, *a*. From a hydraulic standpoint the design is poor, as the water eddies around the corners, causing a great resistance which decreases the flow. Fig. 106, *b*, shows a much better

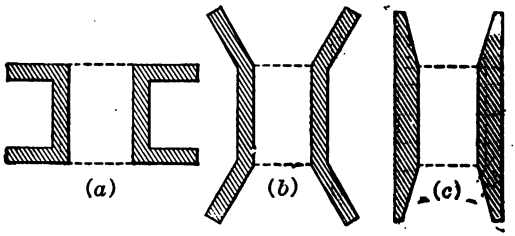


FIG. 106.—TYPES OF CULVERTS.

design in many respects, but much depends on the details of the design as indicated in elements (*b*) and (*d*). As a general thing a good hydraulic design requires complicated and expensive masonry construction, i.e., elements (*b*) and (*d*) are opposed. Design 106, *c*, is sometimes inapplicable because the water is liable to work in behind the masonry during floods and perhaps cause scour. This design uses less masonry than (*a*) or (*b*).

227. Example of arch culvert design. In Plate IV is shown the design for an 8-foot arch culvert according to the standard of the Norfolk and Western R. R. Note that the plan uses the flaring wing walls (Fig. 106, *b*) on the up-stream side (thus protecting the abutments from scour) and straight wing walls (similar to Fig. 106, *c*) on the down-stream end. This economizes masonry and also simplifies the constructive work. Note also the simplicity of the junction of the wing walls with the barrel of the arch, there being no re-entrant angles below the springing line of the arch. The design here shown is but one of a set of designs for arches varying in span from 6′ to 30′.

MINOR OPENINGS.

228. Cattle-guards. (a) **Pit guards.** Cattle-guards will be considered under the head of minor openings, since the old-fashioned plan of pit guards, which are even now defended and

preferred by some railroad men, requires a break in the continuity of the roadbed. A pit about three feet deep, five feet

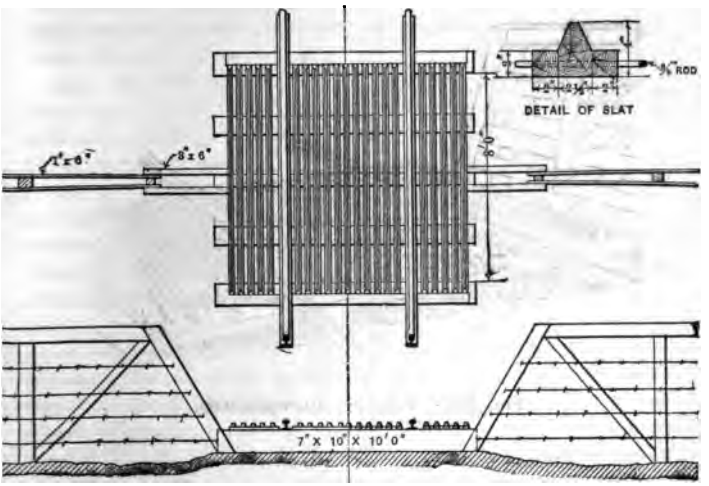


FIG. 107.—CATTLE-GUARD WITH WOODEN SLATS.

long, and as wide as the width of the roadbed, is walled up with stone (sometimes with wood), and the rails are supported on heavy timbers laid longitudinally with the rails. The break in the continuity of the roadbed produces a disturbance in the elastic wave running through the rails, the effect of which is noticeable at high velocities. The greatest objection, however, lies in the dangerous consequences of a derailment or a failure of the timbers owing to unobserved decay or destruction by fire—caused perhaps by sparks and cinders from passing locomotives. The very insignificance of the structure often leads to careless inspection. But if a single pair of wheels gets off the rails and drops into the pit, a costly wreck is inevitable.

(b) **Surface cattle-guards.** These are fastened on top of the ties; the continuity of the roadbed is absolutely unbroken and thus is avoided much of the danger of a bad wreck owing to a possible derailment. The device consists essentially of overlaying the ties (both inside and outside the rails) with a surface on

which cattle will not walk. The multitudinous designs for such a surface are variously effective in this respect. An objection,

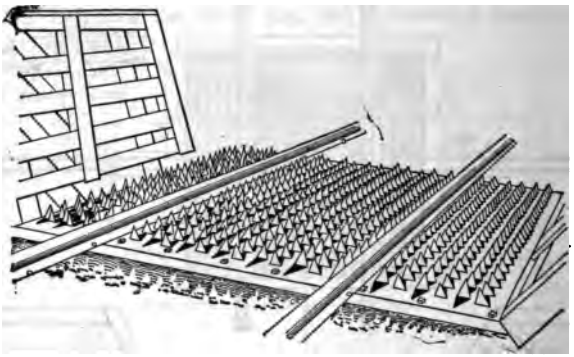


FIG. 108.—SHEFFIELD CATTLE-GUARD.

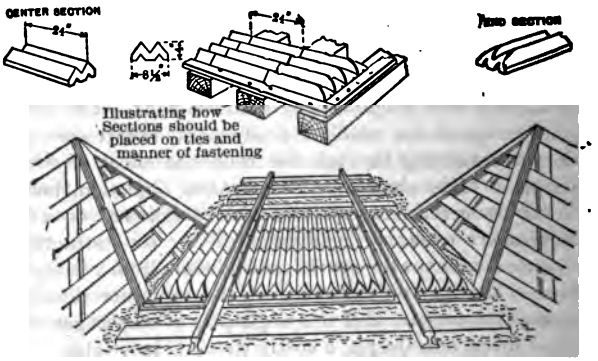


FIG. 109.—CLIMAX CATTLE-GUARD (TILE).

which is often urged indiscriminately against all such designs, is the liability that a brake-chain which may happen to be dragging *may* catch in the rough bars which are used. The bars

are sometimes "home-made," of wood, as shown in Fig. 107. Steel guards may be made as shown in Fig. 108. The general construction is the same as for the wooden bars. The metal bars have far greater durability, and it is claimed that they are more effective in discouraging cattle from attempting to cross.

229. Cattle-passes. Frequently when a railroad crosses a farm on an embankment, cutting the farm into two parts, the railroad company is obliged to agree to make a passageway through the embankment sufficient for the passage of cattle and perhaps even farm-wagons. If the embankment is high enough so that a stone arch is practicable, the initial cost is the only great objection to such a construction; but if an open wooden structure is necessary, all the objections against the old-fashioned cattle-guards apply with equal force here. The avoidance of a grade crossing which would otherwise be necessary is one of the great compensations for the expense of the construction and maintenance of these structures. The construction is sometimes made by placing two pile trestle bents about 6 to 8 feet apart, supporting the rails by stringers in the usual way, the special feature of this construction being that the embankments are filled in behind the trestle bents, and the thrust of the embankments is mutually taken up through the stringers, which are notched at the ends or otherwise constructed so that they may take up such a thrust. The designs for old-rail culverts and arch culverts are also utilized for cattle-passes when suitable and convenient, as well as the designs illustrated in the following section, and the reinforced concrete design of § 225.

230. Standard stringer and I-beam bridges. The advantages of standard designs apply even to the covering of short spans with wooden stringers or with I beams—especially since the methods do not require much vertical space between the rails and the upper side of the clear opening, a feature which is often of prime importance. These designs are chiefly used for culverts or cattle-passes and for crossing *over* highways—providing such a narrow opening would be tolerated. The plans all imply stone abutments, or at least abutments of sufficient stability to withstand all thrust of the embankments. Some of the designs are illustrated in Plate V. The preparation of these standard designs should be attacked by the same general methods as already illustrated in § 190. When computing the required

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CHAPTER VII.

BALLAST.

Purpose and requirements. "The object of the ballast
ransfer the applied load over a large surface; to hold the
work in place horizontally; to carry off the rain-water
he superstructure and to prevent freezing up in winter;
rd means of keeping the ties truly up to the grade line;
) give elasticity to the roadbed." This extremely con-
l statement is a description of an ideally perfect ballast.
alue of any given kind of ballast is proportional to the
to which it fulfills these requirements. The ideally
; ballast is not necessarily the most economical ballast
roads. Light traffic generally justifies something cheaper,
very common error is to use a very cheap ballast when a
additional expenditure would procure a much better
, which would be much more economical in the long run.

Materials. The materials most commonly employed are
and broken stone. In many sections of the country
materials which more or less perfectly fulfill the require-
as given above, are used. The various materials includ-
ne of these special types have been defined by the American
ay Engineering Association as follows:

DEFINITIONS.

ast. Selected material placed on the roadbed for the
se of holding the track in line and surface.

-ballast. Any material of a character superior to that
adjacent cuts, which is spread on the finished sub-grade
: roadbed and below the top ballast, to provide better
ge, prevent upheaval by frost, and better distribute the
ver the roadbed.

-ballast. Any material of a superior character spread
, sub-ballast to support the track structure, distribute the
) the sub-ballast, and provide good initial drainage.

Stone ballast. Stone broken by artificial means into small fragments of specified sizes.

Burnt clay. A clay or gumbo which has been burned into material for ballast.

Chats. Tailings from mills in which zinc, lead, silver and other ores are separated from the rocks in which they occur.

Chert. An impure flint or hornstone occurring in natural deposits.

Cinders. The residue from the coal used in locomotives and other furnaces.

Gravel. Worn fragments of rock, occurring in natural deposits, that will pass through a 2½-inch ring and be retained upon a No. 10 screen.

Gumbo. A term commonly used for a peculiarly tenacious clay, containing no sand.

Sand. Any hard, granular, comminuted rock which will pass through a No. 10 screen and be retained upon a No. 50 screen.

Slag. The waste product, in a more or less vitrified form, of furnaces for reduction of ore. Usually the product of a blast-furnace.

There is still another classification which may or may not be considered as ballast. It is perhaps hardly correct to speak of the natural soils as ballast, yet many miles of cheap railways are "ballasted" with the natural soil, which is then called Mud ballast.

Broken or crushed stone. Rock ballast is specified to be that which will all pass in any position through a 2½-inch ring, but which cannot pass through a ¾-inch mesh. It is most easily handled with forks. This method also has the advantage that when it is being rehandled the fine chips which would interfere with effectual drainage will be screened out. Rock ballast is more expensive in first cost and is also more troublesome to handle, but in heavy traffic especially, the track will be kept in better surface and will require less work for maintenance after the ties have become thoroughly bedded.

Burnt clay. This material has been used in many sections of the country where broken stone or gravel are unobtainable except at a prohibitive cost, and where a suitable quality of clay is readily obtained. This clay should be of "gumbo" variety and contain no gravel. It is sometimes burnt in a kiln, or it is sometimes burnt by piling the clay in long heaps over

a mass of fuel, the pile being formed in such a way that a temporary but effectual kiln is made. It is necessary that a clear, clean fuel shall be used and that the firing shall be done by a man who is experienced in maintaining such a fire until the burning is completed. Such ballast may be burned very hard and it will last from four to six years. The cost of burning varies from 30 to 60 cents per cubic yard, according to the circumstances.

Chats. This is a form of ballast which is peculiar to Southwestern Missouri and Southeastern Kansas. When this material was first used it was obtained from the refuse piles of the mills which treated the zinc and lead ores mined in those regions. With the processes then employed the material was obtained in lumps as large as broken stone, and they were considered to be as valuable as broken stone for ballast. Improvements in the processes of treating the ores have resulted in making this by-product very much smaller grained and of less value as ballast, although it is still considered a desirable form of ballast where it may readily be obtained. It should be noted that it is classed with gravel and cinders in the forms of cross-section shown later.

Chert. This is a form of flint or hornstone which occurs in nodules of a size that is suitable for ballast, and is a very good type of ballast wherever it is found, but its occurrence is comparatively infrequent. It is classed with cemented gravel in the design of cross-sections of ballast.

Cinders. This is one of the most universal forms of ballast, since it is a by-product of every road which uses coal as fuel. The advantages consist in the fairly good drainage, the ease of handling and the cheapness—after the road is in operation. One of the greatest disadvantages is the fact that the cinders are readily reduced to dust, which in dry weather becomes very objectionable. Cinders are usually considered preferable to gravel in yards.

Gravel. This is one of the most common forms of good ballast. There are comparatively few railroads which cannot find, at some place along their line, a gravel pit which will afford a suitable supply of gravel for ballast. Sometimes it is used just as found in the pit, but for Class A and even Class B roads it is usually necessary to screen it. See § 238*a* for specifications.

Sand. Railroads which run along the coast are frequently

ballasted merely with the sand obtained in the immediate neighborhood. One great advantage lies in the almost perfect drainage which is obtained.

Slag. When slag is readily obtainable it furnishes an excellent ballast which is free from dust and perfect in drainage qualities. Slag is classified with crushed rock in the cross-sections shown below, but it should be noted that this only applies to the best qualities of slag, since its quality is quite variable.

Mud ballast. When the natural soil is gravelly so that rain will drain through it quickly, it will make a fair roadbed for light traffic, but for heavy traffic, and for the greater part of the length of most roads, the natural soil is a very poor material for ballast; for, no matter how suitable the soil might be along limited sections of the road, it would practically never happen that the soil would be uniformly good throughout the whole length of the road. Considering that a heavy rain will in one day spoil the results of weeks of patient "surfacing" with mud ballast, it is seldom economical to use "mud" if there is a gravel-bed or other source of ballast anywhere on the line of the road.

233. **Cross-sections.** The required depth of the cross-section to the sub-soil depends largely on the weight of the rolling stock which is to pass over the track. A careful examination of a roadbed to determine the changes which take place under the ties and also an examination of the track and ties during the passage of a heavy train shows that the heavy loads which are now common on railroad tracks force the tie into the ballast with the passage of every wheel load. The effect on the ballast is a greater or less amount of crushing of the ballast. Even the very hardest grades of broken stone are more or less crushed by grinding against each other during the passage of a train. The softer and weaker forms of ballast are ground up much more quickly. One result is the formation of a fine dust which interferes with the proper drainage of water through the ballast. A second result is the compression of the ballast immediately under the tie into the sub-soil. In a comparatively short time a hole is formed under the tie which acts virtually like a pump. With every rise and fall of the tie under each wheel load, the tie actually pumps the water from the surrounding ballast and sub-soil into these various holes. When the

ballast is of such a character that the water does not drain through it easily, the water will settle in these holes long enough to seriously deteriorate the ties. When the track becomes so much out of line or level, or so loose that it needs to be tamped up, the process of tamping has practically the effect of deepening the amount of ballast immediately under the tie, while the sub-soil is forced up between the ties. A longitudinal section of the sub-soil of a track which has been frequently tamped generally has a saw-tooth appearance, and the sub-soil, instead of being a uniform line, has a high spot between each tie, while the ballast is considerably below its normal level immediately under the tie.

234. **Classification of Railroads.** The American Railway Engineering Association has divided railroads into three classes with respect to the standards of construction which should be adopted for ballasting, as well as other details of construction. The three classes are as follows (quoted from the Association Manual):

"Class 'A' shall include all districts of a railway having more than one main track, or those districts of a railway having a single main track with a traffic that equals or exceeds the following:

Freight-car mileage passing over districts per year per mile..... 150000

or,

Passenger-car mileage per annum per mile of district... 10000

with maximum speed of passenger-trains of 50 miles per hour.

"Class 'B' shall include all districts of a railway having a single main track with a traffic that is less than the minimum prescribed for Class 'A' and that equals or exceeds the following:

Freight-car mileage passing over districts per year per mile..... 50000

or,

Passenger-car mileage per annum per mile of district... 5000

with maximum speed of passenger-trains of 40 miles per hour.

"Class 'C' shall include all districts of a railway not meeting the traffic requirements of Classes 'A' or 'B.' "

The classification was adopted on the consideration that *quality* of traffic as well as mere tonnage should determine

the classification of a railroad. For example, it is considered that a road which operates a train at a speed of 50 miles an hour should adopt the first class or Class "A" standards, even though there is but one train per day on that railroad. It likewise means that any road whose traffic makes necessary the construction of a regular double track should adopt the first class specifications.

235. Recommended sections for the several classifications. In Fig. 110 are shown a series of cross-sections which were

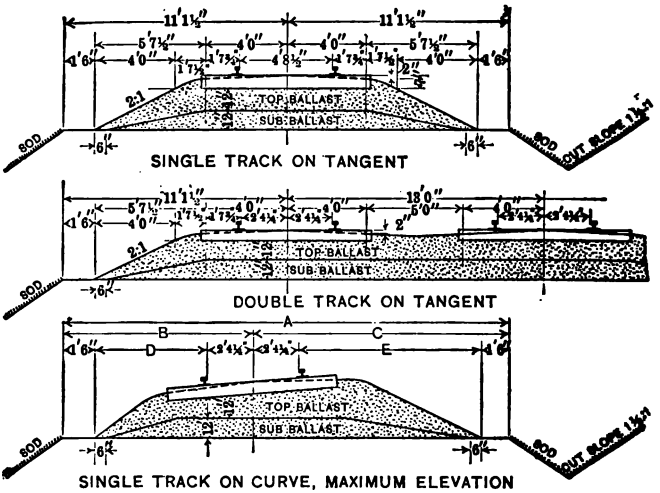
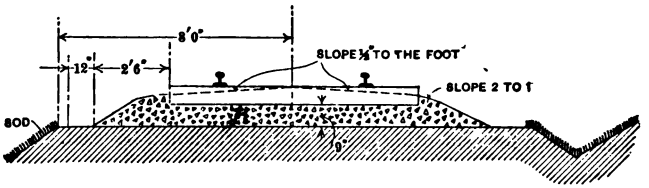


FIG. 110.—CROSS-SECTIONS OF BALLAST FOR CLASS "A" ROADS.

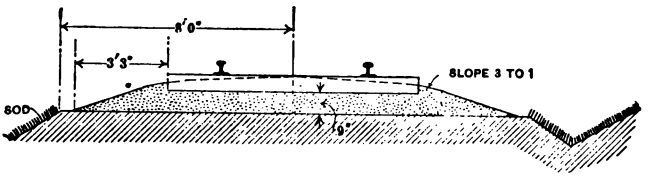
recommended by the A. R. E. A. for Class "A" traffic. It should be noticed that in each case the cross-section of the roadbed from shoulder to shoulder of the roadbed is 22' 3" plus the space between track centers for double track if any. The width of side ditches is merely added to that of the roadbed. The clear thickness of the ballast underneath the ties is made 24 inches. The slope of ½ inch to the foot from the center of the track to the end of the tie, which is common to all the cross-sections, is designed with the idea of allowing a clear space of 1 inch underneath the rail. The ballast is then rounded off

on a curve of 4 feet radius and finally reaches the subsoil on a slope which is 2 : 1.

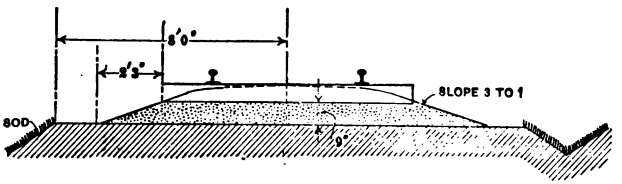
In Fig. 111 are shown a series of cross-sections for various classes of ballast for railroads that belong to Class "B." It



CRUSHED ROCK AND SLAG.



GRAVEL, CINDERS, CHATS, ETC.



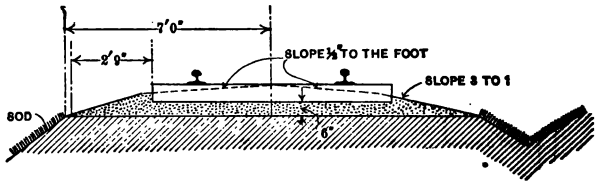
CEMENTING GRAVEL AND CHERT.

FIG. 111.—CROSS-SECTIONS OF BALLAST FOR CLASS "B" ROADS.

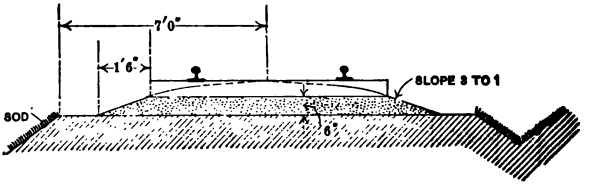
may be noted that the thickness of the ballast under the tie is 9 inches for this class. The width of roadbed between the shoulders, recommended for Class "B" is 16 feet. As before, the width of the ditches is supposed to be added to this width. It should be noted that when using cementing gravel and chert the slope of 3 : 1 is made to begin at the bottom of the tie instead of at a point about 2 inches below the top of the tie. This is done in order to prevent water from accumulating around the end of the tie in a material which is less permeable than the other forms of ballast.

In Fig. 112 are shown two cross-sections for ballast for roads belonging to Class " C." On roads of this class it is assumed that crushed rock will not be used for ballast. The width of roadbed between shoulders is 14 feet, while the depth of ballast underneath the tie is 6 inches.

It should be noticed that the above sections issued by the association do not include any cross-section which is recommended when no special ballast is used other than the natural soil. In such a case a cross-section very similar to the sections shown for cementing gravel and chert should be used. The



GRAVEL, CINDERS, CHATS, ETC.,



CEMENTING GRAVEL AND CHERT.

FIG. 112.—CROSS-SECTIONS OF BALLAST FOR CLASS "C" ROADS.

essential feature of such a section is that the soil, which is probably not readily permeable, should be kept away from the ends of the ties. Specifications for the placing of mud ballast, as well as other forms of ballast, have frequently specified that the ballast should be crowned about 1 inch above the level of the tops of the ties in the center of the track. This feature of any cross-section, although proposed, was rejected by the association, in spite of the fact that when a tie is so imbedded it certainly will have a somewhat greater holding power in the ballast.

236. Proper depth of ballast. The *depth of ballast* is officially defined by the A. R. E. A. as " the distance from the bottom of

the tie to the top of the subgrade." In the recommended sections (Figs. 110 to 112) the depth shown varies from 6 inches to 24 inches. But the Ballast Committee reported in 1915 as a recommended conclusion that "From the data available, it is concluded that with ties 7 in. by 9 in. by 8½ ft., spaced approximately 24 in. to 25.5 ins., center to center, a depth of 24 inches of stone ballast is necessary to produce uniform pressure on the subgrade, and a combination of a lower layer of gravel or cinder ballast, 18 inches to 14 inches, and an upper layer of stone ballast, 6 inches to 10 inches, approximately 24 inches deep in the aggregate, with the same spacing of the ties, will produce nearly the same results." New sections for Class "A" roads which would conform with the above were also recommended. The sections shown in Fig. 110, which are similar to those recommended in 1915, were adopted in 1921. The investigations of the Committee on Track Stresses (see Chap. XXV) have shown why deep ballast is necessary, but the economy of using a second-grade ballast as sub-ballast is possible. As previously stated, old track generally has a depth of ballast under the tie which is greater than the 2 feet recommended—often 3 or 4 feet.

237. Methods of laying ballast. The cheapest method of laying ballast on new roads is to lay ties and rails directly on the prepared subgrade and run a construction train over the track to distribute the ballast. Then the track is lifted up until sufficient ballast is worked under the ties and the track is properly surfaced. This method, although cheap, is apt to injure the rails by causing bends and kinks, due to the passage of loaded construction trains when the ties are very unevenly and roughly supported, and the method is therefore condemned and prohibited in some specifications. The best method is to draw in carts (or on a contractor's temporary track) the ballast that is required under the level of the *bottom* of the ties. Spread this ballast carefully to the required surface. Then lay the ties and rails, which will then have a very fair surface and uniform support. A construction train can then be run on the rails and distribute sufficient additional ballast to pack around and between the ties and make the required cross-section.

The necessity for constructing some lines at an absolute minimum of cost and of opening them for traffic as soon as possible has often led to the policy of starting traffic when there

is little or no ballast—perhaps nothing more than a mere tamping of the natural soil under the ties. When this is done ballast may subsequently be drawn where required by the train-load on flat cars and unloaded at a minimum of cost by means of a "plough." The plough has the same width as the cars and is guided either by a ridge along the center of each car or by short posts set up at the sides of the cars. It is drawn from one end of the train to the other by means of a cable. The cable is sometimes operated by means of a small hoisting-engine carried on a car at one end of the train. Sometimes the locomotive is detached temporarily from the train and is run ahead with the cable attached to it.

238. Cost. The cost of ballast *in the track* is quite a variable item for different roads, since it depends (*a*) on the first cost of the material as it comes to the road, (*b*) on the distance from the source of supply to the place where it is used, and (*c*) on the method of handling. The first cost of cinder or slag is frequently insignificant. A gravel-pit may cost nothing except the price of a little additional land beyond the usual limits of the right of way. Broken stone will usually cost $1 or more per cubic yard. If suitable stone is obtainable on the company's land, the cost of blasting and breaking should be somewhat less than this. The cost of hauling will depend on the distance hauled, and also, to a considerable extent, on the limitations on the operation of the train due to the necessity of keeping out of the way of regular trains. There is often a needless waste in this way. The "mud train" is considered a pariah and entitled to no rights whatever, regardless of the large daily cost of such a train and of the necessary gang of men. "The cost of broken-stone ballast *in the track* is estimated at $1.25 per cubic yard. The cost of gravel ballast is estimated at 60 c. per cubic yard in the track. The cost of placing and tamping gravel ballast is estimated at 20 c. to 24 c. per cubic yard, for cinders 12 c. to 15 c. per cubic yard. The cost of loading gravel on cars, using a steam-shovel, is estimated at 6 c. to 10 c. per cubic yard."—Report Roadmasters' Association, 1885.

238a. Specifications. (Condensed from Am. Rwy. Eng. Assoc. Manual, 1915.) **Broken stone ballast. To be selected** on the basis of maximum (or minimum) figures for the following qualities: (*a*) weight per cubic foot, maximum; (*b*) water absorption in pounds per cubic foot, minimum; (*c*) per cent of

wear, minimum; (*d*) hardness, maximum; (*e*) toughness, maximum; (*f*) cementing value, minimum; (*g*) compression test, maximum. **Gravel ballast.** For Class A railways: Bank gravel which contains more than two (2) per cent dust or forty (40) per cent sand should be washed or screened. Washed or screened gravel should contain not less than twenty-five (25) per cent nor more than thirty-five (35) per cent sand. For Class B railways: Bank gravel which contains more than three (3) per cent dust or sixty (60) per cent sand should be screened or washed. Washed or screened gravel should not contain less than twenty-five (25) per cent nor more than fifty (50) per cent sand. For Class C railways. Any material which makes better track than the natural roadbed may be economically used.

Testing gravel for ballast. Obtain five samples, each about one cubic foot, from various parts of the pit; mix thoroughly; make up a sample of about one cubic foot from the mixture. Sift through a screen, 10 meshes per linear inch, made of No. 24 B. & S. wire; the residue is the "gravel," *G*. Sift the remainder through a screen, 50 meshes per linear inch, made of No. 31 B. & S. wire; the residue is the "sand," *S*. That which passed through the screen is "dust," *D*. The percentage of sand, for example, equals $S \div (G+S+D)$.

CHAPTER VIII.

TIES,

AND OTHER FORMS OF RAIL SUPPORT.

239. Various methods of supporting rails. It is necessary that the rails shall be sufficiently supported and braced, so that the gauge shall be kept constant and that the rails shall not be subjected to excessive transverse stress. It is also preferable that the rail support shall be neither rigid (as if on solid rock) nor too yielding, but shall have a *uniform* elasticity throughout. These requirements are more or less fulfilled by the following methods.

(a) **Longitudinals.** The fundamental idea is to have continous support for the rail rather than to have it act as a continuous girder with numerous supporting points—the ties. In § 264 will be described a system of rails, used to some extent in Europe, having such broad bases that they are self-supporting on the ballast and are only connected by tie-rods to maintain the gauge.

(b) **Cast-iron "bowls" or "pots."** These are castings resembling large inverted bowls or pots, having suitable chairs on top for holding and supporting the rails, and tied together with tie-rods. They will be described more fully later (§ 263).

(c) **Cross-ties of metal or wood.** These will be discussed in the following sections.

240. Economics of ties. The true cost of ties depends on the relative total cost of maintenance for long periods of time. The first cost of the ties delivered to the road is but one item in the economics of the question. Cheap ties require frequent renewals, which cost for the *labor* of each renewal practically the same whether the tie is of oak or of hemlock. Cheap ties make a poor roadbed which will require more track labor to keep even in tolerable condition. The roadbed will require to be disturbed so frequently on account of renewals that the ties never get an opportunity to get settled and to form a smooth roadbed for any length of time. Irregularity in width, thickness, or length of ties is especially detrimental in causing the ballast to act and

wear unevenly. The life of ties has thus a more or less direct influence on the life of the rails, on the wear of rolling stock, and on the speed of trains. These last items are not so readily reducible to dollars and cents, but when it can be shown that the total cost, for a long period of time, of several renewals of cheap ties, with all the extra track labor involved, is as great as or greater than that of a few renewals of durable ties, then there is no question as to the real economy. In the following discussions of the merits of untreated ties (either cheap or costly), chemically treated ties, or metal ties, the true question is therefore of the ultimate cost of maintaining any particular kind of ties for an indefinite period, the cost including the first cost of the ties, the labor of placing them and maintaining them to surface, and the somewhat uncertain (but not therefore nonexistent) effect of frequent renewals on repairs of rolling stock, on possible speed, etc.

WOODEN TIES.

241. Choice of wood. This naturally depends, for any particular section of country, on the supply of wood which is most readily available. Table XXII shows the relative use of the chief varieties in the U. S. Two-thirds of the entire list is white oak, red oak and southern pine. Douglas fir, which grows only in the west, is being transported to the east in increasingly large quantities and is displacing other woods. The use of eastern tamarack, lodge pole pine, western larch, western yellow pine, and hemlock is almost confined to the "western region" — west of the Mississippi river. Redwood was formerly used quite extensively in the west, on account of cheapness and immunity from decay, but the wood is

TABLE XXII.—NUMBER AND KINDS OF CROSS TIES USED BY 78% OF TOTAL MILEAGE OF STEAM RAILROADS IN UNITED STATES IN 1915.

(Bull. 549, U. S. Dept. Agric.).

Kind of wood.	Number of ties.	Per cent.
White oak	30,160,316	34.1
Red oak	15,989,605	18.1
Southern pine	13,226,654	15.0
Douglas fir	6,308,685	7.1
Cypress	4,375,012	4.9
Cedar	4,121,570	4.7
Chestnut	2,666,402	3.0
Eastern tamarack	2,520,475	2.8
Lodge pole pine	1,254,420	1.4
Western larch	1,196,415	1.3
Western yellow pine	1,183,535	1.3
Beech	1,139,457	1.3
Maple	1,062,086	1.2
Hemlock	839,924	1.0
All other	2,454,099	2.8
Total	88,498,655	100.0

too soft. The use of cypress is nearly confined to the west and south, and on the other hand the use of chestnut is nearly confined to the "eastern region"—north of the Ohio and Potomac and east of Chicago.

On the basis of 88,498,655 ties for 78.46% of the mileage, the proportionate total is 112,974,615 ties. 100,000,000 to 125,000,000 ties per year is elsewhere stated to be the normal demand. This means an annual *average* of about 290 ties for each mile of track, including sidings.

242. Durability. The durability of ties depends on the climate; the drainage of the ballast; the volume, weight, and speed of the traffic; the curvature, if any; the use of tie-plates; the time of year of cutting the timber; the age of the timber and the degree of its seasoning before placing in the track; the nature of the soil in which the timber is grown; and, chiefly, for untreated ties, on the species of wood employed. The variability in these items will account for the discrepancies in the reports on the life of various woods used for ties. For example, six records of untreated white oak ties on six different roads gave figures varying from 3 to 14 years. Such a range of values is too wide for practical utilization.

The variability in the actual life of a "group" of ties of nominally the same quality and placed in the track at the same time

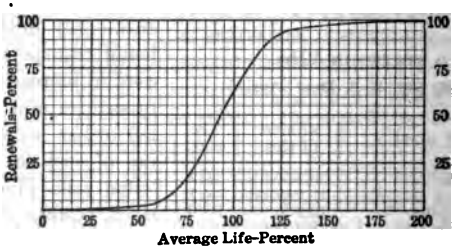


FIG. 112*a*.—RELATIVE ACTUAL LIFE OF TIES OF NOMINALLY UNIFORM QUALITY.

is shown in a study * made by the Forest Products Laboratory, U. S. Forest Service. Records show that there will be, in general,

* "Relation between average life of ties and percentage of renewals," by Mabel E. Thorne, Statistician.

no replacements until after about 30% of the average life of the whole group. Then the replacements will commence and grow more frequent until at the time of the *average* life of the whole group, about 60% will have been replaced. After a time equal to 120% of the average life, about 90% of the ties will have been replaced, but a few of the remainder may stay in the track until nearly or quite 200% of the average life. The law is based on the records of 43 groups of ties comprising 42936 ties, or an average of about 1000 ties per group. The law is substantially true whether applied to short-lived untreated ties or to long-lived treated ties. The law may even be considered as sufficiently established so that when 10% of a "group" of ties have been removed from a track, the time already elapsed may be considered as approximately 70% of the *average* life of the entire group, and the probable life of the remaining 90% of ties may be estimated accordingly.

Some of the softer woods used for ties, such as *cedar* and *redwood*, resist decay very well, but are so soft that they are badly cut by the rail-flanges and do not hold the spikes very well, necessitating frequent respiking. Since the spikes must be driven within certain very limited areas on the face of each tie, it does not require many spike holes to "spike-kill" the tie. On sharp curves, especially with heavy traffic, the wheel-flange pressure produces a side pressure on the rail tending to overturn it, which tendency is resisted by the spike, aided sometimes by rail-braces. Whenever the pressure becomes too great the spike will yield somewhat and will be slightly withdrawn. The resistance is then somewhat less and the spike is soon so loose that it must be redriven in a new hole. If this occurs very often, the tie may need to be replaced long before any decay has set in.

243. Dimensions. The usual dimensions for the best roads (standard gauge) are 8′ to 9′ long, 6″ to 7″ thick, and 8″ to 10″ wide on top and bottom if they are sawed. Hewed ties (with rounded sides) shall have the faces not less than 6 inches wide, but the cross sectional area must not be less than a sawed tie of the same class. Narrow gauge and very-light-traffic roads will reduce these dimensions as much as twenty per cent.

244. Spacing. The Penna. R. R. standard spacing (1921) called for 14, 16, 18 or 20 ties per 33-foot rail, according to the

classification of track. The joints of the two lines of rails are placed "staggered" rather than "opposite" each other. The joints are "suspended" (see § 282) on two ties spaced 20" c. c. There are for each rail length two spaces 20" each and 12, 14, 16 or 18 spaces of $29\frac{2}{3}$", $25\frac{5}{7}$", $22\frac{1}{2}$", or $19\frac{7}{9}$" each.

245. **Specifications.** The specifications for ties are apt to include the items of size, kind of wood, and method of construction, besides other minor directions about time of cutting, seasoning, delivery, quality of timber, etc.

(a) **Size.** The particular size or sizes required will be somewhat as indicated in § 243.

(b) **Kind of wood.** When the kind or kinds of wood are specified, the most suitable kinds that are available in that section of country are usually required.

(c) **Method of construction.** It is generally specified that the ties shall be hewed on two sides; that the two faces thus made shall be parallel planes and that the bark shall be removed. It is sometimes required that the ends shall be sawed off square; that the timber shall be cut in the winter (when the sap is down); and that the ties shall be seasoned for six months These last specifications are not required or lived up to as much as their importance deserves. It is sometimes required that the ties shall be delivered on the right of way, neatly piled in rows, the alternate rows at right angles, piled if possible on ground not lower than the rails and at least ten feet away from the nearest rail, the lower row of ties resting on two ties which are themselves supported so as to be clear of the ground.

(d) **Quality of timber.** The usual specifications for sound timber are required, except that they are not so rigid as for a better class of timber work The ties must be sound, reasonably straight-grained, and not very crooked—one test being that a line joining the center of one end with the center of the middle shall not pass outside of the other end. Splits or shakes, especially if severe, should cause rejection.

Specifications sometimes require that the ties shall be cut from small trees, making what is known as "pole ties" and definitely condemning those which are cut or split from larger trunks, giving two "slab

FIG. 113.—METHODS OF CUTTING TIES.

ties" or four "quarter ties" for each cross-section, as is illustrated in Fig. 113. Even if pole ties are better, their exclusive use means the rapid destruction of forests of young trees.

246. Regulations for laying and renewing ties.—The regulations issued by railroad companies to their track foremen will generally include the following, in addition to directions regarding dimensions, spacing, and specifications given in §§ 242–245. When hewn ties of somewhat variable size are used, as is frequently the case, the largest and best are to be selected for use as joint ties. If the upper surface of a tie is found to be warped (contrary to the usual specifications) so that one or both rails do not get a full bearing across the whole width of the tie, it must be adzed to a true surface along its whole length and not merely notched for a rail-seat. When respiking is necessary and spikes have been pulled out, the holes should be immediately plugged with "wooden spikes," which are supplied to the foreman for that express purpose, so as to fill up the holes and prevent the decay which would otherwise take place when the hole becomes filled with rain-water. Ties should always be laid at right angles to the rails and never obliquely Minute regulations to prevent premature rejection and renewal of ties are frequently made. It is generally required that the requisitions for renewals shall be made by the actual count of the individual ties to be renewed instead of by any wholesale estimates. It is unwise to have ties of widely variable size, hardness, or durability adjacent to each other in the track, for the uniform elasticity, so necessary for smooth riding, will be unobtainable under those circumstances.

After a considerable discussion of the two policies of tie renewals over long continuous stretches of track or of single tie renewals where individually needed, the A. R. E. A. has decided in favor of single tie renewals, as being most economical and producing least track disturbance.

247. Dating nails. These are made of iron or steel, galvanized with zinc. They should be $2\frac{1}{2}$ inches long, $\frac{1}{4}$ inch in diameter, with $\frac{5}{8}$-inch head, which has two figures $\frac{3}{16}$ inch high, denoting the year, which are stamped, by depression, into the head. They should be driven into the upper side of all treated ties, 10 inches inside the rail, on the line side of the track. The use of such dates gives definite knowledge of the life of the tie when it is renewed and a means of studying the effectiveness of the tie treatment.

248. Cost of ties. When railroads can obtain ties cut by farmers from woodlands in the immediate neighorhood, they sometimes advertise a schedule of prices which they will pay, the prices being considerably lower than the prices demanded by dealers. Prices as low as 35 c. were formerly paid directly to tie cutters in tie growing sections, but increasing scarcity has raised the price. A great railway paid $610,713 for 453,000 ties in 1920, an average of $1.31 each. These were of higher grade than the average. The following schedule shows proportionate prices: white oak, $1.39; heart pine, $1.66; chestnut, $1.37; red oak, $1.34; sap pine, $1.19; maple, beech and birch, $1.27.

PRESERVATIVE PROCESSES FOR WOODEN TIES.

249. General principles. Wood has a fibrous cellular structure, the cells being filled with sap or air. The woody fiber is but little subject to decay unless the sap undergoes fermentation. Preservative processes generally aim at removing as much of the water and sap as possible and filling up the pores of the wood with an antiseptic compound. The most common methods all agree in this general process and only differ in the method employed to get rid of the sap and in the antiseptic chemical with which the fibers are filled. One valuable feature of these processes lies in the fact that the softer cheaper woods are more readily treated than are the harder woods and from them a tie can be made which will be as durable as the best (from the standpoint of decay), and, if protected from mechanical wear by tie-plates, will have a very long life. The following woods may be used without preservative treatment: White oak family, long-leaf strict heart yellow pine, cypress, excepting the white cypress, redwood, white cedar, chestnut, catalpa, locust, except the honey locust, walnut and black cherry. The following woods should preferably not be used without preservative treatment: Red oak family, beech, elm, maple, gum, loblolly, short-leaf, Western yellow pine, Norway, North Carolina pine and other sap pines, red fir, spruce, hemlock, and tamarack. It is better to use an excess of chemical rather than not enough. Ties should be grouped before treatment; for example, green ties should not be mixed with seasoned ties, since the treatment should be different. Ties should be air-seasoned before being

treated. When there is time to air-season them at the plant before treatment, they should be piled in groups having the same degree of seasoning, so that they rest on seasoned stringers, the lowest ties at least 6 inches from the ground, which should be thoroughly drained and cleared from weeds, high grass and decaying matter. The ties should not be allowed to over-season or deteriorate. Ties which show signs of checking should be secured with S-irons or bolts to prevent further checking. When ties are to be adzed or bored for the use of tie-plates or screw spikes, the adzing or boring should be done before chemical treatment. Steam seasoning, if excessive, weakens the wood. It should therefore be limited, unless it is imperative to treat green ties because air-seasoned ties are not obtainable.

To do the work, long cylinders, which may be opened at the ends, are necessary. Usually the timbers are run in and out on iron carriages running on rails fastened to braces on the inside of the cylinder. When the load has been run in, the ends of the cylinder are fastened on. The water and air in the pores of the wood are drawn out by subjecting the wood alternately to steam-pressure and to the action of a vacuum-pump. Live steam should be admitted so that a pressure of 20 lbs. is produced within 30 to 50 minutes. This pressure may be maintained from 1 to 5 hours, depending on the condition of the wood, but the pressure should never exceed 20 lbs. A vent should be provided to allow the escape of air and condensed water. After steaming, a vacuum of not less than 24 inches of mercury at sea-level (or correspondingly less for higher altitudes), shall be produced and maintained for half an hour. Then, without breaking the vacuum, the chemical shall be admitted.

250. Creosoting. This process consists in impregnating the wood with creosote oil, a product obtained from coal-gas tar or coke oven tar which shall be free from any tar, including coal-gas tar, oil or residue obtained from petroleum or any other source. The pure creosote oil is strongly recommended by the A.R. E. A., but they recognize that the practice of using other coal tar distillates, when the available supply of creosote is inadequate, is firmly established, and have made specifications accordingly.

It would require about 35 to 50 lbs. of creosote to completely fill the pores of a cubic foot of wood. But it would be impossible to force such an amount into the wood, nor is it necessary or desirable. After one of the vacuum periods, the cylinder is

filled with creosote oil having a temperature of not less than 160° F. The cylinders should be provided with steam coils in order to maintain that temperature during injection. The pressure should immediately be raised to 75 lbs. per square inch, and then by a gradual increase to a maximum of 175 or 200 lbs. or until about 6 to 10 lbs. per cubic foot, or about 21 to 35 lbs. per tie, is absorbed, this amount being indicated by calculations based on gauge readings of the oil in the oil reservoir, taken before and after the introduction and withdrawal of the oil from the cylinder. Owing to variations in the volume of the creosote due to change of temperature during treatment, also to variations in the capacity volume of the cylinder due to change in temperature of the metal, and several other causes, the determination of the volume of the oil actually absorbed by the ties is not simple. Each cylinder must be calibrated by a series of tests, since these causes may easily produce an error of 25% in the nominal results. As a check, the ties on a cylinder tram-car should occasionally be weighed before and after treatment. Even this check will not be conclusive if the ties have been steam seasoned, since steam seasoning usually increases the weight and this increase would be credited as absorption of chemical.

251. **Burnettizing (chloride-of-zinc process).** This process is very similar to the creosoting process except that the chemical is chloride of zinc. The chemical is heated to 140° F. before using. The preliminary treatment of the wood to alternate vacuums and pressure is not continued for quite so long a period as in the creosoting process. Care must be taken, in using this process, that the ties are of as uniform quality as possible, for seasoned ties will absorb much more zinc-chloride than unseasoned (in the same time), and the product will lack uniformity unless the seasoning is uniform. The amount of solution injected shall be equivalent to ½ lb. of dry soluble zinc-chloride per cubic foot of timber. The solution shall be as weak as can be used and still obtain the desired absorption of zinc-chloride, and shall not be stronger than 5%. If the cylinders are provided with steam coils, steam pressure shall be maintained in these coils during treatment. One great objection to burnettized ties is the fact that the chemical is somewhat easily washed out, when the wood again becomes subject to decay. Another objection is the fact that when the solution of zinc-chloride is made strong (over 3%)

the timber is made very brittle and its strength is reduced. The reduction in strength has been shown by tests to amount to $\frac{1}{4}$ to $\frac{1}{10}$ of the ultimate strength, and that the elastic limit has been reduced by about $\frac{1}{7}$.

252. Kyanizing (bichloride-of-mercury or corrosive-sublimate process). This process has been much used, but it is so objectionable, on account of the chemical being such a virulent poison that workmen are sickened by fumes arising from the tanks, that it is no longer included as one of the standard methods.

253. Zinc-tannin process. The last two methods described (as well as some others employing similar chemicals) are open to the objection that since the wood is impregnated with an aqueous solution, it is liable to be washed out very rapidly if the wood is placed under water, and will even disappear, although more slowly, under the action of moisture and rain. Several processes have been proposed or patented to prevent this. By one of these processes the timber is successively subjected to the action of chemicals, each individually soluble in water, and hence readily impregnating the timber, but the chemicals when brought in contact form insoluble compounds which cannot be washed out of the wood-cells. After injecting the zinc-chloride, as before described, the solution is run off and the ties drained for 15 minutes. Then a 2% solution of tannic acid, made from $6\frac{2}{3}$ lbs. of 30% extract of tannin and 100 lbs. of water is run in and maintained at 100 lbs. pressure for one-half hour. Then a solution of glue made by dissolving 2.1 lbs. of glue containing 50% gelatine in 100 lbs. of water is run in and maintained at 100 lbs. pressure for one-half hour. The glue and tannin combine to form an insoluble leathery compound in the cells, which will prevent the zinc-chloride from being washed out.

254. Zinc-creosote emulsion process. The chemical is an emulsion which will leave in the wood an equivalent of 0.4 lb. of dry, soluble zinc-chloride and from 1.25 to 1.5 lbs. of creosote per cubic foot. The zinc-chloride must not be stronger than 3.5%. The emulsion must be effectively mixed in a storage tank and heated to at least 140° F. before it enters the cylinder, where the pressure is raised to 100 lbs. per square inch and maintained there until the required amount of chemical has been absorbed by the wood.

255. Two-injection zinc-creosote process. The zinc-chloride and creosote are injected separately. The zinc-chloride must be as weak as possible (not more than 5%), and yet strong enough

so that the equivalent of 0.3 lb. can be injected per cubic foot. After impregnation, the remaining zinc-chloride is run out and the creosote is forced in and maintained at 100 lbs. pressure until the wood has absorbed about 3 lbs. of oil per cubic foot.

256. Cost of treating. The cost of treating ties by the various methods has been estimated as follows.* The total cost is divided into (1) seasoning; (2) labor; (3) fuel; (4) maintenance and (5) chemicals.

Seasoning. The labor required for air-seasoning, the usual practice, is estimated at from 0.75 c. to 1.5 c. per tie, or is averaged at 1.0 c. per tie. **Labor.** The labor involved in all other handling of the ties is averaged at 6.0 c. per tie. **Fuel** may cost 0.5 c. per tie when natural gas or oil is obtainable and up to 2.0 c. per tie for other scarcer fuels; it is averaged at 1.0 c. per tie. **Maintenance** of the plant is estimated at 1.25 c. to 2.0 c. per tie; as an average it is placed at 1.5 c. for creosoting plants and 1.6 c. for plants using zinc-chloride, since it is more corrosive. **Chemicals.** On the basis of a 7″×9″×8′ tie, having a volume of 3.5 cubic feet, and ½ lb. of zinc-chloride per cubic foot, the amount of $ZnCl_2$ is 1.75 lbs. per tie; at 4c. per pound this would cost 7 c. per tie. Using 10 lbs. of creosote per cubic foot or 35 lbs. per tie, 4.08 gallons (8.58 lbs. per gallon) of creosote would be used per tie. A price of 6 to 10 cents per gallon is quoted for large quantities of creosote. Apparently 6.84 c. per gallon was used in the calculation, since the cost of the creosote was put at 27.9 c. per tie. Summarizing, the cost by the several methods was as given below.

Chemical used.	Quantity per cubic foot.	Seasoning, labor, fuel.	Maintenance of plant.	Chemical cost.	Total.
Creosote	10 lbs.	8.0 c.	1.5 c.	27.9 c.	37.5 c.
"	6 "	8.0 "	1.5 "	16.8 "	26.3 "
Creosote	3 "	8.0 "	1.6 "	15.4 "	25.0 "
$ZnCl_2$	½ "				
Zinc chloride	½ "	8.0 "	1.6 "	7.0 "	16.6 "

Of course the above figures are merely illustrative. Variations in the cost of labor and materials will probably change all these figures. Nothing is included for interest, depreciation, superintendence or profit.

* Bull. No. 118, U. S. Dept. of Agric., Div. of Forestry. Nov., 1912.

257. **Economics of treated ties.** The fact that treated ties are
t universally adopted is due to the argument that the added
e of the tie is not worth the extra cost. If ties can be bought
· 25 c., and cost 25 c. for treatment, and the treatment only
ubles their life, there is apparently but little gained except
e work of placing the extra tie in the track, which is more
less offset by the interest on 25 c. for the life of the untreated
, and the larger initial outlay makes a stronger impression on
e mind than the computed ultimate economy. But when
tilizing some statistics from the Pittsburg, Ft. Wayne &
ıicago Railroad) it is found that white oak ties laid in rock
llast had a life of 10.17 years, and that hemlock ties treated
th the zinc-tannin process and laid in the same kind of ballast
ıted 10.71 years, then the economy is far more apparent.
ıfortunately no figures were given for the cost of these ties
r for the cost of the treatment; but if we assume that the
ıite oak ties cost 75 c. and the hemlock ties 35 c. plus 20 c.
· treatment, there is not only a saving of 20 c. on each tie,
t also the advantage of the slightly longer life of the treated
. In the above case the total life of the two kinds of ties
so nearly the same that we may make an approximation of
eir relative worth by merely comparing the initial cost; but
ually it is necessary to compare the value of two ties one
which may cost more than the other, but will last considerably
ıger. The mathematical comparison of the real value of
o ties under such conditions may be developed as follows:
ıe real cost of a tie, or any other similar item of constructive
ırk, is measured by the cost of perpetually maintaining that
m in proper condition in the structure. It will be here
ıumed that the annual cost of the trackwork, which is assign-
le to the tie, is the same for all kinds of ties, although the
ference probably lies in favor of the more expensive and
ıst durable ties. By assuming this expense as constant, the
naining expense may be considered as that due to the cost
the new ties whenever necessary, plus the cost of placing
ım in the track. We also may combine these two items
one, and consider that the cost of placing a tie in the track,
ıich we will assume at the constant value of 20 c. per tie,
ıardless of the kind of tie, is merely an item of 20 c. in the
ıal cost of the tie. We will assume that T_1 is the present
ıt of a tie, the cost including the preservative treatment if

any, and the cost of placing in the track. The tie is assumed to last n years. At the end of n years another tie is placed in the track, and, for lack of more precise knowledge, we will assume that this cost T_2 equals T_1. The "present worth" of T_2 is the sum which, placed at compound interest, would equal T_2 at the end of n years, and is expressed by the quantity $\frac{T_2}{(1+r)^n}$, in which r equals the rate of interest. Similarly at the end of $2n$ years we must expend a sum T_3 to put in the third tie, and the present worth of the cost of that third tie is expressed by the fraction $\frac{T_3}{(1+r)^{2n}}$. We may similarly express the present worths of the cost of ties for that particular spot for an indefinite period. The sum of all these present worths is given by the sum of a converging series and equals (assuming that all the T's are equal) $\frac{T\times(1+r)^n}{(1+r)^n-1}$. But instead of laying aside a sum of money which will maintain a tie in that particular place in perpetuity, we may compute the annual sum which must be paid at the end of each year, which would be the equivalent. We will call that annual payment A, and then the present worths of all these items are as follows:

For the first payment ..... $\frac{A}{(1+r)}$;

For the second payment ..... $\frac{A}{(1+r)^2}$;

For the third payment ..... $\frac{A}{(1+r)^3}$;

For the nth payment ..... $\frac{A}{(1+r)^n}$.

After the next tie is put in place we have the present worths of the annual payments on the second tie, of which the first one would be

For the $(n+1)$ payment ..... $\frac{A}{(1+r)^{(n+1)}}$.

Similarly after x ties have been put in place the last payment for the x tie would have a present worth $\frac{A}{(1+r)^{nx}}$. The

sum of all these present worths is represented by the sum of a converging series and equals the very simple expression $\frac{A}{r}$. But since the sum of the present worths of these annual payments must equal the sum of the present worths of the payments made at intervals of n years, we may place these two summations equal to each other, and say that

$$A = \frac{r \times T \times (1+r)^n}{(1+r)^n - 1}.$$

Values of A for various costs of a tie T on the basis that r equals 5% have been computed and placed in Table XVIII. To illustrate the use of this table, assume that we are comparing the relative values of two ties, both untreated, one of them a white oak tie which will cost, say 75 c., and will last twelve years, the other a yellow pine tie which will cost, say 35 c., and will last six years. Assuming a charge for each case of 20 c. for placing the tie in the track, we have as the annual charge against the white oak tie, which costs 95 c. in the track, 10.72 c. The pine tie, costing 55 c. in the track and lasting six years, will be charged with an annual cost of 10.48 c., which shows that the costs are practically equal. It is probably true that the track work for maintaining the white oak would be less than that for the pine tie, but since the initial cost of the pine tie is less than that of the oak tie, it would probably be preferred in this case, especially if money was difficult to obtain. It may be interesting to note that if a comparison is made from a similar table which is computed on the basis of compounding the money at 4% instead of 5%, the annual charges would be 10.13 and 10.49 c. for the oak and pine ties respectively, thus showing that when money is "easier" the higher priced tie has the greater advantage.

EXAMPLE 2. Considering again the comparison previously made of a white oak untreated tie which was assumed to cost 75 c., and a hemlock treated tie, which cost 35 c. for the tie and 20 c. for the treatment, the total costs of these ties laid in the track would therefore be 95 c. and 75 c. respectively. These ties had practically the same life (10.17 and 10.71 years), but in order to use the table, we will call it ten years for each tie. The annual charge against the oak tie would therefore

be 12.30 c., while that against the hemlock tie would be 9.72 c. This gives an advantage in the use of the treated tie of 2.58 c. per year, which capitalized at 5% would have a capitalized value of 51.6 c.

The Atchison, Topeka and Santa Fé R. R. has compiled a record of treated pine ties removed in 1897, '98, '99, and 1900, showing that the *average* life of the ties removed had been about 11 years. On the Chicago, Rock Island and Pacific R. R., the average life of a very large number of treated hemlock and tamarack ties was found to be 10.57 years. Of one lot of 21,850 ties, 12% still remained in the track after 15 years' exposure.

It has been demonstrated that much depends on the minor details of the process—whatever it may be. As an illustration, an examination of a batch of ties, treated by the zinc-creosote process, showed 84% in service after 13 years' exposure; another batch, treated by another contractor by the same process (nominally), showed 50% worthless after a service of six years.

METAL TIES.

258. **Extent of use.** In 1894 * there were nearly 35000 miles of "metal track" in various parts of the world. Of this total, there were 3645 miles of "longitudinals" (see § 264), found exclusively in Europe, nearly all of it being in Germany. There were over 12000 miles of "bowls and plates" (see § 263), found almost entirely in British India and in the Argentine Republic. The remainder, over 18000 miles, was laid with metal cross-ties of various designs. There were over 8000 miles of metal cross-ties in Germany alone, about 1500 miles in the rest of Europe, over 6000 miles in British India, nearly 1000 miles in the rest of Asia, and about 1500 miles more in various other parts of the world. Several railroads in this country have tried various designs of these ties, but their use has never passed the experimental stage. These 35000 miles represent about 9% of the total railroad mileage of the world—nearly 400000 miles. They represent about 17.6% of the total railroad mileage, exclusive of the United States and Canada, where they are used but little, except experimentally. In the four years from 1890 to 1894 the use of metal track increased from less than 25000 miles to nearly

* Bulletin No. 9, U. S. Dept. of Agriculture, Div. of Forestry.

35000 miles. This increase was practically equal to the total increase in railroad mileage during that time, exclusive of the increase in the United States and Canada. This indicates a large growth in the percentage of metal track to total mileage, and therefore an increased appreciation of the advantages to be derived from their use.

The above figures were true in 1894. Since then there has been considerable development. In 1915, over one million of the "Carnegie" steel ties, M21 section, had been laid on the Bessemer and Lake Erie R. R. It is now the standard on that road. On several other roads these ties are used extensively and "are not in the nature of test installations." The National Railways of Mexico have adopted as standard a pressed steel tie. The scarcity of tie timber in Mexico, the comparatively light weight of rolling stock and comparatively low speed, combine to favor this form of tie, which is very similar to a tie tried as an experiment by the N. Y. C. & H. R. R. R. in 1892, but which was found unsuitable for their requirements.

259. Forms and dimensions of some metal ties. As shown on Plate VI, the ties have approximately the same external dimensions as wooden ties. Stability in the ballast requires that they shall be heavy, at least as heavy as a wooden tie, and that the shape shall be such that, when surrounded by ballast, they shall be anchored against horizontal or vertical motion. The broad lower flange of the Carnegie tie apparently fulfils the latter requirement. The "Champion" tie, shown on Plate VI, is essentially an inverted T, of $\frac{5}{16}$" metal, with a base 10" wide, and a flange 5" high. Two pairs of white oak blocks, easily renewable, and into which cut spikes or screw spikes may be driven, are higher than the flange and there is therefore no trouble about the insulation of track circuits. The "System Couillet," used in Europe, has some of the same principles, but is much lighter and only serviceable for lighter rolling stock.

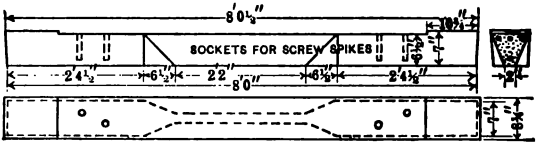
260. Durability. Many metal ties have failed because of breakage, which generally begins at some opening, perhaps a bolt hole, or a place where the metal has been sheared on three sides and bent down on the fourth side to form a lug; the break invariably begins at some *corner*, if the opening has sharp corners. Some metal ties have crushed down immediately under the rail, showing that the design was too light and that there was too little metal there for the traffic it had to carry.

Metal ties are subject to rust, especially when in damp loc ities, such as tunnels; but on the other hand it is in such c fined localities, where renewals are troublesome, that it is esp ally desirable to employ the best and longest-lived ties. Pai tar, etc., have been tried as a protection against rust, but su protection is quickly scraped off and the conditions prevent n renewal of the protection, such as may be done by repaint a bridge, for example. Thirty Carnegie ties, which weigh originally 5213 pounds, were taken from the track after years' service; after the dirt and rust had been removed, th were found to weigh 4912 pounds, a loss of 301 pounds, or average loss of less than 1% per year. A metal tie could p haps lose 35% of its weight by rusting before this cause alo would require its removal.

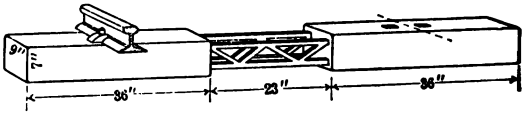
Virtual failures, necessitating removal, are frequently due defects in the device for fastening the rails to the ties. Sor of the designs include a lug, fitting over the base of the rail a held in place by a bolt and nut. These are often jarred loo unless the nuts are held by nutlocks.

Many ties, both steel and concrete, which have abundi strength to support the mere weight of the traffic, are immc ately broken when a derailment causes car wheels or engi drivers to strike them directly. They do not have the tou ness and resiliency of wooden ties to withstand such shocks.

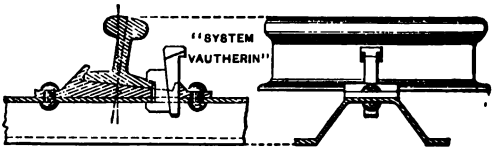
The Carnegie tie is the only steel tie which has been used sufficient quantities and for such a length of time that a rational estimate of its life may be made—except those exp mental types of ties whose life has been so short that they evidently failures. 22400 Carnegie ties were laid on the Dulu Missabe & Northern Rwy. in 1908. In 1916 one tie was remov under special circumstances. In 1919, 30 had failed by cru ing under the rail seat. By 1920, a total of about 100 had fail This is a little over 0.4% after a period of twelve years. T ratio is too small to apply to the curve shown in Fig. 112*a*, § 2 but it indicates a very long average life. Another far less fav able case is that of 384 ties placed in the Erie R. R. in 19 Ten were removed in 1916, eighteen more in 1918, and fourte more in June, 1919. A later report states that the last of th were removed by August, 1919, after an average of over ni years' service. "The majority of them were crushed und the rail seat," which indicates that they were too light for t

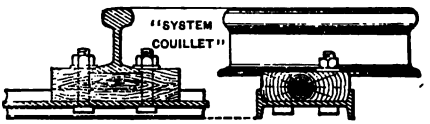


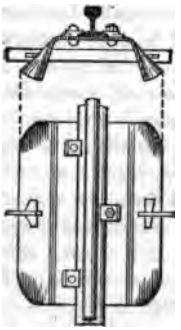
PERCIVAL REINFORCED CONCRETE TIE (1910)



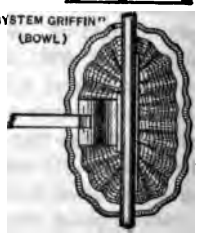
BATES REINFORCED CONCRETE TIE (1912)

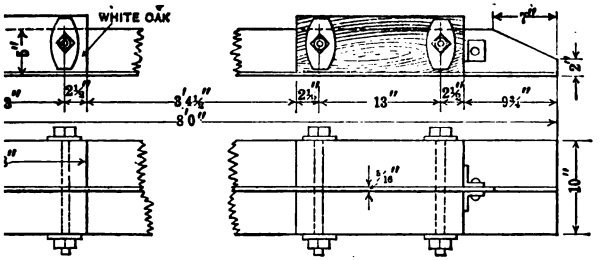




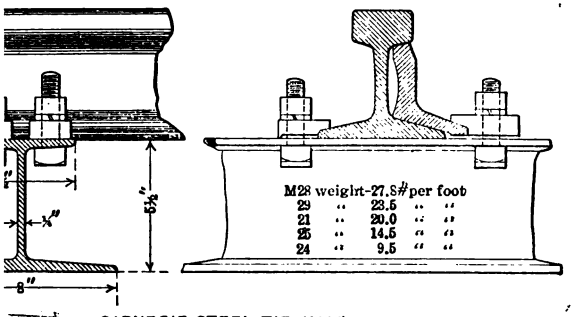


LIVESEY BOWL. (1864)

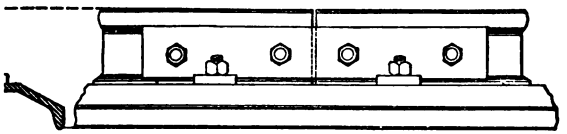




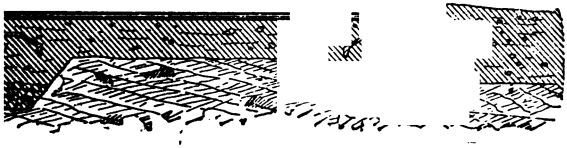
CHAMPION STEEL (1920)



CARNEGIE STEEL TIE (1916)



"SYSTEM KÖSTLIN u. BATTIG"—LONGITUDINAL.



LATE VI.—SOME FORMS OF METAL TIES.

(*Between pp. 292 and 293.*)

work they had to do. Several other roads have made similar reports—a few experimental ties have crushed under the head after a few years' service, evidently because the type chosen (there are five weights) was too light for the weight of the rolling stock.

261. Economics of steel ties. Perhaps the most potent reason for the slow adoption of a substitute for the wooden tie is the plain matter of cost. In spite of the fact that the available supplies of tie timber are being used up at a rate which is several times the rate of renewal of such supplies by growth, the relative cost of steel and wooden ties is such that the steel tie must show a great superiority in order to justify its extra cost. Present prices (1921) are abnormal but are perhaps relatively nearly the same. Assume that a white oak tie costs \$1.40 and that it costs \$1.12 more for spikes and tie plates, and 30 c. more to place it in the track; assume that this tie will last 8 years under a certain class of traffic. Then, by Table XVIII, the annual charge for an initial cost of \$2.82 is $2.82 \times 15.47 = 43.63$ c. The present quoted price for a Carnegie M21 tie, *including fastenings*, is \$5.00; adding 30 c. for placing in track, we have a total of \$5.30. $43.63 \div 5.30 = 8.23$, the *annual* charge in *cents* for each *dollar* of initial expenditure. By interpolation in Table XVIII between 8.27 for 19 years and 8.02 for 20 years, it is seen that the metal tie must have an *average* life of 19 yrs. 2 mo. to equal the economy of the oak tie. The above comparison assumes the substantial equality of cost of track labor and the maintenance of the track fastenings with the two kinds of ties.

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263. Bowls or plates. As mentioned before, over 12000 miles of railway, chiefly in British India and in the Argentine Republic, are laid with this form of track. It consists essentially of large cast-iron inverted "bowls" laid at intervals under each rail and opposite each other, the opposite bowls being tied together with tie-rods. A suitable chair is riveted or bolted on to the top of each bowl so as to properly hold the rail. Being made of cast iron, they are not so subject to corrosion as steel or wrought iron. They have the advantage that when old and worn out their scrap value is from 60% to 80% of their initial cost, while the scrap value of a steel or wrought-iron tie is practically nothing. Failure generally occurs from breakage, the

failures from this cause in India being about 0.4% per annum. They weigh about 250 lbs. apiece and are therefore quite expensive in first cost and transportation charges. There are miles of them in India which have already lasted 25 years and are still in a serviceable condition. Some illustrations of this form of tie are shown in Plate VI.

264. Longitudinals. Although the discussion of longitudinals might be considered to belong more properly to the subject of Rails, yet the essential idea of all designs must necessarily be the *support* of a rail-head on which the rolling stock may run, and therefore this form, unused in this country, will be briefly described here. This form, the use of which is confined almost exclusively to Germany, is being gradually replaced on many lines by metal cross-ties. The system generally consists of a compound rail of several parts, the upper bearing rail being very light and supported throughout its length by other rails, which are suitably tied together with tie-rods so as to maintain the proper gauge, and which have a sufficiently broad base to be properly supported in the ballast. One great objection to this method of construction is the difficulty of obtaining proper drainage especially on grades, the drainage having a tendency to follow along the lines of the rails. The construction is much more complicated on sharp curves and at frogs and switches. Another fundamentally different form of longitudinal is the Haarman compound "self-bearing rail," having a base 12" wide and a height of 8", the alternate sections breaking joints so as to form a practically continuous rail.

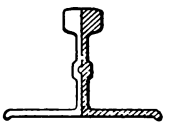


FIG. 114.

Some of the other forms of longitudinals are illustrated in Plate VI.

For a very complete discussion of the subject of metal ties, see the "Report on the Substitution of Metal for Wood in Railroad Ties" by E. E. Russell Tratman, it being Bulletin No. 4, Forestry Division of the U. S. Dept. of Agriculture.

265. Reinforced concrete ties. The wide application of reinforced concrete to various structural purposes, combined with its freedom from decay, has led to its attempted adoption for ties. For several years a standing committee of the Amer. Rwy. Eng. Assoc. has systematically followed the experimental tests on several railroads of numerous substitutes for wooden

ties. Many of these ties are made of metal and have been previously referred to. Others are made of concrete, reinforced with steel. The concrete is not subject to decay but it is so brittle that, when struck by a derailed car or locomotive, it will almost inevitably crack, and after that, its disintegration is a matter of a very short time. The Percival tie, shown on Plate VI, has been tested for several years on some roads having comparatively light traffic. The reports from these roads are encouraging, if not conclusive. The "Bates" tie consists of two concrete blocks, one under each rail, which are connected by a pair of trussed structures of steel. In the center space of of about two feet between the two blocks, the steel is exposed to rust. A report on these ties said that, after seven years service, the exposed trusses were "rusted to a maximum depth of possibly $\frac{1}{16}$" but not to such an extent as to seriously weaken the trusses." It is a common belief that it is essentially impossible to design a concrete tie, even when reinforced with steel, which will have sufficient resiliency to withstand the shocks of rail traffic. Innumerable concrete ties have ignominiously failed after a very short service.

CHAPTER IX.

RAILS.

266. Early forms. The first rails ever laid were wooden stringers which were used on very short tram-roads around coal-mines. As the necessity for a more durable rail increased, owing chiefly to the invention of the locomotive as a motive power, there were invented successively the cast-iron "fish-belly" rail and various forms of wrought-iron strap rails which finally developed into the T rail used in this country and the double-headed rail, supported by chairs, used so extensively in England. The cast-iron rails were cast in lengths of about 3 feet and were supported in iron chairs which were sometimes set upon stone piers. A great deal of the first railroad track of this country was laid with longitudinal stringers of wood placed upon cross-ties, the inner edge of the stringers being protected by wrought-iron straps. The "bridge" rails were first rolled in this country in 1844. The "pear" section was an approach to the present form, but was very defective on account of the difficulty of designing a good form of joint. The "Stevens" section was designed in 1830 by Col. Robert L. Stevens, Chief Engineer of the Camden and Amboy Railroad; although quite defective in its proportions, according to the present knowledge of the requirements, it is essentially the present form. In 1836, Charles Vignoles invented essentially the same form in England; this form is therefore known throughout England and Europe as the Vignoles rail.

267. Present standard forms. The larger part of modern railroad track is laid with rails which are either "T" rails or the double-headed or "bull-headed" rails which are carried in chairs. The double-headed rail was designed with a symmetrical form with the idea that after one head had been worn out by traffic the rail could be reversed, and that its life would be practically doubled. Experience has shown that the wear of the

rail in the chairs is very great; so much so that when one head has been worn out by traffic the whole rail is generally useless.

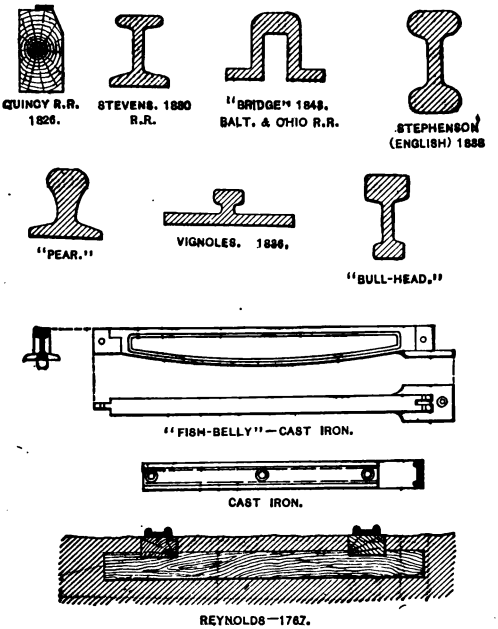


FIG. 115.—EARLY FORMS OF RAILS.

If the rail is turned over, the worn places, caused by the chairs, make a rough track and the rail appears to be more brittle and subject to fracture, possibly due to the crystallization that may have occurred during the previous usage and to the reversal of stresses in the fibers. Whatever the explanation, experience has demonstrated the *fact*. The "bull-headed" rail has the lower head only large enough to properly hold the wooden keys with which the rail is secured to the chairs (see Fig. 116) and furnish the necessary strength. The use of these rails requires the use of two cast-iron chairs for each tie. It is claimed that such track is better for heavy and fast traffic, but it is more

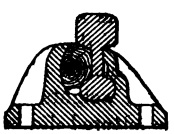


FIG. 116. — BULL-HEADED RAIL AND CHAIR.

expensive to build and maintain. It is the standard form of track in England and some parts of Europe.

Until after 1893 there was a very great multiplicity in the designs of "T" rails as used in this country, nearly every prominent railroad having its own special design, which perhaps differed from that of some other road by only a very minute and insignificant detail, but which nevertheless would require a complete new set of rolls for rolling. This had a very appreciable effect on the cost of rails. In 1893, the American Society of Civil Engineers, after a very exhaustive investigation of the

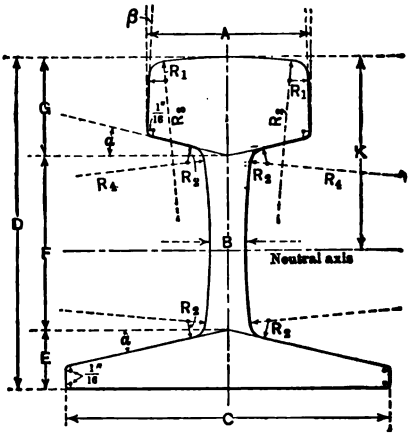


FIG. 117.—STANDARD RAIL SECTIONS.

subject, extending over several years, having obtained the opinions of the best experts of the country, adopted a series of sections which have been very extensively adopted by the railroads of this country.

In 1909 the American Railway Association and the American Railway Engineering Association, by combined action, developed a series of sections. Fig. 117 shows diagrammatically all of these sections and their variations with different weights and systems are shown by the tabular values for the lettered dimensions. It may be noted that the radii of the upper and lower corners of the flanges and of the lower corners of the head are constant ($\frac{1}{16}$") for all weights of rail and for all systems.

TABLE XXIII.—ANGLES AND DIMENSIONS OF STANDARD DESIGNS FOR RAILS.

System.	Branding symbol.	Radii, inches.				Angles.		Weight of rail, lbs. per yard.	Dimensions, inches.							
		Upper corner of head. R_1	Fillet corners. R_2	Top of head. R_3	Side of web. R_4	Bottom of head and top of flange. α	Side of head. β		A	B	C	D	E	F	G	K
American Society of Civil Engineers	A.S.C.E.	$\frac{5}{16}$	$\frac{1}{2}$	12	12	13°	Vert.	60 70 80 90 100	$2\frac{3}{8}$ $2\frac{7}{16}$ $2\frac{5}{8}$ $2\frac{3}{4}$ $2\frac{7}{8}$	$\frac{21}{64}$ $\frac{23}{64}$ $\frac{25}{64}$ $\frac{7}{16}$ $\frac{9}{16}$	$4\frac{1}{4}$ $4\frac{3}{8}$ 5 $5\frac{3}{8}$ $5\frac{3}{4}$	$4\frac{1}{4}$ $4\frac{3}{8}$ 5 $5\frac{3}{8}$ $5\frac{3}{4}$	$\frac{45}{64}$ $\frac{23}{32}$ $\frac{25}{32}$ $\frac{27}{32}$ $\frac{29}{32}$	$2\frac{17}{64}$ $2\frac{19}{64}$ $2\frac{3}{8}$ $2\frac{25}{64}$ $3\frac{9}{64}$	$1\frac{7}{32}$ $1\frac{33}{64}$ $1\frac{9}{16}$ $1\frac{37}{64}$ $1\frac{41}{64}$	2.20 2.40 2.62 2.82 3.02
American Railway Association and	R.A.-A.	$\frac{3}{8}$	$\frac{3}{8}$	14	14	4:1 14° 02′	1:16 3° 35′	60 70 80 90 100	$2\frac{1}{4}$ $2\frac{3}{8}$ $2\frac{1}{2}$ $2\frac{5}{8}$ $2\frac{3}{4}$	$\frac{19}{32}$ $\frac{5}{8}$ $\frac{21}{32}$ $\frac{11}{16}$ $\frac{23}{32}$	4 $4\frac{1}{4}$ $4\frac{1}{2}$ $5\frac{1}{2}$ $5\frac{1}{2}$	$4\frac{1}{2}$ $4\frac{3}{4}$ $5\frac{1}{4}$ $5\frac{1}{2}$ 6	$\frac{13}{16}$ $\frac{27}{32}$ $\frac{29}{32}$ 1 $1\frac{1}{16}$	$2\frac{23}{64}$ $2\frac{1}{2}$ $2\frac{23}{32}$ $3\frac{9}{32}$ $3\frac{1}{4}$	$1\frac{11}{32}$ $1\frac{37}{64}$ $1\frac{7}{16}$ $1\frac{43}{64}$ $1\frac{7}{16}$	2.37 2.55 2.81 3.09 3.25
American Railway Engineering Association	R. A.-B.	$\frac{3}{8}$	$\frac{5}{16}$	12	12	13°	3°	60 70 80 90 100	$2\frac{1}{4}$ $2\frac{3}{8}$ $2\frac{7}{16}$ $2\frac{5}{8}$ $2\frac{31}{32}$	$\frac{21}{32}$ $\frac{11}{16}$ $\frac{43}{64}$ $\frac{15}{16}$ $\frac{15}{16}$	$3\frac{11}{16}$ $4\frac{3}{16}$ $4\frac{15}{16}$ $4\frac{13}{16}$ $5\frac{7}{64}$	$4\frac{3}{16}$ $4\frac{7}{16}$ $4\frac{7}{8}$ $5\frac{11}{16}$ $5\frac{41}{64}$	$\frac{7}{8}$ $\frac{29}{32}$ 1 $1\frac{1}{32}$ $1\frac{7}{64}$	$2\frac{7}{16}$ $2\frac{11}{16}$ $2\frac{29}{32}$ $2\frac{15}{16}$ $2\frac{23}{64}$	$1\frac{1}{32}$ $1\frac{23}{64}$ $1\frac{37}{64}$ $1\frac{25}{32}$ $1\frac{41}{64}$	$2\frac{15}{64}$ $2\frac{43}{64}$ $2\frac{55}{64}$ $2\frac{39}{32}$ $3\frac{7}{64}$
Amer. Rwy. Eng. Assoc. (adopted in 1915)	R.E.	$\frac{1}{2}$	* $\frac{3}{8}$ † $\frac{1}{2}$	14	14	4:1 14° 02′	1:16	100 110 120	$2\frac{1}{4}$ $2\frac{23}{32}$ $2\frac{1}{2}$	$\frac{9}{16}$ $\frac{19}{32}$ $\frac{5}{8}$	$5\frac{3}{8}$ $5\frac{1}{2}$ $5\frac{1}{2}$	6 $6\frac{1}{4}$ $6\frac{1}{2}$	$1\frac{1}{16}$ $1\frac{1}{8}$ $1\frac{1}{16}$	$3\frac{3}{8}$ $3\frac{13}{32}$ $3\frac{7}{16}$	$1\frac{21}{32}$ $1\frac{23}{32}$ $1\frac{25}{32}$	$3\frac{1}{2}$ $3\frac{23}{32}$ $3\frac{31}{32}$
Amer. Rwy. Eng. Assoc. (adopted in 1920)	R.E.	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{4}$	14	14	4:1 14° 02′	1:16	130 140	$2\frac{13}{16}$ 3	$\frac{21}{32}$ $\frac{11}{16}$	6 $6\frac{1}{4}$	$6\frac{3}{4}$ 7	$1\frac{7}{16}$ $1\frac{1}{2}$	$3\frac{11}{16}$ $3\frac{15}{16}$	$1\frac{27}{32}$ $1\frac{29}{32}$	$3\frac{23}{32}$ $3\frac{15}{16}$

* Fillet radius under head. † Fillet radius above base.

The chief features of disagreement among railroad men relate to the radius of the upper corner of the head and the slope of the side of the head. The radius ($\frac{5}{16}$") adopted by the A. S. C. E. for the upper corner (constant for all weights) is a little more than is advocated by those in favor of "sharp corners" who prefer a radius of $\frac{1}{4}$". On the other hand it is much less than is advocated by those who consider that it should be nearly equal to (or even greater than) the larger radius universally adopted for the corner of the wheel-flange. The discussion turns on the relative rapidity of rail wear and the wear of the wheel-flanges as affected by the relation of the form of the wheel-tread to that of the rail. It is argued that sharp rail corners wear the wheel-flanges so as to produce sharp flanges, which are liable to cause derailment at switches and also to require that the tires of engine-drivers must be more frequently turned down to their true form. On the other hand it is generally believed that rail wear is much less rapid when the area of contact between the rail and wheel-flange is small, and that when the rail has worn down, as it invariably does, to nearly the same form as the wheel-flange, the rail wears away very quickly. The A. R. E. A. system uses $\frac{3}{8}$" radius for all rail weights. The "B" sections were proposed to satisfy those that desired that the head should be narrower and deeper than as found in the "A" sections. The A. R. E. A. Manual (1915), suggests that if a section is found to be inadequate because of lack of depth of head, the next heavier section will be found more desirable and economical.

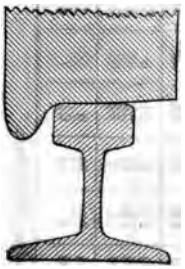


FIG. 118.—RELATION OF RAIL TO WHEEL-TREAD.

268. Weight for various kinds of traffic. The heaviest rails in use weigh 120 to 140 lbs. per yard, and even these are only used on some of the heaviest traffic sections of such roads as the N. Y. Central, the Pennsylvania, the N. Y., N. H. & H., and a few others. Probably the larger part of the mileage of the country is laid with 80- to 90-lb. rails—considering the fact that "the larger part of the mileage" consists of comparatively light-traffic roads and may exclude all the heavy trunk lines. Very light-traffic roads are sometimes laid with 70-lb. rails. Roads with fairly heavy traffic generally use 90- to 100-lb. rails, especially when grades are heavy and there is much and sharp curv-

ature. The tendency on all roads is toward an increase in the weight, rendered necessary on account of the increase in the weight and capacity of rolling stock, and due also to the fact that accumulated operating experience has shown that it is both better and cheaper to obtain a more solid and durable track by increasing the weight of the rail rather than by attempting to support a weak rail by an excessive number of ties or by excessive track labor in tamping. It should be remembered that in buying rails the mere weight is, in one sense, of no importance. The important thing to consider is the STRENGTH and the STIFFNESS. If we assume that all weights of rails have *similar* cross-sections (which is nearly although not exactly true), then, since for beams of similar cross-sections the *strength* varies as the *cube* of the homologous dimensions and the *stiffness* as the *fourth power*, while the area (and therefore the weight per unit of length) only varies as the *square*, it follows that the stiffness varies as the square of the weight, and the strength as the $\frac{3}{2}$ power of the weight. Since for ordinary variations of weight the price per ton is the same, adding (say) 10% to the weight (and cost) adds 21% to the stiffness and over 15% to the strength. As another illustration, using an 80-lb. rail instead of a 75-lb. rail adds only $6\frac{2}{3}$% to the cost, but adds about 14% to the stiffness and nearly 11% to the strength. This shows why heavier rails are more economical and are being adopted even when they are not absolutely needed on account of heavier rolling stock. The stiffness, strength, and consequent durability are increased in a much greater ratio than the cost.

The relation between weight of rail and the weight on the drivers of the locomotives which are to run on it has been briefly expressed by the Baldwin Locomotive Works as "300 pounds of wheel per pound of rail per yard." * This rule may be utilized by making a diagram as shown in Fig. 119. For example, if it is desired to use a type of locomotive with 170,000 lbs. on the drivers and also 75-lb. rails, four pairs of drivers will be needed and such a type of locomotive should be used. By using 95-lb. rails the same weight on the drivers could be placed on three axles. As another example, a Pacific-type locomotive, with 150,000 lbs. on its six drivers, should have a rail with a minimum weight of 83 lbs., or say an 85-lb. rail. Whatever elements are given, the corresponding proper value for the other element may be derived.

* See § 447 (*c*) for expansion of this rule.

269. Effect of stiffness on traction. A very important but generally unconsidered feature of a stiff rail is its effect on tractive force. An extreme illustration of this principle is seen when a vehicle is drawn over a soft sandy road. The constant compression of the sand in front of the wheel has virtually the same effect on traction as drawing the wheel up a grade whose

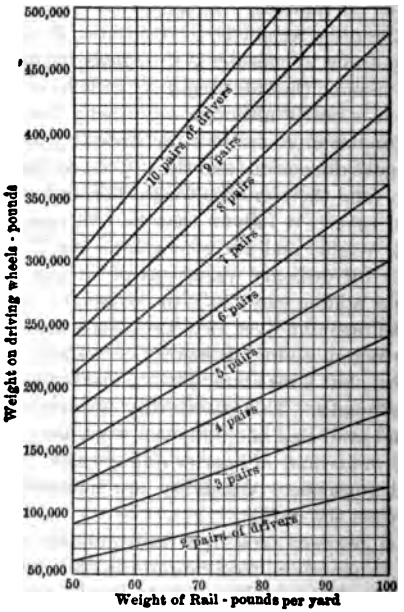


FIG. 119.—CURVES FOR FINDING THE NUMBER OF DRIVERS NEEDED FOR GIVEN WEIGHT ON DRIVING WHEELS AND WEIGHT OF RAILS.

steepness depends on the radius of the wheel and the depth of the rut. On the other hand, if a wheel, made of perfectly elastic material, is rolled over a surface which, while supported with absolute rigidity, is also perfectly elastic, there would be a forward component, caused by the expanding of the compressed metal just behind the center of contact, which would just balance the backward component. If the rail was supported throughout its length by an absolutely rigid support, the high elasticity of the wheel-tires and rails would reduce this form of

resistance to an insignificant quantity, but the ballast and even the ties are comparatively inelastic. When a weak rail yields, the ballast is more or less compressed or displaced, and even though the elasticity of the rail brings it back to nearly its former place, the work done in compressing an inelastic material is wholly lost. The effect of this on the fuel account is certainly very considerable and yet is frequently entirely overlooked. It is practically impossible to compute the saving in tractive power, and therefore in cost of fuel, resulting from a given increase in the weight and stiffness of the rail, since the yielding of the rail is so dependent on the spacing of the ties, the tamping, etc. But it is not difficult to perceive in a general way that such an economy is possible and that it should not be neglected in considering the value of stiffness in rails.

270. Length of rails. The recommended standard length of rails is 33 feet. Several years ago, many roads experimented with 45-foot and even 60-foot rails. The argument in favor of longer rails is chiefly that of the reduction in track-joints, which are costly to construct and to maintain and are a fruitful source of accidents. Mr. Morrison of the Lehigh Valley R. R.* declared that, as a result of extensive experience with 45-foot rails on that road, he found that they are much less expensive to handle, and that, being so long, they can be laid around sharp curves without being curved in a machine, as is necessary with the shorter rails. The great objection to longer rails lies in the difficulty in allowing for the expansion, which will require, in the coldest weather, an opening at the joint of nearly $\frac{3}{4}''$ for a 60-foot rail. The Pennsylvania R. R. and the Norfolk and Western R. R. each laid a considerable mileage with 60-foot rails. The net result is the fixed standard of 33 feet.

271. Expansion of rails. Steel expands at the rate of .0000065 of its length per degree Fahrenheit. The extreme range of temperature to which any rail will be subjected will be about 160°, or say from −20° F. to +140° F. With the above coefficient and a rail length of 60 feet the expansion would be 0.0624 foot, or about $\frac{3}{4}$ inch. But it is doubtful whether there would ever be such a range of motion even if there were such a range of temperature. Mr. A. Torrey, chief engineer of the Mich. Cent. R. R., experimented with a section over 500 feet long, which,

* Report, Roadmasters Association, 1895.

although not a single rail, was made "continuous" by rigid splicing, and he found that there was no appreciable additional contraction of the rail at any temperature below +20° F. The reason is not clear, but the *fact* is undeniable.

The heavy girder rails, used by the street railroads of the country, are bonded together with perfectly tight rigid joints which do not permit expansion. If the rails are laid at a temperature of 60° F. and the temperature sinks to 0°, the rails have a *tendency* to contract .00039 of their length. If this tendency is resisted by the friction of the pavement in which the rails are buried, it only results in a tension amounting to .00039 of the modulus of elasticity, or say 10920 pounds per square inch, assuming 28 000 000 as the modulus of elasticity. This stress is not dangerous and may be permitted. If the temperature rises to 120° F., a tendency to expansion and buckling will take place, which will be resisted as before by the pavement, and a compression of 10920 pounds per square inch will be induced, which will likewise be harmless. The range of temperature of rails which are buried in pavement is much less than when they are entirely above the ground and will probably never reach the above extremes. Rails supported on ties which are only held in place by ballast must be allowed to expand and contract almost freely, as the ballast cannot be depended on to resist the distortion induced by any considerable range of temperature, especially on curves.

272. **Rules for allowing for temperature.** Track regulations generally require that the track foremen shall use iron (*not* wooden) shims for placing between the ends of the rails while splicing them. The thickness of these shims should vary with the temperature. Some roads use such approximate rules as the following: "The proper thickness for coldest weather is $\frac{5}{16}$ of an inch; during spring and fall use $\frac{1}{8}$ of an inch, and in the very hottest weather $\frac{1}{16}$ of an inch should be allowed." This is on the basis of a 30-foot rail. When a more accurate adjustment than this is desired, it may be done by assuming some very high temperature (100° to 125° F.) as a maximum, when the joints should be *tight;* then compute in tabular form the spacing for each temperature, varying by 25°, allowing 0″.0643 (very nearly $\frac{1}{16}$″) for each 25° change. Such a tabular form would be about as follows (rail length 33 feet):

Temperature..	Over 100°	100°–75°	75°–50°	50°–25°	25°–0°	Below 0°
Rail opening..	Close	$\frac{1}{16}''$	$\frac{1}{8}''$	$\frac{3}{16}''$	$\frac{1}{4}''$	$\frac{5}{16}''$

One practical difficulty in the way of great refinement in this work is the determination of the real temperature of the rail when it is laid. A rail lying in the hot sun has a very much higher temperature than the air. The temperature of the rail cannot be obtained even by exposing a thermometer directly to the sun, although such a result might be the best that is easily obtainable. On a cloudy or rainy day the rail has practically the same temperature as the air; therefore on such days there need be no such trouble.

273. Standard specifications. Specifications are constantly varying. They are always a compromise between the wishes of railroad engineers and the interests of rail manufacturers. At present (1921) rail prices are high, the railroads are relatively in a low financial condition, and the specifications are much less rigid than those mutually accepted in 1910. Therefore, instead of quoting verbatim, in this edition, the specifications now current, the general features have been discussed, many of which will probably be modified in future specifications. When buying rails for any road, the latest issue of standard Am. Rwy. Eng. Assoc. specifications should be obtained for reference.

273a. Chemical composition. More than 98% of the composition of steel rails is iron, but the value of the rail, as a rail, is almost wholly dependent upon the large number of other chemical elements which are, or may be, present in very small amounts.

Carbon. Many years ago, when rails were comparatively light and the maximum wheel loads were correspondingly light, the carbon in rails ranged from 0.20% to 0.50%. But the great increase in wheel loads produces a concentrated pressure on the rails which causes the steel to " flow " if the steel is comparatively soft. An increase of a few hundredths of a percent of carbon makes the steel harder but an excess of carbon makes it too brittle. Since heavier wheel loads require heavier rails, more carbon is used in the heavier sections. Since it is safer to use more carbon in open-hearth rails than in Bessemer rails, a higher percentage is so used. The limits at present (1921) are as follows:

Chemical elements.	Bessemer process.		Open-hearth process.		
	Weight, pounds per yard.		Weight, pounds per yard.		
	70 to 84	85 and over	70 to 84	85 to 110	111 and over
Carbon.....	0.40 to 0.50	0.45 to 0.55	0.53 to 0.68	0.62 to 0.77	0.67 to 0.82
Phosphorus, not to exceed	0.10	0.10	0.04	0.04	0.04
Manganese...	0.80 to 1.10	0.80 to 1.10	0.60 to 0.90	0.60 to 0.90	0.60 to 0.90
Silicon, not less than.....	0.10	0.10	0.10	0.10	0.10

Sulphur. Former specifications required that sulphur should not exceed 0.075% in Bessemer rails and 0.06% in open-hearth rails. Manufacturers now demand an excess price if a definite limitation is made but say that it is to their own interests, for other reasons, to have the sulphur within safe limits. As a compromise, no definite limitation is now made. This concession, now allowed by the railroads, illustrates forcibly how the railroads are compelled by financial considerations to relax from the former rigidity of specifications.

When a railroad buys a large order of rails directly from a rail mill, the railroad usually sends an inspector, who is furnished by the manufacturer with chemical analyses of the steel, one for each day and night turn for Bessemer rails or one for each heat of open-hearth rails. Sometimes samples are furnished the inspector, if he is a chemist, and he is given facilities at the mill to make his own check analyses.

273b. Physical requirements. These are increasingly depended on to determine (*a*) ductility or toughness as opposed to brittleness and (*b*) soundness, or its homogeneity and freedom from seams, laminations, cavities, or interposed foreign matter. The ductility is tested by dropping a tup weighing 2000 pounds, which has a striking face with a radius of 5 inches, on a test rail about 5 to 6 feet long, which is supported on two pedestals, also having bearing surfaces with radii of 5 inches, the pedestals being adjustable to spans varying from 3 feet to 4 feet 6 inches. The pedestals are spaced 3 feet for rails weighing 110 pounds per yard or less, and are spaced 4 feet for rails weighing 111 to 140 pounds per yard. The pedestals are firmly secured to an anvil weighing 20,000 pounds which is supported on 20 very heavy springs. Gauge marks, one inch apart for three inches

each side of the center, are marked in the center of the top of the rail. The rails are usually tested with the head in tension, or with the rail inverted. The tup falls from a height of 16 feet on 70- to 79-pound rails, 17 feet on 80- to 90-pound rails, 18 feet on 91- to 110-pound rails and 20 feet on 111- to 140-pound rails. Under such impacts the elongation on one inch of the six-inch scale, marked as above, shall be at least 8%. The permanent set, on a 3-foot chord, is noted for each blow. The test pieces, which do not break under ordinary blows, are nicked and broken so that the interior may be examined for "soundness," or for such flaws as fissures, laminations, cavities, etc. Fissures which are really indicative of structural defects in a rail are sometimes microscopic, even when a specimen is carefully cut from the rail and the surface polished. The defects may be deepened and accentuated by etching the surface with hot concentrated hydrochloric acid.

By agreement between a railroad and a rail manufacturer, the physical test may be made by a **quick-bend machine** instead of a falling weight. Such a machine is essentially a hydraulic press of not less than 350 tons capacity. The bearing supports of the tested rail are flat surfaces, with vertical faces 48 inches apart, of which the inner edges are rounded to a $\frac{1}{2}$-inch radius. The head of the ram has a bearing surface with a radius of five inches. The percentage of elongation before failure may be observed as before.

273c. Classification. Rails are classified as No. 1 and No. 2. No. 1 rails are those with no injurious defects or flaws. No. 2 rails are those which arrive at the straightening presses more crooked than is allowed for No. 1 rails but which, in the judgment of the inspector, may be accepted in spite of this or other minor defects which do not impair their soundness and strength. No. 2 rails must not exceed 5 per cent of the whole order. They must have their ends painted white and have two prick-punch marks on the side of the web near the heat number, near the end of the rail, so placed as not to be covered by the joint bars.

273d. Branding. The name of the manufacturer, the month and year of manufacture, and the weight and type of section of rail shall be rolled in raised letters and figures on one side of the web, where it will not be covered by joint bars. The markings shall be done so effectively that the marks may be read as long as the rails are in service. The type of section is indicated by

A. S. C. E., R. A.–A., R. A.–B., or R. E., to indicate one of the various types elaborated in Table XXIII. Open-hearth rails are branded or stamped O. H., in addition to the other marks.

273e. Dimensions and drilling. The standard length is 33 feet at a temperature of 60° F. Ten per cent of the entire order will be accepted in shorter lengths, varying by one foot from 32 to 25 feet. A variation of $\frac{1}{4}$" from specified lengths is allowed, except that 15% of the order may vary $\frac{3}{8}$" from specified lengths. Drill holes may vary $\frac{1}{32}$" in size and location from the drawings furnished by the railroad company. The recommended position (vertically) in the web is "midway between the intersections of the vertical center line of the rail with the planes of the fishing surfaces of the head and base." The hole centers should be $5\frac{1}{2}$" apart, the first hole center being $2\frac{11}{16}$" from the end of the rail, which allows $\frac{1}{8}$-inch clearance when the rails are bolted together in normal position.

273f. Finishing. Rails must be smooth at the heads, straight in line and surface, and without twists, waves or kinks. The limiting allowable camber in a 33-foot rail is "4 inches for thick base sections and 5 inches for thin base sections." They shall be sawed square at the ends, a variation of not more than $\frac{1}{32}$" being allowed. Burrs must be carefully removed. When a finished rail shows defects at either end or in any drilled hole, the entire rail shall be rejected.

274. Life of rails. There has been a great development since 1900 in the science of manufacturing rails. This is indicated by the decrease in rail "failures." If there is a defect in a rail it will usually break or "fail" before it is worn down. If the defect is serious it will break in a few weeks or months. Minor defects require much longer time to develop. The accompanying tabular form shows the number of rail failures per 100 track miles, after one to five years' service, reported by several railroads of the United States. To appreciate the figures, note that there are 32000 rails 33 feet long in 100 miles of track. The record for rails rolled in 1913 showed that after five years' service, a total of 246.5 per 100 miles, or an average of 0.77%, had failed. Note that the increase of failures per year, after the first year, is regular, as it should be. Note also that there has been a steady improvement in the figures for 3, 4 and 5 years' service, but that since 1914 or 1915 the failures have increased somewhat,

AVERAGE RAIL FAILURES PER 100 TRACK MILES

Year rolled.	Years of service.					
	0	1	2	3	4	5
1908	.....	.....	.....	.....	.....	398.1
1909	.....	.....	.....	.....	224.1	277.8
1910	.....	.....	.....	124.0	152.7	198.5
1911	.....	.....	77.0	104.4	133.3	176.3
1912	.....	28.9	32.1	49.3	78.9	107.1
1913	2.0	12.5	25.8	44.8	69.5	91.9
1914	1.2	8.2	19.8	32.9	50.9	74.0
1915	0.7	8.9	19.0	34.2	53.0	82.4
1916	1.6	11.8	29.2	47.7	70.6	
1917	5.3	21.6	38.9	66.0		
1918	1.6	8.9	27.6			
1919	2.0	14.8				
1920	3.9					

indicating perhaps that war conditions had lowered the quality of the rails. The reports also show that failures are more common using Bessemer than open-hearth rails, and, considering that Bessemer are used in general for lighter service, the ratio against Bessemer would probably be greater for the same service. Bessemer rails cost less than open hearth, and this fact is perhaps the only reason for their use. The percentage of Bessemer rails to the total in 1913 was about 9.1%; in 1918 the percentage was reduced to 2.7%. These figures are based on reports made to the A. R. E. A. Presumably complete statistics (unobtainable) would show a somewhat larger percentage of Bessemer rails, used on small roads which did not make reports, but the above figures show that open-hearth rails are considered to be superior in spite of the higher price.

275. Intensity of pressure on rails. A special committee of the A. R. E. A. made an investigation to determine the intensities of pressure produced by varying wheel loads on the head of a rail and also the amount of permanent deformation or "flow" of the metal. The testing mechanism made it possible to increase the "wheel load" up to 580,000 lbs., a figure about 30 times as great as the greatest working wheel load. The unit intensity of pressure increased with an increase of the wheel load from zero up to about 30,000 lbs. At that figure, which corresponds to an axle load of 60,000 lbs,. or nearly the maximum of present practice, the unit intensity of pressure reached its maximum and remained substantially constant while the wheel load was increased from 30,000 to 580,000 lbs. In other words,

the area of contact increased as fast as the pressure increased. This maximum *average* unit pressure varied considerably with the different rails tested, although it was nearly constant for any one rail. The unit values varied from 105,000 to 160,000 lbs. per square inch.

275a. Flow of metal. The permanent deformation of the metal was measured by noting the reduction in the horizontal and vertical diameters of very small tapering holes drilled into the head of the tested rail slightly below the bearing surface. The testing wheel was caused to roll over the tested rail several hundred thousand times. In one test the initial load was 15,000 lbs., increasing by steps up to 30,000 lbs. Up to a load of 20,000 lbs. no permanent deformation of the holes was observable. With a load of 25,000 lbs. a slight set was observable which grew more rapid when the load was increased to 30,000 lbs. But even then the deformation was not as great as it was in another test when the initial loading was 30,000 lbs. This indicates that the effect on a rail of continued rolling pressure of less than 20,000 lbs., or 40,000 lbs. per axle, is to harden the surface metal and make it better able to withstand wear. This seems to be corroborated by, and also explains, the remarkable wearing qualities of many old rails which were surfaced-hardened by comparatively light loads and which subsequently carried much heavier loads with less wear than new rails.

276. Rail wear on tangents. The weight carried by a single engine driver is often from 24,000 to 30,000 lbs. Each of the eight wheels of a 140,000-lb. capacity coal car, when loaded, carries nearly 25,000 lbs. Such loads will certainly cause a flow of metal, as shown by the laboratory test above described, but a four-weeks' *service* test on the *same* test rails (referred to above) showed a flow of metal as indicated in Fig. 120, the flow being considerably greater than that produced in the laboratory tests with the same weight and number of rollings. The average wheel load in the service test was much less than the maximum in the laboratory tests, but the greater effect was probably due to the great variety of wheel treads making different forms of contact between rail and tread, with occasional great concentration of pressure. But Fig. 120 shows the typical normal wear of a rail on a tangent, the wear being all on the top. The center of pressure is usually about one-half inch inside from the center of the rail and is inclined outward. The wear is approxi-

mately symmetrical with this axis. Fig. 120 also shows the *flow* of metal outside of the original contour, which all occurred on the gauge side. Very soft badly worn rails may show a fin on the outside.

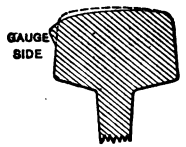


FIG. 120.—RAIL WEAR ON A TANGENT.

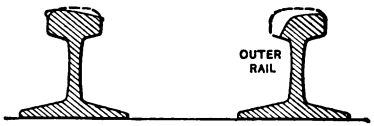


FIG. 121.—RAIL WEAR ON CURVES.

276a. Rail wear on curves. The pressure and grinding action of the wheel flanges against the rails wears away the inner side of the head of the outer rail on curves. If the rail is left in the track and the wear is permitted to continue, the head may be worn to approximately the form shown in Fig. 121. On the other hand, the inner rail is not subjected to any such lateral grinding action, and the rail is worn to substantially the same form as on a tangent, but the wear is more rapid due to the longitudinal slipping—see § 395. If the rails are soft or the traffic very heavy, there will be a flow of metal and a fin will form on the inner edge and perhaps also on the outer edge.

277. Experimental determination of rail wear. Several years ago a series of tests for rail wear were made on the Northern Pacific R. R. by taking up, weighing, and replacing, each year, the several groups of rails under test. Some of these rails were on tangents, the others on curves of various curvature. Some of the rails of each group were made of Bessemer steel, the others of open-hearth steel. No tests were made to determine the loss of weight through mere oxidation. All of the rails were in service for five years and some lasted for six years or more, but the loss in weight during the sixth year was nearly always equal to, and in some cases twice as much as, the loss during the preceding five years. Some of the rails lost over 10% of their weight, or about one-fourth the weight of the head, before being removed. Although the tests were too few to establish any positive laws, some tendencies which may be observed will give at least an approximate idea of the laws of rail wear.

1. The average loss of weight during the first five years on

20 rails on tangents was 0.412 lb. per yard per 10,000,000 tons of traffic.

2. Ten of these same rails were kept in place at least one year longer and during the sixth year lost almost twice as much metal as during the previous five years; in other words, about two-thirds of the entire loss occurred during the sixth year.

3. The average loss of weight during the first five years from 20 rails on a tangent was 0.463 lb. per yard per 10,000 trains. The relation between mere tonnage and number of trains could not be even indicated by so few tests. There is reason to believe that engine drivers are more responsible for rail wear than mere car-wheel tonnage. This practically means that one effect of grade is to increase rail wear, since more (or heavier) engines are needed to haul a given car tonnage.

4. The wear of the outer rail of curves is, of course, far greater than that of the inner rail, but the figures obtained did not seem to follow any rational law, the ratio of outer to inner rail wear varying from 144 to 244%, with an average of 182%.

5. The average rail wear on curves, averaging inner and outer rails, per yard, per degree of curve, per 10,000,000 tons traffic, varied from 0.145 lb. for a 4° 04′ curve down to 0.102 lb. per degree for a 10° 13′ curve. Based on the four curves tested, the results seemed to point to the law that rail wear on curves does *not* increase as fast as the degree of the curve.

6. Although the tests were too few to establish any law, the increase of the mean rail wear on curves with increase in degree of curve was very regular and indicated that the average rail wear on a curve of about 6° 40′ is about twice as great as that on a tangent.

7. The wear on open-hearth rails was almost invariably less than that on Bessemer rails, under identical conditions.

278. Cost of rails. In 1873 the cost of steel rails was about $120 per ton, and the cost of iron rails about $70 per ton. Although the steel rails were at once recognized as superior to iron rails on account of more uniform wear, they were an expensive luxury. The manufacture of steel rails by the Bessemer process created a revolution in prices, and they steadily dropped in price until, many years ago, steel rails were manufactured and sold for $22 per ton. For several years since then the price was very uniform at $28 per ton at the mill. But now (1921) the advantages of open-hearth steel are better appreciated and

a large proportion of rails are being rolled from open-hearth steel, which commands about $2 per ton more. At present (1921) the current prices at Pittsburgh mills run at about $38 per ton for Bessemer and $40 for open-hearth.

There is no longer any demand for iron rails, since the cost of manufacturing them is substantially the same as that of steel rails, while their durability is unquestionably inferior to that of steel rails. Rail quotations are generally on the basis of "long tons" of 2240 lbs.

The freight charge for transporting rails from the mill to the place where used is usually so large that it adds a very appreciable amount to the cost per ton. As an approximation, the freight may be estimated as 0.6 cent per ton-mile, or $3.00 per ton for a haul of 500 miles.

CHAPTER X.

RAIL-FASTENINGS.

RAIL-JOINTS

279. **Theoretical requirements for a perfect joint.** A perfect rail-joint is one that has the *same strength* and *stiffness*—no more and no less—as the rails which it joins, and which will not interfere with the regular and uniform spacing of ties. It should also be reasonably cheap both in first cost and in cost of maintenance. Since the action of heavy loads on an elastic rail is to cause a wave of translation in front of each wheel, any change in the stiffness or elasticity of the rail structure will cause more or less of a shock, which must be taken up and resisted by the joint. The greater the change in stiffness the greater the shock, and the greater the destructive action of the shock. The perfect rail-joint must keep both rail-ends truly in line both laterally and vertically, so that the flange or tread of the wheel need not jump or change its direction of motion suddenly in passing from one rail to the other. A consideration of all the above requirements will show that only a perfect welding of rail-ends would produce a joint of uniform strength and stiffness which would give a uniform elastic wave ahead of each wheel. As welding is impracticable for ordinary railroad work (see § 271), some other contrivance is necessary which will approach this ideal as closely as may be.

280. **Efficiency of any type of rail-joint.** Throughout the middle portion of a rail the rail acts as a continuous girder. If we consider for simplicity that the ties are unyielding, the deflection of such a continuous girder between the ties will be but one-fourth of the deflection that would be found if the rail were cut half-way between the ties and an equal concentrated load were divided equally between the two unconnected ends. The maximum stress for the continuous girder would be but one-half of that in the cantilevers. Joining these ends with rail-joints will give the ordinary "suspended" joint. In order to main-

tain uniform strength and stiffness the rail-joint must supply the deficiency. These theoretical relations are modified to an unkown extent by the uncertain and variable yielding of the ties. Since a theoretically perfect joint is unattainable, on account of the necessity for allowing for expansion, the nearest approach appears to be a joint which, when tested in comparison with a solid rail on an equal span (20 inches), will withstand an equal load before permanent set takes place. Some very thorough tests of several types of joints were made on this basis by the Pennsylvania R. R. at Altoona in 1915. The types tested were plain angle-bars, the "Continuous," the "Bonzano," the "100-per-cent," (see Plate VII) and also the "Duquesne," which is similar to the Bonzano and the 100-per-cent, except that the fin which projects below the rail between the ties has a different form. The "efficiency" of these joints was computed as the ratio of the load carried by the joint when it began to fail (or when permanent set commenced) to the load carried by the solid rail when it began to fail. The efficiencies for these joints, as ordinarily used, tested from 29% to 64%. Tests were also made of "heat-treated" joints (see § 285) which showed efficiencies from 60 to 150% higher than the untreated joints, the efficiencies being, in nearly every case, over 100%. The heat treatment costs about 0.2 c. per pound or say 16 c. for an 80-lb. pair. The added efficiency is so well worth the added cost that the use of heat treated splice bars is becoming more common and may soon become standard.

281. Effect of rail gap at joints. It has been found that the jar at a joint is due almost entirely to the *deflection* of the joint and scarcely at all to the small gap required for expansion. This gap causes a drop equal to the versed sine of the arc having a chord equal to the gap and a radius equal to the radius of the wheel. Taking the extreme case (for a 30-foot rail) of a $\frac{3}{8}''$ gap and a 33" freight-car wheel, the drop is about $\frac{1}{1000}''$. In order to test how much the jarring at a joint is due to a gap between the rails, the experiment was tried of cutting shallow notches in the top of an otherwise solid rail and running a locomotive and an inspection car over them. The resulting jarring was practically imperceptible and not comparable to the jar produced at joints. Notwithstanding this fact, many plans have been tried for avoiding this gap. The most of these plans con-

sist essentially of some form of compound rail, the sections breaking joints. (Of course the design of the compound rail has also several other objects in view.) In Fig. 122 are shown a few of the very many designs which have been proposed. These designs have invariably been abandoned after trial. Another plan, which has been extensively tried on the Lehigh Valley R. R., is the use of mitered joints. The advantages gained by their use are as yet doubtful, while the added expense is unquestionable. The "Roadmasters Association of America" in 1895

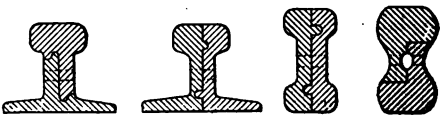


FIG. 122.—COMPOUND RAIL SECTIONS.

adopted a resolution recommending mitered joints for double track, but their use has been abandoned.

282. "Supported," "suspended," and "bridge" joints. A joint is "supported" when a tie is placed immediately under the middle of the joint. The localized traffic stress at the joint must be carried almost exclusively by that one tie and comparatively little is carried by the adjacent ties. A "suspended" joint is located symmetrically between two ties, which share equally the localized stress. Formerly there was a considerable proportion of railroad engineers who favored supported joints, but now the suspended joint is almost universally the standard.

"Bridge"-joints are similar to suspended joints in that the joint is supported on two ties, but there is the important difference that the bridge joint supports the rail from *underneath* and there is no transverse stress in the rail, whereas the suspended joint requires the combined transverse strength of both angle-bars and rail. The "Fisher" bridge joint, now seldom seen, is purely of this type, only two bolts being used to hold the rail ends together. But the principle of supporting the base of the rail is seen in the Wolhaupter, the Weber, the Continuous and the Atlas. See Plate VII. Although some of these forms are in extensive use, the angle-bar (see § 284) is the standard on a large proportion of the mileage of the country.

283. Failures of rail-joints. An instructive report was made in 1915 by an Engineer of Tests of the Pennsylvania R. R. on

an examination of 960 angle-bars, found in a scrap pile, to determine the various causes for their removal from the track. The various causes were classified under five headings, the typical failures being illustrated in Fig. 123. (1) Abrasion on the top fishing surface, the depth of wear varying from $\frac{1}{32}$" to $\frac{1}{16}$" and extending perhaps 8 inches each way from the center. On short 4-hole bars the wear is almost wholly in the center; on the longer 6-hole bars, wear is also found near the ends of the bar. Such wear demonstrates the amount of working and grinding which evidently takes place when a joint is depressed under

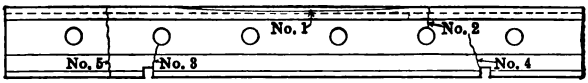


FIG. 123.—DIAGRAM OF TYPES OF BREAKS OF ANGLE-BARS.

traffic. This is the only form of actual *wear* which occurs. 24% of the 960 bars were removed for this cause. (2) When a joint bar is very long, the stresses in the bar may be reversed and there may be tension in the top and a break may start at the top and continue down, usually into a bolt hole. Less than 5% of the failures were of this class. (3) and (4). Usually a crack starts at the bottom and may or may not extend to a bolt hole. Usually the crack starts from a spike slot or from the re-entrant angle at either end of a depending flange. If the cracked bar is permitted to remain in the track, the crack of (2), (3) or (4) develops into a complete break (5). 44% of the 960 bars, or 59% of all but No. 1, were complete breaks.

284. Standard angle-bars. An angle-bar must be so made as to closely fit the rails. The great multiplicity in the designs of rails (referred to in Chapter IX) results in a corresponding variety in the detailed dimensions of the angle-bars. The absolutely essential features required for a fit are (1) the angles of the upper and lower surfaces of the bar where they fit against the rail, and (2) the height of the bar. The bolt-holes in the bar and rail must also correspond. The holes in the angle-bars are elongated or made oval, so that the track-bolts, which are made of corresponding shape immediately under the head, will not be turned by jarring or vibration. The holes in the rails

are made of larger diameter (by about $\frac{1}{16}$") than the bolts, so as to allow the rail to expand with temperature.

In Table XXIV and in Fig. 124 are shown the angles and dimensions for angle-bars to fit the standard rail sections shown in § 267. Note that the dimension a for the angle-bar corresponds with dimension F for the rail and that R_4 and the angle α are the same for both for each type of rail. These dimensions were copied from the 1916 Handbook of the Carnegie Steel Co. Although they correspond perfectly with the rail standards of the

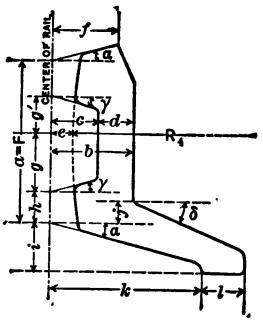


FIG. 124.—STANDARD ANGLE BAR.

A. R. E. A., that association has not yet adopted any such definite standard dimensions for a rail-joint.

The standard drilling for bolt-holes in angle-bar, as adopted by the A. R. E. A. in 1914, is as follows:

For 6-bolt splices, 5 spaces of 5½ inches.
For 4-bolt splices, 3 spaces of 5½ inches.

No definite recommendation was made by the Association as to the total length of angle-bars, but the committee recommended that, on the basis of the above spacing of holes, 24 inches is a satisfactory length for a 4-bolt splice and 32 inches for a 6-bolt splice, in both cases using suspended joints. On this basis, the spacing from the center of the last hole to the end of the bar would be 3¾ inches for the 4-bolt splice and 2¼ inches for the 6-bolt splice.

In Plate VII are shown some of the many designs which have been competing for favor and which have been more or less

TABLE XXIV.—ANGLES AND DIMENSIONS OF STANDARD DESIGNS FOR ANGLE-BARS.

System.		Weight of rail.	R-inches.	Angle or slope ratio.			Dimensions, inches.											
				α	δ	γ	a	b	c	d	e	f	g	g'	h	i	j	k	l
A. S. C. E.		60	12	13°	20°	—	$2\frac{17}{64}$	$1\frac{19}{64}$	$\frac{43}{64}$	$\frac{11}{16}$	$\frac{25}{64}$	$1\frac{3}{16}$	$\frac{7}{16}$	$\frac{7}{16}$	$\frac{89}{128}$	$\frac{43}{64}$	$\frac{21}{128}$	$2\frac{5}{16}$	$\frac{3}{8}$
		70					$2\frac{35}{64}$	$1\frac{27}{64}$	$\frac{47}{64}$		$\frac{27}{64}$	$1\frac{7}{32}$	$\frac{7}{16}$	$\frac{7}{16}$	$\frac{51}{64}$	$\frac{23}{32}$	$\frac{11}{64}$	$2\frac{5}{16}$	$\frac{7}{16}$
		80					$2\frac{5}{8}$	$1\frac{17}{32}$	$\frac{25}{32}$	$\frac{3}{4}$	$\frac{29}{64}$	$1\frac{1}{4}$	$\frac{7}{16}$	$\frac{7}{16}$	$\frac{13}{16}$	$\frac{3}{4}$	$\frac{3}{16}$	$2\frac{3}{8}$	
		90					$2\frac{55}{64}$	$1\frac{5}{8}$	$\frac{13}{16}$	$\frac{13}{16}$	$\frac{15}{32}$	$1\frac{5}{16}$	$\frac{1}{2}$	$\frac{1}{2}$	$1\frac{19}{128}$	$\frac{51}{64}$	$\frac{15}{64}$	$2\frac{15}{16}$	$\frac{7}{16}$
		100					$3\frac{5}{64}$	$1\frac{23}{32}$	$\frac{27}{32}$	$\frac{7}{8}$	$\frac{15}{32}$	$1\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	$1\frac{5}{128}$	$\frac{27}{32}$	$\frac{9}{32}$	$3\frac{1}{8}$	$\frac{1}{2}$
Am. Rwy. Eng. Assoc. and Am. Rwy. Assoc.	A	60	14	14° 02′ 4:1	23°	14° 02′ 4:1	$2\frac{29}{64}$	$1\frac{21}{64}$	$\frac{45}{64}$	$\frac{5}{8}$	$\frac{25}{64}$	$1\frac{1}{8}$	$1\frac{7}{128}$	$\frac{65}{128}$	$\frac{51}{128}$	$\frac{3}{4}$	$\frac{45}{128}$	$2\frac{7}{16}$	$\frac{5}{8}$
		70					$2\frac{1}{2}$	$1\frac{27}{64}$	$\frac{51}{64}$	$\frac{5}{8}$	$\frac{13}{32}$	$1\frac{3}{16}$	$1\frac{7}{64}$	$\frac{33}{64}$	$\frac{25}{64}$	$\frac{27}{32}$	$\frac{23}{64}$	$2\frac{9}{16}$	$\frac{11}{16}$
		80					$2\frac{23}{32}$	$1\frac{17}{32}$	$\frac{7}{8}$	$\frac{21}{32}$	$\frac{27}{64}$	$1\frac{1}{4}$	$1\frac{15}{64}$	$\frac{17}{32}$	$\frac{25}{64}$	$\frac{29}{32}$	$\frac{25}{64}$	$2\frac{3}{4}$	$\frac{3}{4}$
		90					$3\frac{5}{32}$	$1\frac{21}{32}$	$\frac{15}{16}$	$\frac{23}{32}$	$\frac{7}{128}$	$1\frac{9}{32}$	$1\frac{11}{32}$	$\frac{19}{32}$	$\frac{9}{16}$	$\frac{15}{16}$	$\frac{7}{16}$	3	$\frac{13}{16}$
		100					$3\frac{5}{8}$	$1\frac{23}{32}$	$\frac{31}{32}$	$\frac{3}{4}$	$\frac{15}{32}$	$1\frac{3}{8}$	$1\frac{7}{32}$	$\frac{3}{4}$	$\frac{27}{32}$	1	$\frac{15}{32}$	$3\frac{3}{16}$	$\frac{7}{8}$
	B	60	14	14° 02′ 4:1	17°	14° 02′ 4:1	$2\frac{1}{16}$	$1\frac{3}{8}$	$\frac{11}{16}$	$\frac{11}{16}$	$\frac{51}{128}$	$1\frac{1}{16}$	$\frac{55}{64}$	$\frac{3}{4}$	$\frac{11}{64}$	$\frac{3}{4}$	$\frac{7}{32}$	$2\frac{9}{32}$	$\frac{23}{32}$
		70					$2\frac{17}{64}$	$1\frac{9}{16}$	$\frac{13}{16}$	$\frac{3}{4}$	$\frac{53}{128}$	$1\frac{3}{16}$	$\frac{55}{64}$	$\frac{3}{4}$	$\frac{35}{128}$	$\frac{51}{64}$	$\frac{59}{128}$	$2\frac{5}{16}$	$\frac{3}{4}$
		80					$2\frac{15}{32}$	$1\frac{19}{32}$	$\frac{15}{16}$	$\frac{5}{4}$	$\frac{1}{2}\frac{25}{25}$	$1\frac{7}{32}$	$1\frac{1}{128}$	$\frac{27}{32}$	$\frac{29}{128}$	$\frac{13}{16}$	$\frac{31}{128}$	$2\frac{11}{32}$	$\frac{13}{16}$
		90					$2\frac{5}{8}$	$1\frac{23}{32}$	$\frac{31}{32}$	$\frac{13}{16}$	$\frac{15}{32}$	$1\frac{9}{32}$	$1\frac{1}{32}$	$\frac{29}{32}$	$\frac{9}{32}$	$\frac{29}{64}$	$\frac{17}{64}$	$2\frac{105}{128}$	$\frac{27}{64}$
		100					$2\frac{55}{64}$	$1\frac{25}{32}$	$\frac{39}{32}$	$\frac{13}{16}$	$\frac{15}{32}$	$1\frac{21}{64}$	$1\frac{1}{32}$	$\frac{31}{32}$	$\frac{51}{128}$	$\frac{31}{64}$	$\frac{45}{128}$	$3\frac{1}{128}$	$\frac{7}{8}$

extensively tried out for both steam and electric railroad work. While many thousands in the aggregate have been placed on various roads, no one design has succeeded in displacing the angle-bar. There are necessarily as many variations in the details of the angle-bars as there are variations in the sizes of rails, beside other slight variations, but all cross-sections are similar to that shown in Fig. 124. This general design probably represents the majority of all the angle-bars in the country.

285. **Specifications for steel angle-bars.** Formerly these were made of either Bessemer or open-hearth steel. Now (1921), the specifications of the A. E. R. A. require open-hearth steel exclusively. Three grades are used: "high carbon steel," "quenched carbon," and "quenched alloy steel." The special requirements in addition to the usual requirements about accuracy of workmanship, branding, inspection, etc., are as follows: phosphorus not to exceed 0.04%; quenched bars must have carbon between 0.42 and 0.55%, but 1.00% of nickel or 0.35% of chromium will be considered the equivalent of 0.07% of carbon. The physical requirements are:

	High carbon steel.	Quenched carbon.	Quenched alloy steel.
Tensile strength, min., lbs. per sq. in.	85,000	100,000	110,000
Elastic limit, lbs. per sq. in.	.....	70,000	85,000
Elongation, per cent in 2 inches, min.	16%	1,600,000 ÷ tens. str., min. 12%	

All grades: cold bending test, 90°, on arc with diameter three times thickness of tested piece.

All punching, slotting and shaping is to be done at a temperature not less than 800° C. or 1470° F. Quenching shall be done in a bath of oil (or water, if specified) having a temperature of 810° C. (1490° F.) and kept in the bath until cool enough to handle.

TIE-PLATES.

286. **Advantages.** (*a*) As already indicated in § 242, the life of a soft-wood tie is very much reduced by "rail-cutting" and "spike-killing," such ties frequently requiring renewal long before any serious decay has set in. It has been practically demonstrated that the "rail-cutting" is not due to the mere

mately symmetrical with this axis. Fig. 120 also shows the *flow* of metal outside of the original contour, which all occurred on the gauge side. Very soft badly worn rails may show a fin on the outside.

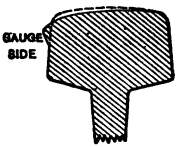


FIG. 120.—RAIL WEAR ON A TANGENT.

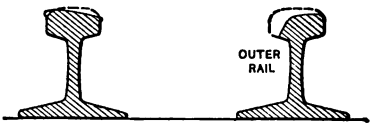
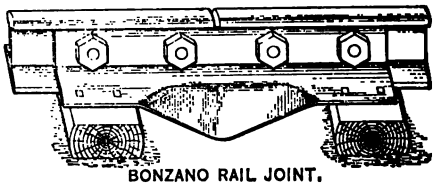


FIG. 121.—RAIL WEAR ON CURVES.

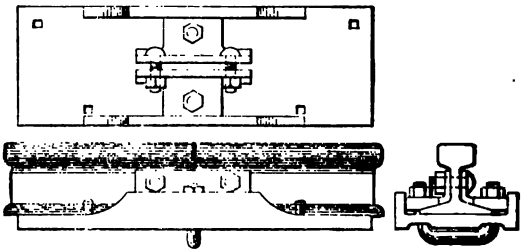
276a. Rail wear on curves. The pressure and grinding action of the wheel flanges against the rails wears away the inner side of the head of the outer rail on curves. If the rail is left in the track and the wear is permitted to continue, the head may be worn to approximately the form shown in Fig. 121. On the other hand, the inner rail is not subjected to any such lateral grinding action, and the rail is worn to substantially the same form as on a tangent, but the wear is more rapid due to the longitudinal slipping—see § 395. If the rails are soft or the traffic very heavy, there will be a flow of metal and a fin will form on the inner edge and perhaps also on the outer edge.

277. Experimental determination of rail wear. Several years ago a series of tests for rail wear were made on the Northern Pacific R. R. by taking up, weighing, and replacing, each year, the several groups of rails under test. Some of these rails were on tangents, the others on curves of various curvature. Some of the rails of each group were made of Bessemer steel, the others of open-hearth steel. No tests were made to determine the loss of weight through mere oxidation. All of the rails were in service for five years and some lasted for six years or more, but the loss in weight during the sixth year was nearly always equal to, and in some cases twice as much as, the loss during the preceding five years. Some of the rails lost over 10% of their weight, or about one-fourth the weight of the head, before being removed. Although the tests were too few to establish any positive laws, some tendencies which may be observed will give at least an approximate idea of the laws of rail wear.

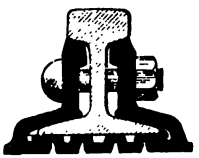
1. The average loss of weight during the first five years on



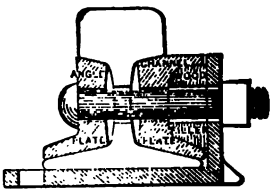
BONZANO RAIL JOINT.



FISHER BRIDGE JOINT.



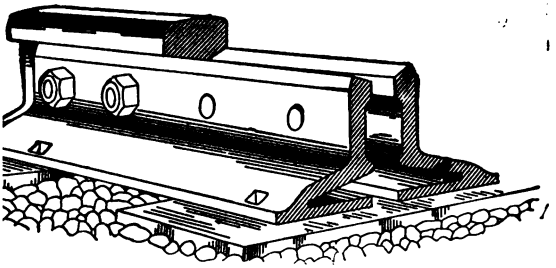
WOLHAUPTER JOINT



WEBER RAIL JOINT.

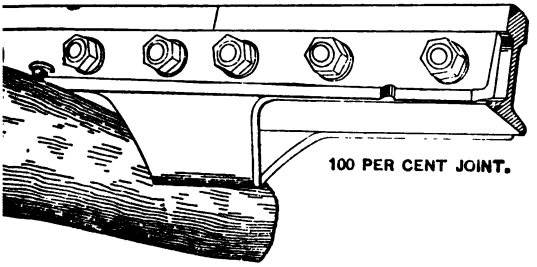
PLATE VII.—SOME FORMS OF RAIL JOINTS.

(Between pp. 320 *and* 321.*)*

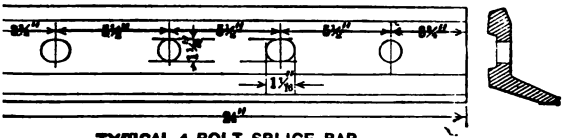


CONTINUOUS RAIL JOINT.

ATLAS SUSPENDED RAIL JOINT.



100 PER CENT JOINT.



TYPICAL 4-BOLT SPLICE BAR

of the rail on the tie, even with a maximum load on but is due to the impact resulting from vibration and ongitudinal working of the rail. It has been proved ; rail-cutting is practically prevented by the use of tie- (*b*) On curves there is a tendency to overturn the outer ; to the lateral pressure on the side of the head. ıduces a concentrated pressure of the outer edge of the the tie which produces rail-cutting and also draws the ikes. Formerly the only method of guarding against by the use of "rail-braces," one pattern of which is ı Fig. 125. But shoulder tie-plates serve the purpose ter and rail-braces are sed for guard rails and ls at switches. (*c*) Driv- es through holes in the ables the spikes on *each* he rail to mutually sup- h other, no matter in ateral) direction the rail l to move, and this prob- ounts in large measure added stability obtained e of tie-plates. (*d*) The pikes, called "necking," y the vertical vibration il against them, is very greatly reduced. (*e*) The cost is ll compared with the value of the added life of the tie, reduction in the work of track maintenance, and the running on the better track which is obtained. It has imated that by the use of tie-plates the life of hard- s is increased from one to three years and the life of ties is increased from three to six years. From the ure of the case, the value of tie-plates is greater when used to protect soft ties.

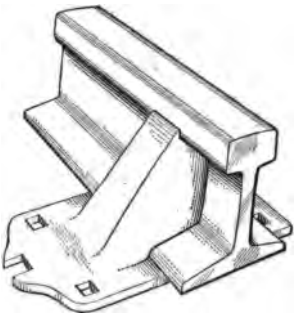


FIG. 125.—ATLAS BRACE K.

lements of the design. The Am. Rwy. Eng. Assoc. has ese principles in its Manual, as follows:

lates shall not be less than 6 inches in width, and as der as consistent with the class of ties to be used." The wide tie presumes heavy traffic and heavy wheel loads refore, a width as great as the face of the tie, up to at at inches, has been recommended.

2. "The length of the plates [parallel with the length of the tie] shall not be less than the safe-bearing area of the ties divided by the width of the plate, and, when made for screw spikes, shall be so shaped as to provide proper support for the screw spikes." 335 lbs. per square inch is declared to be, by test, the minimum safe-bearing load. Tie-plates sometimes sink quickly and deeply into the tie, thus proving that the area is inadequate for the wheel loads and traffic on them.

3. "The thickness of the plate shall be properly proportioned to the length." Tie-plates have been used as thin as $\frac{3}{16}$ inch, but it is now being realized that the real function of the plate is to be a *bearing* plate which shall distribute the load, rather than a mere surface plate which shall protect the tie from abrasion. The Track Committee of the A. R. E. A. recommended that the plates should be at least $\frac{5}{8}$ inch thick under either edge of the rail. Although the Association refused to concur, the discussion developed the fact that the thin plates formerly used have been found to be too thin and that thicker plates are more satisfactory.

4. Height of shoulder. The height of "at least ½-inch" was recommended in the 1915 Manual. The Track Committee has since then recommended that the height should "not be less than $\frac{1}{4}$" nor more than $\frac{3}{8}$".

5. "Where treated ties are used or where plates are for screw spikes, a flat-bottom plate is preferable. Where ribs of any kind are used on base of plate, these shall be few in number and not to exceed $\frac{1}{4}$ inch in depth." This specification is in direct contrast to the older designs which had been corrugations and even "claws" which were forced deeply into the tie, in order to anchor the plate immovably to the tie. But experience has proved that these corrugations hasten deterioration. In spite of this, the type using claws (see Fig. 126) is still the standard on some roads.

6. "Punching must correspond to the slotting in the splice-bars and, where advisable, may be so arranged that the plates may be used for joints. Spike holes may be punched for varying widths of rail base where the slotting will permit such punching without the holes interfering with each other and when the plate is of such design that the additional holes will not impair the strength of the plate."

Tie-plates are variously made of steel, wrought iron and malleable iron. Tie-plates are peculiarly subject to rust, especially

s an effect of brine drippings from refrigerator cars. The omparative immunity from rust of malleable iron explains its se for this purpose. The specifications for steel and wrought ·on are similar to other physical tests for such a metal when oughness rather than high ultimate strength is desired. The ıalleable iron tie-plates have lugs cast on them for testing pur-oses. When this lug is broken off, it must not break easily, as ast iron, but must show toughness. The fracture must show a arrow band of white metal on the surface, the center portion

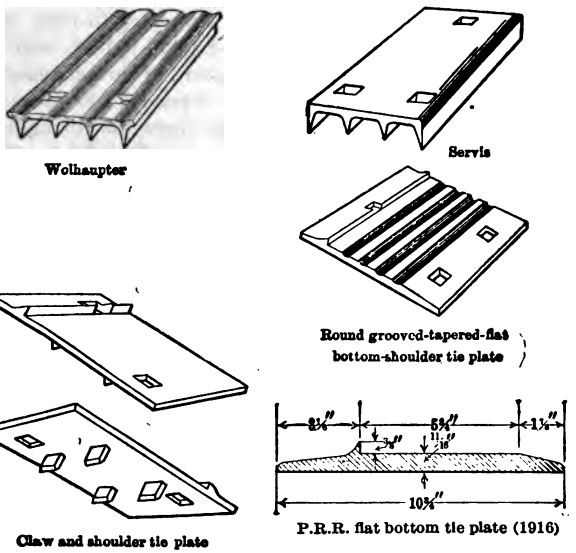


FIG. 126.—VARIOUS FORMS OF TIE-PLATES.

eing dark and fiberless. The plates must, when tested, bend ıfficiently to prove thorough annealing.

The holes in a tie-plate should be about $\frac{1}{16}''$ larger than the ze of the intended spike. The length of the plate, perpendicular o the rail, should be $8\frac{1}{2}$ to 11 in., the extension on the outside f the rail base being $\frac{3}{4}''$ to $1\frac{1}{4}''$ more than that on the inside. 'or very heavy traffic the thickness should be $\frac{5}{8}''$ to $\frac{3}{4}''$; for lighter raffic they may be as thin as $\frac{3}{8}''$. Flat-bottom plates should e at least $\frac{1}{2}''$ thick; corrugated plates, being somewhat stiffer,

may be thinner for the same service. The tie-plates over the joint ties must be somewhat longer than the intermediates, in order to allow for the extra length from out to out of the angle-plates.

288. Method of setting. A very important detail in the process of setting the tie-plates on the ties is that the plates should be rigidly attached to the ties in their intended position during the process of setting. If tie-plates with flat bottoms are used, the surface of the tie must be adzed, so that it is not only plane but level, so that there will be no danger that the plate will rock on the tie. When using tie-plates which are corrugated on the under surface, it is necessary to force them into the tie until the under side of the plate is flush with the surface of the tie. This requires a pressure of several thousand pounds. Sometimes trackmen have depended on the easy process of waiting for passing trains to force the corrugations into the tie until the plate is in its intended position. Until the plates are finally set the spikes cannot be driven home, and this apparently cheap and easy process generally results in loose spikes and rails. The best method for new work is to drive the plates into the tie before setting the tie in position. A tie-plate gauge holds both tie-plates in their proper relative position, and both plates may be driven by the use of heavy beetles. When it is necessary to place the plate under the rail and drive it in, it is somewhat difficult to drive it by striking the plate with a swage on each side of the rail alternately. When it is struck on one side, the other side flies up unless held down by a wedge driven between the plate and the rail on the other side of the rail. A straddler, which straddles the rail somewhat like an inverted U, is very useful for this purpose, since it makes it possible to strike the head of the straddler and force down both sides of the plate at once. The Southern Pacific Railroad Company has rigged up a small pile-driver on a hand-car, which is used in connection with a straddler to drive the tie-plates into position. Some western railroads have even adopted the process of rigging up a flat car with a machine which will press the tie-plates into place in the ties before the ties are placed in the track.

SPIKES.

289. Requirements. The rails must be held to the ties by a fastening which will not only give sufficient resistance, but which

ll retain its capacity for resistance. It must also be cheap d easily applied. The ordinary track-spike fulfills the last uirements, but has comparatively small resisting power, com- red with screws or bolts. Worse than all, the tendency to

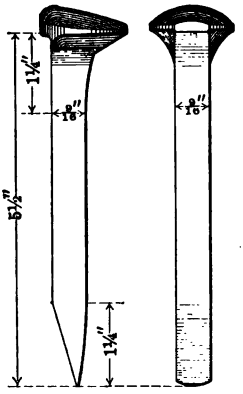


FIG. 127.

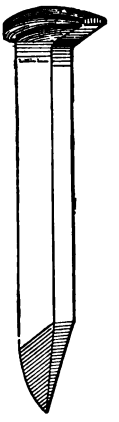


FIG. 128.

tical vibration in the rail produces a series of upward pulls on spike that soon loosens it. When motion has once begun capacity for resistance is greatly reduced, and but little more ration is required to pull the spike out so much that redriving necessary. Driving the spike to place again in the same hole of small value except as a very temporary expedient, as its ding power is then very small. Redriving the spikes in new es very soon "spike-kills" the tie. Many plans have been ised to increase the holding power of spikes, such as making m jagged, twisting the spike, swelling the spike at about the ter of its length, etc. But it has been easily demonstrated t the fibers of the wood are generally so crushed and torn by ving such spikes that their holding power is less than that of plain spike, and the durability is greatly diminished.

The ordinary spike (see Fig. 127) is made with a square cross- tion which is uniform through the middle of its length, the er 1¼ in. tapering down to a chisel edge, the upper part swelling to the head. The Goldie spike (see Fig. 128) aims to improve form by reducing to a minimum the destruction of the

fibers. To this end, the sides are made smooth, the edges are clean-cut, and the point, instead of being chisel-shaped, is ground down to a pyramidal form. Such fiber-cutting as occurs is thus accomplished without much crushing, and the fibers are thus pressed away from the spike and slightly downward. Any tendency to draw the spike will, therefore, cause the fibers to press still harder on the spike and thus increase the resistance.

A series of tests made by a committee of the A. R. E. A. and reported to the 1914 Convention, established some very valuable conclusions with respect to the use of the ordinary cut spike. Spikes with sharp pyramidal points and with various degrees of bluntness, and also the ordinary chisel-pointed spike, were driven into ties and other timbers and were withdrawn by a testing machine. Then the timbers were cut so as to expose the holes to their full length, so that the crushing of the fibers by the spike driving could be observed. A series of photographs illustrated this feature. In some cases the spikes were driven into $\frac{7}{8}$-in. bored holes, some of which were $2\frac{1}{2}$ ins. deep, but the most of them were 4 ins. deep. In other cases, the spikes were driven without previous boring. The following conclusions were unmistakable.

1. The spike with a pyramidal point about 1 in. long (virtually the "Goldie" design Fig. 128), has greater holding power, not only when it first begins to yield, but also afterward while the spike is being drawn out.

2. The long-pointed spikes crushed the fiber far less than any other type.

3. The chisel-pointed spike, virtually as shown in Fig. 127, and which is the type now in most common use, has the least holding power and is more destructive in crushing the fibers.

4. Spikes driven into $\frac{7}{8}$-in. bored holes have greater holding power than when driven without boring, and the crushing of the fiber is much less. This indicates the very real economy in boring holes where the life of the tie is an economical consideration.

290. Driving. The holding power of a spike depends largely on how it is driven. If the blows are eccentric and irregular in direction, the hole will be somewhat enlarged and the holding power largely decreased. The spikes on each side of the rail in any one tie should not be directly opposite, but should be staggered. Placing them directly opposite will tend to split the tie, or at least decrease the holding power of the spikes. The direction of staggering should be reversed in the two pairs

of spikes in any one tie (see Fig. 129). This will tend to prevent any twisting of the tie in the ballast, which would otherwise loosen the rail from the tie.

291. Screw spikes. The D., L. & W. R. R. began the general use of screw spikes for all new work and for extensive track renewals in 1910. In five years they used over 12,000,000 screw spikes. The design is shown in Fig. 130. From a report made

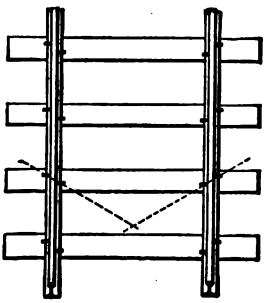


FIG. 129.—SPIKE-DRIVING.

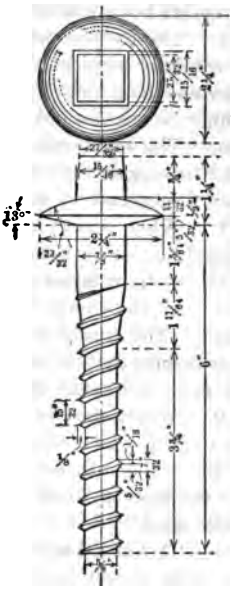


FIG. 130.—SCREW SPIKE, D. L. & W. R. R.

by Mr. G. J. Ray, Chief Engineer, to the A. R. E. A., the following facts and conclusions are deduced:

1. The use of screw spikes, in conjunction with suitable tie-plates, is almost a necessity in order to fully utilize the durability of a treated tie. A treated tie is seldom removed on account of decay in the body of the tie. Its destruction is generally due to "spike-killing," rail cutting, or to the decay which comes immediately after mechanical injury to the wood under the rail. Screw spikes and tie-plates largely prevent this mechanical injury.

2. "As a rule, with woods which it will pay to treat, the poorer the quality of the timber the more elaborate and expensive the fastening must be if the mechanical life of the tie is made to approach the life of the treated timber."

3. "Tie-plates should be used on all ties where screw spikes are used."

4. "Four holes should be provided for screw spikes, so that two extra holes will be available if needed."

5. "The size of screw spikes and the design of the thread should be carefully considered before a screw spike is adopted. Thereafter no changes should be made; otherwise the new screw spikes cannot be used in old holes without damaging the wood fiber."

6. "The screw-spike head should have tapering sides to prevent turning in the wrench socket after the size of the head has been diminished by rust."

7. "When screw spikes are fully seated, no further strain should be put on them, as this will tend to destroy the threads in the wood or injure the spikes."

8. "All ties should be bored at the treating plant before treatment. This can be done while the ties are being adzed, and not only insures that the holes are bored sufficiently deep, but provides for good treatment of all wood adjacent to the spike holes."

9. "Where the ties are bored before treatment, the track must be to proper gauge before the ties can be placed."

10. "The holes for screw spikes should be of proper dimensions for the class of wood used, with due regard to the size of screw spike used."

11. "A limited number of holes can be bored with one bit, after which its size will diminish so as to make it unfit for a hole of a given size." [The paper nowhere makes any statement as to the size of the bored hole in comparison with the diameter of the screw. The bored hole should have *about* the same diameter as the diameter of the screw at the base of the screw thread, but the hardness of the wood requires some variation, since, if the hole is too small, it will be impossible to turn the screw. The exact diameter must be determined for each kind of wood and must be strictly maintained.]

12. "Holes should be bored somewhat deeper than the length of the screw spike. There is no serious objection to boring the holes clear through the ties."

13. "Not only is the lateral and vertical resistance of a screw spike greater than that of a cut spike when both are first applied, but the lateral and vertical resistance of a loose screw spike is considerably greater than the lateral and vertical resistance of a loose cut spike."

14. "When the threads in the tie are entirely destroyed, a screw lining (any one of several different varieties) may be used with good results."

15. "All ties should be bored and adzed before treatment. This insures good gauge, a perfect bearing for the tie-plates and good treatment under the rail seat and around the screw-spike holes."

16. "In placing screw spikes, they should be driven by hammer only sufficient to make the threads take hold. If rigid instructions are not carried out, laborers will continually overdrive spikes and thus destroy the wood fibers near the top of the holes."

17. "The best results with the screw spikes can be expected in new construction, and where the number of screw spikes in tie renewals predominate over cut spikes."

18. "The use of screw spikes for the past five years has not made it necessary to increase the number of sectionmen per mile of track."

19. "Whether or not it will pay to use screw spikes will depend upon the cost of ties, their probable life and the amount of traffic."

292. **"Wooden spikes."** Among the regulations for track-laying given in § 246, mention was made of wooden "spikes," or plugs, which are used to fill up the holes when spikes are withdrawn. The value of the policy of filling up these holes is unquestionable, since the expense is insignificant compared with the loss due to the quick and certain decay of the tie if these holes are allowed to fill with water and remain so. But the method of making these plugs is variable. On some roads they are "hand-made" by the trackmen out of otherwise useless scraps of lumber, the work being done at odd moments. This policy, while apparently cheap, is not necessarily so, for the hand-made plugs are irregular in size and therefore more or less inefficient. It is also quite probable that if the trackmen are required to make their own plugs, they would spend time on these very cheap articles which could be more profitably employed otherwise. Since the holes made by the spikes are larger at the top than they are near the bottom, the plugs should *not* be of uniform cross-section but should be slightly wedge-shaped. The "Goldie tie-plug" (see Fig. 131) has been designed to fill these requirements. Being machine-made, they are uniform in size; they are of a shape which will best fit the hole; they can be furnished of any desired wood, and at a cost which makes it a wasteful economy to attempt to cut them by hand.

FIG. 131.

TRACK-BOLTS.

293. **Essential requirements.** The track-bolts must have sufficient strength and must be screwed up tight enough to hold the angle-plates against the rail with sufficient force to develop the full transverse strength of the angle-bars. On the other hand the bolts should not be screwed so tight that slipping may not take place when the rail expands or contracts with temperature. It would be impossible to screw the bolts tight enough to prevent slipping during the contraction due to a considerable fall of temperature on a straight track, but when the track is curved, or when expansion takes place, it is conceivable that the resistance of the ties in the ballast to lateral motion may be less than the resistance at the joint. A test to determine this resistance was made by Mr. A. Torrey, chief engineer of the Mich. Cent. R. R., using 80-lb. rails and ordinary angle-bars, the bolts being screwed up as usual. If required a force of about 31000 to 35000 lbs. to start the joint, which would be equivalent to the stress induced by a change of temperature of about 22°. But if the central angle of any given curve is small, a comparatively small lateral component will be sufficient to resist a compression of even 35000 lbs. in the rails. Therefore there will ordinarily be no trouble about having the joints screwed too tight. The vibration caused by the passage of a train reduces the resistance to slipping. This vibration also facilitates an objectionable feature, viz., loosening of the nuts of the track-bolts. The bolt is readily prevented from turning by giving it a form which is *not* circular immediately under the head and making corresponding holes in the angle-plate. See Fig. 132. Note also the elongated and the round bolt holes in the standard angle bar shown on Plate VII. Half the nuts are thus on either side of the rail and the danger that *all* the bolts of a joint might be simultaneously sheared off by a derailment is somewhat minimized.

"As a rule, as large track-bolts should be used as the rail and splice-bars will permit." [From 1915 Manual, A. R. E. A.] There is always some danger that a trackman may stretch a bolt beyond its elastic limit. A pull of 100 lbs. on a 33-inch track wrench will induce a stress of about 45000 lbs. per square inch in a $\frac{7}{8}$-inch track bolt. The same work on a 1-inch bolt would produce a stress of about 35000 lbs. per square inch. In order to

obtain the necessary toughness, bolts must be made of low-carbon steel or of nickel-steel, untreated or heat-treated. When made of carbon steel, specifications require an elastic limit of at least 35,000 lbs. per square inch but at the same time an elongation of 25% in 2 inches and a reduction of area of at least 50%. A harder steel would have a higher elastic limit, but would not be sufficienlty ductile. Higher elastic limits, with sufficient ductility, may be obtained by using untreated nickel or other alloy steel (at least 45,000 lbs. per square inch), or heat-treated nickel or other alloy steel (at least 75,000 lbs. per square inch). The elastic limit shall not be less than 50% of the ultimate. Added strength can only be obtained by using larger bolts or a more expensive metal.

294. Design of track-bolts. In Fig. 132 is shown a common design of track-bolt. In its general form this represents the bolt used on nearly all roads, being used not only with the common angle-plates, but also with many of the improved designs of rail-joints. The variations are chiefly a general increase in size to correspond with the increased weight of rails, besides variations in detail dimensions which are frequently unimportant. The diameter is usually $\frac{3}{4}''$ to $\frac{7}{8}''$; 1″ bolts are used for 100-lb. rails. As to length, the bolt should not extend more than $\frac{1}{2}''$ outside of the nut when it is screwed up. If it extends farther than this it is liable to be broken off by a possible derailment at that point. The lengths used vary from $3\frac{1}{4}''$, which may be used with 60-lb. rails, to 5″, which is required with 100-lb. rails. The length required depends somewhat on the type of nut-lock used.

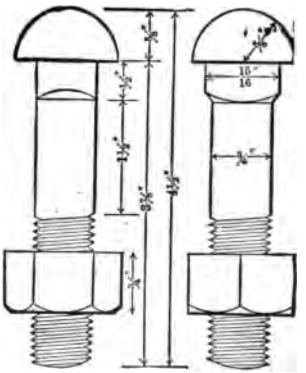


FIG. 132.—TRACK-BOLT.

NUT-LOCKS.

295. Design of nut-locks. The designs for nut-locks may be divided into three classes: (*a*) those depending entirely on an

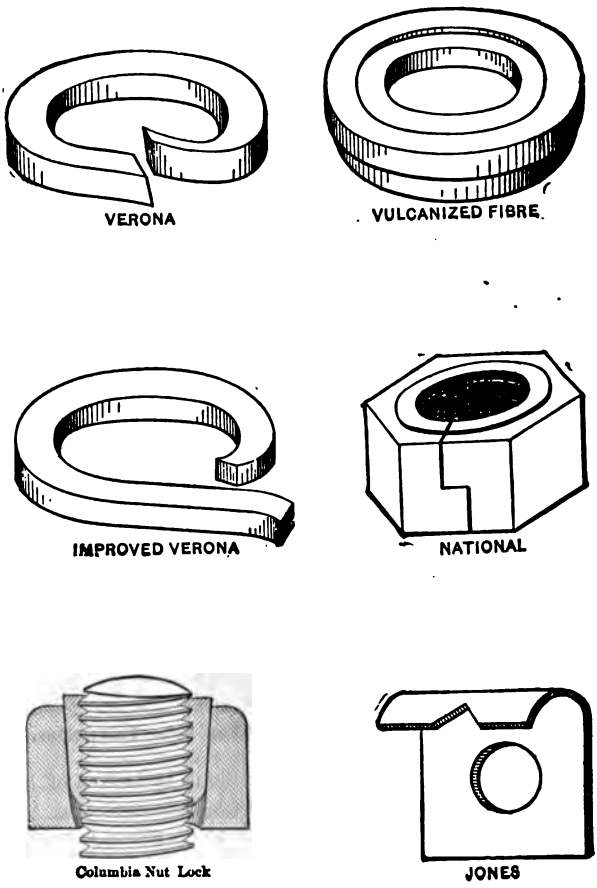


FIG. 133.—TYPES OF NUT-LOCKS.

elastic washer which absorbs the vibration which might otherwise induce turning; (*b*) those which jam the threads of the bolt and nut so that, when screwed up, the frictional resistance is too great to be overcome by vibration; (*c*) the "positive" nut-locks—those which mechanically hold the nut from turning. Some of the designs combine these principles to some extent. The "vulcanized fiber" nut-lock is an example of the first class. It consists essentially of a rubber washer which is protected by an iron ring. When first placed this lock is effective, but the rubber soon hardens and loses its elasticity and it is then ineffective and worthless. Another illustration of class (*a*) is the use of wooden blocks, generally 1″ to 2″ oak, which extend the entire length of the angle-bar, a single piece forming the washer for the four or six bolts of a joint. This form is cheap, but the wood soon shrinks, loses its elasticity, or decays so that it soon becomes worthless, and it requires constant adjustment to keep it in even tolerable condition. The "Verona" nut-lock is another illustration of class (*a*) which also combines some of the positive elements of class (*c*). It is made of tempered steel and, as shown in Fig. 133, is warped and has sharp edges or points. The warped form furnishes the element of elastic pressure when the nut is screwed up. The steel being harder than the iron of the angle-bar or of the nut, it bites into them, owing to the great pressure that must exist when the washer is squeezed nearly flat, and thus prevents any *backward* movement, although forward movement (or tightening the bolt) is not interfered with. The "National" nut-lock is a type of the second class (*b*), in which, like the "Harvey" nut-lock, the nut and lock are combined in one piece. With six-bolt angle-bars and 30-foot rails, this means a saving of 2112 pieces on each mile of single track. The "National" nuts are open on one side. The hole is drilled and the thread is cut slightly smaller than the bolt, so that when the nut is screwed up it is forced slightly open and therefore presses on the threads of the bolt with such force that vibration cannot jar it loose. Unlike the "National" nut, the "Harvey" nut is solid, but the form of the thread is progressively varied so that the thread pinches the thread of the bolt and the frictional resistance to turning is too great to be affected by vibration.

The "Columbia" nut-lock is a two-piece nut, both parts of which must turn simultaneously. As shown in the figure, one

section wedges into the other. The greater the tension in the bolt, the greater the wedging action and the greater the friction to prevent turning.

The "Jones" nut-lock, belonging to class (*c*), is a type of a nut-lock that does not depend on elasticity or jamming of screw-threads. It is made of a thin flexible plate, the square part of which is so large that it will not turn after being placed on the bolt. After the nut is screwed up, the thin plate is bent over so that the re-entrant angle of the plate engages the corner of the nut and thus mechanically prevents any turning. The metal is supposed to be sufficiently tough to endure without fracture as many bendings of the plate as will ever be desired. Nut-locks of class (*c*) are not in common use.

The above types have been discussed in order to show the development of the various devices. With but few exceptions, the standard nut-lock is a steel spring ring of the same general class as the Verona. The A. R. E. A. have prepared specifications for such nut-locks which include the following:

"After the finished nut-lock has been subjected for one hour to pressure sufficient to compress it flat and has been released, its reaction shall be not less than two-thirds its height or thickness of section, provided thickness is less than width of section. If the section is square, the reaction must be not less than one-half its thickness. If height or thickness of section is more than width, the reaction shall be not less than the width of the section. The internal diameters naturally affect the percentage of reaction, and the above specifications apply to nut-locks of internal diameters from $\frac{13}{16}$ in. to 1 $\frac{5}{16}$ ins. Owing to the difficulty of establishing a common rate of percentage that shall be uniformly applicable to any internal diameter of any nut-lock of any section it has been sought to cover the matter as above. Amount and durability of reactionary power under constant pressure is the true test of any spiral spring nut-lock. The percentage of reaction increases proportionately with the increased internal diameter of any given section."

"With one end of the finished nut-lock secured in a vise, and the opposite end twisted to 45 degrees, there must be no sign of fracture. When further twisted until broken, the fracture must show a good quality of steel."

CHAPTER XI.

SWITCHES AND CROSSINGS.

SWITCH CONSTRUCTION.

296. Essential elements of a switch. Flanges of some sort are a necessity to prevent car-wheels from running off from the rails on which they may be moving. But the flanges, although a neccssity, are also a source of complication in that they require some special mechanism which will, when desired, guide the wheels out from the controlling influence of the main-line rails. This must either be done by raising the wheels high enough so that the flanges may pass *over* the rails, or by breaking the continuity of the rails in such a way that channels or "flange spaces" are formed *through* the rails. An ordinary stub-switch breaks the continuity of the main-line rails in three places, two of them at the switch-block and one at the frog. The Wharton switch avoids two of these breaks by so placing inclined planes that the wheels, rolling on their flanges, will surmount these inclines until they are a little higher than the rails. Then the wheels on the side toward which the switch runs are guided over and across the main rail on that side This rise being accomplished in a short distance, it becomes impracticable to operate these switches except at slow speeds, as any sudden change in the path of the center of gravity of a car causes very destructive jars both to the switch and to the rolling stock. The other general method makes a break in one main rail (or both) at the switch-block. In both methods the wheels are led to one side by means of the "lead rails," and finally one line of wheels passes *through* the main rail on that side by means of a "frog." There are some designs by which even this break in the main rail is avoided, the wheels being led *over* the main rail by means of a short *movable* rail which is on occasion placed across the main rail, but such designs have not come into general use.

297. Frogs. Frogs are provided with two channel-ways or "flange spaces" through which the flanges of the wheels move.

Each channel cuts out a parallelogram from the tread area. Since the wheel-tread is always wider than the rail, the wing rails will support the wheel not only across the space cut out by the channel, but also until the tread has passed the point of the frog and can obtain a broad area of contact on the tongue of the frog. This is the theoretical idea, but it is very imperfectly

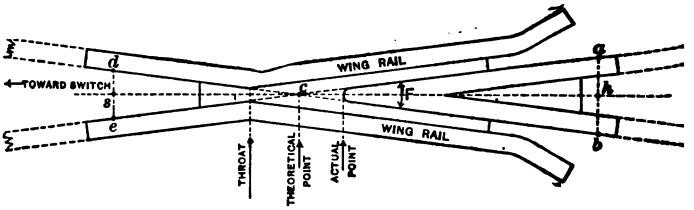


FIG. 134.—DIAGRAMMATIC DESIGN OF FROG.

realized. The wing rails are sometimes subjected to excessive wear owing to "hollow treads" on the wheels—owing also to the frog being so flexible that the point "ducks" when the wheel approaches it. On the other hand the sharp point of the frog will sometimes cause destructive wear on the tread of the wheel. Therefore the tongue of the frog is not carried out to the sharp theoretical point, but is purposely somewhat blunted. But the break which these channels make in the continuity of the tread area becomes extremely objectionable at high speeds, being mutually destructive to the rolling stock and to the frog. The jarring has been materially reduced by the device of "spring frogs"—to be described later. Frogs were originally made of cast iron—then of cast iron with wearing parts of cast steel, which were fitted into suitable notches in the cast iron. This form proved extremely heavy and devoid of that elasticity of track which is necessary for the safety of rolling stock and track at high speeds. The present standard practice is to build the frog up of pieces of rails which are cut or bent as required. There are always four pieces for single-pointed frogs. Usually they are assembled by bolts running through the rail webs, which are properly separated by rolled steel filler blocks. Sometimes they are enclosed by clamps held in place by wedges. Sometimes the rails are bolted or riveted to a base plate. For the hardest service, the wearing parts are made of manganese

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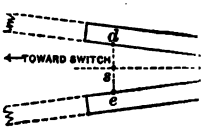

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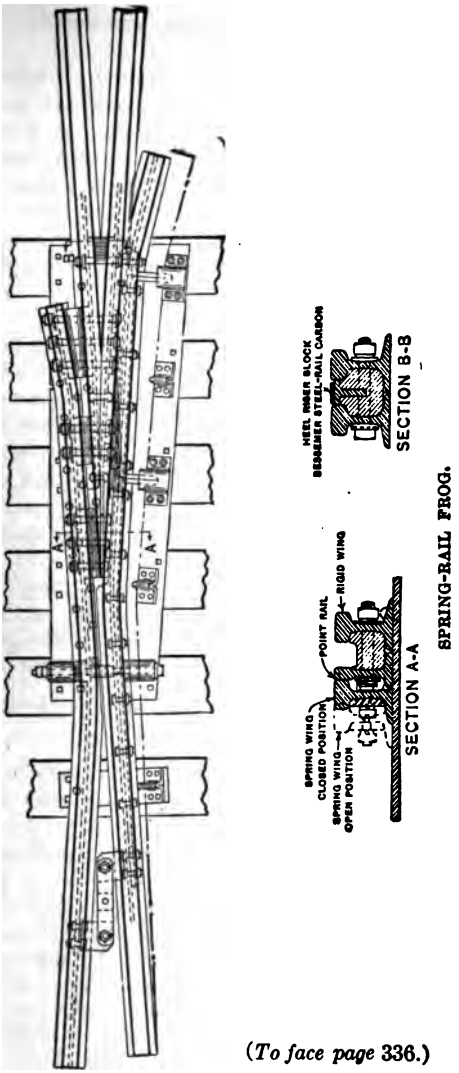

(*To face page* 336.)

PLATE VIII.—SOME TYPES OF FROGS.
(As made by Ramapo Iron Works.)

section wedges into the other. The greater the tension in the bolt, the greater the wedging action and the greater the friction to prevent turning.

The "Jones" nut-lock, belonging to class (*c*), is a type of a nut-lock that does not depend on elasticity or jamming of screw-threads. It is made of a thin flexible plate, the square part of which is so large that it will not turn after being placed on the bolt. After the nut is screwed up, the thin plate is bent over so that the re-entrant angle of the plate engages the corner of the nut and thus mechanically prevents any turning. The metal is supposed to be sufficiently tough to endure without fracture as many bendings of the plate as will ever be desired. Nut-locks of class (*c*) are not in common use.

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"With one end of the finished nut-lock secured in a vise, and the opposite end twisted to 45 degrees, there must be no sign of fracture. When further twisted until broken, the fracture must show a good quality of steel."

farther rail end, besides producing such a stress in the construction that a breakage is rendered quite likely, and such a breakage might have very serious consequences.

300. **Point switches.** The essential principle of a point switch is illustrated in Fig. 136. As is shown, one main rail and also one of the switch-rails is unbroken and immovable. The other main rail (from A to F) and the corresponding portion of the other lead rail are substantially the same as in a stub switch. A portion of the main rail (AB) and an equal length of the opposite lead rail (usually 16.5 to 22 feet long) are fastened together by tie-rods. The end at A is jointed as usual and the other end is pointed, both sides being trimmed down so that the feather edge at B includes the web of the rail. In order to retain in it as much strength as possible, the point-rail is raised so that it rests on the base of the stock-rail, one side of the base of the point-rail being nearly cut away. As may be seen in Fig. 137, although the influence of the point of the rail in moving the wheel-flange away from the stock-rail is really zero at that point, yet the rail has all the strength of the web, more than one-half that of the base, and is also reinforced. The planing runs back in *straight* lines, until at about six or seven feet back from the point the full width of the head is obtained. The full width of the base will only be obtained at about 13 feet from the point. The A. R. E. A. standard switch rail is always cut on the basis that the distance between gauge lines at the heel of the switch (the distance MN in Fig. 143) is $6\frac{1}{4}$ inches and that the "point" is $\frac{1}{4}$ inch wide. Then, using four standard lengths, 11, $16\frac{1}{2}$, 22 and 30 feet, the angles vary from 2° 36′ 19″ to 0° 57′ 18″ as shown in Table III.

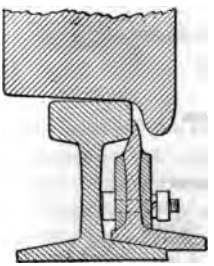


FIG. 137.

301. **Switch-stands.** The simplest and cheapest form is the "ground lever," which has no target. The radius of the circle described by the connecting-rod pin is precisely one-half the throw. From the nature of the motion the device is practically

Each channel cuts out a parallelogram from the tread area. Since the wheel-tread is always wider than the rail, the wing rails will support the wheel not only across the space cut out by the channel, but also until the tread has passed the point of the frog and can obtain a broad area of contact on the tongue of the frog. This is the theoretical idea, but it is very imperfectly

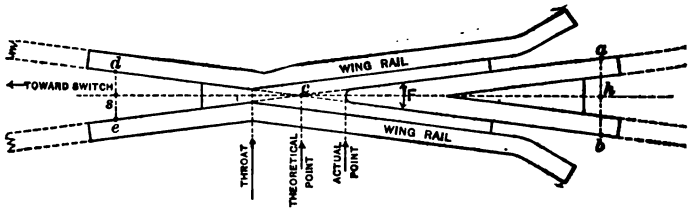


FIG. 134.—DIAGRAMMATIC DESIGN OF FROG.

realized. The wing rails are sometimes subjected to excessive wear owing to "hollow treads" on the wheels—owing also to the frog being so flexible that the point "ducks" when the wheel approaches it. On the other hand the sharp point of the frog will sometimes cause destructive wear on the tread of the wheel. Therefore the tongue of the frog is not carried out to the sharp theoretical point, but is purposely somewhat blunted. But the break which these channels make in the continuity of the tread area becomes extremely objectionable at high speeds, being mutually destructive to the rolling stock and to the frog. The jarring has been materially reduced by the device of "spring frogs"—to be described later. Frogs were originally made of cast iron—then of cast iron with wearing parts of cast steel, which were fitted into suitable notches in the cast iron. This form proved extremely heavy and devoid of that elasticity of track which is necessary for the safety of rolling stock and track at high speeds. The present standard practice is to build the frog up of pieces of rails which are cut or bent as required. There are always four pieces for single-pointed frogs. Usually they are assembled by bolts running through the rail webs, which are properly separated by rolled steel filler blocks. Sometimes they are enclosed by clamps held in place by wedges. Sometimes the rails are bolted or riveted to a base plate. For the hardest service, the wearing parts are made of manganese

placed base to base, to say nothing of the $\frac{3}{4}''$ required for spikes, it becomes necessary to cut the flange of the guard-rail. The length of the rail should be 16 feet 6 inches, the middle portion being straight for a length of 3 feet 6 inches, and the ends, each being 6 feet 6 inches long, curved out so that the side of the rail head at each end is 4 inches from the main rail head, when the flange-way at the center is $1\frac{3}{4}$ inches. See Fig. 141.

MATHEMATICAL DESIGN OF SWITCHES.

In all of the following demonstrations regarding switches, turnouts, and crossovers, the lines are assumed to represent the *gauge-lines*—i.e., the lines of the *inside* of the head of the rails.

304. Design with circular lead-rails. The simplest method is to consider that the lead-rails curve out from the main track-

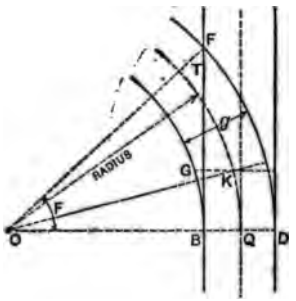


Fig. 142.

rails by arcs of circles which are tangent to the main rails and which extend to the frog-point F. The simple curve from D to F is of such radius that $(r+\frac{1}{2}g)$ vers $F=g$, in which F = the frog angle g = gauge, L = the "lead" (BF), and r = the radius of the *center* of the switch-rails.

$$\therefore r+\tfrac{1}{2}g=\frac{g}{\text{vers } F}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

Also, $BF \div BD = \cot \frac{1}{2}F$; $BD=g$; $BF=L$.

$$\therefore L=g \cot F. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

Also, $$L=(r+\tfrac{1}{2}g)\sin F; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

$$QT=2r\sin \tfrac{1}{2}F. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

These formulæ involve the angle F. As shown in Table III, the angles (F) are always odd quantities, and their trigonometric functions are somewhat troublesome to obtain closely with ordinary tables. The formulæ may be simplified by substituting the frog-number n, from the relation that $n=\frac{1}{2}\cot\frac{1}{2}F$. Since

$$r-\tfrac{1}{2}g=L\cot F \quad \text{and} \quad r+\tfrac{1}{2}g=L\operatorname{cosec} F,$$

then $$r=\tfrac{1}{2}L(\cot F+\operatorname{cosec} F)$$

$$=\tfrac{1}{2}g\cot\tfrac{1}{2}F(\cot F+\operatorname{cosec} F)$$

$$=\tfrac{1}{2}g\cot^2\tfrac{1}{2}F, \text{ since } (\cot a+\operatorname{cosec} a)=\cot\tfrac{1}{2}a$$

$$=2gn^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

Also, $$L=2gn, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (74)$$

from which $$r=n\times L. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (75)$$

These extremely simple relations may obviate altogether the necessity for tables, since they involve only the frog-number and the gauge. On account of the great simplicity of these rules, they are frequently used as they are, regardless of the fact that the curve is never a uniform simple curve from switch-block to frog. In the first place there is a considerable length of the gauge-line within the frog, which is straight unless it is purposely curved to the proper curve while being manufactured, which is seldom if ever done—except for the very large-angled frogs used for street-railway work, etc. It is also doubtful whether the switch-rails (BA, Fig. 135) are bent to the computed curve when the rails are set for the switch. The switch-rails of point switches are *straight*, thus introducing a stretch of straight track which is about one-fifth of the total length of the lead-rails. The effect of these modifications on the length and radius of the lead-rails will be developed and discussed in the following sections.

The throw (t) of a stub switch depends on the weight of the rail, or rather on the width of its base. The throw must be at

least $\frac{3}{4}''$ more than that width. The head-block should therefore be placed at such a distance from the heel of the switch (B) that the versed sine of the arc equals the throw. These points *must* be opposite on the two rails, but the points on the two rails where these relations are exactly true will not be opposite. Therefore, instead of considering either of the two radii $(r+\frac{1}{2}g)$ and $(r-\frac{1}{2}g)$, the mean radius r is used. Then (see Fig. 142)

$$\text{vers } KOQ = t \div r,$$

and the length of the switch-rails is

$$QK = r \sin KOQ. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$

Stub-switches are generally used with large frog angles. For small frog angles (large frog-numbers) the values of QK are so great that the length of rail left unspiked is too great for a safe track. If this were obviated by spiking down a portion of the lead the theoretical accuracy of the switch would be lost.

The use of stub switches may now be considered obsolete. But the above demonstration has been retained in this edition for its educational value as an introduction to the more complicated method which is now the standard.

305. Standard design, using straight frog-rails and straight point-rails. It becomes necessary in this case to find a curve which shall be tangent to both the point-rail and the frog-rail. The curve therefore begins at M, its tangent making an angle of α (varying from 0° 52′ to 2° 36′) with the main rail, and runs to J. $FJ = W =$ the length of the " wing-rail " from the theoretical point of the frog (F) to the toe, J or J'. $FK = K =$ the length from the theoretical point to the heel of the frog. $MN = H =$ the " heel distance," or the distance of the gauge line of the switch-rail at the heel from the gauge line of the main track rail.

The central angle of the curve equals $(F-a)$. The angle of the chord JM with the main rails is therefore

$$\tfrac{1}{2}(F-a)+a = \tfrac{1}{2}(F+a);$$

$$JM = \frac{g - W \sin F - H}{\sin \frac{1}{2}(F+a)};$$

$$r+\tfrac{1}{2}g=\frac{JM}{2\sin\tfrac{1}{2}(F-a)}$$

$$=\frac{g-W\sin F-H}{2\sin\tfrac{1}{2}(F+a)\sin\tfrac{1}{2}(F-a)}$$

$$=\frac{g-W\sin F-H}{\cos a-\cos F}; \quad \ldots\ldots\ldots \quad (77)$$

$$DN=s\cos\alpha, \quad \ldots\ldots\ldots \quad (78)$$

in which S = length of switch-rail.

$$BF=L=JM\cos\tfrac{1}{2}(F+a)+W\cos F+S\cos\alpha$$

$$=(g-W\sin F-H)\cot\tfrac{1}{2}(F+a)+W\cos F+S\cos\alpha. \quad (79)$$

It may be more simple, if $(r+\frac{1}{2}g)$ has already been computed, to write

$$L=2(r+\tfrac{1}{2}g)\sin\tfrac{1}{2}(F-a)\cos\tfrac{1}{2}(F+a)+W\cos F+S\cos\alpha$$

$$=(r+\tfrac{1}{2}g)(\sin F-\sin a)+W\cos F+S\cos\alpha. \quad \ldots\ldots \quad (80)$$

The above equations for L give the distance from the actual (blunt) point of the switch-rail to the *theoretical* point of the frog. The lead (L') given in Table III is the distance from the actual point of the switch-rail to the actual (blunt) point of the frog. The difference (L'—L) is the "frog bluntness," which in each case equals the width of the frog point ($\frac{1}{2}$ inch = .04166 foot) multiplied by the frog number. The values of the frog bluntness for the various frogs is given in the second column of Part B, Table III.

The value of $MN=H$ has been standardized by the A. R. E. A. as $6\frac{1}{4}$ inches for all lengths of switch-rail and for all values of α. The point of the switch-rail (at D) is invariably $\frac{1}{4}$-inch thick. When it is necessary to calculate MN for other standards of construction, it may be computed (calling S = length of switch-rail) to be

$$MN=S\sin\alpha+(\text{thickness of point of switch rail}).$$

The length to the blunt point of the frog ($W=FJ$) is given for each frog in the third column of Table III, Part B. The several values of F and α are also given in Table III. g is the gauge $=4$ feet $8\frac{1}{2}$ inches $=4.7083$ feet.

The solution of Eq. 77–80 for various frog angles will give a series of "theoretical leads," as given in Table III. Part B. The "closure rails," between the switch points and the frog, will invariably have such odd total lengths that there must be at least one rail cutting (and some wastage of rail) for each

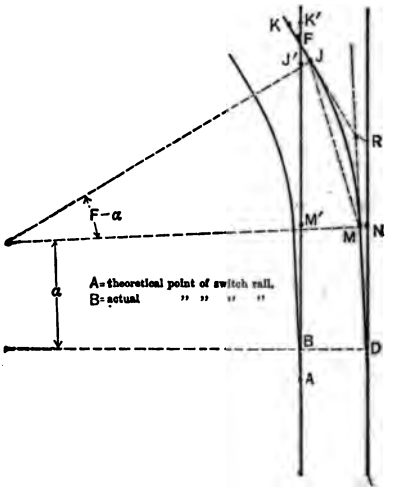


FIG. 143.

closure length. By shortening the radius of the connecting curve very slightly and inserting a very short length of tangent either between the curve and switch-rail at M, or between the curve and frog-rail at J, all of which will change very slightly the length of lead, the closure lengths can be made such that one rail cutting can be eliminated, and yet the combinations of curves and tangents are mathematically perfect. The detailed method of computing these combinations is tedious and will not be elaborated here, but a series of results developed by the A. R. E. A. is given under the heading of "practical leads" in Table III. Part C.

The above computations and tabular values assume that the two switch points (at B and D) are directly opposite. This would always mean that the straight rail (BF) is somewhat shorter than the curved rail from D to F. In the maximum case the difference is less than 4 inches. Therefore, assuming that rails are obtainable at even-foot lengths down to 27 feet, or 24 feet for a No. 4 frog switch, the system of practical leads never requires more than one rail cutting. But even this is sometimes avoided by using for the straight-rail closure the same number and lengths of uncut rails as are specified for the closure of the curved part. The chief effect of this is that the point of the switch-rail will be located a few inches below its normal position at B and that the gauge at the switch-point will be slightly widened when the switch is open. This effect is possibly an advantage rather than a disadvantage.

306. Design for a turnout from the OUTER side of a curved track. Fig. 144 is a diagram of what the construction would be

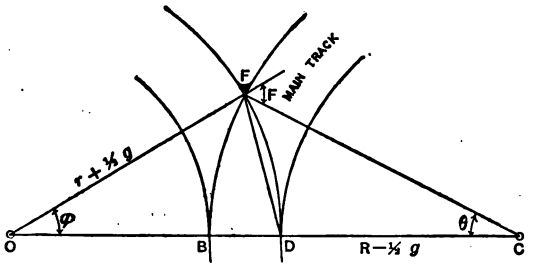


FIG. 144.

if the switch-rails were circular throughout. Before the invention of point switches and when stub switches were in universal use, the lead-rails were considered to be circular, both for straight and for curved main track. If Eqs. 70 and 75 and the corresponding Eqs. 77 to 80 are solved for any given frog, it is found that the lead, when using straight switch-rails and straight frog-rails, is considerably less than when using circular lead-rails throughout; also the curvature is considerably sharper. But stub-rail switches are obsolete and the mathematical solutions used for them cannot be utilized, even approximately, for point switches. If such a diagram as Fig. 144 is worked out in detail, as has been done in previous editions, it is found that

(*a*) the lead (*BF*) is almost identical with that computed from Eq. 70 or 74, when the main line is straight.

(*b*) the degree of curve (*d*) of the circular switch-rails would be *very nearly* equal to the degree of curve (*d'*) of the circular switch-rails for a straight track minus the degree of curve (*D*) of the main track; or, $d = d' - D$.

These statements are more exactly true when the degree of curvature of the main track is small. Even for a 10° curve on the main track the errors are not large. It has been found to be a needless refinement to compute the precise mathematical properties of the switch-rails from a curved main track, any more than as given by the two principles stated above. Therefore

(*a*) the length of the lead is assumed to be the same as that for a straight track, using the same frog, and

(*b*) the degree of curve of the switch-rails is found as stated above—in principle (*b*). As the curvature of the main track sharpens, the curvature of the switch-rails becomes less until they become straight. For still sharper main track, the center of curvature is on the same side. This is illustrated in Fig. 145, if we consider the sharper curved track to be the main track and the easier curve the switch. The above rule is still applicable, the algebraic sign of the result showing the location of the center.

307. **Design for a turnout from the INNER side of a curved track.** As in the previous section, Fig. 145 illustrates the dia-

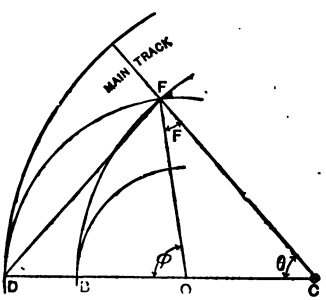


FIG. 145.

gram for circular lead rails. It may be shown that the degree of the turnout (*d*) is *nearly* the *sum* of the degree of the main

track (D) and the degree (d') of a turnout from a straight track when the frog angle is the same. The discrepancy in this case is somewhat greater than in the other, especially when the curvature of the main track is sharp. If the frog angle is also large, the curvature of the turnout is excessively sharp. If the frog angle is very small, the liability to derailment is great. Turnouts to the inside of a curved track should therefore be avoided, unless the curvature of the main track is small.

308. Connecting curve from a straight track. The "connecting curve" is the track lying between the frog and the side

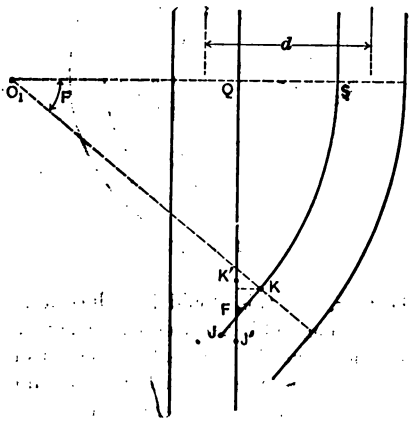


FIG. 146.

track where it becomes parallel to the main track (KS in Fig. 146 or 147). Call d the distance between track centers. The angle $KO_1S = F$ (see Fig. 146). Call r' the radius of the connecting curve. Then

$$(r' - \tfrac{1}{2}g) = \frac{d - g - K \sin f}{\text{vers } F}; \quad \text{(81)}$$

$$FQ = (r' - \tfrac{1}{2}g) \sin F + K \cos f \quad \text{(82)}$$

In these equations (and in several that follow) K is the distance from the theoretical point of the frog to the heel. The length, for each standard frog, is found in Table III, Part B.

309. Connecting curve from a curved track to the OUTSIDE. When the main track is curved, the required quantities are the radius of the connecting curve from K to S, Fig. 147, and its length or central angle.

The accuracy of all these computations on switches and frogs in curved main track is vitiated by the fact that the frog-rails are straight. The design might be mathematically more perfect if the main track curve were transformed into two curves on either side of the frog which had centers separated as far as the

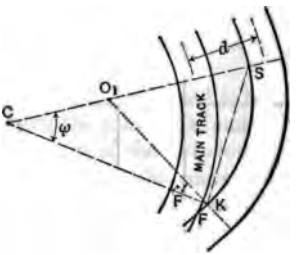


FIG. 147.

length of the frog, but this would introduce a very great and needless complication and is never done. The more simple solution is to consider that the frog-rail is a chord of the original curve, which (*a*) narrows the track gauge by an amount equal to the middle ordinate of that chord and which (*b*) is not tangent to the curve at either end. For all ordinary curvature neither of these theoretical defects is vitally objectionable or even appreciable. In Fig. 147 KC is practically perpendicular to one frog-rail and KO_1 is exactly perpendicular to the other frog-rail. Therefore, the angle CKO_1 equals the frog angle F. While the following calculations are amply precise for practical purposes, the discrepancy from strict mathematical accuracy should be noted and properly valued.

In the triangle CSK

$$CS+CK : CS-CK :: \tan \tfrac{1}{2}(CKS+CSK) : \tan \tfrac{1}{2}(CKS-CSK);$$

but $\frac{1}{2}(CKS+CSK)=90-\frac{1}{2}\psi$; and, since the triangle O_1SK is isosceles, $\frac{1}{2}(CKS-CSK)=\frac{1}{2}F$;

$$\therefore 2R+d+K\sin F : d-g-K\sin F :: \cot \tfrac{1}{2}\psi : \tan \tfrac{1}{2}F$$
$$:: \cot \tfrac{1}{2}F : \tan \tfrac{1}{2}\psi;$$

$$\therefore \tan \tfrac{1}{2}\psi = \frac{2n(d-g-K\sin F)}{2R+d+K\sin F}. \quad . \quad . \quad . \quad . \quad . \quad (83)$$

From the triangle CO_1K we may derive

$$r-\tfrac{1}{2}g : R+\tfrac{1}{2}g+K\sin F :: \sin\psi : \sin(F+\psi);$$

$$r-\tfrac{1}{2}g = (R+\tfrac{1}{2}g+K\sin F)\frac{\sin\psi}{\sin(F+\psi)}. \quad . \quad . \quad . \quad . \quad (84)$$

Also

$$KS = 2(r-\tfrac{1}{2}g)\sin\tfrac{1}{2}(F+\psi). \quad . \quad . \quad . \quad . \quad . \quad . \quad (85)$$

310. Connecting curve from a curved track to the INSIDE. As above, it may readily be deduced from the triangle CKS (see Fig. 148) that

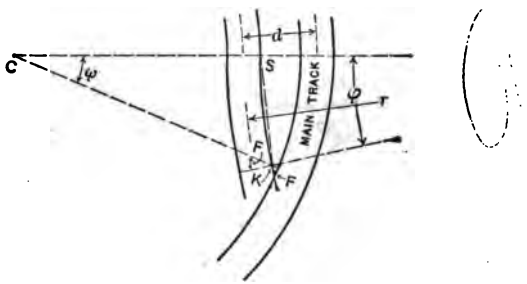


FIG. 148.

$$CK+CS : CK-CS :: \tan\tfrac{1}{2}(CSK+CKS) : \tan\tfrac{1}{2}(CSK-CKS);$$
$$(2R-d-K\sin F) : (d-g-K\sin F :: \cot\tfrac{1}{2}\psi : \tan\tfrac{1}{2}F;$$

$$\tan\tfrac{1}{2}\psi = \frac{2n(d-g-K\sin F)}{2R-d-K\sin F}. \quad . \quad . \quad . \quad . \quad (86)$$

From triangle CO_1K,

$$O_1K : CK :: \sin\psi : \sin(F-\psi);$$
$$(r-\tfrac{1}{2}g) : (R-\tfrac{1}{2}g-K\sin F) :: \sin\psi : \sin(F-\psi);$$

$$(r-\tfrac{1}{2}g) = (R-\tfrac{1}{2}g-K\sin F)\frac{\sin\psi}{\sin(F-\psi)}. \quad . \quad . \quad (87)$$

Also

$$KS = 2(r - \tfrac{1}{2}g) \sin \tfrac{1}{2}(F - \psi). \quad . \quad . \quad . \quad . \quad (88)$$

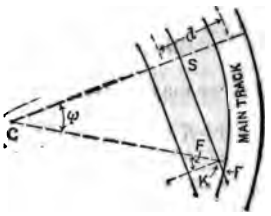


FIG. 149.

Two other cases are possible. (*a*) r may increase until it becomes infinite (see Fig. 149), then $F = \psi$. In such a case we may write, by substituting in Eq. 86,

$$2R - d - K \sin F = 4n^2(d - g - K \sin F). \quad . \quad . \quad (89)$$

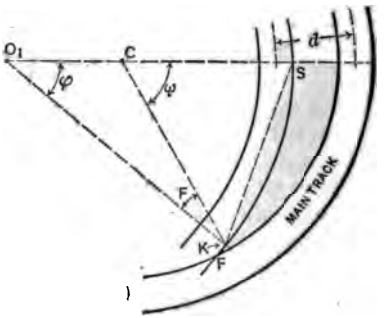


FIG. 150.

This equation shows the value of R which renders this case possible. (*b*) ψ may be greater than F. As before (see Fig. 150).

$$(2R - d - K \sin F) : (d - g - K \sin F) :: \cot \tfrac{1}{2}\psi : \tan \tfrac{1}{2}F;$$

$$\tan \tfrac{1}{2}\psi = \frac{2n(d - g - K \sin F)}{2R - d - K \sin F}$$

the same as Eq. 86, but

$$(r + \tfrac{1}{2}g = (R - \tfrac{1}{2}g - K \sin f) \frac{\sin \psi}{\sin (\psi - F)}. \quad . \quad . \quad (90)$$

Problem. To find the dimensions of a connecting curve running to the INSIDE of a curved main track; number 9 frog, 4° 30′ curve, $d = 13'$, $g = 4'\ 8\frac{1}{2}''$.

Solution.

[Eq. 86] $d = 13.000$ $K = 10'\ 0''$ $K \sin F = 1.108$ $\log 2n = 1.2552\bar{7}$

5.816 $g = 4.708$

7.184 5.816

$\log 7.184 = 0.8563\bar{6}$

$R = 1273.6$ $2R - d - K \sin F = 2533.1$

$2R = 2547.2$ $\log = 3.40365$

$(d + K \sin F) = 14.108$ $\text{co-log} = 6.59635$

$\text{co-log} = 6.59635$

$\log \tan \frac{1}{2}\psi = 8.70799$

$\frac{1}{2}\psi = 2°\ 55'\ 20''$

$\psi = 5°\ 50'\ 40''$

$F = 6°\ 21'\ 35''$

$F - \psi = 0°\ 30'\ 55''$

Since $F > \psi$, we must use Eq. 87, rather than Eq. 90.

$\frac{1}{2}g = 2.354$ $R - \frac{1}{2}g - K \sin F = 1270.1$ $\log = 3.1038\bar{4}$

$K \sin F = 1.108$ $(F - \psi) = 1855''$; $\log = 3.2683\bar{4}$ $\log \sin \psi = 9.0078\bar{7}$

sum $= 3.462$ 4.68557

$7.9539\bar{1}$

$\text{co-log} = 2.04608$ $\text{co-log} = 2.0460\bar{8}$

$r - \frac{1}{2}g = 14381.2$ $4.1577\bar{9}$

$r = 14383.5$

$d = 0°\ 24'$

[Eq. 88].

$\frac{1}{2}(F - \psi) = 927.5''$; $\log = 2.9673\bar{1}$ 2 $0.3010\bar{3}$

$4.6855\bar{7}$ $r - \frac{1}{2}g$ $4.1577\bar{9}$

$\sin \frac{1}{2}(F - \psi) = 7.6528\bar{9}$ $7.3528\bar{9}$

$KS = 129.33$ $2.1117\bar{1}$

311. Crossover between two parallel straight tracks. (See Fig. 151.) The turnouts are as usual. The cross-over track may be straight, or it may be a reversed curve. The reversed curve shortens the total length of track required, but is somewhat objectionable. The first method requires that both frogs must be equal. The second method permits unequal frogs, although equal frogs are preferable. The length of straight crossover track is F_1T.

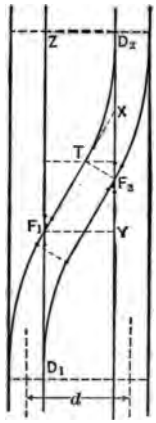


FIG. 151.

$$F_1T \sin F_1 + g \cos F_1 = d - g;$$

$$F_1T = \frac{d-g}{\sin F_1} - g \cot F_1. \quad . \quad . \quad (91)$$

The total distance along the track may be derived as follows:

$$DZ = D_1F_1 + D_2F_2 + F_2Y$$
$$= DF_1 + D_2F_2 + XY - XF_2;$$

$$XY = (d-g) \cot F_1;$$

$$XF_2 = g \div \sin F_2;$$

$$\therefore D_1Z = 2D_1F_1 + (d-g)\cot F_1 - \frac{g}{\sin F_2}. \quad (92)$$

312. Crossover between two parallel curved tracks. Using a straight connecting curve. This solution has limitations. If one frog (F_1) is chosen, F_2 must be determined, being a function of F_1. If F_1 is less than some limit, depending on the width (d) between the parallel tracks, this solution becomes impossible. In Fig. 152 assume F_1 as known. Then $K_1N = g \sec F_1$. In the triangle NOK_2 we have

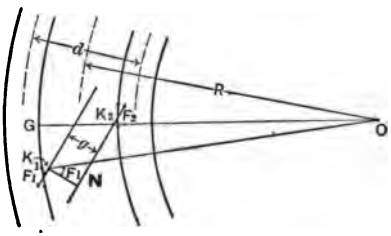


FIG. 152.

$$\sin NK_2O : \sin K_2NO :: NO : K_2O;$$
$$\sin K_2NO = \cos F_1; \quad NK_2O = 90° + F_2;$$
$$\therefore \sin NK_2O = \cos F_2.$$

$$NO = R + \tfrac{1}{2}d - \tfrac{1}{2}g - K_1 \sin F_1 - g \sec F_1; \quad K_2O = R - \tfrac{1}{2}d + \tfrac{1}{2}g + K_2 \sin F_2;$$

$$\therefore \cos F_2 = \cos F_1 \frac{R + \tfrac{1}{2}d - \tfrac{1}{2}g - K_1 \sin F_1 - g \sec F_1}{R - \tfrac{1}{2}d + \tfrac{1}{2}g + K_2 \sin F_2}. \quad . \quad (93)$$

The solution of this equation involves the frog angle F_2, which is the angle sought, but there is little error in considering in this solution that $K_2 \sin F_2$ is numerically equal to $K_1 \sin F_1$ and solving accordingly. If the computed value of F_2 is very different from F_1, it would be more precise to recompute Eq. 93 by substituting for $K_2 \sin F_2$ the more exact quantities obtainable from the first trial solution. The relative position of the frogs F_1 and F_2 may be determined as follows:

$$NO_2K = 180° - (90° - F_1) - (90° + F_2) = F_1 - F_2.$$

Then $$GF_1 = 2(R + \tfrac{1}{2}d - \tfrac{1}{2}g) \sin \tfrac{1}{2}(F_1 - F_2) + K_1 \cos F_1. \quad . \quad (94)$$

There is a theoretical, but practically inappreciable, inaccuracy in Eq. 94, since the chord GF_1 is really the sum of two chords of which one is the chord from the point G to the point where ON produced intersects the gauge line. After locating G, the point radially opposite, on the outer gauge line of the inner track, may be located, from which the frog-point F_2 is located at a distance of $K_2 \cos F_2$. Note that these frog-points referred to are the *theoretical* points. Due allowance must be made during location for the "frog bluntness."

In general, the value of F_2 computed from Eq. 93 is *not* the angle of any standard number-frog, and a strict compliance with theory would require that the frog should be made to order. This is needlessly expensive and the nearest size frog may generally be used without appreciable error.

Example. A crossover between parallel tracks on a 6° curve, the track spacing d being 13 feet. F_1 assumed a No. 9 frog.

[Eq. 93]

R = 955.37	$\frac{1}{2}g$ = 2.35	K_1 = 10 ft.	
$\frac{1}{2}d$ = 6.5	$K_1 \sin F_1$ = 1.11	$\sin F_1$ = .11077	
961.87	$g \sec F_1$ = 4.74		
− 8.20	8.20		
953.67			log = 2.97940
R = 955.37			
$\frac{1}{2}g$ = 2.35			
$K_2 \sin F_2$ = 1.11 (assumed = to $K_1 \sin F_1$)			
958.83			
$-\frac{1}{2}d$ = −6.5			
952.33			log = 2.97879
			0.00061
		log cos F_1 . .	9.99732
F_2 = 5° 35′ 30″		log cos 5° 35′ 30″	9.99793

This angle is within 8 minutes of the angle of a No. 10 frog, which could be used without appreciable error. The point K_2 would be shifted laterally .023 foot, or about ¼ inch, but there would be no visible irregularity in alinement.

$$NOK_2 = F_1 - F_2 = 6° 21' 35'' - 5° 35' 30'' = 0° 46'.$$

[Eq. 94]

$$R + \tfrac{1}{2}d = 961.87$$
$$-\tfrac{1}{2}g = -2.35$$
$$959.52 \quad \log = 2.98205$$
$$\sin \tfrac{1}{2}NOK_2 = \sin 0° 23' \quad \log = 7.82545$$
$$12.84 \quad \log = 1.10750$$
$$K_1 \cos F_1 = 9.94$$
$$GF_1 = 22.78$$

It is instructive to note that if the same crossover problem is worked out for a straight track, as in § 311, using No. 9 frogs on both tracks, the distance between frog points, measured parallel with the track, is nearly the same as in the above problem, especially when the distance 12.84, measured on the outer track, is reduced by bringing it in to the center line. This is analogous to the statement, previously made, that the lead of a switch on a curved track is nearly the same as that for a straight track.

It is theoretically possible to find two standard frog angles which may be so located that the connecting curve consists of straight lines and circular curves, which connect tangentially, making perfect alinement, but such methods are very complicated and the above method is sufficiently exact for practical purposes.

313. **Practical rules for switch-laying.** A consideration of the previous sections will show that the formulæ are comparatively simple when the lead-rails are assumed as circular; that they become complicated, even for turnouts from a straight main track, when the effect of straight frog and point rails is allowed for, and that they become hopelessly complicated when allowing for this effect on turnouts from a curved main track. It is also shown (§ 306) that the length of the lead is practically the same whether the main track is straight or is curved with such curves as are commonly used, and that the degree of curve of the lead-rails from a curved main track may be found with close approximation by mere addition or subtraction. From this it may be assumed that if the length of lead (L) and the

radius of the lead-rails (r) are computed from Eq. 77 and 80 for various frog angles, the same leads may be used for curved main track; also, that the degree of curve of the lead-rails may be found by addition or subtraction, as indicated in § 306, and that the approximations involved will not be of practical detriment. In accordance with this plan Table III has been computed from Eq. 77, 78 and 80. The *leads* there given may be used for all main tracks, straight or curved. The table gives the degree of curve of the lead-rails for *straight* main track; for a turnout to the *inside, add* the degree of curve of the main track; for a turnout to the *outside, subtract* it.

But there are complications resulting from practical and economical switch construction. A committee of the A. R. E. A., in 1921, adopted certain standards in details, which, when applied to Eqs. 77 to 80 give the values for switch dimensions as quoted in the second section of Table III. They adopted four lengths of switch-rails. In each case the " point " is always ¼" thick. The gauge line at the other end is always to be placed 6¼" from the gauge line of the main rail, and the planing is so done that when in this position the switch-rail lies against the main rail. Therefore the angle α is always an angle whose sine equals 6 inches (or 0.5 foot) divided by the length of the switch-rail in feet. In Fig. 153, the point D is not on the gauge line of the main rail but at a point ¼" away from it, and the point M 6¼" away from it. The straight rail BF consists of a point-rail at one end, the "closure rails," and one of the toe rails of the frog at the other end. The closure rails will in general consist of one rail cut to a computed length and one or more rails from 24 to 33 feet long, the lengths being in even feet. The curved rail DF will also consist of a point-rail, a frog toe-rail, and one or more lengths of closure rail, but the closure rails in this case are slightly longer than those for the straight rail. Since it is always practically easier to measure to the " actual point " of a frog (see Fig. 134), rather

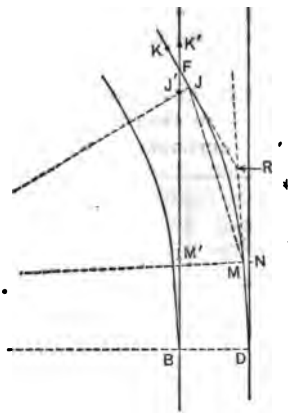


FIG. 153.

than to the theoretical point, Table III gives the distance L', which is the distance $L = BF$, plus the "frog bluntness," which is found by multiplying $\frac{1}{2}''$ (=0.0417 foot) by the frog number.

The curvature for a curved switch-rail (for a straight track) is most readily determined by measuring off a series of ordinates whose origin is at the switch-point D, Fig. 153, the points being the center and the quarter points of the actual curve. More accurately, the origin is on the gauge line of the main rail, opposite D, which is $\frac{1}{4}''$ from the gauge line. These ordinates, as computed on the basis of "practical leads," by the A. R. E. A. committee, are quoted below. It should be remembered that the system of practical leads usually involves a very short tangent adjacent to either M or J, and that the line MJ for "practical leads" is not entirely an arc.

TABLE XXV. — RECTANGULAR COORDINATES TO THE QUARTER AND CENTER POINTS ON THE GAUGE SIDE OF CURVED RAIL, REFERRED TO POINT OF SWITCH-RAIL AS ORIGIN.

Frog No.	Measured along main rail.			Measured perpendicular to main rail.		
	X	X_1	X_2	Y	Y_1	Y_2
5	17.92	24.83	31.75	0.97	1.69	2.69
6	19.19	27.37	35.56	1.03	1.79	2.83
7	26.71	36.92	47.12	0.98	1.72	2.76
8	28.10	39.71	51.31	1.005	1.77	2.80
9	28.75	40.98	53.19	1.02	1.76	2.75
10	30.28	44.05	57.81	1.04	1.79	2.78
11	40.74	56.47	72.19	1.08	1.84	2.87
12	43.99	60.65	77.28	1.15	1.90	2.91
14	41.10	60.21	79.31	1.08	1.87	2.91
15	52.00	74.00	96.00	1.03	1.81	2.86
16	53.23	76.46	99.69	1.04	1.83	2.89
18	54.73	79.46	104.19	1.06	1.86	2.91
20	57.75	85.50	113.25	1.10	1.91	2.95

If the position of the switch-block is definitely determined, then the rails must be cut accordingly; but when some freedom is allowable (which never need exceed 16.5 feet and may require but a few inches), one rail-cutting may be avoided. Mark on the rails at B, F, and D; measure off the length DN and locate the point M at the distance $6\frac{1}{4}''$ from N. If the frog must be placed during the brief period between the running times of

trains, it will be easier to joint up to the heel of the frog (the point K', Fig. 153), a piece of rail, the farther end of which will just reach the next joint and also joint up to the toe of the frog the straight closure rail and the point-rail. Then, when all is ready, the rails are loosened from the ties back to B, the joint beyond the frog is removed and the whole rail back to B is swung outward. The new combination is shoved into place and spiked, even the point-rail being temporarily spiked to hold it in place as a main track rail, until the other switch-rail and the tie rods can be placed. When the frog is thus in place, the point J becomes located. The curved closure rails, as called for in Table III, should prove to be just long enough, when properly curved, to fill in the gap between M and J. Using the proper pairs of values for X and Y as given above, the three values of X may be measured on the main track rail from the point D, and the corresponding offsets will give points on the curved switch-rail. The old main track rail which was bent outward from B may be utilized as the other switch-rail and set to gauge from the rail just located.

Example.—Given a main track on a 4° curve—a turnout to the outside, using a No. 9 frog; gauge 4′ 8½″; $W = 6'.00$; $H = 6\frac{1}{4}''$; $S = 16'\ 6''$ and $a = 1°\ 44'\ 11''$ Then for a *straight* track r would equal 605.18 [$d = 9°\ 28'\ 42''$]. For this curved track d will be nearly $9°\ 29' - 4° = 5°\ 29'$, or r will be 1045.3. L' for a *straight* track would be 72.28, and is here considered to be the same. The closure rails have a total arc length of 49.59, and will here be taken the same. Note that the curved and straight closure rails each have odd lengths which are made by one cut of a 33-foot rail. This avoids all rail waste and also one rail-cutting and the boring of holes.

314. Slips. Track movements in crowded yards are facilitated by using "slips" (see Fig. 154), which may be "single" or "double." The crossing of two rails is done either by operating two movable rails or by using fixed "frogs," but a comparison of the continuity of the running rails, using ordinary frogs (see Fig. 134) and these frogs, will show their radical difference. These slips can be used for frog angles from No. 6 to No. 15. The levers are so connected that the several operations necessary to set the rails for any desired train movement are accomplished by one motion.

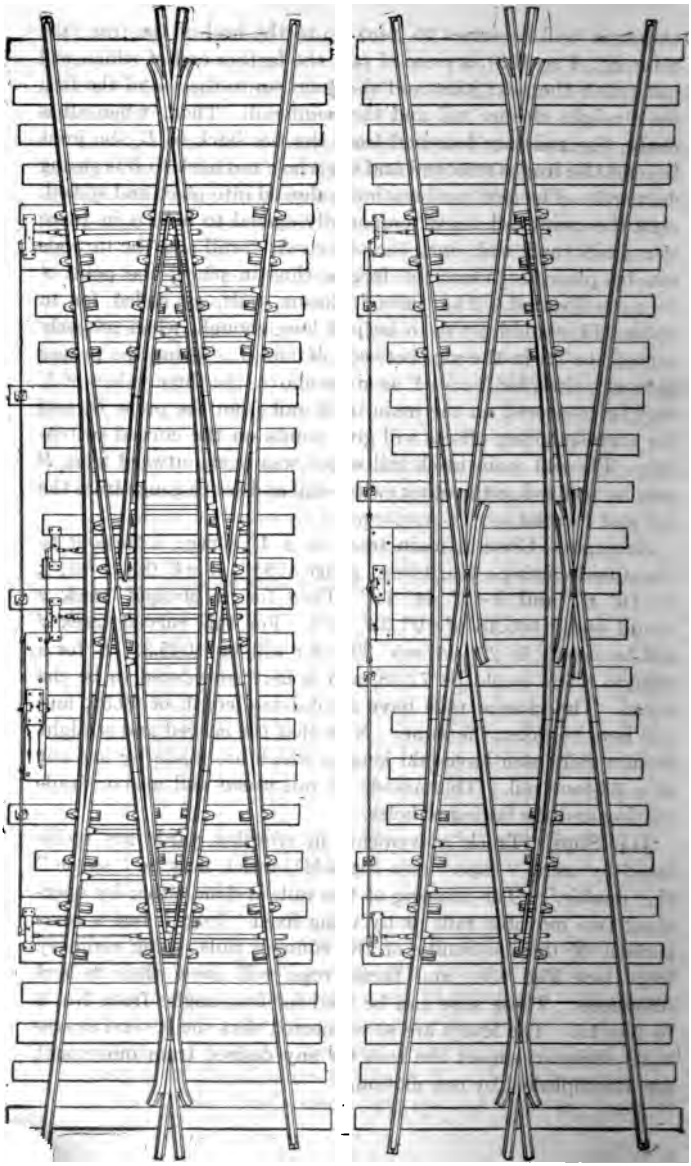


FIG. 154.—SINGLE AND DOUBLE SLIPS.

CROSSINGS.

315. **Two straight tracks.** When two straight tracks cross each other, four frogs are necessary, the angles of two of them being supplementary to the angles of the other. Since such crossings are sometimes operated at high speeds, they should be

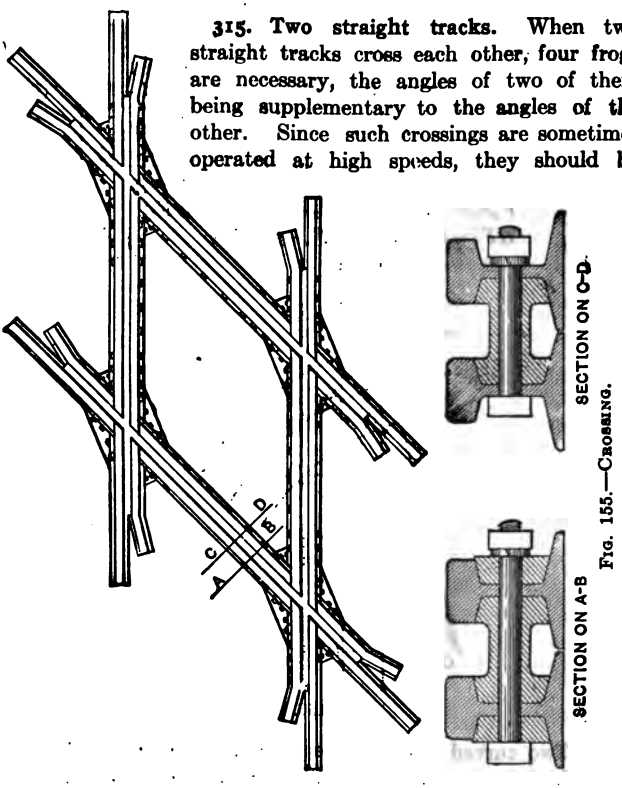


FIG. 155.—CROSSING.

very strongly constructed, and the angles should preferably be 90° or as near that as possible. The frogs will not in general be "stock" frogs of an even number, especially if the angles are large, but must be made to order with the required angles as measured. In Fig. 155 are shown the details of such a crossing. Note the fillers, bolts, and guard-rails.

316. **One straight and one curved track.** Structurally the crossing is about the same as above, but the frog angles are all unequal. In Fig. 156, R is known, and the angle M, made by

the center lines of the tracks at their point of intersection, is also known. $M = NCM$. $NC = R \cos M$.

$$(R-\tfrac{1}{2}g)\cos F_1 = NC+\tfrac{1}{2}g; \quad \therefore \cos F_1 = \frac{R\cos M+\tfrac{1}{2}g}{R-\tfrac{1}{2}g}.$$

Similarly
$$\cos F_2 = \frac{R\cos M+\tfrac{1}{2}g}{R+\tfrac{1}{2}g}, \quad \cos F_3 = \frac{R\cos M-\tfrac{1}{2}g}{R+\tfrac{1}{2}g}, \quad \cos F_4 = \frac{R\cos M-\tfrac{1}{2}g}{R-\tfrac{1}{2}g}. \qquad (95)$$

$$F_3F_4 = (R+\tfrac{1}{2}g)\sin F_3 - (R-\tfrac{1}{2}g)\sin F_4;$$
$$HF_4 = (R-\tfrac{1}{2}g)(\sin F_4 - \sin F_1). \qquad (96)$$

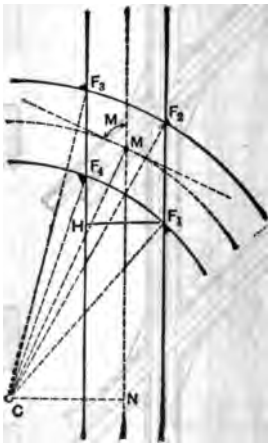


FIG. 156.

317. **Two curved tracks.** The four frogs are unequal, and the angle of each must be computed. The radii R_1 and R_2 are known; also the angle M. r_1, r_2, r_3 and r_4 are therefore known by adding or subtracting $\frac{1}{2}g$, but the lines are so indicated for brevity. Call the angle $MC_1C_2 = C_1$, the angle $MC_2C_1 = C_2$, and the line $C_1C_2 = c$. Then

$$\tfrac{1}{2}(C_1+C_2) = 90° - \tfrac{1}{2}M$$

and
$$\tan \tfrac{1}{2}(C_1-C_2) = \cot \tfrac{1}{2}M \frac{R_2-R_1}{R_2+R_1} \quad . \quad . \quad . \quad . \quad (97)$$

C_1 and C_2 then become known and

$$c = C_1C_2 = R_2\frac{\sin M}{\sin C_1} \quad . \quad . \quad . \quad . \quad . \quad (98)$$

In the triangle $F_1C_1C_2$, call $\frac{1}{2}(c+r_1+r_4)=s_1$; $s_2=\frac{1}{2}(c+r_2+r_4)$;

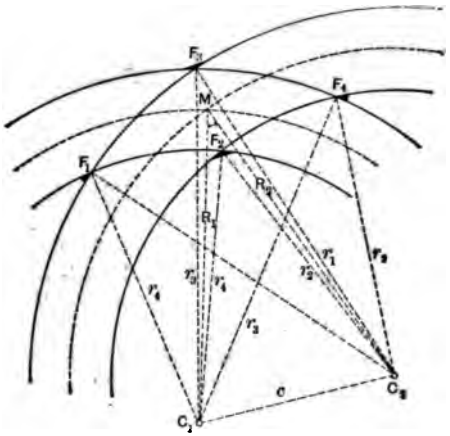


FIG. 157.

$s_3=\frac{1}{2}(c+r_1+r_3)$; and $s_4=\frac{1}{2}(c+r_2+r_3)$. Then, by formula 29, Table XIV,

$$\left.\begin{aligned} &\text{vers } F_1=\frac{2(s_1-r_1)(s_1-r_4)}{r_1r_4}.\\ \text{Similarly}\quad &\text{vers } F_2=\frac{2(s_2-r_2)(s_2-r_4)}{r_2r_4},\\ &\text{vers } F_3=\frac{2(s_3-r_1)(s_3-r_3)}{r_1r_3},\\ &\text{vers } F_4=\frac{2(s_4-r_2)(s_4-r_3)}{r_2r_3}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (99)$$

$$\sin C_1C_2F_4=\sin F_4\frac{r_3}{c};$$

$$\sin C_1C_2F_2=\sin F_2\frac{r_4}{c};$$

$$\therefore\ F_2C_2F_4=C_1C_2F_4-C_1C_2F_2, \quad . \quad . \quad . \quad . \quad . \quad (100)$$

$$\sin F_1C_1C_2=\sin F_1\frac{r_1}{c};$$

$$\sin F_2C_1C_2=\sin F_2\frac{r_2}{c},$$

$$\therefore\ F_1C_1F_2=F_1C_1C_2-F_2C_1C_2; \quad . \quad . \quad . \quad . \quad (101)$$

from which the chords F_1F_2 and F_2F_4 are readily computed.

F_1F_3 and F_2F_4 are nearly equal. When the tracks are straight and the gauges equal, the quadrilateral is equilateral.

Problem. Required the frog angles and dimensions for a crossing of two curves ($D_1=4°$; $D_2=3°$) when the angle of their tangents at the point of intersection $=62°\ 28'$ (the angle M in Fig. 157).

Solution

$R_1=1432.7$; $R_2=1910.1$;

$r_1=R_2+\frac{1}{2}g=1910.1+2.35=1912.45$;

$r_2=R_2-\frac{1}{2}g=1910.1-2.35=1907.75$;

$r_3=R_1+\frac{1}{2}g=1432.7+2.35=1435.05$;

$r_4=R_1-\frac{1}{2}g=1432.7-2.35=1430.35$.

Eq. 97.

$\log\cot\frac{1}{2}M=0.21723$

$R_2-R_1=477.4$; $\log=2.67888$

$R_2+R_1=3342.8$; $\log=3.52411$; co-log $=\overline{6.47589}$

$\frac{1}{2}(C_1-C_2)=13°\ 15'\ 07''$; $\tan 13°\ 15'\ 07''=9.37200$

$\frac{1}{2}(C_1+C_2)=58°\ 46'$ $\quad[\frac{1}{2}(C_1+C_2)=90°-\frac{1}{2}M]$

$C_1=72°\ 01'\ 07''$

$C_2=45°\ 30'\ 53''$

Eq. 98.

$\log R_2=3.28105$

$\log\sin M=9.94779$

$\log\sin C_1=9.97825$; co-log $=\overline{0.02175}$

$c=C_1C_2=1780.7$; $\log C_1C_2=3.25059$

Eq. 99.

$c=1780.7$	$c=1780.7$	$c=1780.7$	$c=1780.7$
$r_1=1912.45$	$r_2=1907.75$	$r_1=1912.45$	$r_3=1907.75$
$r_4=1430.35$	$r_4=1430.35$	$r_3=1435.05$	$r_3=1435.05$
$2\vert 5123.50$	$2\vert 5118.80$	$2\vert 5128.20$	$2\vert 5123.50$
$s_1=2561.75$	$s_2=2559.40$	$s_3=2564.10$	$s_4=2561.75$
$s_1-r_1=649.30$	$s_2-r_2=651.65$	$s_3-r_1=651.65$	$s_4-r_3=654.00$
$s_1-r_4=1131.40$	$-r_4=1129.05$	$s_3-r_3=1129.05$	$s_4-r_3=1126.70$

$\log 2=0.30103$

(s_1-r_1); $\log 649.30=2.81244$

(s_1-r_4); $\log 1131.40=3.05361$

$r_1=1912.45$; $\log=3.28159$; co-log $=6.71841$

$r_4=1430.35$; $\log=3.15544$; co-log $=6.84456$

$F_1=62°\ 25'\ 31''$; $\log\text{vers}\ 62°\ 25'\ 31''=9.73006$

$\log 2=0.30103$

(s_2-r_2); $\log 651.65=2.81401$

(s_2-r_4); $\log 1129.05=3.05271$

$r_2=1907.75$; $\log=3.28052$; co-log $=6.71948$

$r_4=1430.35$; $\log=3.15544$; co-log $=6.84456$

$F_2=62°\ 33'\ 55''$; $\log\text{vers}\ 62°\ 33'\ 55''=9.73180$

$\log 2 = 0.30103$

$(s_2 - r_1)$; $\log 651.65 = 2.81401$

$(s_3 - r_3)$; $\log 1129.05 = 3.05271$

$r_1 = 1912.45$; $\log = 3.28159$; co-log $= 6.71841$

$r_3 = 1435.05$; $\log = 3.15686$; co-log $= 6.84313$

$F_3 = 62° 21' 57''$; log vers $62° 21' 57'' = 9.72930$

$\log 2 = 0.30103$

$(s_4 - r_2)$; $\log 654.00 = 2.81558$

$(s_4 - r_3)$; $\log 1126.70 = 3.05181$

$r_2 = 1907.75$; $\log = 3.28052$; co-log $= 6.71948$

$r_3 = 1435.05$; $\log = 3.15686$; co-log $= 6.84313$

$F_4 = 62° 30' 14''$; log vers $62° 30' 14'' = 9.73103$

As a check, the *mean* of the frog angles $= 62° 27' 54'$, which is within $6''$ of the value of M.

Eq. 100.

$\log \sin F_4 = 9.94794$

$\log r_3 = 3.15686$

$\log c = 3.25059$; co-log $c = 6.74940$

$C_1C_2F_4 = 45° 37' 51''$; $\sin C_1C_2F_4 = 9.85421$

$\log \sin F_2 = 9.94818$

$\log r_4 = 3.15544$

co-log $c = 6.74940$

$C_1C_2F_2 = 45° 28' 17''$; $\sin C_1C_2F_2 = 9.85303$

$F_2C_2F_4 = 45° 37' 51'' - 45° 28' 17'' = 0° 09' 34''$.

$\log 2 = 0.30103$

$\log r_2 = 3.28052$

$\frac{1}{2}(0° 09' 34'') = 0° 04' 47''$; $\log \sin = \begin{pmatrix} 4.68557 \\ 2.45788 \end{pmatrix}$

$F_2F_4 = 5.309$; $\log F_2F_4 = 0.72500$

Eq. 101.

$\sin F_1 = 9.94763$

$\log r_1 = 3.28159$

co-log $c = 6.74940$

$F_1C_1C_2 = 72° 10' 22''$; $\sin F_1C_1C_2 = 9.97863$

$\sin F_2 = 9.94818$

$\log r_2 = 3.28052$

co-log $c = 6.74940$

$F_2C_1C_2 = 71° 57' 38''$; $\sin F_2C_1C_2 = 9.97811$

$F_1C_1F_2 = 72° 10' 22'' - 71° 57' 38'' = 0° 12' 44''$.

$\log 2 = 0.30103$

$\log r_4 = 3.15544$

$\frac{1}{2}(0° 12' 44'') = 0° 06' 22''$; $\log \sin = \begin{pmatrix} 4.68557 \\ 2.58206 \end{pmatrix}$

$F_1F_2 = 5.298$; $\log F_1F_2 = 0.72411$

As a check, F_2F_4 and F_1F_2 are very nearly equal, as they should be.

The foregoing problems on switches, connecting curves and crossings cover only a few of the most common of the problems encountered by the engineer. For the solution of a far wider range of problems, the engineer is referred to "Track Formulæ and Tables," by S. S. Roberts. [Wiley & Sons.]

CHAPTER XII.

MISCELLANEOUS STRUCTURES AND BUILDINGS.

WATER-STATIONS AND WATER-SUPPLY.

318. Location. The water-tank on the tender of a locomotive has a capacity of from 3000 to 10000 gallons—sometimes less, rarely very much more. The consumption of water is very variable, and will correspond very closely with the work done by the engine. On a long down grade it is very small; on a ruling grade, going up, using full stroke, an engine with 28-in. cylinders, 30-in. stroke, 180 lbs. boiler pressure, will use 4.59 lbs. of steam, or water, per stroke or 18.36 pounds per revolution. With 63-in. drivers, the circumference is 16.5 feet and there will be 320 revolutions per mile. The engine will use 5875 lbs. or 700 gallons of water per mile. This engine has a tank capacity of 9000 gallons, which would permit running about 12 miles at full stroke. But it is very rare that a locomotive must work for such long distances at full stroke. After starting and attaining full normal speed, the valves may be set to cut off at one-fourth stroke, or even at one-fifth or one-sixth for high speed running. With ordinary grades, such an engine might average 200 gallons per mile, in both directions. A quoted numerical case is that of a 106-ton engine using 7,500,000 gallons during an annual mileage of 45000 miles. This means an average of 167 gallons per mile. Observations were taken in 1910, on the N. Y. Central R.R., where the grades are moderate, showing that the heavy passenger trains of eight to twelve cars consumed 80 to 100 gallons of water per mile and that freight trains of about fifty loaded cars consumed from 110 to 130 gallons per mile. These figures are far less than those given above, but the grades on the N. Y. Central are very light.

Freight engines, running at lower speeds and longer cut-off, require more frequent water-tanks than passenger engines. Even before a road is built, the water-tank requirements and the minimum spacing may be computed on the basis of the steam consumption (see § 454), of the locomotives with which it is expected to handle the estimated traffic of the road. Usually tanks will be located at intervals of 10 to 20 miles.

In the early history of some of the Pacific railroads it was necessary to attach one or more tank-cars to each train in order to maintain the supply for the engine over stretches of 100 miles and over where there was no water. Since then water-stations have been obtained at great expense by boring artesian wells. The individual locations depend largely on the facility with which a sufficient supply of suitable water may be obtained. Streams intersecting the railroad are sometimes utilized, but if such a stream passes through a limestone region the water is apt to be too hard for use in the boilers. More frequently wells are dug or bored. When the local supply at some determined point is unsuitable, and yet it is necessary to locate a water-station there, it may be found justifiable to pipe the water several miles. The construction of municipal water-works at suitable places along the line has led to the frequent utilization of such supplies. In such cases the railroad is frequently the largest single consumer and obtains the most favorable rates. When possible, water-stations are located at regular stopping points and at division termini.

319. **Required qualities of water.** Chemically pure water is unknown except as a laboratory product. The water supplied by wells, springs, etc., is always more or less charged with calcium and magnesium carbonates and sulphates, as well as other impurities. The evaporation of water in a boiler precipitates these impurities to the lower surface of the boiler, where they sometimes become incrusted and are difficult to remove. The protection of the iron or steel of a boiler from the fierce heat of the fire depends on the presence of water on the other side of the surface, which will absorb the heat and prevent the metal from assuming an excessively high temperature. If the water side of the metal becomes covered or incrusted with a deposit of chemicals, the conduction of heat to the water is much less free, the metal will become more heated and its deterioration or destruction will be much more rapid. An especially common effect is the production of leaks around the joints between tubes and tube-sheets and the joints in the boiler-plates. Such injury can only be prevented by the application of one (or more) of three general methods—(*a*) the mechanical cleaning of the boilers, (*b*) the chemical purification of the water before its introduction into the boiler, and (*c*) the use of some "boiler compound" which is introduced directly into the boiler and which

causes precipitation of the harmful ingredients as non-incrusting solids which can be readily blown out.

320. **Mechanical cleaning,** as a sole dependence is impracticable except in the comparatively rare localities where the water is so "soft" that no incrusting deposits will be made and such precipitation as does take place is of such a character that it is removable by blowing out the boiler. There are many railroads, especially the smaller ones, which do not give any chemical treatment to any of their engine water-supply, and yet which are not fortunate enough to obtain even approximately soft water. The only method by which such roads can prevent a great waste of heat and the rapid deterioration of boiler tubes and sheets is by frequent mechanical cleaning.

321. **Chemical purification** before the water enters the boiler has the advantage of removing the troublesome ingredients, leaving nothing further to be done except the occasional removal, by blowing out, of the suspended matter or harmless matter precipitated by boiling. Sodium carbonate is the most common reagent. It is commercially sold as "soda crystals, sal soda, washing soda, Scotch soda, concentrated crystal soda, sesquicarbonate of soda, crystal carbonate of soda, black ash, soda ash and pure alkali." Although often chemically impure, it can now readily be obtained with a purity of 97 to 99%. The chemicals which are most common as incrustants are calcium and magnesium carbonates and sulphates. The effect of sodium carbonate on calcium sulphate is to produce soluble sodium sulphate—which is non-incrustant—and calcium carbonate, which precipitates into a sludge at the bottom of the water softener tank. The action on magnesium sulphate is similar. When this is done in a purifying tank, the purified water is drawn off from the top of the tank and supplied pure to the engines. The precipitants are drawn off from the settling-basin at the bottom of the tank. This purification, which makes no pretense of being chemically perfect, may be accomplished for a few cents per 1000 gallons. There are manufacturers which make a specialty of machinery, working more or less automatically, which introduces into the raw water a measured amount of chemical which, by analysis, has been calculated to be necessary with that particular quality of water. In spite of the automatic features, such machinery needs constant attention, and the water, both raw and treated, needs frequent analysis to

insure efficiency, since the character of the raw water may change.

Sodium hydrate, or "caustic soda," has the same general chemical effect as sodium carbonate, and acts more quickly and powerfully, but its caustic nature makes it somewhat objectionable to handle. Common lime, barium hydrate, and many other chemicals are also more or less used.

In the following tabular form is given the quantities of reagents required per unit of scaling or corroding substance held in solution, the table being copied from the 1915 Manual of the Amer. Rwy. Eng. Assoc. "Where the commercial product is not chemically pure, the proportion of reagents should be increased to correspond with an equivalent quantity of pure reagent. Given the analysis of a water, the pounds of incrusting or corrosive matter held in solution per 1000 gallons can be obtained by dividing the grains per gallon of each substance by seven, or the parts per 100,000 by twelve. In order to ascertain the full amount of lime necessary, the amount of free carbonic acid contained in the water should be determined, as well as the solids contained in solution, since this free acid must be eliminated in

TABLE XXVI. QUANTITY OF PURE REAGENTS REQUIRED TO REMOVE ONE POUND OF INCRUSTING OR CORROSIVE MATTER FROM THE WATER.

Incrusting or corrosive substance held in solution.	Amount of reagent (pure).	Foaming matter increased
Sulphuric acid	0.57-lb. lime plus 1.08 lbs. soda ash	1.45 lbs.
Free carbonic acid	1.27 lbs. lime	None
Calcium carbonate	0.56-lb. lime	None
Calcium sulphate	0.78-lb. soda ash	1.04 lbs.
Calcium chloride	0.96-lb. soda ash	1.05 "
Calcium nitrate	0.65-lb. soda ash	1.04 "
Magnesium carbonate	1.33 lbs. lime	None
Magnesium sulphate	0.47-lb. lime plus 0.88 lb. soda ash	1.18 lbs.
Magnesium chloride	0.59-lb. lime plus 1.11 lbs. soda ash	1.22 "
Magnesium nitrate	0.38-lb. lime plus 0.72-lb. soda ash	1.15 "
Calcium carbonate	3.15 lbs. barium hydrate	None
Magnesium carbonate	3.76 lbs. barium hydrate	None
Magnesium sulphate	2.62 lbs. barium hydrate	None
Calcium sulphate *	2.32 lbs. barium sulphate	None

* In precipitating the calcium sulphate, there would also be precipitated 0.74 lb. of calcium carbonate or 0.31 lb. of magnesium carbonate, the 2.32 lbs. of barium hydrate performing the work of 0.41 lb. of lime and 0.78 lb. of soda ash, or for reacting on either magnesium or calcium sulphate, 1 lb. of barium hydrate performs the work of 0.18 lb. of lime plus 0.34 lb. of soda ash, and the lime treatment can be correspondingly reduced.

order to obtain efficient treatment of water and reduce scaling matter to the minimum."

322. Foaming and priming. This phenomenon is the foaming or frothing of the water for a considerable height above its normal level in the boiler. The rapid flow of steam into the steam pipe in the dome mechanically carries some of this froth into the steam pipe and causes water to accumulate in the steam pipe and also in the cylinders, with considerable resulting loss in efficiency. Foaming in treated water is largely due to the presence of sodium salts as a result of treatment for incrusting sulphates, and this constitutes one of the objections to the use of soda in treating water. The presence of suspended matter in the water aggravates and even causes foaming. The constant withdrawal of the water from the boiler leaves these suspended solids in the boiler and they keep accumulating until the concentrations reach a critical point, which is about 100 grains per gallon. Beyond this point foaming will be experienced unless the water is changed, which is done by a systematic blowing-off and an occasional complete blowing-down and washing. But blowing-off involves the wastage of water which has been heated to boiler temperature and which has, perhaps, been chemically treated. Even the raw water costs something, perhaps several cents per 1000 gallons. The blowing-off required to keep the concentration below the proper limit may be so excessive that some anti-foaming agent may be necessary. The required effect is physical rather than chemical, the object being to reduce the surface tension, which is done chiefly by the use of oils, petroleum and castor oil being used. Tannic acids are also used for such a purpose.

323. Boiler compounds. Chemical treatment at special plants along the road is unquestionably the most efficient method, but it is costly. The use of boiler compounds, often patented, obviates the erection of any plant, but, since the water at each water-supply station has its own characteristics and it is impracticable to vary the chemicals used at each supply-station according to the character of the water, the treatment is very imperfect. Minute instructions to enginemen to introduce definite amounts of chemical at each water-station have proved unsatisfactory and impractical. Sometimes the chemical is mixed with enough water to partially suspend it and then it is thrown into the tender tank, this method having the advantage that a considerable part of the precipitation takes place promptly and the sludge

never enters the boiler. Sometimes a siphon attached to the feed-pipe outside of the injector, or, perhaps, a special injector, leads from a reservoir in which the chemical, suspended in water, has been placed. Sometimes a stick or "brick" of the chemical is placed directly in the boiler, through a hand-hole, during one of its periodical cleanings. In spite of the inefficiency of the method, 70% of replies to a circular inquiry reported the use of some kind of boiler compound. The chemicals used, some of which are patented compounds, are in general the same as those used in the outside chemical plants. Sodium carbonate is the most common constituent.

324. **Tanks. Height above rail.** Whatever the source, the water must be led or pumped into tanks which are supported on columns so that the bottoms of the tanks are high enough above the track to force a flow of 2500 gallons of water per minute through a 12-inch spout. The frictional resistance in the pipes, elbows, valves, etc., are such that, allowing that the spout is 12 feet above the rail, the bottom of the tank should be about 16 feet above the rail. If the water flows from the tank into a "stand-pipe," see § 327, there is additional frictional resistance, to allow for which the height of the support or "tower" is increased to perhaps 30 feet. The standard heights for towers are 16, 20 and 30 feet. **Sub-structure.** A standard plan, recommended by the Water Service committee of the A. R. E. A., is to support such tanks on twelve 12″×12″ posts, arranged in a double cross, four posts in each line, each post resting on a concrete footing. The posts are suitably cross-braced as in trestle work, and are surmounted by cast iron caps. These support 12″×14″ timber caps, which carry 4″×14″ joists, spaced 14″, which are immediately under the bottom of the tank. **Size.** Two sizes of tanks are standard. The "16×24" has a net height inside of 15′ 4″ and a net inside diameter of 24′ 0″. Although the capacity, brimming full, would be nearly 52,000 gallons, it is called a "50,000-gallon" tank since the outlet pipe must be several inches below the top. The "20×30" tank has a net inside diameter of 30′ 0″ and net height of 19′ 4″. It will contain 100,000 gallons when the water depth is slightly less than 19 feet. Since it is found that the 100,000-gallon tank costs but 10% more than a 75,000-gallon tank, the committee recommended that the 50,000-gallon and the 100,000-gallon tanks should be considered the two standard sizes.

Details. Cylindrical tanks are recommended, rather than tapered. The staves are machine-dressed so that the edges have the proper bevel toward the tank axis, and the outside is dressed to the proper convex cylindrical surface so that the hoops have a bearing for the full width of the stave. The "croze," $2\frac{5}{8}''$ wide and $\frac{5}{8}''$ deep, into which the bottom planks, 3" thick, slightly beveled at the ends, are inserted for a tight joint, is 4" above the bottom of the staves. When the jointing edges are properly made, the tank will be water-tight without any plugging or caulking, which should not be permitted. The weight of the tank should be transmitted through the bottom planks and in no case by means of the staves. Round hoop-rods, rather than elliptical or flat, are recommended. They should be made of refined double-rolled wrought iron. Each hoop should have three sections for 16×24 tanks and four sections for 20×30 tanks. On the basis of a maximum working stress of 12,500 pounds per square inch on the area at the base of the screw threads, the safe working load in pounds is as follows:

$\frac{3}{4}''$, 3750; $\frac{7}{8}''$, 5250; $1''$, 6875; $1\frac{1}{8}''$, 8625.

The spacing of hoops may be computed from the formula:

$$\text{Spacing in inches} = \frac{\text{safe load for the given hoop in pounds}}{2.6 \text{ diameter (ft.)} \times \text{depth in feet}}.$$

In the above formula, "depth" means the distance from top of stave to location of hoop. One hoop should be placed within two inches of the top and two hoops around the bottom opposite the croze. One of these is assumed to take up the bursting pressure due to the swelling of the bottom planks when water soaked, and that it does not withstand water pressure. The spacing should never exceed 21 inches. Hoop "lugs," made of cast or malleable iron, are used to connect the sections of the hoops. Each end of each rod should be threaded for $4\frac{1}{2}''$ and be provided with two hexagon nuts.

325. Pumping. (*a*) **Steam-pumps.** When coal is very cheap or "when 100 lbs. of coal in the pumphouse is cheaper than one gallon of fuel oil in the storage tank," and especially when steam can be procured from the railroad repair-shop plant, direct-acting steam pumps may be preferable and more economical, but they always require skilled attendance. (*b*) **Gasoline-engines.** These have been so highly developed in recent years that they are very efficient and are nearly "fool-proof," so that they may be oper-

The amount scooped up is practically proportional to the depth of immersion when that depth is over 2¾ inches. **Heating.** The water must be heated in winter to prevent freezing. There are two general methods: (*a*) Live steam is forced into the tank through nozzles about 40 feet apart; (*b*) a "circulatory system" by which steam is forced into a water main which feeds the tank in such a way that the water is in constant circulation through the main, into the tank and then back again into the main to be reheated. For the climatic conditions of the N. Y. Central R. R. a steam capacity of 100 H. P. is considered essentail to heat 7000 sq. ft. of tank surface, which means about 4400 lineal feet of 19-inch tank, or two good-length tanks on a double track. On account of the great amount of water splashed over the track and its scouring action on any ordinary ballast, a large item in the cost of an installation is the reconstruction of the track. The certainty of quick freezing in winter, at least in high latitudes, demands that a drainage system, to carry away the spilled water, shall be effective and thorough. Scouring is prevented by a pavement of cobbles, 6-inch quarry spalls, or large flat stones, laid over the ballast. [illegible] of large stones under [illegible] st facilitates draina[illegible] cross drain[illegible] ins laid [illegible] or furth[illegible] ferrec[illegible] W. [illegible]

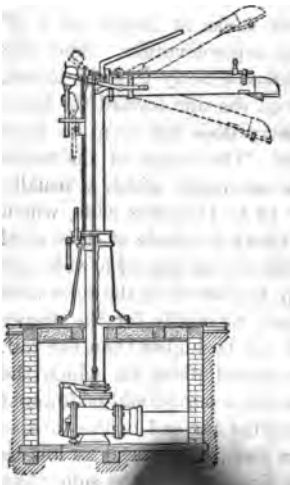


FIG. 159 [illegible] Way, XIV,

327. Stand[illegible] are usual[illegible] who make a [illegible] such track ordinarily be [illegible] nish a corr[illegible] Fig. 159 is sho[illegible] nufacture[illegible] Attention is ca[illegible] tion of H[illegible] for holding the [illegible] the trac[illegible]

a cheaper source of power than steam, gasolene or fuel oil. The electric motor either operates a centrifugal pump, or a slow-speed motor is direct-connected to a triplex reciprocating pump.

A Committee of the Amer. Rwy. Eng. Assoc. reported in 1915 the comparative cost (see Table XXVIII) of pumping 240,000 gallons per day of 10 hours. By comparing the data with that of any given locality a fair idea of relative costs and of the proper choice for that particular station may be made.

326. **Track tanks.** These are chiefly required as one of the means of avoiding delays during fast-train service. A trough, made of steel plate, is placed between the rails on a stretch of *perfectly level* track. A scoop on the end of a pipe is lowered from under the tender into the tank while the train is in motion. The rapid motion scoops up the water, which then flows into the tender tank. They should preferably be located on tangents, although the Penn. R. R. has track tanks at Atglen on a 2° curve where the track has 4 inches superelevation. Since the inside width of the tank (19″) is almost exactly $\frac{1}{3}$ of the gauge, the water is about $1\frac{1}{2}$ inches deeper on the side toward the inner rail, but this much lack of symmetry does not seem to have interfered with successful operation. The length of the tanks varies from 1200 to 2500 feet; the net inside width is usually 19 inches. The scoops are usually 12 to 13 inches wide, which gives allowance for swaying. The tanks are made of sheet steel $\frac{3}{16}$″ to $\frac{1}{4}$″ thick. The usual cross-section is that of a wide and shallow U, 19″ wide, 6″ to $7\frac{1}{2}$″ deep, reinforced on the sides with angles. The ties are usually dapped, especially for the deeper tanks, so that the upper edges will not be higher than the rail. At each end there is a double inclined plane on which the scoops may slide without catching if the scoop should be lowered too soon or if it is not raised before the far end of the tank is reached. Experiments have shown that, at a speed as low as 20 m.p.h., more water is wasted by slopping over the sides than the amount collected by the scoop. At a speed of 45 to 50 m.p.h. the amount wasted becomes minimum and the amount scooped up becomes maximum. At higher speeds the amount scooped up decreases and the wastage increases. The best results show a wastage of at least one-eighth of the total. These same tests showed that at 45 to 50 m.p.h. the 13″ scoop in a 19″ tank will scoop up about 625 gallons per inch of immersion per 1000 feet of tank, or say 2500 gallons per 1000 feet for a 4-inch immersion.

The amount scooped up is practically proportional to the depth of immersion when that depth is over 2½ inches. **Heating.** The water must be heated in winter to prevent freezing. There are two general methods: (*a*) Live steam is forced into the tank through nozzles about 40 feet apart; (*b*) a "circulatory system" by which steam is forced into a water main which feeds the tank in such a way that the water is in constant circulation through the main, into the tank and then back again into the main to be reheated. For the climatic conditions of the N. Y. Central R. R. a steam capacity of 100 H. P. is considered essentail to heat 7000 sq. ft. of tank surface, which means about 4400 lineal feet of 19-inch tank, or two good-length tanks on a double track. On account of the great amount of water splashed over the track and its scouring action on any ordinary ballast, a large item in the cost of an installation is the reconstruction of the track. The certainty of quick freezing in winter, at least in high latitudes, demands that a drainage system, to carry away the spilled water, shall be effective and thorough. Scouring is prevented by a pavement of cobbles, 6-inch quarry spalls, or large flat stones, laid over the ballast. A layer of large stones under the ballast facilitates drainage to numerous cross drains and to longitudinal drains laid between the tracks. For further details the student is referred to a monograph by Geo. W. Vaughan, Eng. Main. of Way, N. Y. Central R. R., in Vol. XIV, Proc. Am. Rwy. Eng. Assoc.

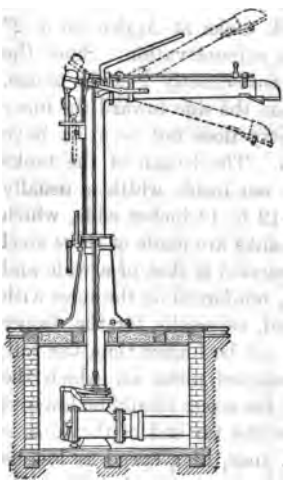


FIG. 159.—STAND-PIPE.

327. Stand-pipes. These are usually manufactured by those who make a specialty of such track accessories, and who can ordinarily be trusted to furnish a correctly designed article. In Fig. 159 is shown a form manufactured by the Sheffield Car Co. Attention is called to the position of the valve and to the device for holding the arm parallel to the track when not in use so that

it will not be struck by a passing train. When a stand-pipe is located between parallel tracks, the strict requirements of clearance demand that the tracks shall be bowed outward slightly. If the tracks were originally straight, they may be shoved over by the trackmen, the shifting gradually running out at about 100 feet each side of the stand-pipe. If the tracks were originally curved, a slight change in radius will suffice to give the necessary extra distance between the tracks.

BUILDINGS.

328. **Station platforms.** These are most commonly made of planks at minor stations. Concrete is used in better-class work, also paving brick. An estimate of the cost of a platform of paving brick laid at Topeka, Kan., was $4.89 per 100 square feet when laid flat and $7.24 per 100 square feet when laid on edge. The curbing cost 30 cents per linear foot. Cinders, curbed by timbers or stone, bound by iron rods, make a cheap and fairly durable platform, but in wet weather the cinders will be tracked into the stations and cars. Three inches of crushed stone on a cinder foundation is considered to be still better, after it is once thoroughly packed, than a cinder surface.

Elevation.—The elevation of the platform with respect to the rail has long been a fruitful source of discussion. Some roads make the platforms on a level with the top of the rail, others 3 inches above, others still higher. As a matter of convenience to the passengers, the majority find it easier to enter the car from a high platform, but experience proves that accidents are more numerous with the higher platforms, unless steps are discarded altogether and the cars are entered from level platforms, as is done on elevated roads. As a railroad must generally pay damages to the stumbling passenger, they prefer to build the lower platform. Convenience requires that the rise from the platform to the lowest step should not be greater than the rise of the car steps. This rise is variable, but with the figures usually employed the application of the rule will make the platform 5 ins. to 15 ins. above the rail.

Position with respect to tracks.—Low platforms are generally built to the ends of the ties, or, if at the level of the top of the rail, are built to the rail head. Car steps usually extend 2 ft. 6 ins. from the track center and are 14 ins. to 24 ins. above

the rail. The platform must have plenty of clearance, and when the platform is high its edge is generally required to be 5 ft. 6 ins. from the track center.

329. Minor stations. The Amer. Rwy. Eng. Assoc. recommend one general waiting room (without reference to separate waiting room for colored people), for a passenger station of medium size for the following reasons: (See 1915 Manual, p. 187).

(1) It permits the general waiting room to be properly proportioned.

(2) It permits proper development of a retiring room for women, with private entrance to the lavatory.

FIG. 160.—DIVISION OF FLOOR AREA RECOMMENDED FOR PASSENGER STATIONS WITH ONE GENERAL WAITING ROOM.

(3) It readily admits of the other rooms being properly proportioned.

(4) It permits ease of access from the agent's office to the trains, to the baggage room and to the waiting room.

(5) It permits the ticket office to be of proper size and location for general office purposes.

(6) It admits of the station being contracted in size without detriment to facilities.

(7) It offers economy in heating.

In the Southern States a separate waiting room for colored people is provided and is sometimes even required by law. The older design, combining a residence for the agent with the station, is now obsolete for new construction, although many such still exist. "Combination stations" (for both passenger and freight business) were formerly quite popular for very small stations and

are still considered desirable when all responsible freight and passenger business must be handled by one man. But it is desirable to separate them whenever the volume of business will justify the employment of two responsible men.

In Gillette's Handbook of Cost Data (1910 ed.), is given in detail the cost of several station buildings. Such figures can be utilized when unit prices are given or can be derived. For example, in one case the building was 24×60 ft., exclusive of platforms; there was no masonry foundation nor plastering. The summary was as follows:

Materials.	Total.	Per cent.	Per sq. ft. of floor.
30,057 ft. B. M. at $13.23 (aver.)	$296.97	33.2	21 ft. B. M.
20 M shingles at $1.10	22.00	2.4	
Millwork	55.75	6.1	3.9 cents
Hardware	37.50	4.1	2.6 "
23 gal. paint at 70 cents	16.10	1.8	1.1 "
1100 brick, at $8.00 per M	8.80	1.0	
Total materials	$437.12	48.6	30.4 "
Labor:			
176.2 days' labor, building at $2.32	$406.38	45.3	28.2 cents
2 days' labor, put up ladders, at $2.50	5.00	0.6	
14 days' labor, painting at $1.75	24.50	2.8	1.7 cents
4 days' labor, building chimney, at $4.00	16.00	1.8	
8 days' labor, filling cinders, at $1.20	8.50	0.9	
Total labor	$460.38	51.4	31.9 "
Total, materials and labor	$897.50	100.0	
Freight, 55 tons, 200 miles ½c. ton-m.	55.00		
Tools (excessive in this case)	38.50		
Grand total	$990.00		68.8 cents

The cost of lumber was very low and even the unit cost of labor (carpenters, $2.50; masons, $4.00; average of all, $2.32), were lower than must frequently be paid. But the figures can be utilized by noting the percentages of the various items to the total and applying local unit costs for material and labor. The total cost per square foot ($0.688), is abnormally low, partly because of no masonry foundation nor cellar, which would add 40 to 50 cents per square foot. Note also that no expenses were included for lighting, plumbing, or heating—except a chimney.

FREIGHT HOUSES.

330. Two types. The freight house, or freight room, at a station where the business is small, is merely a small ordinary building or a room attached to the station building. As the business

becomes larger, efficient operation requires that two types of buildings must be designed—the inbound and the outbound freight house. These types agree in requiring certain details in common, but there are also differences.

331. **Fire-risk.** A small freight house in the country usually has a minimum of actual fire-risk and of valuable freight stored at any one time. This may justify an inexpensive type of frame building which is in no sense fireproof. On the other hand, a building in the heart of a city, closely surrounded by other buildings and stored with a large amount of valuable freight, justifies an expensive type of fireproof construction. The term "fireproof" is only relative. Certain devices and added expenditures will reduce more and more the probability of destructive fires. Certain principles of construction which reduce fire-risk are as follows: (*a*) Use of noncombustible materials for floor, side walls and roof; (*b*) avoidance of space under wooden main floor, between foundations, where combustible rubbish may accumulate; (*c*) fire-walls dividing large houses so that there is not more than 5000 square feet of floor between fire-walls; fire-walls to be never more than 200 feet apart; (*d*) minimum number of doors through a fire-wall; no door larger than 80 square feet; all doors fireproof and automatically self-closing; (*e*) fireproofing protection of walls and roof for at least five feet each side of a fire-wall; (*f*) provision for fire stand-pipes and hose racks not more than 150 feet apart; the stand-pipe should run up about 8 feet above floor where there should be 50 feet of 2-inch linen hose in a hose rack; the valve should be in a pit (*always* accessible), and so far below floor level that there is little or no danger of freezing, since freight houses are ordinarily not heated.

332. **Dimensions.** A freight house usually has a track on one side and a vehicle driveway on the other, the floor being utilized for the more or less temporary storage of freight, which in this case is always in "less than carload" (L. C. L.) lots, carload shipments being transferred directly between cars and vehicles. Since small shipments can usually be loaded into cars (outbound shipments) with less delay than the delivery of freight to vehicles (inbound shipments), the required space for outbound shipments can be less than that for inbound. Experience has shown that for outbound freight only, a width of 30 feet is desirable; for both outbound and inbound, the width may be 30 to 40 feet;

for inbound only it should be 40 to 60 feet. Too great a width needlessly increases the amount of hand-trucking. The length is indefinite and should correspond to the amount of business to be handled. Freight houses are usually single-storied, except where galleries or partial second stories are built to accommodate offices, file and stationery rooms, toilet and locker rooms, the room for " over, short and damaged " freight and the cooperage room for repairing broken packages.

333. **Platforms.** The platform on the track side should preferably be 8 to 10 feet wide, which will avoid the necessity of spotting cars with their doors directly in front of freight-house doors. The platform should be not more than 4 feet above the top of the rail. Even this would be too high to permit opening the doors of refrigerator cars, which swing outward. An occasional refrigerator car could be handled, even with a high platform, by opening the doors before placing the car. The M. C. B. standard, for regular use of refrigerator cars, is " not more than 3 ft. 8 ins." The P. R. R. standard is 3 ft. 5 ins. The minimum distance from track center to edge of platform is 5 ft. 9 ins. The P. R. R. standard is 6 ft. 1½ ins. If there is a platform on the driveway side, it should be 3 to 4 feet above the driveway level. At an outbound house, where the freight is delivered from the vehicle into the freight house, the height should be not more than 3 feet. Platforms should slope away from the house with a grade of about 1 in. to 8 ft. for drainage.

334. **Floors.** The designed floor loading should be 250 lbs. per square foot. In § 347 are described several types of floors suitable for engine houses, many of which are also suitable for freight houses. In selecting a type, it should be remembered that hand-trucking is apt to be concentrated along certain rather narrow paths and that this wears out the floor surface, requiring premature renewals along these paths, unless these paths are overlaid with iron or steel plates. When a solid type of floor is used (supported on sub-soil), the flooring should be independent of the side walls, which avoids trouble due to floor settlement. For inbound freight houses the floor should slope about 1 inch in 8 feet from the track side toward the driveway side, the slope continuing to the outer edge of the driveway platform, since this is in the direction of traffic and aids it, but the track platform must slope the other way for drainage. For outbound freight houses, the slope is exactly reversed.

335. **Doors.** Ordinary swinging doors are unsuitable. Lifting doors, counterbalanced, which sometimes fold as they lift, are used. Rolling metal shutters are, perhaps, most satisfactory, but are expensive. Sliding doors require that a guarded space be made so that stored freight does not interfere with the sliding. They also limit the possible total door width to less than half the side of the house. All lifting types permit opening up the whole side of the house (if desired), except the space occupied by the posts. Continuous doors are particularly necessary when there is no platform between the house and the track. Doors should be at least 8 feet high. On the track side this is sufficient, since the car door cannot be higher. On the driveway side a greater height might be desirable.

336. **Roofs projecting over platforms.** These are desirable as a protection when loading or unloading during storms. That over the driveway platform should be at least 10 feet above the platform or 14 feet above the driveway. When not forbidden by State laws, the roof may be extended beyond the edge of the track platform, but it should be, at least, 17 feet above the rail and 18 inches from the track center, thus leaving a walking space on top of the car.

337. **Lighting.** Daylight lighting should be obtained by windows through the side-walls above the doors, or by vertical sashes in a monitor roof, which will also provide for ventilation. Skylights, especially when nearly flat, are expensive both for construction and for maintenance. Artificial lighting should be obtained from electricity, with wires run according to the strictest specifications of the National Board of Underwriters. Platforms should be illuminated. A series of push plugs should be placed along the platform wall face, from which extension cords with bulbs may be run to light car interiors.

338. **Scales.** Outbound houses need scales, with capacity of 8000 lbs., to weigh outgoing freight. "From 50 to 80 feet apart is good practice."

339. **Ramps.** These are slopes from the driveway level to the car level which facilitate the loading or unloading of agricultural implements and all heavy vehicles running on their own wheels. They are usually built at the end of an extension of the platform, with as low a grade as the circumstances will permit.

"Buildings and Structures of American Railroads," by Walter

G. Berg, although now (1916) somewhat old, contains many plans, showing considerable detail, of station and other buildings. "Railroad Structures and Estimates" by J. W. Orrock, also shows some plans.

340. **Section houses.** These are houses built along the right-of-way by the railroad company as residences for the trackmen. The liability of a wreck or washout at any time and at any part of the road, as well as the convenience of these houses for ordinary track labor, makes it all but essential that the trackmen should live on the right-of-way of the road, so that they may be easily called on for emergency service at any time of day or night. This is especially true when the road passes through a thinly settled section, where it would be difficult if not impossible to obtain suitable boarding places. It is in no sense an extravagance for a railroad to build such houses. Even from the direct financial standpoint the expense is compensated by the corresponding reduction in wages, which are thus paid partly in free house rent. And the value of having men on hand for emergencies will often repay the cost in a single night. Where the country is thickly settled the need for such houses is not so great, and railroads will utilize or perhaps build any sort of suitable house, but on Southern or Western roads, where the need for such houses is greater, standard plans have been studied with great care, so as to obtain a maximum of durability, usefulness, comfort, and economy of construction. (See Berg's Buildings, etc., noted above.) On Northwestern roads, protection against cold and rain or snow is the chief characteristic; on Southern roads good ventilation and durability must be chiefly considered. Such houses may be divided into two general classes—(*a*) those which are intended for trackmen only and which may be built with great simplicity, the only essential requirements being a living room and a dormitory, and (*b*) those which are intended for families, the houses being then distinguished as "dwelling-houses for employees."

ENGINE HOUSES.*

341. **Form.** When not more than three or four engines are to be housed at once and when no turntable is to be provided,

* Condensed and abbreviated from Committee Report, Am. Ry. Eng. Assoc., 1915.

the rectangular form is preferable. All large engine houses are "circular," with a turntable at the center of the circle, except some very large houses, which are really repair shops, where it seems advisable to install a transfer table.

342. **Doors.** The clear opening should be not less than 13 feet wide by 16 feet high. The doors should fold outward and should have such a design that a pilot door may be inserted.

343. **Length.** The length of stall along the center line of the track should be 15 feet greater than the overall length of the longest locomotive, which will provide a walkway behind the tender, a trucking space in front of the pilot and a sufficient distance in which to stop the engine so that the side rods will be in any desired position.

344. **Materials of construction.** Wood was formerly very commonly used, but it is too inflammable. The walls should be made of brick, stone, or plain concrete—not reinforced, at least "for that portion of the wall directly in line of track where engine is liable to run into it." The roof is the difficult problem, since wood is inflammable and iron or steel, even for framing, is very rapidly corroded by coal gas from the engines. Reinforced concrete is the only thoroughly satisfactory material but "when the roof is of reinforced concrete, the columns and roof beams should be of the same material," i. e., it is useless to support a reinforced concrete slab on steel beams.

345. **Engine pits.** These "should be not less than 60 feet in length, with convex floor, with drainage toward the turntable. The walls and floors may be of concrete. Proper provision should be made for the support of the jacking timbers." The engine should stand with its tender toward the turntable.

346. **Smokejacks.** Locomotives leave an engine house under their own steam, which requires starting their fires considerably beforehand, and the smoke must be removed. The precise position of the locomotive on the track is variable, since it must be adjusted to the place where the side rods are in a proper position for repairs. A smokejack is essentially a funnel whose base is at the minimum height above the track which will give the smokestack a proper clearance. The base should be 42 inches wide and long enough for the adjustment as stated above, which means at least 10 feet. The sides should slope upward gradually to a flue whose area should be not less than 7 square feet. There should be a drip trough around the base of the jack.

naterial should be "non-combustible," but the choice is esome. Sheet iron, even when heavily painted, corrodes y. Wood, covered with "fireproof paint," has been tried. ron has been tried but is exceedingly heavy as well as ex- e. Asbestos is being used on several important roads. ted. designs, of which there are several, are used on the ty of roads.

Floors. (*a*) **Stone screenings.** Subsoil should be good; t spots cleaned out and filled with good material; subsoil Foundation of cinders or gravel, 6 ins. thick. Top coat, es of stone screenings, perhaps mixed with a little clay or oil, the surface being thoroughly rolled. Special founda- or machinery necessary. Surface is not good for heavy ng. (*b*) **Planks.** Subsoil same as above; 6 ins. cinders vel, with 4"×6" creosoted sleepers, spaced about 3 feet, ded in upper surface of cinders; then 3-inch plank. special foundations for machinery and at jacking-up are necessary. (*c*) **Creosoted wood-block.** The wood , 4 ins. deep, fiber vertical, should be laid on a 1-inch n coat of sand which is supported by a 6-inch layer of te. A 6-inch layer of cinders, as specified above, is also nended as a bed for the concrete, but this may depend on aracter of the subsoil. The joints should be filled with tic mastic, and an expansion joint 1 inch wide should be ed every 50 feet. (*d*) **Wood floor on concrete.** Sleepers, about 3 feet, trapezoidal, 4-inch top, 6-inch bottom, 4 deep, embedded in a 6-inch layer of concrete, so that the s project ½ inch above concrete. Then layer of 2-inch covered with 1⅛-inch maple flooring. (*e*) **Brick.** Same except that bricks are used in place of wood block. (*f*) te. Same foundation as above; 6-inch course of concrete d with 1-inch surface coat (1:2) laid on before base has nitial set. (*g*) **Asphalt.** Same as (*f*) except that surface 1½ inches of rock mastic. Expert workmen are needed isfactorily mixing and laying the asphalt, but the floor is

Drop pits are necessary, where pairs of truck, driving and wheels may be dropped from their journals and removed ıe engine for repairs or renewals.

Heating. The primary object of heating is to thaw out ines so that they may be returned to service as quickly

as possible, rather than to heat the building, whose general temperature should be kept at 50° to 60°. Therefore heat should be concentrated at the pits. Hot air should be forced through permanent ducts, preferably laid under the floor. The outlets should have dampers, which may be closed when men are working in the pits. Fresh air should be drawn from outdoors and no recirculation permitted. The air should be heated by passing over coils containing exhaust steam, supplemented by live steam, if necessary. The air passes out of the building through annular openings around the smokejacks, and also through openings between the wall plates and the roof rafters. These openings should extend entirely around the building.

350. Window lighting. Skylights are undesirable because of preponderant disadvantages. The windows in the outer walls should be as large, wide and as high as safe construction will permit, the sill not more than 4 feet from the floor. Windows should be placed over the locomotive doors. Windows set into locomotive doors cause heavy maintenance charges on the doors.

351. Electric lighting. Numerous lights should be provided to avoid shadows. Plugged outlets for incandescent lights in alternate spaces between pits should be provided.

352. Piping. Pipes for air, steam and water supply should be provided, and where desired, piping for a washout and refilling system should be installed. Where this system is installed, the blow-off lines should be led to a central reservoir; where it is not used, the blow-off lines should be led outside the house. The steam outlet should be located near the front end of the boiler. The blow-off pipe, the air, the washout and refilling water and the cold water connections should be near the front end of the firebox. Connections need only be provided in alternate spaces between stalls.

353. Tools. There should ordinarily be facilities provided for hand tools and for the location of a few machine tools, preferably electrically driven.

354. Hoists. Hoists with differential blocks are generally used for handling heavy repair parts, and suitable provision should be made for supporting them.

355. Turntables. The turntable should be long enough to balance the engine when the tender is empty. The deck form is preferable to the through form. Power should be provided at turntables having great service. Electric power is best and least

expensive when it is available. Compressed air, supplied either by a pumping plant or by the locomotive itself, is sometimes used. The turntable pit should be thoroughly drained and preferably paved. The circle wall should be of concrete or brick, with proper supports and fastenings for rails on the coping. The circle rail should preferably bear directly on concrete base. The use of wood ties and tie-plates supported by masonry is desirable for the circle rail under some conditions. Easy access to the parts of a turntable for the oiling of bearings, painting and inspection should be provided in the design of the turntable pit, unless ample provision is made in the turntable itself.

LOCOMOTIVE COALING STATIONS.

356. Hand shoveling. For roads of the smallest traffic, particularly at terminals where locomotives lie overnight, hand shoveling direct from coal cars or from platforms provided with a jib crane and one-ton buckets, is the most economical.

357. Locomotive crane. A locomotive crane, equipped with buckets, provides an efficient method of transferring coal from the coal car to a tender, particularly when the crane can be profitably employed at other times.

358. Coaling trestle. This method requires a trestle with an approach not exceeding 5%, so that coal may fall from bottom-dumping cars into a pocket and then be discharged through chutes into the tender on a track on either side of the trestle. This method is satisfactory when two coaling tracks are sufficient and when there is available space for the approach track.

359. Coal conveyors. When more than two coaling tracks are essential, a conveyor system may be preferable. The coal is brought to the plant in bottom-dumping gondola cars, which dump the coal on to a conveyor which conveys it up and drops it into the bin, from which it may fall either into the tender or into an elevated conveyor car which runs it across a system of parallel tracks and dumps it into a tender, spotted there for the purpose. Incidentally, such a plant usually has also an **ash conveyor** onto which ashes are dumped from the engine. This conveyor carries the ashes to a place where the conveyor buckets dump them into a waiting gondola car, which when full is hauled away.

360. **Oil houses** * should be fireproof and should be separated from other buildings. Above ground there should be a masonry building, 20′×40′, or perhaps less, with one fireproof door and one or more windows, having wire glass. This room contains a row of pumps, one for each kind of oil; also a series of inlet pipes in the floor leading to tanks in the basement. The floor should be 4 feet above the track rail outside and there should be a

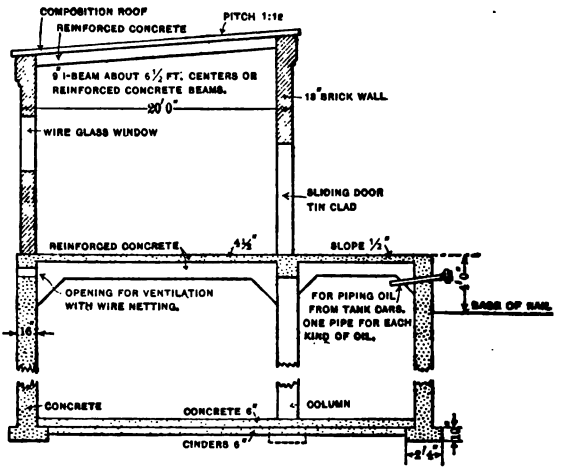


FIG. 161.—CROSS-SECTION OF TYPICAL OIL-HOUSE.

platform between the house and the track. The storage space for oil is entirely in the basement and includes the area under the floor and also the area under the platform. The height depends on the required storage space for tanks. A series of pipes, one for each kind of oil, pass through the outer vertical face of the platform, for the convenient emptying of tank cars into the storage tanks. The inlet pipes through the floor are only for small quantities of oil drawn from barrels.

The delivery system from the storage tanks to the faucets should be such that the oil can be delivered quickly and measured automatically. The delivery should also be such that there will

* Condensed from the Manual of the Am. Rwy. Eng. Assoc., 1915 Ed.

be a minimum of dripping at the faucet and that the dripping may drain back to the storage tanks. Openings for ventilation should be provided above the level of the top of the tanks. Lighting, when required, should be by electricity and heating by steam. For fire protection purposes a live-steam line should be run to the oil storage space, controlled by a valve outside the house.

361. Section tool houses. For small-traffic roads these should be 10′×14′, the short dimension parallel with the track, with double swinging doors, swinging out on the end nearest the track. For roads of larger traffic the dimension parallel with the track should be 18 to 20 feet and the other dimension 12 to 14 feet. There should be a sliding door, 8 feet in clear, at extreme end, on track side, to permit the storing of hand car. A sliding wooden shutter (instead of glass) may serve as a window for fair weather. It should *not* be made so convenient and comfortable that it will become a lounging place for trackmen in stormy or wintry weather. The building should be of wooden frame construction, resting on wooden posts, or on masonry piers if the location can be considered permanent. Drop siding on the sides and some kind of prepared roofing will usually be most economical.

362. Sand houses. Sand is a necessity in the operation of locomotives. Ordinarily it is obtained in a more or less moist and caked condition. It must be made thoroughly dry, so that it will flow readily through a pipe having sufficient slope. The plant consists essentially of a "wet storage bin," about 12′×16′, which adjoins a "drying room" of about the same size. This room contains a screen, which is usually necessary to screen out the coarser particles; also a furnace to dry the sand, and a coal bin. For small traffic roads it may be sufficient to store the dry sand in a bin or even in buckets which are lifted by hand to the engine. For heavier traffic it may be justifiable to raise the sand to a bin or hopper whose lowest point is at least 22 feet above the rail, from which the sand may flow through a jointed pipe, somewhat similar to a water-supply pipe, directly into the sand box on the engine. Of course the bottom of the hopper must have sufficient slope so that the sand will always flow over it. The sand is hoisted to the hopper, either by some mechanical conveyor system, or is forced through a pipe by compressed air. The building should be located about 8 feet from the nearest track center.

363. Ash pits. A locomotive must dump the ashes from its ash pan at frequent intervals. The operation is usually timed to be done at terminal or divisional points, just before taking on water, coal, etc. These several plants are, therefore, grouped together in the yard. When there are no facilities for removing ashes by a conveyor at the same time that coal is being loaded on to the tender (see §§ 356–359), the ashes are dumped into a pit. The poorest roads dump them on the track under the engine, but this burns the ties, is dangerous, and is uneconomical, since they must be immediately removed. The simplest form of ash pit is made by dropping the ties about a foot, and then laying the rails on a pair of stringers about 12″×12″. The stringers and ties must be covered with sheet iron to protect them from hot ashes. The capacity of such a pit is so small that the ashes must be removed quite frequently, which must usually be done by hand shoveling over the side of a gondola car on an adjacent track. The next development is a deeper pit, with concrete walls. Even then, the rails must be fastened to longitudinal wooden stringers, protected with sheet iron, or to cast-iron chairs which are embedded in the concrete. The ashes may be shoveled out by hand after the locomotive has passed, or they may be dropped from the ash pan into buckets or small cars, which run on a narrow track at the bottom of the pit, and which may be lifted out by a jib crane. Another development is to widen the pit, running one rail on one wall and the other rail on a series of cast-iron columns. The pit has much greater capacity and the ashes may be hoisted out at any time, even if the locomotive is still on the ash track. Great economy in the disposal of ashes is obtained when it is practicable to construct a depressed track, with its track center about 14 feet away from the ash track and 9 feet or more lower. The ashes may then be dropped onto a platform about 3 feet below the ash track, the platform extending to the top of a vertical retaining wall whose face is 5 ft. 6 ins. from the center of the depressed track, and from there the ashes are easily shoveled over the side of a gondola car placed on the lower track. No lifting of the ashes by hand is necessary. As in the previous plan, one rail of the ash track is supported by a wall, while the rail toward the depressed track is supported on cast-iron columns. The platform space is thus 10 to 11 feet wide.

Ashes should be quenched promptly after being deposited,

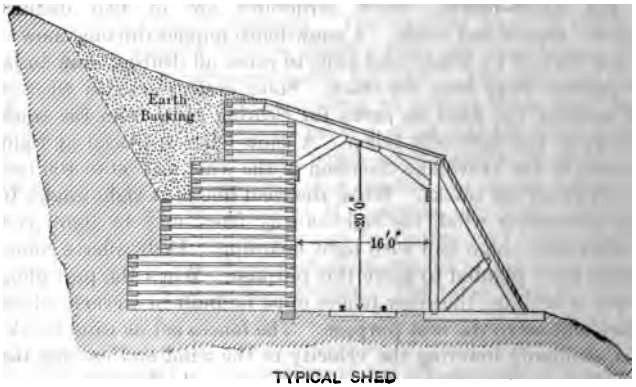
so as to reduce their heating effect even on metal and masonry. This requires a hose and a water supply. The pits should be graded so as to drain to a sump, which should have an overflow sufficiently above the bottom so that periodical cleaning out will suffice to keep the drain pipe from getting clogged with detritus from the ashes.

SNOW STRUCTURES.

364. Snow-fences. Snow structures are of two distinct kinds—fences and sheds. A snow-fence implies drifting snow—snow carried by wind—and aims to cause all drifting snow to be deposited away from the track. Some designs actually succeed in making the wind an agent for clearing snow from the track where it has naturally fallen. A snow-fence is placed at right angles to the prevailing direction of the wind and 50 to 100 feet away from the tracks. When the road line is at right angles to the prevailing wind, the right-of-way fence may be built as a snow-fence—high and with tight boarding. Hedges have sometimes been planted to serve this purpose. When the prevailing wind is oblique, the snow fences must be built in sections where they will serve the best purpose. The fences act as wind breakers, suddenly lowering the velocity of the wind and causing the snow carried by the wind to be deposited along the fence. Portable fences are frequently used, which are placed (by permission of the adjoining property owners) outside of the right-of-way. If a drift forms to the height of the portable fence the fence may be replaced on the top of the drift, where it may act as before, forming a still higher drift. When the prevailing wind runs along the track line, snow-fences built in short sections on the sides will cause snow to deposit around them while it scours its way along the track line, actually clearing it. Such a method is in successful operation at some places on the White Mountain and Concord divisions of the Boston & Maine Railroad. Snow-fences, in connection with a moderate amount of shoveling and plowing, suffice to keep the tracks clear on railroads not troubled with avalanches. In such cases snow-sheds are the only alternative.

365. Snow-sheds. These are structures which will actually keep the tracks clear from snow regardless of its depth outside. Fortunately they are only necessary in the comparatively rare situations where the snowfall is excessive and where the snow

is liable to slide down steep mountain slopes in avalanches. These avalanches frequently bring down with them rocks, trees and earth, which would otherwise choke up the road-bed and render it in a moment utterly impassable for weeks to come. The sheds are usually built of 12″×12″ timber framed in about the same manner as trestle timbering; the "bents" are sometimes placed as close as 5 feet, and even this has proved insufficient to withstand the force of avalanches. The sheds are there



TYPICAL SHED

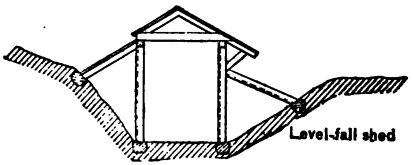


FIG. 162.—SNOW-SHEDS—CANADIAN PACIFIC RAILROAD.

fore so designed that the avalanche will be *deflected* over them instead of spending its force against them. Although these sheds are only used in especially exposed places, yet their length is frequently very great and they are liable to destruction by fire. To confine such a fire to a limited section, "fire-breaks" are made—i.e., the shed is discontinued for a length of perhaps 10 feet. Then, to protect that section of track, a V-shaped deflector will be placed on the uphill side which will deflect all descending material so that it passes over the sheds. Solid cri

work is largely used for these structures. Fortunately suitable timber for such construction is usually plentiful and cheap where these structures are necessary. Sufficient ventilation is obtained by longitudinal openings along one side immediately under the roof. "Summer" tracks are usually built outside the sheds to avoid the discomfort of passing through these semi-tunnels in pleasant weather. The fundamental elements in the design of such structures is shown in Fig. 162, which illustrates some of the sheds used on the Canadian Pacific Railroad.

FENCES.

366. Wire fences. The following is condensed from the conclusions adopted by the Amer. Rwy. Eng. Assoc. and incorporated in their 1915 Manual. The recommended standard right-of-way fence is a wire fence, supported on wood or concrete posts. The wiring is to consist of five to nine longitudinal strands, with vertical stay wires spaced 12 to 24 inches apart. The longitudinal and vertical wires are to be locked or fastened with a mechanical lock which will prevent slipping either longitudinally or vertically, or the wires shall be electrically welded. The wire shall be galvanized so as to stand the following test: "The galvanizing shall consist of an even coating of zinc, which shall withstand one-minute immersion tests in a solution of commercial sulphate of copper crystals and water, the specific gravity of which shall be 1.185 and whose temperature shall be from 60° to 70° F. Immediately after each immersion the sample shall be washed in water and wiped dry. If the zinc is removed, or a copper-colored deposit formed at the end of the fourth immersion, the lot of material from which the sample is taken shall be rejected. The fence shall be so fabricated as not to remove the galvanizing or impair the tensile strength of the wire." Electrically welded fencing should be galvanized after it has been fabricated.

367. Types. Class A fence has 9 horizontal smooth wires whose spacing, starting at the ground, is 5, 4, 4½, 5, 5½, 6, 7, 8 and 9 inches. To make it "hog-tight" the bottom space (5″) is reduced to 3 inches and a barbed wire is inserted midway in the 3-inch space. The top and bottom smooth wires are No. 7 gauge wire and the 7 intermediate wires are No. 9. The vertical stay wires, spaced 12 inches, shall be No. 9 gauge.

Class B fence has 7 horizontal wires, with vertical wires spaced 18 inches—all wires No. 9 gauge. The spacing, starting at the ground, is 7, 6½, 7, 7½, 8, 8½ and 9 inches.

Class C fence has 5 horizontal wires, with vertical wires spaced 24 inches—all wires No. 9 gauge. The spacing, starting at the ground, is 9, 7½, 8, 8½ and 9 inches.

Class D fence has 5 horizontal wires and no vertical stay wires, the wires being No. 9 gauge. The spacing, starting at the ground, is 10, 10, 10, 12 and 12 inches.

368. Posts. End, corner, anchor and gate posts shall be at least 8 feet long and set 3 feet 4 inches in the ground, even if blasting must be resorted to. Intermediate posts shall be at least 7 feet long and set 2 feet 4 inches in the ground. Where rock is encountered at intermediate post holes, the intermediate posts, if of wood and not more than two in succession, may be set on sills, 6″×6″×4′ 0″, braced on both sides by braces 2″×6″×3′ 0″. End, corner, anchor and gate posts, when of wood, shall be 8 inches in diameter at the small end; when of concrete, shall be 6 inches square at the top, 8 inches square at the base and shall be reinforced with four ⅜-inch square twisted rods. Intermediate wood posts shall be at least 4 inches in diameter at the small end; intermediate concrete posts shall be 4 inches thick at the top, 5½ inches at the bottom and reinforced with three (or four, depending on design) ¼-inch square twisted rods.

369. Braces. End, corner, anchor and gate posts shall be braced by 4″×4″ sawed lumber, or round posts at least 4 inches in diameter, or by concrete struts, 4″×4″, reinforced with four ¼-inch twisted rods. The strut braces shall extend from a point about 12″ below the top of the braced post to a point about 12″ from the ground line at the adjacent intermediate post. In addition, a tie, made of a double strand of No. 9 galvanized soft wire, looped around the end, corner, anchor or gate post near the ground line, and around the next intermediate or line post about 12 inches from the top, shall be put on and twisted until the top of the next intermediate or line post is drawn back about 2 inches.

370. Concrete posts. These are recommended. They may be made of one part of cement to four parts of pit gravel; or one part cement, two parts sand and four parts of stone of low absorption or screened gravel, the aggregate in any case being not less than ¼″ nor more than ½″. The molds should be oiled

or soaped and should be vibrated while concrete is poured to make the concrete more compact. The concrete should have a "quaking" consistency. The pouring should not be done out of doors in freezing weather. The concrete should not be exposed to sun, should be sprinkled every day for 8 or 10 days and should have 90 days for curing. They should be packed in sawdust or straw for shipment. Posts are usually made tapering and the cross-section is variously a square, a rectangle, or an isosceles triangle, the corners being chamfered. The reinforcement should be placed not more than ½" from the surface and should be wired by bands spaced about 12". The fencing is sometimes fastened to the posts merely by wires tied tightly about the post or may be fastened to metal lugs which are embedded in the soft concrete during molding.

371. **Construction details.** Wood posts shall be **anchored** by gaining and spiking two cleats, 2"×6"×2' 0", on the side of the post below the ground line. **Staples** shall be 1 inch long for hard wood, and 1½ inch for soft wood, made of No. 9 galvanized steel wire. They shall be driven diagonally with the grain of the wood, the top wires double-stapled. Staples, No. 9 wire, 1 inch long, weigh 108 to the pound; 1½ inch long, 72 to the pound.

Wire. No. 7 wire is 0.177 inch in diameter, weighs 439 pounds to the mile, or 12.05 feet to the pound. No. 9 wire is 0.148 inch in diameter, weighs 306 pounds to the mile or 17.24 feet to the pound. Smooth wire is preferable to barbed. A heavy smooth wire or a plank should be used at the top of a barbed-wire fence. Wires shall be placed on the side of the post away from the track. **Splicing** shall be done as follows: "The ends of the wires shall be carried 3 inches past the splicing tools and wrapped around both wires backward from the tool for at least five turns, and after the tool is removed, the space occupied by it shall be closed by pulling the ends together." After erection, wood posts should be sawed off, on a one-fourth pitch, the high side being next to the wire and 2 inches above it.

Gates should be hinged to swing away from the track; should be at least 12 feet wide and 4 feet 6 inches above the ground; should swing shut by gravity, and the free end should overlap the post so that it cannot be swung open toward the track. **All-metal construction** is preferable.

SIGNS.

372. **Highway signs.** The crossing sign recommended by the Amer. Rwy. Eng. Assoc. is essentially as follows: Two wooden blades, 12 inches wide, 8 feet long, with mitered ends, are placed diagonally, with an angle of 50° between the blades, on an 8″×8″×16′ 0″ wooden post sunk 4 feet in the ground. The lower 9 feet is painted black, the upper 7 feet white. The blades are painted white with black letters and a ½-inch black border around the blades. The border and lettering is on both sides. The lettering is Egyptian style 9 inches high with the exception of the connecting terms, as "for the" in the recommended sign, which should be 4 inches high. The recommended wording is "RAILROAD CROSSING" on one blade and "LOOK OUT FOR THE LOCOMOTIVE" on the other blade. The width of band of the letters is 1¼ inches. If two railroads parallel each other within 400 feet, another blade marked "TWO CROSSINGS" should be added. The laws in some states prescribe what the lettering shall be.

373. **Trespass signs.** The specifications for these signs are applicable to many other public warnings which must be displayed. A cast-iron plate, ¼ inch thick, stiffened on the back by ⅜-inch diagonal cast ribs and having the letters and border cast on the front by raising the surface about ⅛ inch, is set on an iron post 10 feet long, which is embedded 2 feet in a block of concrete, which serves as foundation. The letters should be about 2 inches high. A socket is cast on the rear side of the plate of such dimensions that it will set on the pipe and be fastened with a ½-inch set screw. The posts may be made of 2½-inch wrought iron pipe or of good second-hand boiler tubes, which should be filled with cement grout. The face of the letters and the borders should be painted black while the background is painted white. The tablet will usually be about 30 inches wide by 18 inches high with rounded corners, although the dimensions will vary in accordance with the lettering to be placed on it. The following trespass signs frequently need to be displayed:

RAILROAD PROPERTY TRESPASSING FORBIDDEN UNDER PENALTY OF LAW

DANGER DO NOT TRESPASS ON THE RAILROAD

DANGER
DO NOT
TRESPASS ON THIS
BRIDGE

374. **Marker posts.** Mile posts are most economically made, considering their durability, of skeletonized cast iron. The post is made up of two slabs of cast iron ½ inch thick, 8 feet long, the width tapering from 10 inches to 12 inches, the two slabs being formed in one piece and connected at intervals by ½-inch webs and a top and bottom plate. They should be set 3 feet 6 inches in the ground and have a 4-inch slab of concrete or a heavy, flat stone as a base. The mile post numbers should be cast in raised letters on the face, the letters being 4½ inches high. The two faces should be at right angles with each other and should each stand at an angle of 45° with the track. They should be set at least 8 feet from the gauge line of the nearest rail and 11 feet away, where it is practicable. The numbers should be so set that, on approach, the distance to the terminus or division point beyond will be indicated.

The separating line between divisions is indicated to track men by an iron sign, called a division post, which is structurally the same as that of the mile posts. The two divisions are indicated by raised lettering on the faces of the posts. Of course there must be a variation in the lettering or numbering and a special post must be cast for each location of division post or mile post.

Whistle signs are made similarly except that there is but one slab, suitably reinforced with ribs, and which faces in the desired direction. The letter W 7½ ins. high is cast in raised letters near the top. The ring sign is made similarly by using the letter R. The separating line between sections is indicated to the trackmen by a cast-iron sign, called a section post, which is made similarly to the Trespass Signs, except that the tablet is much smaller. Such a sign will have two consecutive numbers, for example, 24–25, to indicate that the sign is at the separating line between section 24 and section 25.

375. **Bridge warning.** When possible the headroom beneath overhead bridges is made at least 22 ft., which will make it safe for a trainman to stand on the top of a freight car which is

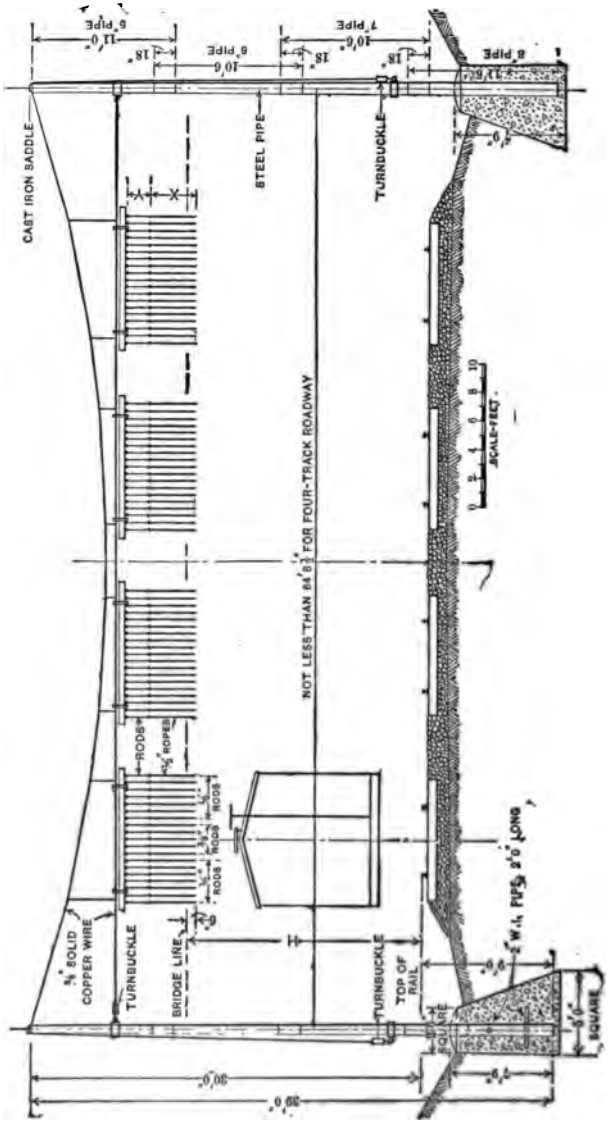


FIG. 163.—BRIDGE WARNING. Penn. R. R. Standard (1911).

passing under the bridge, but it is not always possible to have that amount of headroom. Under such circumstances, a warning for trainmen is necessary. These are made by suspending "ticklers," which are a series of ropes spaced 6 ins. apart which are suspended over the track at a sufficient distance from the bridge or tunnel so that the trainman shall have sufficient warning if he is struck by the dangling ropes. For a single track road the tickler may be suspended from a horizontal arm fastened to a pole planted at least 10 ft. from the track center, the arm being braced by a tie from the top of the pole and also by a short strut underneath. When several tracks are to be spanned, two poles will be used and a catenary cable, between the tops of the poles, supports a horizontal cable by means of a pair of suspenders over each track. The standard on the Pennsylvania Railroad has 19 ticklers 6 ins. apart over each track. The bottoms of the several ropes are 6 ins. below the bottom line of the bridge, the ropes having a length varying from 3 ft. to 5 ft. 3 ins. The ropes are fastened to $\frac{1}{4}$ in. or $\frac{3}{8}$ in. iron rods which swing on ring-bolts which are run through a wooden arm or hanger. The distance from the warning to the bridge or tunnel should be about 100 to 200 ft., depending somewhat on the grade, since that affects the time of the average freight train in passing the interval.

CHAPTER XIII.

YARDS AND TERMINALS.

376. Value of proper design. A large part of the total cost of handling traffic, particularly freight, is that incurred at terminals and stations. It amounts to about 15% of the total operating expenses of a railroad. Freight arrives at any one of the hundreds of thousands of freight stations of the country, to be shipped to any other one of those stations. It may consist of a single package or several carloads of bulk freight. It may have to be transferred from car to car, or the car itself transferred from road to road. In any case, the classification and handling of the freight, whether in individual packages or in carloads, is complicated and expensive and any device for reducing the labor of handling such freight, or which saves time in doing it, has a definite money value. Assume that an improvement in the design of the yard will permit a saving of the use of one switching engine, or for example, that the work may be accomplished with three switching engines instead of four. Assuming a daily cost of $25, we have in 313 working days an annual saving of $7825, which, capitalized at 5%, gives $156,500, enough to reconstruct any ordinary yard.

377. Definitions. (Compiled from Proc. Amer. Rwy. Eng. Assoc.)

Yard. A system of tracks within defined limits provided for making up trains, storing cars, and other purposes, over which movements not authorized by timetable or by train order may be made, subject to prescribed signals, rules and regulations.

Receiving yard. A yard for receiving trains.

Classification yard. A yard in which cars are classified or grouped in accordance with requirements.

Departure or forwarding yard. A yard in which cars are assembled in trains for forwarding.

Storage yard. A yard in which cars are held awaiting disposition.

Summit or hump yard. A yard in which the movement of cars is accomplished by pushing them over a summit, beyond which they run by gravity.

Body track. Each of the parallel tracks of a yard, upon which cars are switched or stored.

Ladder track. A track connecting successively the body tracks of a yard.

Lead track. An extended track connecting either end of a yard with the main track.

Running track. A track reserved for movement through a yard.

Crossover track. A track connecting two adjacent tracks.

Stub track. A track connected with another at one end only.

Spur track. A stub track of indefinite length diverging from a main line or track.

House track. A track alongside of (or entering) a freight house; used for cars receiving or delivering freight at the house.

Team track. A track where freight is transferred directly between cars and wagons.

378. **General principles.** It should be recognised at the start that at many places an ideally perfect yard is impossible, or at least impracticable, generally because ground of the required shape or area is practically unobtainable. But there are some general principles which may and should be followed in every yard and other ideals which should be approached as nearly as possible. Nevertheless every yard is an independent problem.

Body tracks should be spaced 13 feet to 14 feet center to center, under ordinary conditions, and where they are parallel to main track or other important running track, the first body track should be spaced not less than 15 feet center to center from such main or important track.

Ladder tracks should be spaced not less than 15 feet center to center from any parallel track. Frogs of greater angle than No. 8 should not be generally used, and the angle between the ladder track and body tracks will be governed by the distance on ladder tracks required for a turnout.

To facilitate train movements the connections of **lead tracks** with the main track should be interlocked.

Running tracks should be provided for movements in either direction to enable yard engines to pass freely from one position of the yard to the other; also to enable road and yard engines to pass to and from the engine house and other points where facilities are provided.

Crossover tracks should be located at most convenient points where they will least interfere with regular movements.

Caboose tracks should be so located, where conditions permit, that cabooses can be placed on and removed from trains in the order of their arrival, and should be so constructed that cabooses can be dropped by gravity onto the rear of trains made up for departure.

Scale tracks should be so located that weighing can be done with least delay and without drilling over scale. Where many cars are to be weighed they should pass separately over the scale by gravity, being weighed while in motion.

Coaling, ashpit, sand and engine tracks should be located on the route leading to and from the engine house and should provide sufficient storage for the reception of engines by the hostler. They should be so arranged that water, coal and sand can be taken and ashes disposed of in convenient rotation, and that switching engines may clean fires, take coal, water and sand and pass around waiting engines.

Bad-order tracks. Where cars are classified, one or more classification tracks, easy of access, should be provided for setting off cars in bad order, from which they may be readily removed to the repair tracks.

Repair tracks should preferably be connected at both ends and have a maximum capacity of about 15 cars each, spaced alternately 16 feet and 24 feet center to center and be connected conveniently to bad-order tracks.

Icing tracks should be so located that the work of shifting out, icing and classifying cars for movement can be performed in the least time.

The **Main tracks** of both single and double track roads should be located, if it is possible to so arrange, on the outside of yard, and the engine house, coaling station, etc., should be centrally located.

The **Coach cleaning yard** should be located near the terminal station. The tracks should be of sufficient length to hold full trains, with a car cleaners' repair and supply building adjacent thereto.

Roadways. Where the freight house is on one side and a wall on the other, the minimum width of roadway should be 30 feet; but where a freight house is on one side and a team track or another freight house is on the other, the minimum clear width of roadway should be 40 feet.

A **Transfer Station** should be located at a point where traffic

is concentrated and where a necessity exists for consolidating freight into a less number of cars for movement to a certain destination, or for separating and reloading freight into a greater number of cars or into system cars for further movement to final delivery.

The car capacity of freight tracks should be computed on the basis of 42 feet for each car.

Frogs. Although not absolutely necessary, there is an advantage in having all frog numbers and switch dimensions uniform. No. 8 frogs are recommended. Sharper-angled frogs make easier riding, less resistance and less chance of derailment, but on the

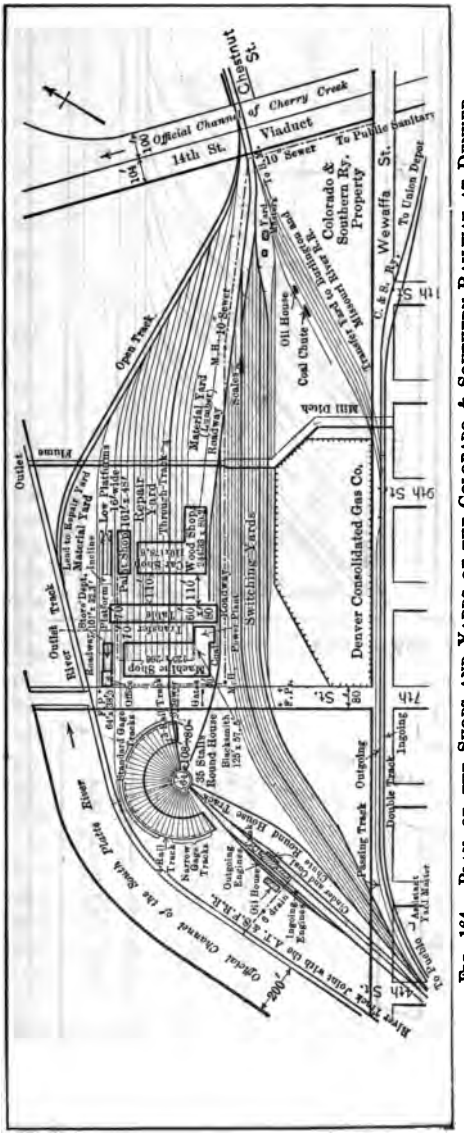


FIG. 164.—PLAN OF THE SHOPS AND YARDS OF THE COLORADO & SOUTHERN RAILWAY AT DENVER.

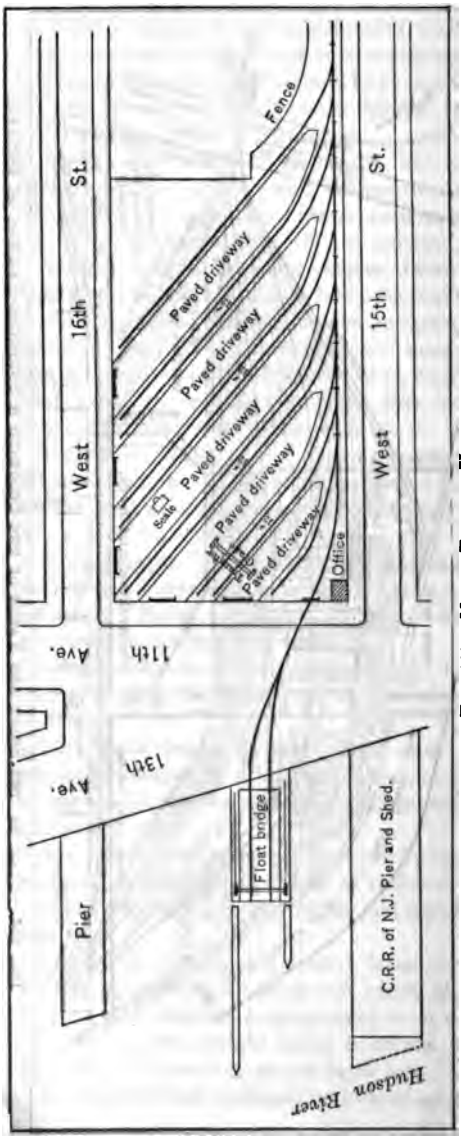


FIG. 165.—MINOR FREIGHT YARD.

other hand require longer leads and more space. No. 7 and even No. 6 frogs are sometimes used on account of economy of space, but they have the disadvantages of greater tractive resistance, greater wear and tear on track and rolling stock, and greater danger of derailment.

The design of an existing yard is best studied by first picking out the ladder tracks and the through tracks which lead from one division of the yard to another. These are tracks which must always be kept open for the passage of trains, in contradistinction to the tracks on which cars may be left standing, even though it is only for a few mo-

ments, while drilling is being done. Such a set of tracks, which may be called the skeleton of the yard, is shown by heavy lines in Fig. 164. Each line indicates a pair of rails. The tracks of the storage yards are shown by the lighter lines.

379. Minor freight yards. Fig. 165 illustrates a freight yard on the New York harbor front to which cars are brought on floats. Ten team tracks for the transfer of freight between cars and teams have been provided in a very limited space. Great ingenuity is often required to obtain the desired facilities without the use of excessively sharp curvature. The limiting radius which will permit cars to pass a curve without adjacent corners touching is about 175 feet. Extension coupler bars, although inconvenient, will make possible the use of still sharper curves.

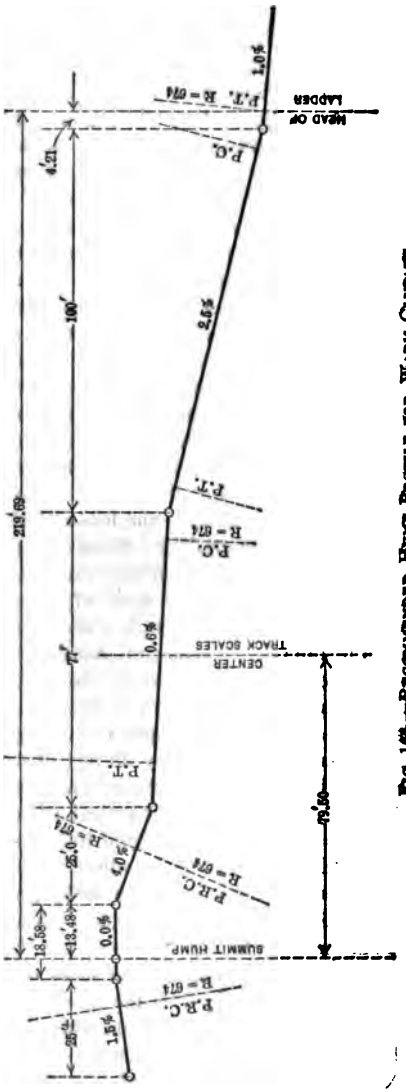


FIG. 165.—RECOMMENDED HUMP PROFILE FOR WARM CLIMATE.

380. Hump yards. The operation of hump yards makes it possible to develop the necessary potential energy for car movement by a switching engine with the maximum of economy, while the classification is accomplished in the minimum of time. The cars are pushed up the grade and over the summit, from which they begin immediately to descend on a grade which is preferably 4%. As each "cut" of one or more cars reaches the 4% grade, gravity accelerates its motion and it separates automatically from the cars behind it. Each cut then passes down the ladder track until it reaches the particular body track on which it is desired to be run. **Grades.** In Chapter XVI, it is elaborated that track resistance is greater in winter than in summer, and also that it is much greater on switch tracks than on straight unbroken track. The difference between cold-weather and warm-weather resistance is so great that the length or rate of the acceleration grade required to furnish the necessary energy varies with the temperature or climate. The Amer. Rwy. Eng. Assoc. in 1917 adopted three typical profiles for humps, designed for "cold, moderate and warm climates." The designs also include the location of track scales (see § 382) which modify the grading. Some of the grades are only nominal since the transition from one grade to another requires such long vertical curves (see §§ 84–87) that they occupy the entire length of the nominal grades, and the profile over the hump, and for some distance beyond, consists of a series of compounded vertical curves. For example, the profile which is recommended for warm climates is shown in Fig. 166. Nominally the summit is reached by a short length of 1.5% grade, with a level grade at the summit followed by 25 feet of 4% down-grade and then 77 feet of 0.6% down-grade, on which is located the track scales. But Fig. 166 shows that a vertical curve of 674 feet radius starts from the 1.5% grade, is tangent to the level grade at the summit, and reaches the 4% grade, where it reverses into an up-curving 674-ft. curve which joins the 0.6% grade. The recommended profiles for "moderate and cold" climates can be constructed, similar to that in Fig. 166 from the data in the tabular form. Note that the length or steepness of the acceleration grades and of the ladder track is increased as the climate is colder. If the grades are too low the cars will not reach their desired destinations; if too steep, there must be an unnecessary use of brakes or a destructive bumping of cars on the body tracks. Never-

Locality.	Hump-level length.	Accel. grade.	Scale grade.	Accel. grade.	Ladder track.	Radius vert. curve.
Warm climate	18.58′	25′ 4%	77′ 0.6%	100′ 2.5%	1.0%	674′
Moderate "	28.75′	37.5′ 4	89′ 0.8	100′ 3.0	1.25	1040′
Cold "	39.30′	50′ 4	100′ 1.0	100′ 4.0	1.5	1428.6′

theless, as will be shown in § 438, the actual resistance of cars through switches is so variable that an excess of power *must* be provided to prevent the stalling of *some* cars before they reach their destination. The grade from the receiving track to the hump should be such that one engine can push the maximum train over the hump. Since empty cars have a greater tractive resistance per ton than loaded cars, they require a steeper grade to maintain the same velocity, and, therefore, when tracks are set aside for the use of empty cars, the grade leading to such empty tracks should be increased if possible. **Operation.** To operate such a hump efficiently, the yard clerk makes up a triple (or quadruple) list for each freight train arriving at the yard for distribution. One of these lists is given to the man cutting off the cars at the top of the hump, and one to the towerman, if the switches are operated from the tower, or one to each switch tender if the switches are hand-operated. Each list contains in the first column the consecutive number of the cut, in the second column the number of the track on which that cut of cars is to be placed, and in the third column the number of cars cut. Cut No. 1 is the first car (or cars) to go over the hump. A brakeman, or " rider," accompanies each car, or group of cars. To avoid the great waste of time required for these riders to walk back to the hump, it has been found economical in some large yards to have a track for the exclusive use of a car, especially fitted for easy jumping on or off, operated, perhaps, by a switching engine, or possibly by gasoline, which picks up the riders and carries them back to the hump. The aggregate time saved justifies the expenditure. The **scale grade** has been designed in each case so that each car will pass over the scale with a maximum velocity of four miles per hour, which means that the car shall be entirely on the scale platform for a minimum time of three seconds. Although the grade over the scales may be as high as 1% for motion weighing, the weighing mechanism must be installed on a level plane and the weighing rails are blocked up to the desired grade.

381. Ladder tracks. Twenty-seven types of ladder tracks are shown in the 1917 Committee report to the A. R. E. A., but nearly one-half of the ladders reported in actual use belong to type *a*, Fig. 166*a*, and about one-half of the remainder belong to types *b* and *c*. The other twenty-four types are chiefly expansions and developments of the three types shown. Note that in types *a* and *c*, the switches are, in each case, in a straight line along

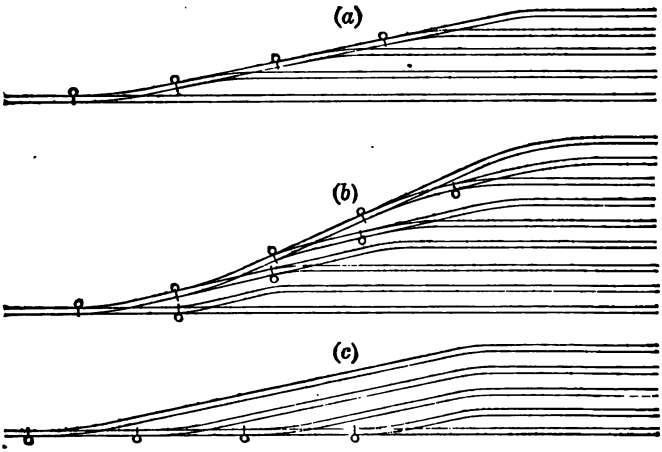


FIG. 166a.—TYPES OF LADDER TRACKS.

one of the tracks, which simplifies the working of the switches, whether they are worked from a tower or on the ground by hand.

382. Track scales. The standard design for a hump yard, § 380, shows a track-scale grade, as an integral part of the design, located just beyond the hump. It has been found that it is practicable to weigh cars with sufficient accuracy while the cars are in motion, provided the speed does not exceed 4 miles per hour, or 5.87 feet per second, and provided that the length of the scale is such that the car is entirely and alone on the scale for a minimum of three seconds. This condition will be fulfilled when the scale is 17.6 feet longer than the distance from front to rear axle of the car. Scales with lengths of 50, 56 and 60 feet are considered standard. The sensibility reciprocal is the weight required to be added or removed from the live rails to turn the beam from a horizontal position of equilibrium in the

center of the trig loop to a position of equilibrium at either limit of its travel; such weight shall not exceed 50 lbs. in any case. The **tolerance** to be allowed on the first field test, after installation corrections, of all new railroad track scales, shall not exceed $\frac{1}{20}$ of 1%, or 50 lbs. per 100,000 lbs. for any position of the test-car load on the scale. The minimum test-car load shall be 30,000 lbs. **Location.** The scale should be elevated above the other tracks of the yard so that surface drainage shall not drain into the pit. The location of the scale near a hump summit fits in with this requirement. The **foundations** should be made of concrete. The finished floor of the pit should be at least 7 feet below the base of the rails; the floor should be at least 6 inches thick and as much thicker as a soft sub-soil might demand. The concrete of the walls and floor should be effectively waterproofed to exclude sub-soil water. A sump, with provision for drainage outfall, should be provided to dispose of any rainfall or other drainage which might accumulate in the pit. **Approach.** There should be at least 50 feet of tangent track on each approach. The approach tracks should be carried on approach walls or piers extending 15 to 25 feet from the end walls of the pit, so that accurate line and surface of the approach tracks is maintained and so that the approach rails may be absolutely anchored against creeping. **Dead rails,** offsetted 16″ from the live rails, will carry cars over the scale pit, when so desired, without any stress or influence on the scale mechanism. One dead rail may be supported on the side wall of the pit and the other on pedestals or on transverse floor beams which are spaced (usually) 2′ 6″ and which are independent of the weighing platform. Details must conform to the somewhat varying plans of various manufacturers.

383. Transfer cranes. These are almost an essential feature for yards doing a large business. The transportation of built-up girders, castings for excessively heavy machinery, etc., which weigh 5 to 30 tons and even more, creates a necessity for machinery which will easily transfer the loads from the car to the truck and *vice versa*. An ordinary "gin-pole" will serve the purpose for loads which do not much exceed 5 tons. A fixed framework, covering a span long enough for a car track and a team space, with a trolley traveling along the upper chord, is the next design in the order of cost and convenience. Increasing the span so that it covers two car tracks and two team spaces

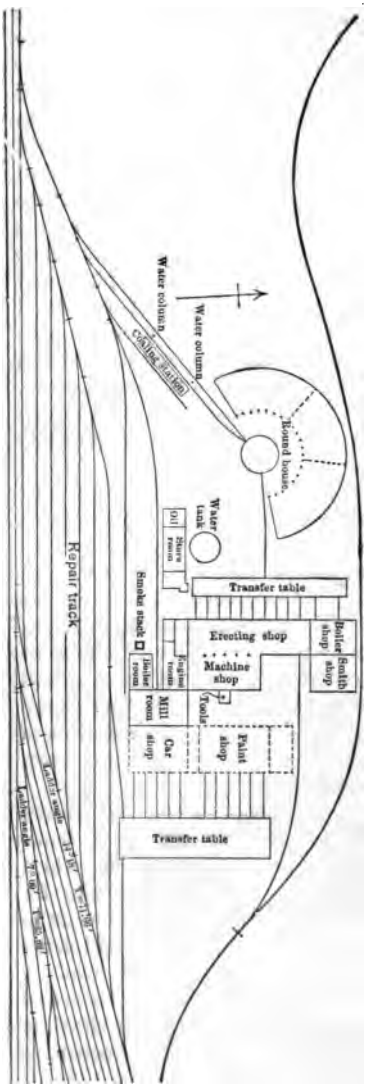
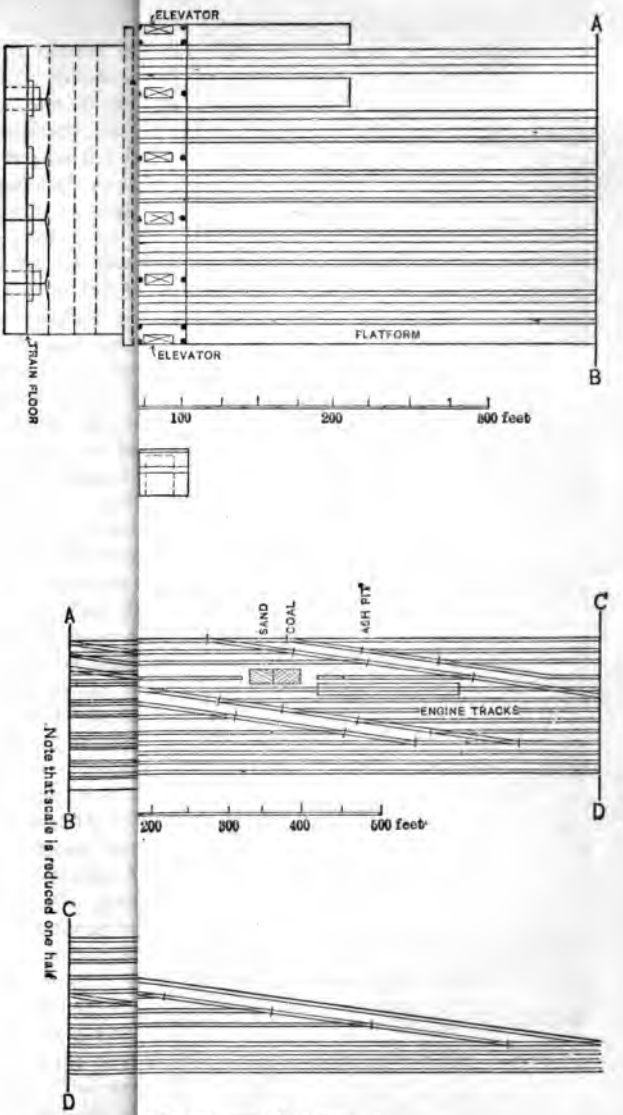


FIG. 167.—ENGINE YARD AND SHOPS, URBANA, ILL.

will very materially increase the capacity. Making the frame movable so that it travels on tracks which are parallel to the car tracks, giving the frame a longitudinal motion equal to two or three car lengths, and finally operating the raising and traveling mechanism by power, the facility for rapidly disposing of heavy articles of freight is greatly increased. Of course only a very small proportion of freight requires such handling, and the business of a yard must be large or perhaps of a special character to justify and pay for the installation of such a mechanism. A transfer crane, evidently of the fixed type, is indicated in Fig. 165.

384. Engine Yards or Terminals. These should be located so that there is easy access to both the main line and the various yards, with the fewest possible reverse or conflicting movements. The yards must contain all the tracks, buildings, structures, and facilities which are necessary for the maintenance, care, and storage of locomotives and for providing them

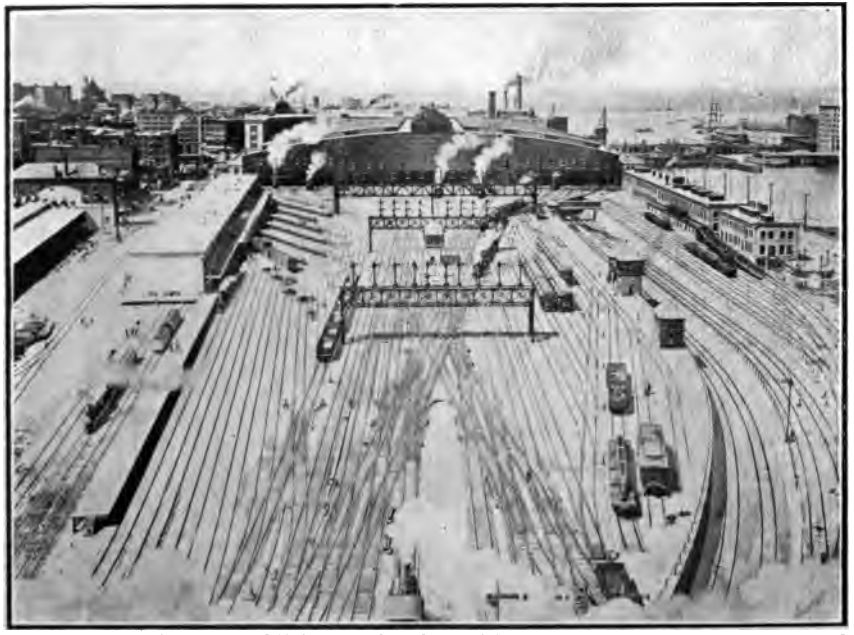


ND PASSENGER TERMINAL.

ed supplies. The supplies are fuel, water, sand, ow, etc. Ash-pits are generally necessary for the economical disposition of ashes; engine-houses for the storage of engines and as a place where can be quickly made. A turntable is another all requirement. The arrangement of all these engine yard should properly depend on the form In general they should be grouped together and near as possible to the place where through he trains just brought in and where they couple ed outgoing trains, so that all unnecessary run- r be avoided. Switching engines should be able , take their supplies and pass around waiting In Figs. 164, 167 and 167*a* are shown designs e studied with reference to the relative arrange- rd facilities.

nger terminals. The word **terminal** is applied railway station at an actual terminus, beyond is are run, but also to an important intermediate trains are assembled, assorted, classified and two types are called dead-end and through term- n. Rwy. Eng. Assoc. has adopted standard plans ese two types. Even when there is good reason some of the details, certain principles should be ar as possible. Some of these principles, which ly to both types, are as follows:

d terminals. See Fig. 167*a*. The track level and raised above the street level, so as to permit any oss streets to run *under* the tracks. A **ramp** on is indicated in the section of the terminal building. serves a pair of tracks whose centers are 28′ ng 5′ 6″ from the track center to the edge of the platforms themselves are 17′ wide. The length ns vary from about 600 to over 1100 feet, but the ir number should depend on the extent of business . The intermediate platforms are protected for of their length by "butterfly" roofs upported lumns, the roofs draining inward to lo udinal center, which discharge into leaders alo e t o sets of **ladder tracks,** with single or t with each one of the platform



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SOUTH BOSTON TERMINAL.

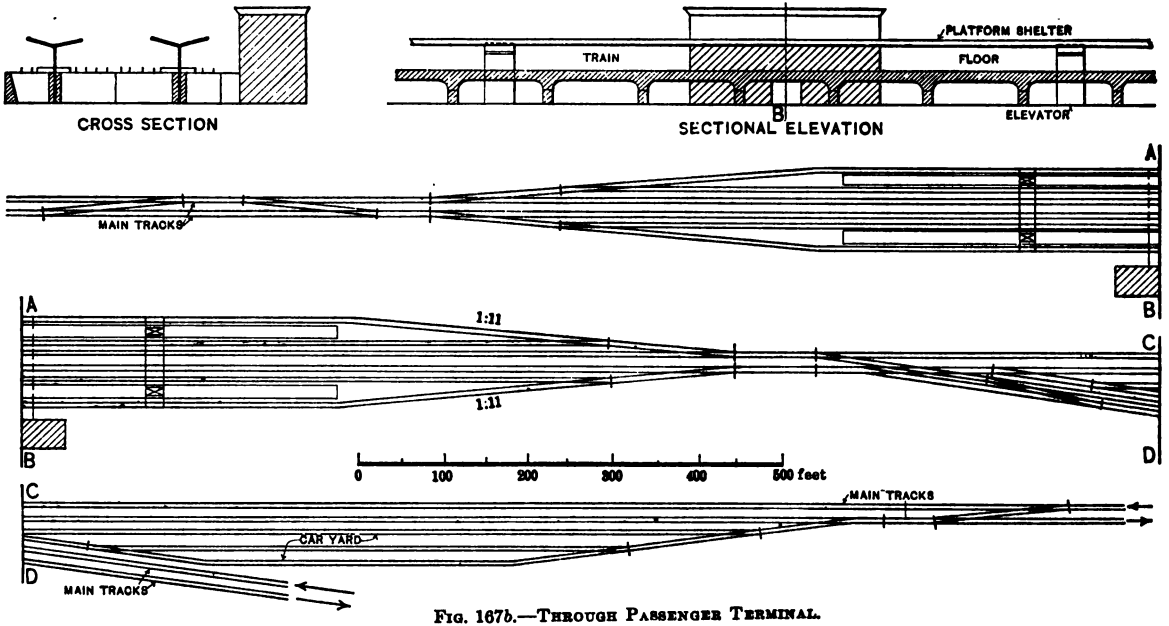


FIG. 167b.—THROUGH PASSENGER TERMINAL.

either main track may be directly connected with any platform track. The space under the tracks, and at the street level, is utilized for rooms for baggage, mail and express, which are carried to the track level by elevators, one to each platform. The coach cleaning yard has a series of parallel tracks 13′ to 18′ apart c.c. between a pair of parallel ladder tracks. The engine yard has a sand and coaling station, ash-pit with ash-car track, oil-house, water tank, engine supply house, turntable, shop and shop-yard tracks.

(b) **Through terminals.** See Fig. 167*b*. As above, the train floor level is above the street level. A passage way runs transversely under the tracks, from which two pairs of stairways run in each direction to the station platforms. As before, the baggage, mail and express rooms are on the street level, under the tracks, and connect with the platforms by elevators. The two middle tracks are main tracks, which may be used by any trains, through freight or passenger, which do not stop at the station. The two platforms each have two tracks, one on each side. The three tracks of each group run into one main track for each direction of movement, at either end of each platform. The two main tracks are connected by two crossovers, arranged for direct and reversed movement. The figure also shows an arrangement of switches for the junction of a branch line with the main line, with three car-yard tracks in the Y of the junction.

stations located so frequently that only one train will be allowed between two consecutive offices at once, it virtually becomes a block system even though it is not called such. When such a system is adhered to rigidly, it is called an *absolute block* system. But when operating on this system, a delay of one train will necessarily delay every other train that follows closely after. A portion, if not all, of the delay to subsequent trains may be avoided, although at some loss of safety, by a system of permissive blocking. By this system an operator may give to a succeeding train a "clearance card" which permits it to pass into the next block, but at a reduced speed and with the train under such control that it may be stopped on very short notice, especially near curves. One element of the danger of this system is the *discretionary* power with which it invests the signalmen, a discretion which may be wrongfully exercised. A modification (which is a fruitful source of collisions on single-track roads) is to order two trains to enter a block approaching each other, and with instructions to pass each other at a passing siding at which there is no telegraph-station. When the instructions are properly made out and literally obeyed, there is no trouble, but every thousandth or ten thousandth time there is a mistake in the orders, or a misunderstanding or disobedience, and a collision is the result. The telegraph line, a code of rules, a corps of operators, and signals under the immediate control of the operators, are all that is absolutely needed for the simple manual system.

387. Development of the manual system. One great difficulty with the simple system just described is that each operator is practically independent of others except as he may receive general or specific orders from a train-dispatcher at the division headquarters. Such difficulties are somewhat overcome by a very rigid system of rules requiring the signalmen at each station to keep the adjacent signalmen or the train-dispatcher informed of the movements of all trains past their own stations. When these rules (which are too extensive for quotation here) are strictly observed, there is but little danger of accident, and a neglect by any one to observe any rule will generally be apparent to at least one other man. Nevertheless the safety of trains depends on *each* signalman doing his duty, and a little carelessness or forgetfulness on the part of any one man may cause an accident. The signaling between stations *may* be done by

CHAPTER XIV.

BLOCK SIGNALING.

GENERAL PRINCIPLES.

385. Two fundamental systems. The growth of systems of block signaling has been enormous within the last few years—both in the amount of it and in the development of greater perfection of detail. The development has been along two general lines: (*a*) the *manual*, in which every change of signal is the result of some definite action on the part of some signalman, but in which every action is so controlled or limited or subject to the inspection of others that a mistake is nearly, if not quite, impossible; (*b*) the *automatic*, in which the signals are operated by mechanism, which cannot set a wrong signal as long as the mechanism is maintained in proper order. The fundamental principles of the two systems will be briefly outlined, after which the chief details of the most common systems will be pointed out.

386. Manual systems. Small traffic roads are usually operated on the basis of the "train-order system." A "train dispatcher" controls the movement of every train on his division and telegraphs orders to men (who are frequently station agents) at various points along the line, who transmit these orders to the trainmen as the trains reach these points. A train-order signal station, whether at a regular traffic station or in a special cabin, has "train-order signals" which, when in the stop position, inform the engineman and conductor that they are to receive orders at the telegraph office; the clear position informs them that there are no orders for them. When more than one train is allowed on a single track between two consecutive train-order stations, the engineman and conductor of each train has strict orders with reference to the other train, for example, that the trains are to pass at some siding where there is no telegraphic station. A very strict code of rules has been developed which, when literally followed, ensures safety of operation, but these rules cannot eliminate the human element, or the liability of personal negligence or error. When such a system is applied to a double-track road, or even to a single-track road, with train-order signal

stations located so frequently that only one train will be allowed between two consecutive offices at once, it virtually becomes a block system even though it is not called such. When such a system is adhered to rigidly, it is called an *absolute block* system. But when operating on this system, a delay of one train will necessarily delay every other train that follows closely after. A portion, if not all, of the delay to subsequent trains may be avoided, although at some loss of safety, by a system of permissive blocking. By this system an operator may give to a succeeding train a "clearance card" which permits it to pass into the next block, but at a reduced speed and with the train under such control that it may be stopped on very short notice, especially near curves. One element of the danger of this system is the *discretionary* power with which it invests the signalmen, a discretion which may be wrongfully exercised. A modification (which is a fruitful source of collisions on single-track roads) is to order two trains to enter a block approaching each other, and with instructions to pass each other at a passing siding at which there is no telegraph-station. When the instructions are properly made out and literally obeyed, there is no trouble, but every thousandth or ten thousandth time there is a mistake in the orders, or a misunderstanding or disobedience, and a collision is the result. The telegraph line, a code of rules, a corps of operators, and signals under the immediate control of the operators, are all that is absolutely needed for the simple manual system.

387. Development of the manual system. One great difficulty with the simple system just described is that each operator is practically independent of others except as he may receive general or specific orders from a train-dispatcher at the division headquarters. Such difficulties are somewhat overcome by a very rigid system of rules requiring the signalmen at each station to keep the adjacent signalmen or the train-dispatcher informed of the movements of all trains past their own stations. When these rules (which are too extensive for quotation here) are strictly observed, there is but little danger of accident, and a neglect by any one to observe any rule will generally be apparent to at least one other man. Nevertheless the safety of trains depends on *each* signalman doing his duty, and a little carelessness or forgetfulness on the part of any one man may cause an accident. The signaling between stations *may* be done by

ordinary telegraphic messages or by telephone, but is frequently done by electric bells, according to a code of signals, since these may be readily learned by men who would have more difficulty in learning the Morse code.

In order to have the signalmen mutually control each other, the "controlled manual" system has been devised. The first successful system of this kind which was brought into extensive use is the "Sykes" system, of which a brief description is as follows: Each signal is worked by a lever; the lever is locked by a latch, operated by an electro-magnet, which, with other necessary apparatus, is inclosed in a box. When a signal is set at danger, the latch falls and locks the lever, which cannot be again set free until the electro-magnet raises the latch. The magnet is energized only by a current, the circuit of which is closed by a "plunger" at the *next* station ahead; just above the plunger is an "indicator," also operated by the current, which displays the words *clear* or *blocked*. (There are variations on this detail.) When a train arrives at a block station (*A*), the signalman should have previously signaled to the station *ahead* (*B*) for permission to free the signal. The man ahead (*B*) pushes in the "plunger" on his instrument (assuming that the previous train has already passed him), which electrically opens the lock on the lever at the previous station (*A*). The signal at *A* can then be set at "safety." As soon as the train has passed *A* the signal at *A* must be set at "danger." A further development is a device by which the mere passage of the train over the track for a few feet beyond the signal will automatically throw the signal to "danger." After the signal once goes to danger, it is automatically locked and cannot be released except by the man in advance (*B*), who will not do so until the train has passed him. The "indicator" on *B*'s instrument shows "blocked" when *A*'s signal goes to danger after the train has passed *A*, and *B*'s plunger is then locked, so that he cannot release *A*'s signal while a train is in the block. As soon as the train has passed *A*, *B* should prepare to get his signals ready by signaling ahead to *C*, so that if the block between *B* and *C* is not obstructed, *B* may have his signals at "safety" so that the train may pass *B* without pausing. The student should note the great advance in safety made by the Sykes system; a signal cannot be set free except by the combined action of two men, one the man who actually operates the signal and

the other the man at the station ahead, who frees the signal electrically and who by his action certifies that the block immediately ahead of the train is clear.

A still further development makes the system still more "automatic" (as described later), and causes the signal to fall to danger or to be kept locked at danger, if even a single pair of wheels comes on the rails of a block, or if a switch leading from a main track is opened.

388. Permissive blocking. "Absolute" blocking renders accidents due to collisions almost impossible unless an engineer runs by an adverse signal. The signal mechanism is usually so designed that, if it gets out of order, it will inevitably fall to "danger," i.e., as described later, the signal-board is counterbalanced by a weight which is much heavier. If the wire breaks, the counterweight will fall and the board will assume the horizontal position, which always indicates "danger."* But it sometimes happens that when a train arrives at a signal-station, the signalman is unable to set the signal at safety. This may be because the previous train has broken down somewhere in the next block, or because a switch has been left open, or a rail has become broken, or there is a defect of some kind in the electrical connections. In such cases, in order to avoid an indefinite blocking of the whole traffic of the road, the signalman may give the engineer a "caution-card" or a "clearance card," which authorizes him to proceed slowly and with his train under complete control into the block and through it if possible. If he arrives at the next station without meeting any obstruction it merely indicates a defective condition of the mechanism, which will, of course, be promptly remedied. Usually the next section will be found clear, and the train may proceed as usual. On roads where the "controlled manual" system has received its highest development, the rules for permissive blocking are so rigid that there is but little danger in the practice, unless there is an absolute disobedience of orders.

389. Automatic systems. By the very nature of the case, such systems can only be used to indicate to the engineers of trains something with reference to the passage of previous

* This was written on the basis of the older system, in which the semaphore swings through the *lower* right-hand quadrant. The most recent practice swings the semaphore through the *upper* right-hand quadrant. A break in the wire holding the semaphore vertical will cause it to fall to horizontal position without the aid of a counterweight.

trains. The complicated shifting of switches and signals which is required in the operation of yards and terminals can only be accomplished by "manual" methods, and the only automatic features of these methods consist in the mechanical checks (electric and otherwise), which will prevent wrong combinations of signals. But for long stretches of the road, where it is only required to separate trains by at least one block length, an automatic system is generally considered to be more reliable. As expressed forcibly by a railroad manager, "an automatic system does not go to sleep, get drunk, become insane, or tell lies when there is any trouble." The same cannot always be said of the employés of the manual system.

The basic idea of all such systems is that when a train passes a signal-station (A), the signal automatically assumes the "danger" position. This may be accomplished electrically, pneumatically, or even by a direct mechanism. When the train reaches the end of the block at B and passes into the next one, the signal at B will be set at danger and the signal at A will be set at safety. The lengths of the blocks are usually so great that the only practicable method of controlling from B a mechanism at A is by electricity, although the actual motive power at A may be pneumatic or mechanical. At one time the current from A to B was run only through wires. This method has the very positive advantage of reliability, definite resistance to the current, and small probability of short-circuiting or other derangement. But now all such systems use the rails for a track circuit and this makes it possible to detect the presence of a single pair of wheels on the track anywhere in the block, or an open switch, or a broken rail. Any such circumstances, as well as a defect in the mechanism, will break or short-circuit the current and will cause the signal to be set at danger. To prevent an indefinite blocking of traffic owing to a signal persistently indicating danger, most roads employing such a system have a rule substantially as follows: When a train finds a signal at danger, after waiting one minute (or more, depending on the rules), it may proceed slowly, expecting to find an obstruction of some sort; if it reaches the next block without finding any obstruction and finds the next signal clear, it may proceed as usual, but must promptly report the case to the superintendent. Further details regarding these methods will be given later. See § 394.

390. "Distant" signals. The close running of trains that is required on heavy-traffic roads, especially where several branches combine to enter a common terminal, necessitates the use of very short blocks. A heavy train running at high speed can hardly make a "service" stop in less than 2000 feet, while the curves of a road (or other obstructions) frequently make it difficult to locate a signal so that it can be seen more than a few hundred feet away. It would therefore be impracticable to maintain the speed now used with heavy trains if the engineer had no foreknowledge of the condition in which he will find a signal until he arrives within a short distance of it. To overcome this difficulty the "distant" signal was devised. This is placed about 1800 or 2000 feet from the "home" signal, and is interlocked with it so that it gives the *same* signal. The distant signal is frequently placed on the same pole as the home signal of the previous block. When the engineer finds the distant signal "clear," it indicates that the succeeding home signal is also clear, and that he may proceed at full speed and not expect to be stopped at the next signal; for the distant signal cannot be cleared until the succeeding home signal is cleared, which cannot be done until the block succeeding that is clear. A clear distant signal therefore indicates a clear track for two succeeding blocks. When the engineer finds the distant signal blocked, he need not stop (providing the home signal is clear). It simply indicates that he must be prepared to stop at the next home signal and must reduce speed if necessary. It may happen that by the time he reaches the succeeding home signal it has already been cleared, and he may proceed without stopping. This device facilitates the rapid running of trains, with no loss of safety, and yet with but a moderate addition to the signaling plant.

391. "Advance" signals. It sometimes becomes necessary to locate a signal a few hundred feet short of a regular passenger-station. A train might be halted at such a signal because it was not cleared from the signal-station ahead—perhaps a mile or two ahead. For convenience, an "advance" signal may be erected immediately beyond the passenger-station. The train will then be permitted to enter the block as far as the advance signal and may deliver its passengers at the station. The advance signal is interlocked with the home signal back of it, and cannot be cleared until the home signal is cleared and

the entire block ahead is clear. In one sense it adds another block, but the signal is entirely controlled from the signal station back of it.

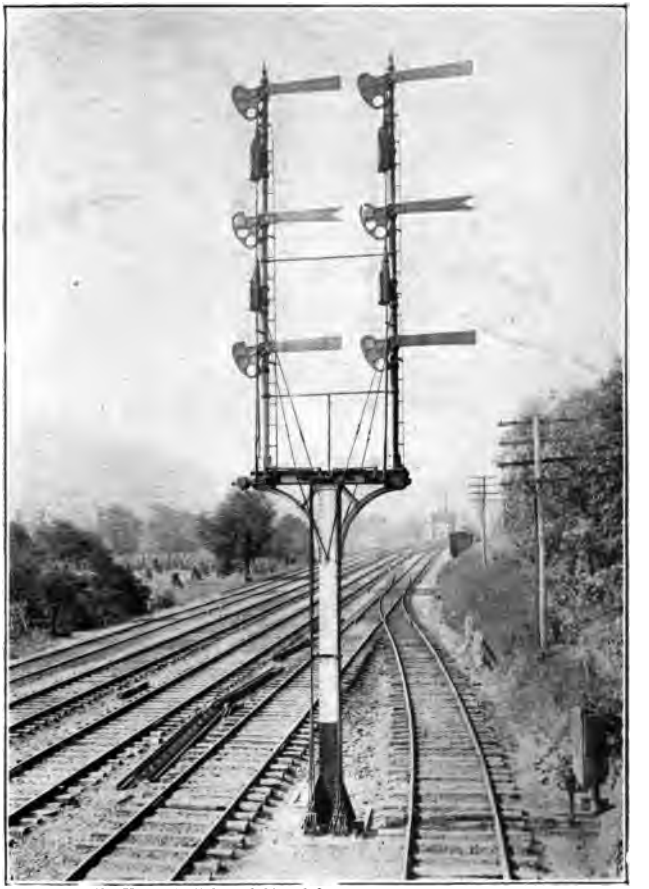
MECHANICAL DETAILS.

308. Signals. The primitive signal is a mere cloth flag. A better signal is obtained when the flag is suspended in a suitable place from a fixed horizontal support, the flag weighted at the bottom, and so arranged that it may be drawn up and out of sight by a cord which is run back to the operator's office. The next step is the substitution of painted wood or sheet metal for the cloth flag, and from this it is but a step to the standard semaphore on a pole, as is illustrated in Fig. 168. The simple flag, operated for convenience with a cord, is the signal employed on thousands of miles of road, where they perhaps make no claim to a block-signal system, and where the trains are run on the "train-order system."

Semaphore boards. These are about 5 feet long, 8 inches wide at one end, and tapered to about 6 inches wide at the hinge end. The boards are fastened to a casting which has a ring to hold a red glass which may be swung over the face of a lantern, so as to indicate a red signal. "Distant" signal-boards usually have their ends notched or pointed; the "home" signal-boards are square ended. The boards are always to the *right* of the hinge when a train is approaching them. The "home" signals are generally painted red and the "distant" signals green, although these colors are not invariable. The backs of the boards are painted white. Therefore any signal-board which appears on the *left* side of its hinge will also appear *white*, and is a signal for traffic in the opposite direction, and is therefore of no concern to an engineman.

Poles and bridges. When the signals are set on poles, they are always placed on the right-hand side of the track. When there are several tracks, four or more, a bridge is frequently built and then each signal is over its own track. The signals for two tracks, operated in the same direction, may be placed on one pole by having a cross-piece which supports two "masts," see Fig. 168. In that figure the signals on the left-hand mast control the second track at the left of the signal; those on the right-hand mast control the track just to the left of the signal.

(*To face page* 418.)



Courtesy of the Union Switch and Signal Co.

FIG. 168.—SEMAPHORES.

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Courtesy of the Union Switch and Signal Co.

FIG. 169.—SIGNAL BRIDGE.

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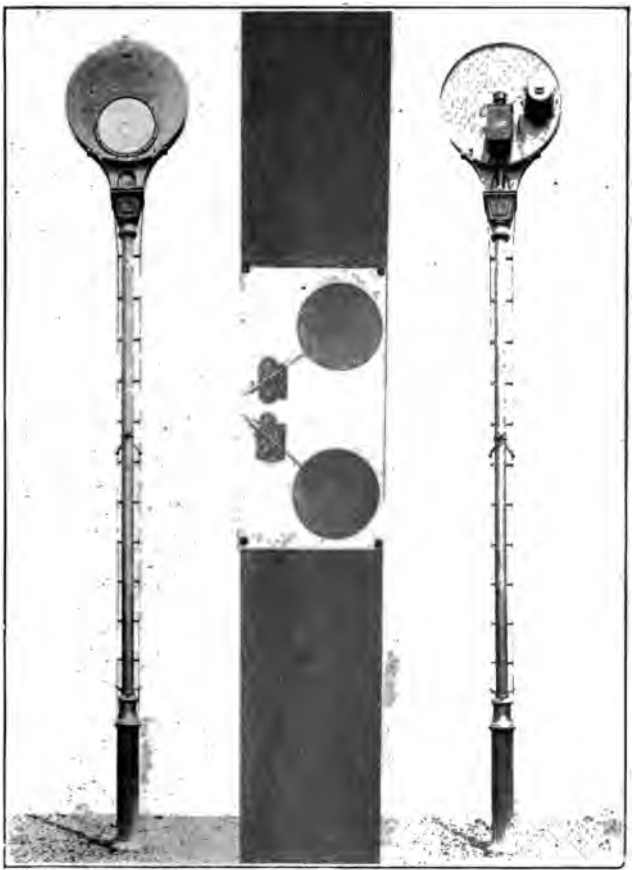


Courtesy of the Union Switch and Signal Co.

FIG. 170.—"BANJO" SIGNALS.

A train movement, from the switch track at the right of the signal on to the main track, is controlled by the "dwarf" signal at the right of the switch track. The signals controlling the two tracks at the extreme left are not shown. The building at the left of the track in the extreme background is apparently the signal tower controlling this signal.

In Fig. 169 is shown a "bridge" and the two signals (home and distant), for each track. The two pairs of signals on the two right-hand poles are extended to the right and show that the movement of trains on those tracks is away from the observer. The darkness of the blades in the picture shows that they are painted dark, probably orange or red. The other blades show light (because painted white), and extend to the left but would appear to the right to an engineman on either left-hand track coming toward the observer. Incidentally the picture shows, over the two right-hand tracks, the ropes of a "tickler" (see § 375), to protect brakemen on the tops of cars which will enter the tunnel shown in the background.

"Banjo" signals. This name is given to a form of signal, illustrated in Fig. 170, in which the indication is taken from the *color* of a round disk inclosed with glass. The great argument in their favor is that they may be worked by an electric current of low voltage, which is therefore easily controlled; that the mechanism is entirely inside of a case, is therefore very light, and is not exposed to the weather. The argument urged against them is that it is a signal of *color* rather than *form* or *position*, and that in foggy weather the signal cannot be seen so easily; also that unsuspected color-blindness on the part of the engineman may lead to an accident. Notwithstanding these objections, this form of signal is used on thousands of miles of line in this country.

393. Wires and pipes. Signals are usually operated by levers in a signal-cabin, the levers being very similar to the reversing-lever of a locomotive. The distance from the levers to the signals is, of course, very variable, but it is sometimes 2000 feet. The connecting-link for the most distant signals is usually No. 9 wire; for nearer signals and for all switches operated from the cabin it may be 1-inch pipe. When not too long, one pipe will serve for both motions, forward and back. When wires are used, it is sometimes so designed (in the cheaper systems) that one wire serves for one motion, gravity being de-

pended on for the other, but now all good systems require two wires for each signal.

Compensators. Variations of temperature of a material with as high a coefficient as iron will cause very appreciable difference of length in a distance of several hundred feet, and a dangerous lack of adjustment is the result. To illustrate: A fall of 60° F. will change the length of 1000 feet of wire by

$$1000 \times 60 \times .0000065 = 0.39 \text{ foot} = 4.68 \text{ inches.}$$

A much less change than this will necessitate a readjustment of length, unless automatic compensators are used. A compensator for pipes is very readily made on the principle illustrated in Fig. 171. The problem is to preserve the distance between a and d constant regardless of the temperature. Place the compensator half-way between a and d, or so that $ab = cd$. A fall of temperature contracts ab to ab'. Moving b to b' will cause c to move to c', in which $bb' = cc'$. But cd has also shortened to $c'd$; therefore d remains fixed in position.

The regulations of the Am. Rwy. Eng. Assoc. require that "A compensator shall be provided for each pipe line over fifty (50) feet in length and under eight hundred (800) feet, with crank-arms eleven by thirteen (11×13) inch centers. From eight hundred (800) to twelve hundred (1200) feet in length, crank-arms shall be eleven by sixteen (11×16) inch centers. Pipe lines over twelve hundred (1200) feet in length shall be provided with an additional compensator.

"Compensators shall have one sixty (60) degree and one one hundred and twenty (120) degree angle-cranks and connecting link, mounted in cast iron base, having top of center pins supported. The distance between center of pin-holes shall be twenty-two (22) inches."

The compensator should be placed in the middle of the length when only one is used. When two are used they should be placed at the quarter points. Note that in operating through a compensator the *direction* of motion changes; i.e., if a moves to the right, d moves to the left, or if there is compression in ab there is tension in cd, and *vice versa*. Therefore this form of compensator can only be used with pipes which will withstand compression. It has seemed impracticable to design an equally satisfactory compensator for wires, although there are several designs on the market.

ıe change of length of these bars is so great that allowance ; be made for the temperature at the time of installation. he basis of 50° as the mean temperature, the pipes are so sted that the distance between the points *b* and *c* of Fig. 171 ade greater or less than 22 inches, according to the tem- ture of installation. For example, if the temperature were ınd the length of the piping were 900 feet, the length of the ı should be adjusted so that *bc* is *less* than 22 inches by an ınt equal to $900 \times (80° - 50°) \times .0000065 = 0.1755$ feet =

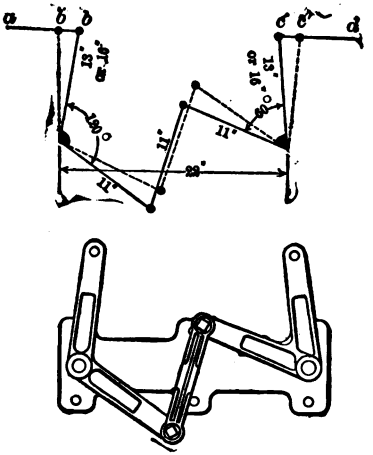


FIG. 171.—STANDARD PIPE COMPENSATOR.

inches. The length should therefore be 19.9 inches in- of 22 inches. If the mean temperature was very different in Florida) some higher temperature should be taken as al, so that the extreme range above and below the normal be approximately the same.

des around curves and angles. When wires are required ss around curves of large angle, pulleys are used, and a ı of chain is substituted for the wire. For pipes, when urve is easy the pipes are slightly bent and are guided gh pulleys. When the angle is sharper, "angles" are

The operation of these details is self-evident from an :tion of Fig. 172.

394. **Track circuit for automatic signaling.** The fundamental principle of the track circuit method of indicating a track obstruction or breakage, using direct current, is as follows: A current of low potential is run from a battery at one end of a section through one line of rails to the other end of the section, then through a relay, and then back to the battery through the other line of rails. To avoid the excessive resistance which would occur at rail joints which may become badly rusted, a wire

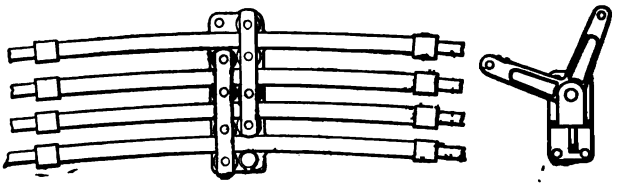


FIG. 172.—DEFLECTING-RODS AND ANGLE.

suitably attached to the rails is run around each joint. In order to insulate the rails of one section from the rails at either end and yet maintain the rails structurally continuous, the ends of the rails at these dividing points are separated by an insulator and the joint pieces are either made of wood or have some insulating material placed between the rails and the ordinary metal joint. The bolts must also be insulated. When the relay is energized by a current, it closes a local circuit at the signal-station, which will set the signal there at "safety." The resistance of the relay is such that it requires nearly the whole current to work it and to keep the local circuit closed. Therefore, when there is any considerable loss of current from one rail to the other, the relay will not be sufficiently energized, the local circuit will be broken, and the signal will automatically fall to danger. This diversion of current from one rail to the other before the current reaches the relay may be caused in several ways: the presence of a pair of wheels on the rails anywhere in the section will do it; also the breakage of a rail; also the opening of a switch anywhere in the section; also the presence of a pair of wheels on a siding between the "fouling point" and the switch. (The "fouling point" of a siding is that point where the rails first commence to approach the main track.) In Fig. 173 is shown all of the above details as well as some others.

At *A*, *B*, and the "fouling point" are shown the insulated joints. The batteries and signals are arranged for train motion to the *right*. When a train has passed the points near *A*, where the wires leave the rails for the relay, the current from the "track battery" at *B* will pass through the wheels and axles, and although no electrical connection is broken, so much current will be shunted through the wheels and axles that the weak current still passing through the relay is not strong enough to energize it against its spring and the "signal-magnet" circuit is broken, and the signal *A* goes to "danger." At the turnout the rails between the fouling point and the switch are so connected (and insulated) that a pair of wheels on these rails will produce the same effect as a pair of the main track. This is to guard against the effect of a car standing too near the switch, even though it is not on the main track. When the train passes *B*, if there is no other interruption of the current, the track battery at *B* again energizes the relay at *A*, the signal-magnet circuit at *A* is closed, and the signal is drawn to "safety."

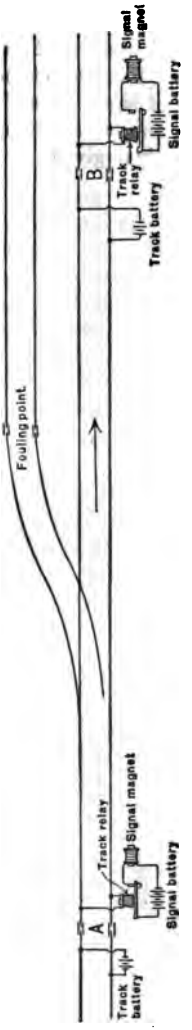


FIG. 173.

About 1903 the application of *alternating current* to signaling circuits was invented. This not only permits the substitution of a. c. circuit for track batteries, but also makes it possible to utilize the track circuit method to indicate obstructions or rail breakages even when the track is the return circuit for an electrified road. But an explanation of this development would be too long for this text-book. It is

given in a 548-page book called "Alternating Current Signaling," published by the Union Switch & Signal Co., Swissvale, Pa.

This chapter also omits all references to "interlocking plants," which are essential features of the operation of large terminal yards. Even an elementary treatment of the present development of signaling and interlocking would require a large textbook, and, therefore, nothing more than the above brief outline will be here given.

CHAPTER XV.

ROLLING-STOCK.

(It is perhaps needless to say that the following chapter is in no sense a course in the design of locomotives and cars. Its chief idea is to give the student the elements of the construction of those vehicles which are to use the track which he may design—to point out the mutual actions and reactions of vehicle against track and to show the effect on track wear of variations in the design of rolling-stock. The most of the matter given has a direct practical bearing on track-work, and it is considered that all of it is so closely related to his work that the civil engineer may study it with profit. For "Stresses in Track," see Chap. XXV.)

WHEELS AND RAILS.

395. Effect of rigidly attaching wheels to their axles. The wheels of railroad rolling-stock are invariably secured rigidly to the axles, which therefore revolve with the wheels. The chief reason for this is to avoid excessive wear between the axles and the wheels.

Any axle must always be somewhat loose in its journals. A sidewise force P (see Fig. 174) acting against the circumference of the wheel will produce a much greater pressure on the axle at S and S', and if the wheel moves on the axle, the wear at S and S' will be excessive. But when the axle is fitted to the wheel with a "forced fit" and does not revolve, the mere *pressure* produced at S is harmless. When two wheels are fitted tight to an axle, as in Fig. 175, and the axle revolves in the journals aa, a sidewise pressure of the rail against the wheel flange will only produce a slight and harmless increase of the journal pressure Q, although at Q there is sliding contact. Twist-

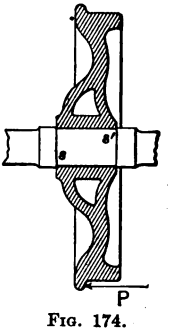


FIG. 174.

ing action in the journals is thus practically avoided, since a small pressure at the journal-boxes at each end of the axle suffices to keep the axle truly in line.

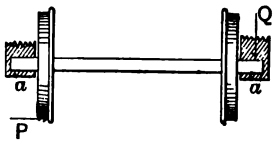


FIG. 175.

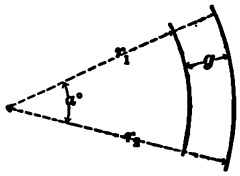


FIG. 176.

On the other hand, when the wheels are rigidly attached to their axles, both wheels must turn together, and when rounding curves, the inner rail being shorter than the outer rail, one wheel must slip by an amount equal to that difference of length. The amount of this slip is readily computable:

$$\text{Longitudinal slip} = \frac{2\pi a°}{360°}(r_2 - r_1) = \frac{2\pi g}{360°}a° = Ca°, \quad . \quad (102)$$

in which C is a constant for any one gauge, and g = the track gauge $= (r_2 - r_1)$. For standard gauge (4.708) the slip is .08218 foot per degree of *central angle*. This shows that the longitudinal slipping around any curve of any given central angle will be *independent of the degree of the curve*. The constant (.08218) here given is really somewhat too small, since the true gauge that should be considered is the distance between the lines of tread on the rails. This distance is a somewhat indeterminate and variable quantity, and probably averages 4.90 feet, which would increase the constant to .086. The slipping may occur by the inner wheel slipping ahead or the outer wheel slipping back, or by both wheels slipping. The total slipping will be constant in any case. The slipping not only consumes power, but wears both the wheels and the rail. But even these disadvantages are not sufficient to offset the advantages resulting from rigid wheels and axles.

396. Effect of parallel axles. Trucks are made with two or three parallel axles (except as noted later), in order that the axles shall mutually guide each other and be kept approximately

perpendicular to the rails. If the curvature is very sharp and the wheel-base comparatively long (as is notably the case for four-wheeled street cars passing around street corners), the front

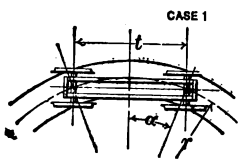


FIG. 177.

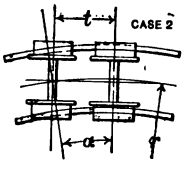


FIG. 178.

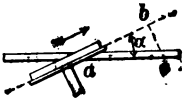


FIG. 179.

and rear wheels will stand at the same angle (a) with the track, as shown in Fig. 177, which also applies to easy curvature whenever the rear outer wheel-flange is forced against the rail, which is claimed by some to be the normal position. Others claim that for ordinary curvature the rear axle will take a position normal to the curve, as shown in Fig. 178. But it is certain that track irregularities cause the rear wheels to sway within the limits of the play of the gauge and that the angle α varies. For Case 1, $\sin \alpha = t \div 2r$; for Case 2, $\sin \alpha = t \div r$.

When the two parallel axles are on a curve (as shown), the wheels tend to run in a straight line. In order that they shall run on a curve they must slip laterally. The principle is illustrated in an exaggerated form in Fig. 179. The wheel *tends* to roll from a toward b. Therefore in passing along the track from a to c it must actually slip laterally an amount bc which equals $ac \sin a$. The lateral slipping *per unit* of distance traveled therefore equals $\sin a$. For Case 1, both front wheels slip laterally toward the curve center, and both rear wheels slip laterally away from the center. For Case 2, both front wheels slip laterally toward the center, but the slip per unit of forward distance is only one-half that of Case 1, while the rear axle, being radial does not slip laterally at all. Neither Case 1 nor Case 2 (nor any other combination) is constantly applicable.

From the above it might be inferred that the flanges of the forward wheels will have much greater wear than those of the rear wheels. Since cars are drawn in both directions about equally, no difference in flange wear due to this cause will occur, but locomotives (except switching-engines) run forward almost

exclusively, and the excess wear of the front wheels of the pilot- and tender-trucks is plainly observable.

For a given curve the angle a (and the accompanying resistance) is evidently greater the greater the distance between the axles. On the other hand, if the two axles are very close together, there will be a tendency for the truck to twist and the wheels to become jammed, especially if there is considerable play in the gauge. The flange friction would be greater and would perhaps exceed the saving in lateral slipping. A general rule is that the axles should never be closer together than the gauge; usually it is considerably more.

Although the slipping per unit of length along the curve varies directly as the degree of curvature, the length of curve necessary to pass between two tangents is inversely as the degree of curve, and the total slipping between the two tangents is independent of the degree of curve. Therefore when a train passes between two tangents, the total slipping of the wheels on the rails, longitudinal and lateral, is a quantity which depends only on the central angle and is independent of the radius or degree of curve.

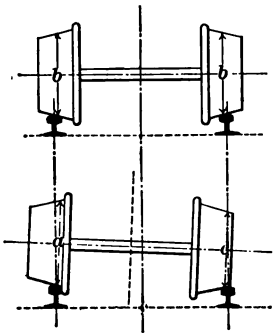


FIG. 180.

397. Effect of coning wheels. The wheels are always set on the axle so that there is some "play" or chance for lateral motion between the wheel-flanges and the rail. The treads of the wheel are also "coned." This coning and play of gauge are shown in an exaggerated form in Fig. 180. When the wheels are on a tangent, although there will be occasional oscillations from side to side, the normal position will be the symmetrical position in which the circles of tread bb are equal. When centrifugal force throws the wheel-flange against the rail, the circle of tread a is larger than b, and much larger than c; therefore the wheels will tend to roll in a circle whose radius equals the slant height of a cone whose elements would pass through the unequal circles a and c. If this radius equaled the radius of the track, and if the axle were free to assume a radial position, the wheels would roll freely on the rails without any

slipping or flange pressure. Under such ideal conditions, coning would be a valuable device, but it is impracticable to have all axles radial, and the radius of curvature of the track is an extremely variable quantity. It has been demonstrated that with parallel axles the influence of coning diminishes as the distance between the axle increases, and that the effect is practically inappreciable when the axles are spaced as they are on locomotives and car-trucks. The coning actually used is very slight (see Chapter XV, § 420) and has a different object. It is so slight that even if the axles were radial it would only prevent the slipping on a very light curve—say a 1° curve.

398. Effect of flanging locomotive driving-wheels. If all the wheels of all locomotives were flanged it would be practically impossible to run some of the longer types around sharp curves. The track-gauge is always widened on curves, and especially on sharp curves, but the widening would need to be excessive to permit a consolidation locomotive to pass around an 8° or 10° curve if all the drivers were flanged. The action of the wheels on a curve is illustrated in Figs. 181, 182, and 184. All small truck-wheels are flanged. The rear drivers are always flanged and four-driver engines usually have all the drivers flanged. Consolidation engines have only the front and rear drivers flanged. Mogul and ten-wheel engines have one pair of drivers blank. On Mogul engines it is always the middle pair. On ten-wheel engines, when used on a road having sharp curves, it is preferable to flange the front and rear driving-wheels and use a "swing bolster" (see § 399); when the curvature is easy, the middle and rear drivers may be flanged and the truck made with a rigid center. The blank drivers have the same total width as the other drivers and of course a much wider tread, which enables these drivers to remain on the rail, even though the curvature is so sharp that the tread overhangs the rail considerably.

399. Action of a locomotive pilot-truck. The purpose of the pilot-truck is to guide the front end of a locomotive around a curve and to relieve the otherwise excessive flange pressure that would be exerted against the driver-flanges. There are two classes of pilot-trucks—(*a*) those having fixed centers and (*b*) those having shifting centers. This second class is again subdivided into two classes, which are radically different in their action—(b_1) four-wheeled trucks having two parallel axles

and (b_2) two-wheeled trucks which are guided by a "radius-bar." The action of the four-wheeled fixed-centered truck (a) is shown in Fig. 181. Since the center of the truck is forced

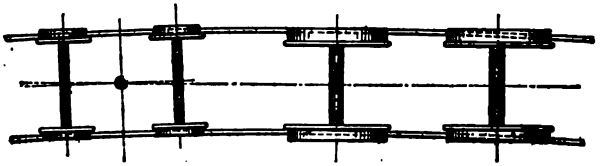


FIG. 181.—FIXED CENTER PILOT-TRUCK.

to be in the center of the track, the front drivers are drawn away from the outer rail. The rear outer driver tends to roll *away* from the outer rail rather than toward it, and so the effect

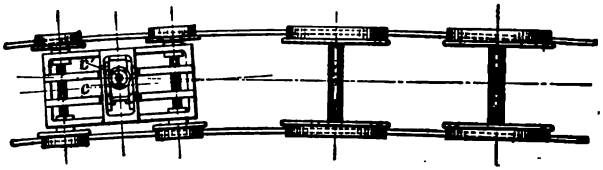


FIG. 182.—FOUR-WHEELED TRUCK—SHIFTING CENTER.

of the truck is to relieve the driver-flanges of any excessive pressure due to curvature. The only exception to this is the case where the curvature is sharp. Then the front inner driver may be pressed against the *inner* rail, as indicated in Fig. 181. This limits the use of this type of wheel-base on the sharper curves.

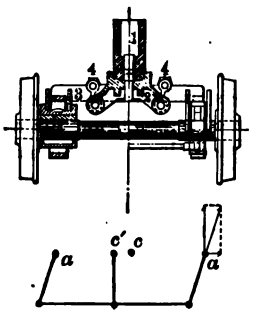


FIG. 183.—ACTION OF SHIFTING CENTER.

The next type—(b_2) four-wheeled trucks with shifting centers—is much more flexible on sharp curvature; it likewise draws the front drivers away from the outer rail. The relative position of the wheels is shown in Fig. 182, in which c' represents the position of center-pin and c the displaced truck center. The structure and action of the truck is shown in Fig. 183. The "center-pin" (1) is supported on the "truck-bolster" (2), which is hung by the "links" (4) from the "cross-ties" (3). The links are therefore

in tension and when the wheels are forced to one side by the rails the *links* are inclined and the front of the engine is drawn inward by a force equal to the weight on the bolster times the tangent of the angle of inclination of the links. This assumes that all links are vertical when the truck is in the center. Frequently the opposite links are normally inclined to each other, which somewhat complicates the above simple relation of the forces, although the general principle remains identical.

The two-wheeled pilot-truck with shifting center is illustrated in Fig. 184. The figure shows the facility with which

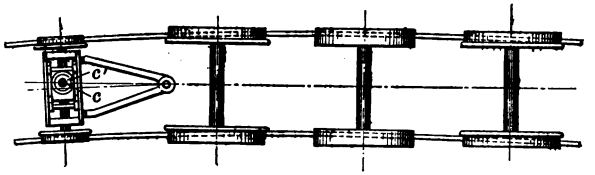


FIG. 184.—TWO-WHEELED TRUCK—SHIFTING CENTER.

an engine with long wheel-base may be made to pass around a comparatively sharp curve by omitting the flanges from the middle drivers and using this form of pilot-truck. As in the previous case, the eccentricity of the center of the truck relative to the center-pin induces a centripetal force which draws the front of the engine inward. But the swing-truck is not the only source of such a force. If the "radius-bar pin" were placed at O' (see Fig. 185), the truck-axle would be radial. But the radius-bar is always made somewhat shorter than this, and the pin is placed at O, a considerable distance ahead of O', thus creating a tendency for the truck to run toward the inner rail and draw the front of the locomotive in that direction. This tendency will be objectionably great if the radius-bar is made too short, as has been practically demonstrated in cases when the radius-bar has been subsequently lengthened with a resulting improvement in the running of the engine. This type of pilot truck is used on both Mogul and Consolidation locomotives and explains why these long engines can so easily operate on sharp curves.

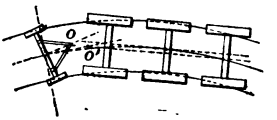


FIG. 185.—ACTION OF TWO-WHEELED TRUCK.

400. Types of locomotive wheel-bases. The variations in locomotive service have developed all conceivable types as to total weight, ratio of total weight to weight on drivers, types of running gear, relation of steaming capacity to tractive power, etc. The method of classification on the basis of the running gear is very simple. The number of wheels on both rails of the pilot truck, if any, is placed as the first of three numbers. If there is no pilot truck, the character 0 is used. This is followed by the number of drivers and then by the number of trailing wheels, if any. For example, a Pacific type engine has four wheels on the pilot truck, six driving wheels, and two trailing wheels under the rear of the boiler. The wheel-base is symbolized as 4–6–2. The most common types of locomotives, with their popular names and wheel base symbols, are

American	4–4–0	Consolidation	2–8–0
Columbia	2–4–2	Mikado	2–8–2
Atlantic	4–4–2	Mastodon	4–8–0
Mogul	2–6–0	Santa Fe	2–10–2
Prairie	2–6–2		
Ten-wheel	4–6–0	Mallet	A–B–B–A
Pacific	4–6–2	A = truck wheels, usually	2 or 0
Six-wheel switcher	0–6–0	B = drivers, varying from	4 to 10

The "Mallet" type of locomotive is one which combines sufficient flexibility to operate on ordinary railroad curves, wheel loads on the drivers which are not excessive, a very great increase in the total tractive power and yet operated by one engineman. In one respect it is like coupling two or three locomotives together, but the saving consists in reducing the number of enginemen and firemen which would be needed to run the two or three locomotives. Excluding freak variations, they are usually "four-cylinder compounds," one pair of cylinders discharging into the other pair and then exhausting. This type has from five to ten driving axles and has a length of engine wheel-base up to about 60 ft., but this wheel-base is flexible, so that it will bend on a curved track. Sometimes the boiler is made flexible by having a set of accordion-shaped steel rings forming a joint in the boiler shell. The boiler itself is on one side of this flexible joint and the feed-water heater, the reheater, and perhaps the superheater are on the other side of the joint. In this case each half of the flexible boiler is carried on a frame supported by one of the sets of driving wheels, the two frames being connected by a suitable joint. The boiler shell is made rigid; one end is rigidly attached to the frame carrying the high-pressure cylinders and

the other end is supported on a bearing on the truck frame which carries the low-pressure cylinders and the drivers operated by them. The low-pressure truck frame swings around a pivot in the fixed frame. This flexibility has been made so great that these locomotives are operated successfully on 20° curves. The Baldwin Locomotive Works have developed this type still further by building a locomotive for the Erie R. R. which has three wheel frames, mutually flexible with each other, the third frame being under the tender. Each wheel frame has eight driving wheels. The total load carried by the twenty-four drivers is 761,600 lbs. or an average of 31,733 lbs. per driver. There are six cylinders of equal size. The two cylinders on the center frame use high-pressure steam and exhaust into the other four cylinders. The total weight of locomotive and tender is 853,050 lbs. On a test trip it pulled a train with a total length of 8547 ft. or 1.6 miles, the total weight of the train being 18,338 tons. The maximum draw-bar pull, registered by the dynamometer car, was 130,000 lbs. The adhesion between the drivers and the rails must have been considerably more. Such engines are chiefly used for hauling long trains of slow-speed freight. Their boilers cannot produce steam fast enough to develop their enormous tractive power at high speeds and the power falls off rapidly with increase in speed. They are frequently equipped with automatic stokers for burning coal, or with oil-burning outfits, since the great amount of power developed can only be produced by the consumption of a corresponding amount of fuel, and a fireman would be physically incapable of shoveling coal as rapidly as the production of such an amount of power would demand.

LOCOMOTIVES.

GENERAL STRUCTURE.

401. Frame. The frame or skeleton of a locomotive consists chiefly of a collection of forged wrought-iron bars, as shown in Figs. 186 and 187. These bars are connected at the

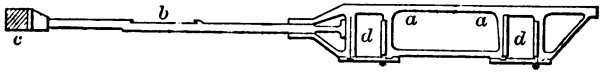


FIG. 186.—ENGINE-FRAME.

front end by the "bumper" (*c*), which is usually made of wood.

A little further back they are rigidly connected at *bb* by the cylinders and boiler-saddle. The boilers rest on the frames at *aaaa* by means of "pads," which are bolted to the fire-box, but which permit a free expansion of the boiler along the frame. This expansion is sometimes as much as $\frac{5}{16}''$. On a "consolidation" engine (frame shown in Fig. 187) it is frequently

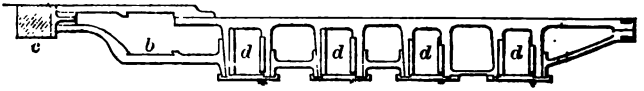


FIG. 187.—ENGINE-FRAME—CONSOLIDATION TYPE.

necessary to use vertical swing-levers about 12″ long instead of "pads." The swinging of the levers permit all necessary expansion. At the back the frames are rigidly connected by the iron "foot-plate." The driving-axles pass through the "jaws" *dddd*, which hold the axle-boxes. The frame-bars have a width (in plan) of 3″ to 4″. The depth (at *a*) is about the same. Fig. 186 shows a frame for an "American" type of locomotive; Fig. 187 shows a frame for a "Consolidation" type (see § 400).

402. Boiler. A boiler is a mechanism for transferring the latent heat of fuel to water, so that the water is transformed from cold water into high-pressure steam, which by its expansion will perform work. The efficiency of the boiler depends largely on its ability to do its work rapidly and to reduce to a minimum the waste of heat through radiation. The boiler contains a fire-box (see Fig. 188), in which the fuel is burned. The gases of consumption pass from the fire-box through the numerous boiler-tubes into the "smoke-box" *S* and out through the smoke-stack. The fire-box consists of an inner and outer shell separated by a layer of water 3″ to 5″ thick. The exposure of water-surface to the influence of the fire is thus very complete. The efficiency of this transferal of heat is somewhat indicated by the fact that, although the temperature of the gases in the fire-box is probably from 3000° to 4000° F., the temperature in the smoke-box is generally reduced to 500° to 600° F. If the steam pressure is 180 lbs., the temperature of the water is about 380° F., and, considering that heat will not pass from the gas to the water unless the gas is hotter than the water, the water evidently absorbs a large part of the theoretical maximum. Nevertheless gases at a temperature of

600° F. pass out of the smoke-stack and such heat is utterly wasted.

The tubes vary from $1\frac{3}{4}''$ to $2''$, inside diameter, with a thickness of about $0''.10$ to $0''.12$. The aggregate cross-sectional

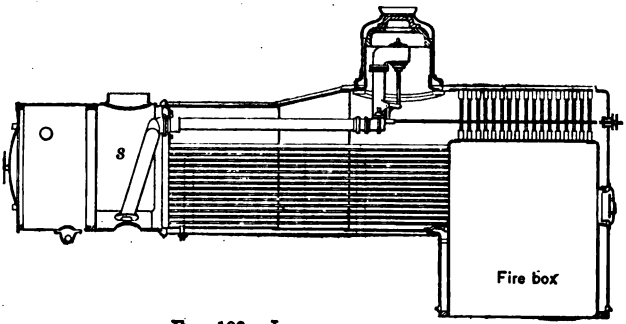


FIG. 188.—LOCOMOTIVE-BOILER.

area of the tubes should be about one-eighth of the grate area. The number will vary from 140 to 375. The length varies from 11′ to 21′, but the length is virtually determined by the type and length of engine.

403. Fire-box. The fire-box is surrounded by water on the four sides and the top, but since the water is subjected to the

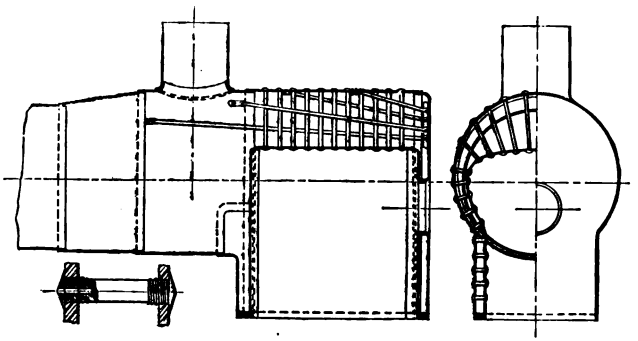


FIG. 189. FIG. 190.

boiler pressure, the plates, which are $\frac{5}{16}''$ to $\frac{5}{8}''$ thick, must be stayed to prevent the fire-box from collapsing. This is easily accomplished over the larger part of the fire-box surface by

having the outside boiler-plates parallel to the fire-box plates and separated from them by a space of 3″ to 5″. The plates

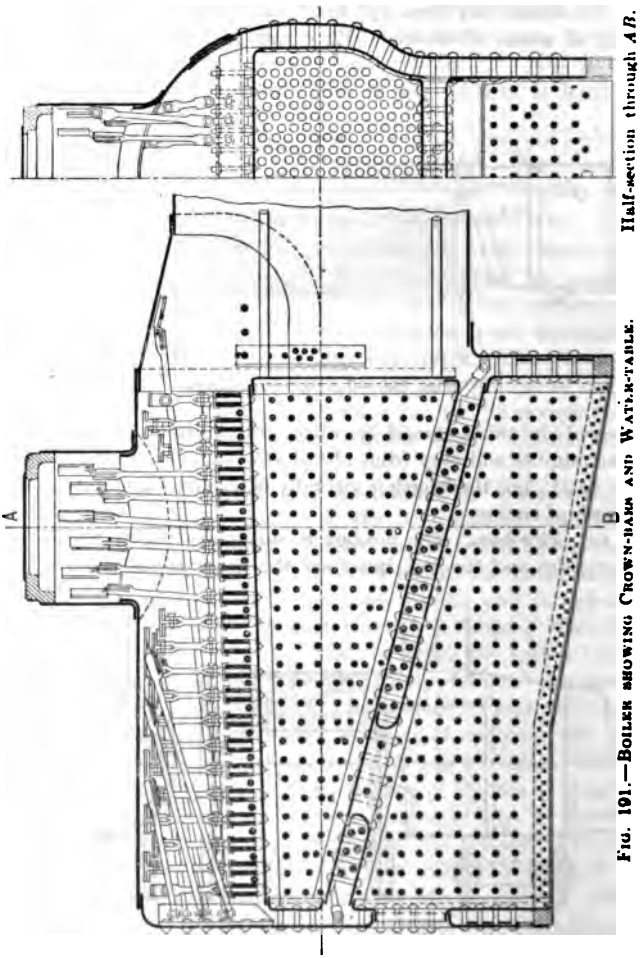
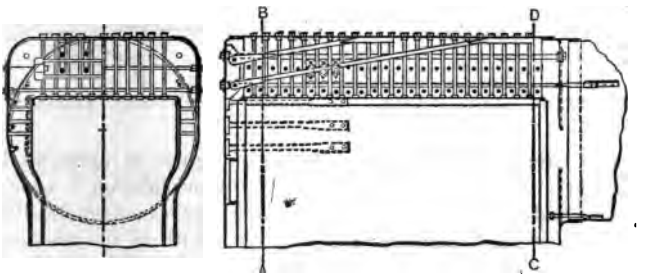


Fig. 191.—Boiler showing Crown-bars and Water-table. Half-section through *AB*.

are then mutually held by "stay-bolts." See Fig. 189. These are about $\frac{7}{8}$″ in diameter and spaced 4″ to 4½″. The $\frac{3}{16}$″ hole, drilled 1¼″ deep, indicated in the figure, will allow the escape

of steam if the bolt breaks just behind the plate, and thus calls attention to the break. The stay-bolts are turned down to a diameter equal to that at the root of the screw-threads. This method of supporting the fire-box sheets is used for the two sides, the entire rear, and for the front of the fire-box up to the boiler-barrel. The "furnace tube-sheet"—the upper part of the front of the fire-box—is stayed by the tubes. But the top of the fire-box is troublesome. It must always be covered with water so that it will not be "burned" by the intense heat. It must therefore be nearly, if not quite, flat. There are three general methods of accomplishing this.



Thru *AB* | Thru *CD*
Half-sections.

FIG. 192.—"BELPAIRE" FIRE-BOX.

(a) **Radial stays.** This construction is indicated in Fig. 190. Incidentally there is also shown the diagonal braces for resisting the pressure on the back end of the boiler above the fire-box. It may be seen that the stays are not perpendicular to either the crown-sheet or the boiler-plate. This is objectionable and is obviated by the other methods.

(b) **Crown-bars.** These bars are in pairs, rest on the side furnace-plates, and are further supported by stays. See Fig. 191.

(c) **Belpaire fire-box.** The boiler above the fire-box is rectangular, with rounded corners. The stays therefore are perpendicular to the plates. See Fig. 192.

Fire-brick arches. These are used, as shown in Fig. 193, to force all the gases to circulate through the upper part of the fire-box. Perfect combustion requires that all the carbon shall be turned into carbon dioxide, and this is facilitated by the forced circulation.

Water-tables. The same object is attained by using a water-table instead of a brick arch—as shown in Fig. 191. But it has the further advantages of giving additional heating-surface and avoiding the continual expense of maintaining the bricks. One feature of the design is the use of a number of steam-jets which force air into the fire-box and assist the combustion.

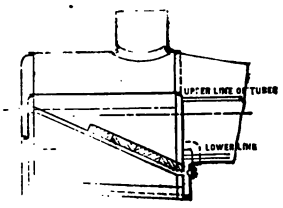


FIG. 193.—FIRE-BRICK ARCH.

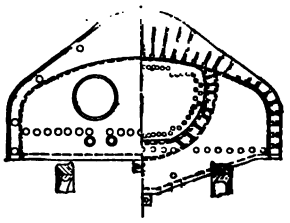


FIG. 194.—WOOTTEN FIRE-BOX.

404. **Area of grate.** The older types of engines, as represented by the "American," "Mogul" or "Consolidation" type, always had the fire-box set between the drivers, which practically meant that the maximum effective inside width of the fire-box was limited to about 3 ft. 5 ins. for standard-gauge locomotives. The maximum distance over which a fireman can properly control a fire is perhaps 10 to 11 ft., but such extreme lengths are objectionable. The grate area was thus quite definitely limited. The Wootten fire-box, illustrated in Fig. 194, obtained a fire-box eight feet wide by raising it above the level of the drivers, as shown, but this required that the drivers should be objectionably small in diameter, except for low-speed engines, or that the fire-box would be set objectionably high. The last difficulty has been solved by engines of the "Columbia," "Atlantic," "Pacific," "Mikado," and "Santa Fe" types, all of which have a pair of trailing wheels, 36 to 45 ins. in diameter, set back of the driving wheels and under the fire-box, which may thus be widened to 7 or 8 ft., the entire fire-box being placed back of the driving wheels.

405. **Superheaters.** Inside of a boiler the steam has a temperature corresponding to its pressure. For example, if the pressure is 180 lbs., the temperature is about 379° F. When the steam of a locomotive is superheated, the steam is conducted from the throttle to the cylinders through pipes which are pur-

posely placed in the path of the flue gases on their way to the smokestack. A simple form of superheater is a series of tubes and drums located in the smokebox. Here the temperature is perhaps 600° F., which is sufficient to heat the steam from 30° to 50° above the boiler temperature and to produce substantial economies. In another more effective but more costly type a considerable number of the ordinary $2\frac{1}{4}$-inch boiler tubes are replaced by $5\frac{1}{2}$-inch tubes, inside of each of which is a pipe loop extending from the smokebox headers to within a short distance of the fire-box, where the temperature approaches the fire-box temperature, which is perhaps 2000° F. The live steam passes through these loops and is so heated that, even after it reaches the cylinder, it has a superheat of 150° to 200° over the boiler temperature, but since its pressure is substantially the boiler pressure, the *quantity* (or weight) of steam required to fill the cylinder at that temperature and pressure is much less than the quantity of steam at the same pressure but lower temperature. Superheating also has the advantage of making the steam more dry and of preventing condensation in the cylinders until the steam has lost in temperature at least the amount of its superheat. Superheating is chiefly advantageous for use with passenger engines, when they must work at high power for long, continuous runs. An economy of 15 to 25% in coal consumption (and even 30% in some tests), can ordinarily be obtained by the use of superheaters, but the economy is somewhat offset by the additional cost for installation and for subsequent repairs and maintenance.

406. Reheaters. A reheater is substantially the same as a superheater in its general principle of construction. When steam has been exhausted from a high-pressure cylinder, the temperature and pressure are both considerably lower than their boiler values. If the steam is to be again used, an economy is obtained and the steam is dried by passing it through a reheater. They are generally used on Mallet engines to reheat the steam in its passage from the high-pressure to the low-pressure cylinders.

407. Coal consumption. No form of steam-boiler (except a boiler for a steam fire-engine) requires as rapid production of steam, considering the size of the boiler and fire-box, as a locomotive. The combustion of coal per square foot of grate per hour for stationary boilers averages about 15 to 25 lbs. and seldom exceeds that amount. An ordinary maximum for a

locomotive is 125 lbs. of coal per square foot of grate-area per hour, and in some recent practice 220 lbs. have been used. Of course such excessive amounts are wasteful of coal, because a considerable percentage of the coal will be blown out of the smoke-stack unconsumed, the draft necessary for such rapid consumption being very great. The only justification of such rapid and wasteful coal consumption is the necessity for rapid production of steam. The best quality of coal is capable of evaporating about 14 lbs. of water per pound of coal, i.e., change it from water at 212° to steam at 212°; the heat required to change water at ordinary temperatures to steam at ordinary working pressure is (roughly) about 20% more. From 6 to 9 lbs. of water per pound of coal is the average performance of ordinary locomotives, the efficiency being less with the higher rates of combustion. Some careful tests of locomotive coal consumption gave the following figures: when the consumption of coal was 50 lbs. per square foot of grate-area per hour, the rate of evaporation was 8 lbs. of water per pound of coal. When the rate of coal consumption was raised to 180, the evaporation dropped to 5 lbs. of water per pound of coal. It has been demonstrated that the efficiency of the boiler is largely increased by an increased length of boiler-tubes. The actual consumption of coal per mile is of course an exceedingly variable quantity, depending on the size and type of the engine and also on the work it is doing—whether climbing a heavy grade with its maximum train-load or running easily over a level or down grade. A test of a 50-ton engine, running without any train at about 20 to 25 miles per hour, showed an average consumption of 21 lbs. of coal per mile. Statistics of the Pennsylvania Rail road show a large increase (as might be expected, considering the growth in size of engines and weight of trains) in the average number of pounds of coal burned per *train*-mile—some of the figures being 55 lbs. in 1863, 72 lbs. in 1872, and nearly 84 lbs. in 1883. Figures are published showing an average consumption of about 10 lbs. of coal per passenger-car mile, and 4 to 5 lbs. per freight-car mile. But these figures are always obtained by dividing the total consumption per train-mile by the number of cars, the coal due to the weight of the engine being thrown in. Wellington developed a rule, based on the actual performance of a very large number of passenger-trains, that the number of pounds of coal per mile = 21.1 + 6.74 times the number of passenger-cars. The amount of coal assigned

to the engine agrees remarkably with the test noted above. For freight-trains the amount assigned to the engine should be much greater (since the engine is much heavier), and that assigned to the individual cars much less, although the great increase in freight-car weights in recent years has caused an increase in the coal required per car.*

There is a physical limit to the amount of coal which can be shovelled into a firebox by a fireman. Tests have shown that the average fireman can handle about 4000 lbs. of coal per hour and keep up such work almost indefinitely. For a short time he can shovel coal at the rate of 80 or 90 lbs. per minute, and this may be necessary to keep up steam while the train is going over some hump, but it must be followed by some relief which will make the average about the same. **Automatic stokers** have been devised for locomotives which can feed as much as 6000 lbs. of coal per hour when the grate area is less than 70 square feet and up to 8000 lbs. per hour when the grate area is 70 square feet or over. These are necessary on some of the most powerful locomotives in order to produce steam fast enough to develop their maximum capacity.

408. Oil-burning locomotives. In 1912 over one-sixth of all the locomotives west of the Mississippi River used oil as fuel. Some of the advantages in using oil are as follows: (1) the British thermal units in one pound of oil vary from about 19,000 to 21,000; those in a pound of coal vary from perhaps 14,000 for the very best down to 5000 for the poorer grades of lignite found in the western parts of the United States, and this means a great reduction in the cost of carrying and storing fuel, measured in heat units; (2) the cost of handling fuel is reduced and that of disposing of ashes is eliminated; (3) engine repairs are reduced in many respects, although it is said that the increased cost of fire-box repairs, due to the intense heat of the oil flame, offsets any reduction in other items; (4) the fires can be more easily controlled and waste of heat reduced during stoppages or when drifting down grade; (5) wayside fires due to sparks are altogether eliminated; (6) there is a practical limitation (see § 407), to the amount of coal that one fireman can feed to a fire; but there is no such limitation when using oil; (7) there is an equality in cost of heat units when a 42-gallon barrel of oil, weighing 7.3 lbs. per gallon, costs 60 cents and a ton (2000 lbs.) of coal, having

* See Chap. XVIII for further discussion of relation of coal consumed topower produced.

two-thirds as many heat units per pound, costs $2.61, or 4.35 times as much. The other items of difference almost invariably favor the oil and might make it more desirable even when the ratio of cost seemed to favor the coal. The extensive use of oil west of the Mississippi River is due to the fact that in many localities a very suitable quality of crude oil is plentiful and cheap while coal is expensive and of low calorific power.

409. Heating-surface. The rapid production of steam requires that the hot gases shall have a large heating-surface to which they can impart their heat. From 50 to 75 square feet of heating-surface is usually designed for each square foot of grate-area. A more recently used rule is that there should be from 60 to 70 square feet of tube heating-surface per square foot of grate-area for bituminous coal. 40 or 50 to 1 is more desirable for anthracite coal. Almost the whole surface of the fire-box has water behind it, and hence constitutes heating-surface. Although this surface forms but a small part of the total (nominally), it is really the most effective portion, since the difference of temperature of the gases of combustion and the water is here a maximum, and the flow of heat is therefore the most rapid. The heating-surface of the tubes varies from 85 to 93% of the total, or about 7 to 15 times the heating-surface in the fire-box. By dividing the total weight of a well-designed engine (exclusive of tender) by the number of square feet of heating-surface (fire-box and tubes), we get a quotient which varies from 60 to 80 or over. For example, a light engine, weighing only 96,450 lbs. had a total heating surface of 1449 square feet, or about 67 lbs. per square foot. On the other hand, a Mikado engine, weighing 297,500 lbs., had 4359 square feet of heating surface, or 68 lbs. per square foot.

410. Loss of efficiency in steam pressure. The effective work done by the piston is never equal to the theoretical energy contained in the steam withdrawn from the boiler. This is due chiefly to the following causes:

(*a*) The steam is "wire-drawn," i.e., the pressure in the cylinder is seldom more than 85 to 90% of the boiler pressure. This is due largely to the fact that the steam-ports are so small that the steam cannot get into the cylinder fast enough to exert its full pressure. Partially closing the throttle, so that the steam will be used less rapidly, also wire-draws the steam.

(*b*) **Entrained water.** Steam is always drawn from a dome

placed over the boiler so that the steam shall be as far above the water-surface as possible, and shall be as dry as possible. In spite of this the steam is not perfectly dry and carries with it water at a temperature of, say, 361°, and pressure of 140 lbs. per square inch. When the pressure falls during the expansion and exhaust, this hot water turns into steam and absorbs the necessary heat from the hot cylinder-walls. This heat is then carried out by the exhaust and wasted.

(*c*) The **back pressure** of the exhaust-steam, which depends on the form of the exhaust-passages, etc. This amounts to from 2 to 20% of the power developed.

(*d*) **Clearance-spaces.** When cutting off at full stroke this waste is considerable (7 to 9%), but when the steam is used expansively the steam in these clearance-spaces expands and so its power is not wholly lost.

(*e*) **Radiation.** In spite of all possible care in jacketing the cylinders, some heat is lost by radiation.

(*f*) **Radiation into the exhaust-steam.** This is somewhat analogous to (*b*). Steam enters the cylinder at a temperature of, say, 361°; the walls of the cylinder are much cooler, say 250°; some heat is used in raising the temperature of the cylinder-walls; some steam is vaporized in so doing; when the exhaust is opened the temperature and pressure fall; the heat temporarily absorbed by the cylinder-walls is reabsorbed by the exhaust-steam, re-evaporating the vapor previously formed, and thus a certain portion of heat-energy goes through the cylinder without doing any useful work. With an early cut-off the loss due to this cause is very great.

The sum of all these losses is exceedingly variable. They are usually less at lower speeds. The loss in *initial pressure* (the difference between boiler pressure and the cylinder pressure at the beginning of the stroke) is frequently over 20%, but this is not all a net loss With an early cut-off the average cylinder pressure for the whole stroke is but a small part of the boiler pressure, yet the horse-power developed may be as great as, or greater than that developed at a lower speed, later cut-off, and higher average pressure

411. Tractive power The work done by the two cylinders during a complete revolution of the drivers evidently = area of pistons × average steam pressure × stroke × 2 × 2. The resistance overcome evidently = tractive force at circumference of

drivers times distance traveled by drivers (which is the circumference of the drivers) Therefore

$$\text{Tractive force} = \left\{ \frac{\text{area pistons} \times \text{average steam pressure} \times \text{stroke} \times 2 \times 2.}{\text{circumference of drivers}} \right.$$

Dividing numerator and denominator by π (3.1415), we have

$$\text{Tractive force} = \left\{ \frac{(\text{diam piston})^2 \times \text{average steam pressure} \times \text{stroke}}{\text{diameter of driver}} \right., \quad . \quad (103)$$

which is the usual rule Although the rule is generally stated in this form, there are several deductions In the first place the net effective area of the piston is less than the nominal on account of the area of the piston-rod. The ratio of the areas of the piston-rod and piston varies, but the effect of this reduction is usually from 1.3 to 1.7%. No allowance has been made for friction—of the piston, piston-rod, cross-head, and the various bearings This would make a still further reduction of several per cent. Nevertheless the above simple rule is used, because, as will be shown, no great accuracy can be utilized.

The maximum draw bar pull is limited by the adhesion between the driving wheels and the rails. This is usually about one-fourth of the weight. The use of sand may increase it to one-third. But this ratio is important only when starting or at very low speeds. The adhesion is always ample for the much lower cylinder power which can be developed at higher speeds. This is considered more fully in Chapter XVIII.

RUNNING GEAR.

412. Equalizing-levers. The ideal condition of track, from the standpoint of smooth running of the rolling stock, is that the rails should always lie in a plane surface. While this condition is theoretically possible on tangents, it is unobtainable on curves, and especially on the approaches to curves when the outer rail is being raised. Even on tangents it is impossible to *maintain* a perfect surface, no matter how perfectly the track may have been laid. In consequence of this, the points

of contact of the wheels of a locomotive, or even of a four-wheeled truck, will not ordinarily lie in one plane. The rougher and more defective the track, the worse the condition in this respect. Since the frame of a locomotive is practically rigid, and the frame rests on the driver-axles through the medium of springs at each axle-bearing, the compression of the springs (and hence the pressure of the drivers on the rail) will be variable if the bearing-points of the drivers are not in one plane. This variable pressure affects the tractive power and severely strains the frame. Applying the principle that a tripod will stand on an uneven surface, a mechanism is employed which

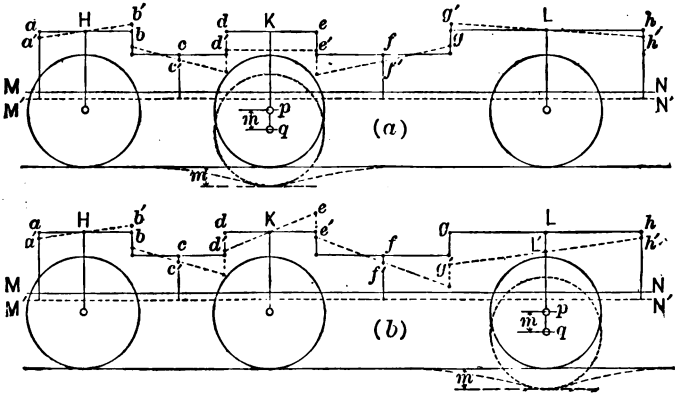


FIG. 195.—ACTION OF EQUALIZING-LEVERS.

virtually supports the locomotive on three points, of which one is usually the center-bearing of the forward truck. On each side the pressure is so distributed among the drivers that even if a driver rises or falls with reference to the others, the load carried by each driver is unaltered, and that side of the engine rises or falls by one *n*th of the rise or fall of the single driver, where *n* represents the number of wheels. The principle involved is shown in an exaggerated form in Fig. 195. In the diagram, *MN* represents the normal position of the frame when the wheels are on line. The frame is supported by the hangers at *a*, *c*, *f*, and *h*. *ab*, *de*, and *gh* are horizontal levers vibrating about the points *H*, *K*, and *L*, which are supported by the axles. While it is *possible* with such a system of levers to make

MN assume a position not parallel with its natural position, yet, by an extension of the principle that a beam balance loaded with equal weights will always be horizontal, the effect of raising or lowering a wheel will be to move MN parallel to itself. It only remains to determine *how much* is the motion of MN relative to the rise or drop of the wheel.

The dotted lines represent the positions of the wheels and levers when one wheel drops into a depression. The wheel center drops from p to q, a distance m. L drops to L', a distance m (see Fig. 195, b); M drops to M', an unknown distance x; therefore $aa'=x$; $bb'=x$; $cc'=x$; $dd'=3x=ee'$; $ff'=x$; $\therefore\ gg'=5x$; $hh'=x$; $LL'=\frac{1}{2}(gg'+hh')=\frac{1}{2}(6x)=m$; $\therefore\ x=\frac{1}{3}m$; i.e., MN drops, parallel to itself, $1/n$ as much as the wheel drops, where n is the number of wheels. The resultant effect caused by the simultaneous motion of two wheels with reference to the third is evidently the algebraic sum of the effects of each wheel taken separately.

The practical benefits of this device are therefore as follows:

(a) When any driver reaches a rough place in the track, a high place or a low place, the stress in all the various hangers and levers is unchanged.

(b) The motion of the frame (represented by the bar MN in Fig. 195) is but $1/n$ of the motion of the wheel, and the jar and vibration caused by a roughness in the track is correspondingly reduced.

The details of applying these principles are varied, but in general it is done as follows:

(a) American and ten wheeled types. Drivers on each side form a system. The center-bearing pilot-truck is the third point of support. The method is illustrated in Fig. 196.

(b) Mogul and consolidation types. The front pair of drivers is connected with the two-wheeled pilot-truck (as illustrated in Fig. 197) to form one system. The remaining drivers on each side are each formed into a system.

The device of equalizers is an American invention. Until recently it has not been used on foreign locomotives. The necessity for its use becomes less as the track is maintained with greater perfection and is more free from sharp curves. A locomotive not equipped with this device would deteriorate very rapidly on the comparatively rough tracks which are usually found on light-traffic roads. It is still an open ques-

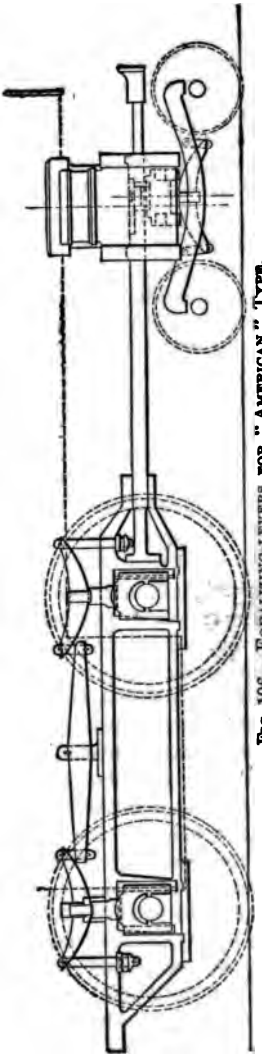


FIG. 196.—EQUALIZING-LEVERS FOR "AMERICAN" TYPE.

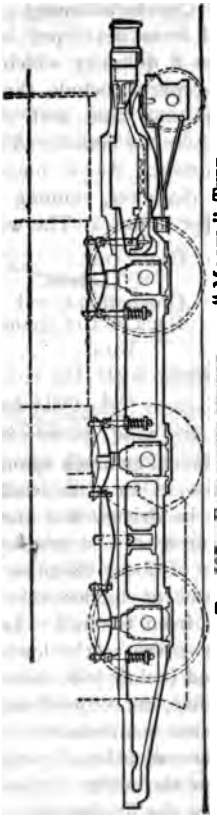


FIG. 197.—EQUALIZING-LEVERS FOR "MOGUL" TYPE.

ion to what extent the neglect of this device is responsible for he statistical fact that average freight-train loads on foreign

trains are less in proportion to the weight on the drivers than is the case with American practice. The recent increasing use of this device on foreign heavy freight locomotives is perhaps an acknowledgment of this principle.

413. **Counterbalancing.** At very high velocities the centrifugal force developed by the weight of the rotating parts becomes a quantity which cannot be safely neglected. These rotating parts include the crank-pin, the crank-pin boss, the side rod, and that part of the weight of the connecting-rod which may be considered as rotating about the center of the crank-driver. As a numerical illustration, a driving-wheel 62″ in diameter, running 60 miles per hour, will revolve 325 times per minute. The weights are:

Crank-pin	110 lbs.
" boss	150 "
One-half side rod	240 "
Back end of connecting-rod (56%)	190 "
Total	690 lbs.

If the stroke is 24″, the radius of rotation is 12″, or 1 foot. Then

$$\frac{Gv^2}{gr} = \frac{690 \times 4\pi^2 1^2 \times 325^2}{32.2 \times 1 \times 60^2} = 24821 \text{ lbs.},$$

which is half as much again as the weight on a driver, 16000 lbs. Therefore if *no* counterbalancing were used, the pressure between the drivers and the rail would always be less (at any velocity) when the crank-pin was at its highest point. At a velocity of about 48 miles per hour the pressure would become zero, and at higher velocities the wheel would actually be thrown from the rail. As an additional objection, when the crank-pin was at the lowest point, the rail pressure would be increased (velocity 60 miles per hour) from 16000 lbs. to nearly 41000 lbs., an objectionably high pressure. These injurious effects are neutralized by "counterbalancing." Since all of the above-mentioned weights can be considered as concentrated at the center of the crank-pin, if a sufficient weight is so placed in the drivers that the center of gravity of the eccentric weight is diametrically opposite to the crank-pin, this centrifugal force can be wholly balanced. This is done by filling up a portion of the space between the spokes. If the center of gravity of the counterbalancing weight is 20″ from the center, then, since the crank-pin radius is 12″, the required weight would be $690 \times \frac{12}{20} = 414$ lbs.

In addition to the effect of these revolving parts there is the effect of the sudden acceleration and retardation of the reciprocating parts. In the engine above considered the weights of these reciprocating parts will be:

Front end of connecting-rod (44%)	150 lbs.
Cross-head	174 "
Piston and piston-rod	300 "
Total	624 lbs.

Assume as before that the reciprocating parts may be considered as concentrated at one point, the point P of the diagram in Fig. 198. Since the motion of P is horizontal only, the force required to overcome its inertia at any point will exactly equal the *horizontal component* of the force required to overcome the inertia of an *equal* weight at S revolving in a circular path. Then evidently the horizontal component of the force required to keep W in the circular path will exactly balance the force required to overcome the inertia of P. Of course $W=P$. But a smaller weight W', whose weight is inversely proportional to its radius of rotation, will evidently accomplish the same result. In the above numerical case, if the center of gravity of the counterweights is 20″ from the center, the required weight to completely counterbalance the reciprocating parts would be $624\times\frac{12}{20}=374.4$ lbs. This counterweight need not be all placed on the driver carrying the main crank-pin, but can be (and is) distributed among all the drivers. Suppose it were divided between the two drivers in the above case. At 60 miles per hour such a counterweight would produce an additional pressure of 11211 lbs. when the counterweight was down, or a lifting force of the same amount when the counterweight was up. Although this is not sufficient to lift the driver from the rail, it would produce an objectionably high pressure on the rail (over 27000 lbs.), thus inducing just what it was desired to avoid on account of the eccentric rotating parts. Therefore a compromise must be made. Only a portion (one half to three fourths) of the weight of the reciprocating parts is balanced. Since the effect of the rotating

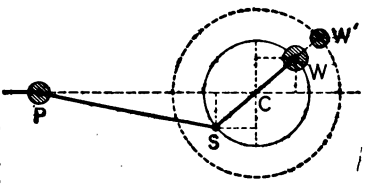


FIG. 198.—ACTION OF COUNTERBALANCE.

weights is to cause variable pressure on the rail, while the effect of the reciprocating parts is to cause a horizontal wobbling or "nosing" of the locomotive, it is impossible to balance both. Enough counterweight is introduced to partially neutralise the effect of the reciprocating parts, still leaving some tendency to horizontal wobbling, while the counterweights which were introduced to reduce the wobbling cause some variation of pressure. The vertical or horizontal pressure developed by the unbalanced rotating and reciprocating parts is called the **dynamic augment.**

An additional injurious effect on the track of the dynamic augment is due to the fact that the center of gravity of the side rod is several inches outside of the vertical plane in which the counterweight revolves, and that the center of gravity of the main rod, or connecting rod, is still further outside. The dynamic augment will be increased by the ratio of the distance between these planes of rotation to the distance between the centers of the companion drivers. This ratio averages about 11% for the side rods and for the part of the pin within the side rod; the corresponding figure for the main rod is about 23%. The physical effect of the dynamic augment on the stresses produced in the track is further discussed in Chapter XXV.

By using hollow piston-rods of steel, ribbed cross-heads, and connecting- and side-rods with an I section, the weight of the reciprocating parts may be greatly lessened without reducing their strength, and with a decrease in weight the effect of the unbalanced reciprocating parts and of the "excess balance" (that used to balance the reciprocating parts) is largely reduced.

Current practice is somewhat variable on three features:

(*a*) The proportion of the weight of the connecting-rod which should be considered as revolving weight.

(*b*) The proportion of the total reciprocating weight that should be balanced.

(*c*) The distribution among the drivers of the counterweight to balance the reciprocating parts.

The principal rules which have been formulated for counterbalancing may be stated as follows, although there is considerable variation in the figures used in rules 2 and 3.

1. Each wheel should be balanced correctly for the revolving parts connected with it.

2. *In addition,* introduce counterbalance sufficient for 50% of the weight of the reciprocating parts for ordinary engines, increasing this to 75% when the reciprocating parts are excessively heavy (as in compound locomotives) or when the engine is light and unable to withstand much lateral strain or when the wheel-base is short.

3. Consider the weight of the connecting-rod as ½ revolving and ½ reciprocating when it is over 8 feet long; when shorter than 8 feet, consider $\frac{6}{10}$ of the weight as revolving and $\frac{4}{10}$ as reciprocating.

4. The part of the weight of the connecting-rod considered as revolving should be entirely balanced in the crank-driver wheel.

5. The "excess balance" should be divided equally among the drivers.

6. Place the counterbalance as near the rim of the wheel as possible and also as near the outside of the wheel as possible in order that the center of gravity shall be as near as possible opposite the center of gravity of the rods, etc., which are all outside of even the plane of the face of the wheel.

In Fig. 199 is shown a section of a locomotive driver with the cavities in the casting for the accommodation of the lead which is used for the counterbalance weight. Incidentally several other features and dimensions are shown in the illustration.

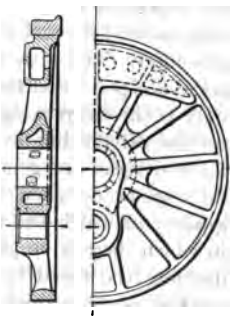


FIG. 199.—SECTION OF LOCOMOTIVE-DRIVER.

414. Mutual relations of the boiler power, tractive power, and cylinder power for various types. The design of a locomotive includes three *distinct* features which are varied in their mutual relations according to the work which the engine is expected to do.

(a) The boiler power. This is limited by the rate at which steam may be generated in a boiler of admissible size and weight. Engines which are designed to haul very fast trains which are comparatively light must be equipped with very large grates and heating surfaces so that steam may be developed with great rapidity in order to keep up with the very rapid consumption

Engines for very heavy freight work are run at very much lower velocity and at a lower piston speed in spite of the fact that more strokes are required to cover a given distance and the demand on the boiler for *rapid* steam production is not as great as with high-speed passenger-engines. The capacity of a boiler to produce steam is therefore limited by the limiting weight of the general type of engine required. Although improvements may be and have been made in the design of fire-boxes so as to increase the steam-producing capacity without adding proportionately to the weight, yet there is a more or less definite limit to the boiler power of an engine of given weight.

(b) **The tractive power.** This is limited by the possible driver adhesion. The absolute limit of tractive adhesion between a steel-tired wheel and a steel rail is about one-third of the pressure, but not more than one-fourth of the weight on the drivers can be depended on for adhesion and wet rails will often reduce this to one fifth and even less. The tractive power is therefore absolutely limited by the practicable weight of the engine. In some designs, when the maximum tractive power is desired, not only is the entire weight of the boiler and running gear thrown on the drivers, but even the tank and fuel-box are loaded on. Such designs are generally employed in switching-engines (or on engines designed for use on abnormally heavy mountain grades) in which the maximum tractive power is required, but in which there is no great tax on the boiler for *rapid* steam production (the speed being always very low), and the boiler and fire-box, which furnish the great bulk of the weight of an engine, are therefore comparatively light, and the requisite weight for traction must, therefore, be obtained by loading the drivers as much as possible. On the other hand, engines of the highest speed cannot possibly produce steam fast enough to maintain the required speed unless the load be cut down to a comparatively small amount. The tractive power required for this comparatively small load will be but a small part of the weight of the engine, and therefore engines of this class have but a small proportion of their weight on the drivers; generally have but two driving-axles and sometimes but one.

(c) **Cylinder power.** The running gear forms a mechanism which is simply a means of transforming the energy of the boiler into tractive force and its power is unlimited, within the practical conditions of the problem. The power of the running

ear depends on the steam pressure, on the area of the piston, n the diameter of the drivers, and on the ratio of crank-pin adius to wheel radius, or of stroke to driver diameter. It s always possible to increase one or more of these elements y a relatively small increase of expenditure until the cylinders re able to make the drivers slip, assuming a sufficiently great esistance. Since the power of the engine is limited by the ower of its weakest feature, and since the running gear is the nost easily controlled feature, the power of the running gear or the "cylinder power") is always made somewhat excessive n all well-designed engines. It indicates a badly designed ngine if it is stalled and unable to move its drivers, the steam ressure being normal. If it is attempted to use a freight-ngine on *fast* passenger service, it will probably fail to attain he desired speed on account of the steam pressure falling. he tractive power and cylinder power are superabundant, but he boiler cannot make steam as fast as it is needed for high peed, especially when the drivers are small. The practical esult would be a comparatively low speed kept up with a forced re. If it is attempted to use a high-speed passenger-engine n heavy freight service, the logical result is a slipping of the rivers until the load is reduced. The boiler power and cylinder ower are ample, but the weight on the drivers is so small that he tractive power is only sufficient to draw a comparatively mall load.

These relations between boiler, cylinder, and tractive power re illustrated in the following comparative figures referring o a fast passenger-engine, a heavy freight-engine, and a switch-ng-engine. The weights of the passenger- and freight-engines re about the same, but the passenger-engine has only 74% *

Kind.	Cylinders.	Total W ght.	Wt. on Driv'rs	Heating Surface, sq. ft.	Grate area sq. ft.	Steam Pressure in Boiler.	Stroke. / Diam. Driver.
ast passenger.	19″×24″	126700	81500	1831.8	26.2	180	$\frac{24}{78}$ = .31
eavy freight.	20″×24″	128700	112600	1498.3	31.5	140	$\frac{24}{50}$ = .48
vitcher.....	19″×24″	109000	109000	1498.0	22.8	160	$\frac{24}{50}$ = .48

* Computed from Eq. 103.

of the tractive power of the freight. But the passenger-engine has 22% more heating-surface and can generate steam much faster; it makes less than two-thirds as many strokes in covering a given distance, but it runs at perhaps twice the speed and probably consumes steam much faster. The switch-engine is lighter in total weight, but the tractive power is a little greater than the freight and much greater than the passenger-engine. While the heating-surfaces of the freight- and switching engines are practically identical, the grate area of the switcher is much less; its speed is always low and there is but little necessity for rapid steam development.

While these figures show the general tendency for the relative proportions, and in this respect may be considered as typical, there are large variations. The recent enormous increase in the dead weight of passenger-trains has necessitated greater tractive power. This has been provided sometimes by using the "Pacific" type, which combines rapid steaming capacity and great tractive power. On the other hand, the demand for fast-freight service, and the possibility of safely operating such trains by the use of air-brakes, has required that heavy freight-engines shall be run at comparatively high speeds, and that requires the rapid production of steam, large grate areas and heating surfaces. But in spite of these variations, the normal standard for passenger service is a four-driver engine carrying about two-thirds of the weight of the engine on the drivers, which are very large; the normal standard for freight work is an 8-driver engine with perhaps 90% of the weight on the drivers, which are small, but which must have the pony truck for such speed as it uses; and finally the normal standard for switching service has all the weight on the drivers and has comparatively low steam-producing capacity.

415. Life of Locomotives. The life of locomotives (as a whole) may be taken as about 800000 miles or about 22 to 25 years. While its life should be and is considered as the period between its construction and its final consignment to the scrap pile, parts of the locomotive may have been renewed more than once. The boiler and fire-box are especially subject to renewal. The milcage life is much longer than formerly. This is due partly to better design and partly to the custom of drawing the fires less frequently and thereby avoiding some of the destructive strains caused by extreme alternations of

heat and cold. Recent statistics give the average annual mileage on twenty-three leading roads to be 41000 miles.

CARS.

416. Capacity and size of cars. The capacity of freight-cars has been enormously increased of late years. In 1870 the usual live-load capacity for a box-car was about 20000 lbs. In 1916, out of 58299 box cars owned by the Pennsylvania R. R., 32923 or 56% had a capacity of 100000 or over; 49597 or 85% had a capacity 70000 or over; only 555, less than 1%, had a capacity of less than 60000 lbs., and the most of these were refrigerator cars or cars for special service. The Norfolk & Western R. R. had (in 1916), 750 gondola drop-bottom coal cars, each with a nominal capacity of 180000 lbs.; their length is 46 feet 10¾ inches, and the extreme width 10 feet 4½ inches. These cars are carried on six-wheel trucks. The usual width of freight-cars is about 9 to 10 feet, while parlor-cars and sleepers are generally 10 feet wide and sometimes 11 feet. The highest point of a train is usually the smokestack of the locomotive, which is generally 15 feet above the rails and occasionally over 16 feet. A sleeping-car usually has the highest point of the car about 14 feet above the rails. Box-cars are usually about 8 feet high (above the sills), with a total height of 13 to 14 feet. Some furniture and automobile cars, whose unit live load per cubic foot of space is not high, have a total height of over 15 feet. The *average* length of freight cars, as required in the design of freight yards, is now considered to be 42 feet; the allowance for each car was formerly 40 feet. The P. R. R. standards vary between 38 feet 1 inch and 44 feet 6 inches in length. Day coaches have an extreme length varying from 45 to 80 feet. An 80-foot all-steel coach weighs about 118000 lbs. and has a seating capacity of 88. Allowing the high average weight of 150 lbs., the maximum live load would be 13200 lbs., a little over 11% of the dead load, which shows that the tractive force required to haul the car will be almost constant, whether the car is full or empty. A dining-car may weigh 150000 lbs. and a sleeper even more. The weight of the 25 or 30 passengers it *may* carry is hardly worth considering in comparison.

417. Stresses to which car-frames are subjected. A car is structurally a truss, supported at points at some distance from the ends and subjected to transverse stress. There is,

therefore, a change of flexure at two points between the trucks. Besides this stress the floor is subjected to compression when the cars are suddenly stopped and to tension when in ordinary motion, the tension being greater as the train resistance is greater and as the car is nearer the engine. The shocks, jars, and sudden strains to which the car-frames are subjected are very much harder on them than the mere static strains due to their maximum loads if the loads were quiescent. Consequently any calculations based on the static loads are practically valueless, except as a very rough guide, and previous experience must be relied on in designing car bodies. As evidence of the increasing demand for strength in car-frames, it has been recently observed that freight-cars, built some years ago and built almost entirely of wood, are requiring repairs of wooden parts which have been *crushed* in service, the wood being perfectly sound as regards decay.

418. The use of metal. The use of metal in car construction

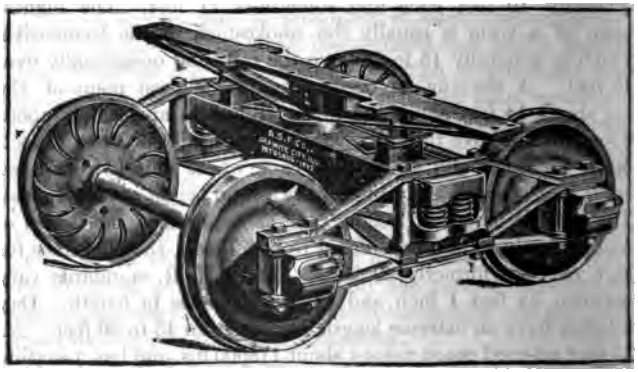


FIG. 201.

is very rapidly increasing. The demand for greater strength in car-frames has grown until the wooden framing has become so heavy that it is found possible to make steel frames and trucks at a small additional cost, the steel frames being twice as strong and yet reducing the dead weight of the car about 5000 lbs., a consideration of no small value, especially on roads having heavy grades. Another reason for the increasing use of metal is the great reduction in the price of rolled or pressed

100,000-lb. Box Car.

Steel Coal Car.

Wooden Box Car; Steel Frame.

Fig. 200.—Some Heavy Freight Cars.

(*To face page* 456.)

steel, while the cost of wood is possibly higher than before. The advocates of the use of steel advise steel floors, sides, etc. For box-cars a wooden floor has advantages. For ore and coal-cars an all-metal construction has advantages. (Fig. 200.) In Germany, where steel frames have been almost exclusively in use for many years, they have not yet been able to determine the normal age limit of such frames; none have yet *worn* out. The life is estimated at 50 to 80 years

Brake beams are also best made of metal rather than wood, as was formerly done. Metal brake-beams are generally used on cars having air-brakes, as a wooden beam must be excessively large and heavy in order to have sufficient rigidity.

Truck-frames (see Fig. 201), which were formerly made principally of wood, are now largely made of pressed steel. It makes a reduction in weight of about 3000 lbs. per car. The increased durability is still an uncertain quantity.

419. Draft gear. The enormous increase in the weight and live load capacities of rolling stock have necessitated a corresponding development in draft gear. Even within recent years, "coal-jimmies," carrying a few tons have been made up into trains by dropping a chain of three big links over hooks on the ends of the cars. But the great stresses due to present loadings would tear such hooks from the cars or tear the cars apart if such cars were used in the make-up of long heavy trains as now operated. The next stage in the development of draft gear was the invention of the "spring coupler," by which the energy due to a sudden tensile jerk or the impact of compression may be absorbed by heavy springs and gradually imparted to the car body. Such devices, for which there are many designs, seemed to answer the purpose for cars of 25 to 40 tons capacity. The use of 100,000-pound steel cars soon proved the inadequacy of even spring couplers. The friction-draft gear was then invented. The general principle of such a gear is that, when acting at or near its maximum capacity, it harmlessly transforms into heat the excessive energy developed by jerks or compression. There are several different designs of such gear, but the general principle underlying all of them may be illustrated by a description of the Westinghouse draft gear. The gear employs springs which have sufficient stiffness to act as ordinary spring-couplers for the ordinary pushing and pulling of train operations. Sections of the gear are shown in Fig. 202,

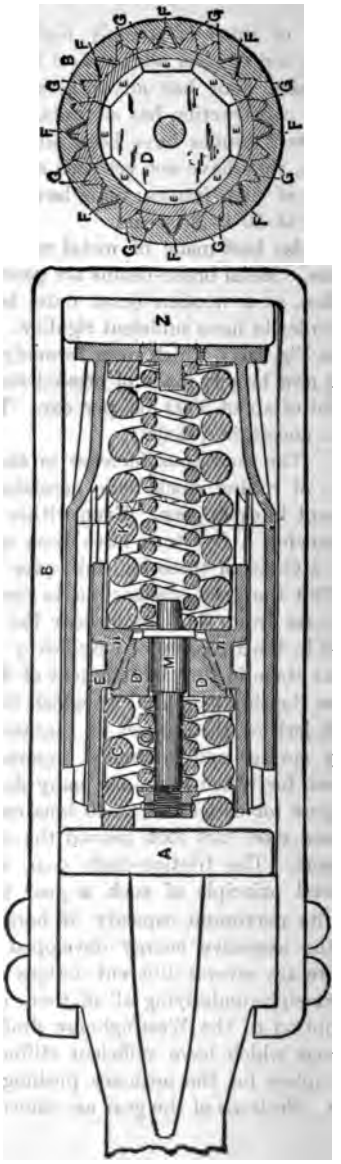
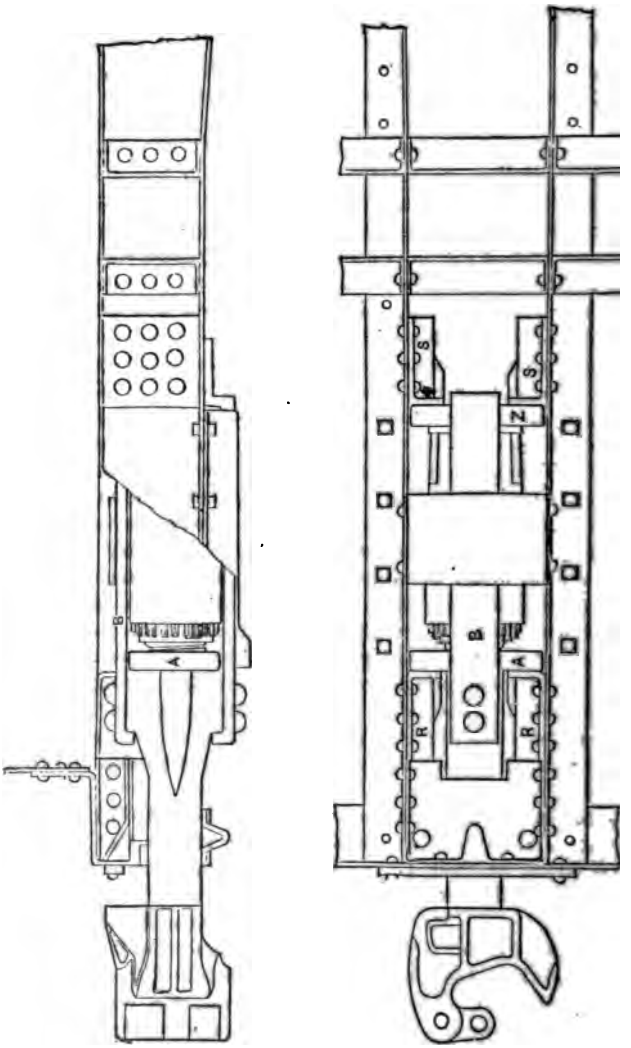


FIG. 202.—WESTINGHOUSE DRAFT GEAR. DETAILS.

while the method of its application to the framing of a car of the pressed steel type is shown in Fig. 203, *a* and *b*. When the draft gear is in tension the coupler, which is rigidly attached to *B*, is drawn to the left, drawing the follower *Z* with it. Compression is then exerted through the gear mechanism to the follower *A* which, being restrained by the shoulders *RR*, against which it presses, causes the gear to absorb the compression. The coil-spring *C* forces the eight wedges *n* against the eight corresponding segments *E*. The great compression of these surfaces against the outer shell produces a friction which retards the compression of the gear. The total possible movement of the gear, as determined by an official test, was 2.42 inches, when the maximum stress was 180,000 pounds. The work done in producing this stress amounted to 18,399 foot-pounds. Of this total energy 16,666 foot-pounds, or over 90%, represents the amount of energy absorbed and dissipated as heat by the frictional gear. The remaining 10% is given back by the recoil. The main release spring *K* is used for returning the segments and friction strips to their normal position after the force to close them has been removed. It also gives additional capacity to the entire mechanism. The auxiliary spring *L* releases the wedge *D*, while the release pin *M* releases the pressure of the auxiliary spring *L* against the wedge during frictional operation. If we omit from the above design the frictional features and consider only the two followers *A* and *Z*, separated by the springs *C* and *K*, acting as one spring, we have the essential elements of a spring-draft gear. In fact, this gear acts exactly like a spring-draft gear for all ordinary service, the frictional device only acting during severe tension and compression.

420. Gauge of wheels and form of wheel-tread.—In Fig. 204 is shown the standard adopted by the Master Car Builders' Association at their twentieth annual convention. Note the normal position of the gauge-line on the wheel-tread. In Fig. 118, § 267, the relation of rail to wheel-tread is shown on a smaller scale. It should be noted that there is no definite position where the wheel-flange is absolutely "chock-a-block" against the rail. As the pressure increases the wheel mounts a little higher on the rail until a point is soon reached when the resistance is too great for it to mount still higher. By this means is avoided the shock of unyielding impact when the car


A
B
S
S
N
B
A
R
R

sways from side to side. When the gauge between the inner faces of the wheels is greater or less than the limits given in the figure, the interchange rules of the Master Car Builders' Association authorize a road to refuse to accept a car from another road for transportation. At junction points of railroads inspectors are detailed to see that this rule (as well as many others) is complied with in respect to all cars offered for transfer.

TRAIN-BRAKES.

421. Introduction. Owing to the very general misapprehension that exists regarding the nature and intensity of the action of brakes, a complete analysis of the problem is considered justifiable. This misapprehension is illustrated by the common notion (and even practice) that the effectiveness of braking a car is proportional to the brake pressure, and therefore a brakeman is frequently seen using a bar to obtain a greater leverage on the brake-wheel and using his utmost strength to obtain the maximum pull on the brake-chain while the car is skidding along with locked wheels.

When a vehicle is moving on a track with a considerable velocity, the mass of the vehicle possesses kinetic energy of translation and the wheels possess kinetic energy of rotation. To stop the vehicle, this energy must be destroyed. The rotary kinetic energy will vary from about 4 to 8% of the kinetic energy of translation, according to the car loading (see § 435). On steam railroads brake action is obtained by pressing brake-shoes against car-wheel treads. As the brake-shoe pressure increases, the brake-shoes retard with increasing force the rotary action of the wheels. As long as the wheels do not slip or "skid" on the rails, the adhesion of the rails forces them to rotate with a circumferential velocity equal to the train velocity. The retarding action of the brake-shoe checks first the rotative kinetic energy (which is small), and the remainder develops a *tendency* for the wheel to slip on the rail. Since the rotative kinetic energy is such a small percentage of the total, it will hereafter be ignored, except as specifically stated, and it will be assumed for simplicity that the only work of the brakes is to overcome the kinetic energy of translation. The possible effect of grade in assisting or preventing retardation, and the effect of all other track resist-

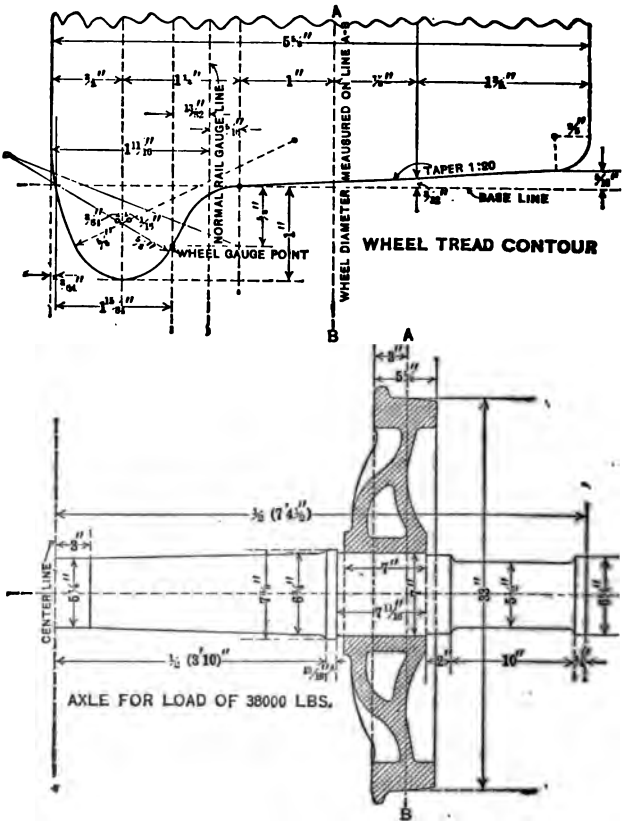


FIG. 204.—M. C. B. STANDARD WHEEL-TREAD AND AXLE. (1918.)

ances, is also ignored. The amount of the developed force which retards the train movement is limited to the possible adhesion or *static* friction between the wheel and the rail. When the friction between the brake-shoe and the wheel exceeds the adhesion between the wheel and the rail, the wheel skids, and then the friction between the wheel and the rail at once drops to a much less quantity. It must therefore be remembered at the outset that the retarding action of brake-shoes on wheels as a means of stopping a train is absolutely limited by the possible static friction between the braked wheels and the rails.

422. Laws of friction as applied to this problem. Much of the misapprehension regarding this problem arises from a very common and widespread misstatement of the general laws of friction. It is frequently stated that friction is independent of the velocity and of the unit of pressure. The first of these so-called laws is not even approximately true. A very exhaustive series of tests were made by Capt. Douglas Galton on the Brighton Railway in England in 1878 and 1879, and by M. George Marié on the Paris and Lyons Railway in 1879, with trains which were specially fitted with train-brakes and with dynagraphs of various kinds to measure the action of the brakes. Experience proved that variations in the condition of the rails (wet or dry), and numerous irregularities incident to measuring the forces acting on a heavy body moving with a high velocity, were such as to give somewhat discordant results, even when the conditions were made as nearly identical as possible. But the tests were carried so far and so persistently that the general laws stated below were demonstrated beyond question, and even the numerical constants were determined as closely as they may be practically utilized. These laws may be briefly stated as follows:

(*a*) The coefficient of friction between cast-iron brake-blocks and steel tires is about .3 when the wheels are "just moving"; it drops to about .16 when the velocity is about 30 miles per hour, and is less than .10 when the velocity is 60 miles per hour. These figures fluctuate considerably with the condition of the rails, wet or dry.

(*b*) The coefficient of friction is greatest when the brakes are first applied; it then reduces very rapidly, decreasing nearly one third after the brakes have been applied 10 seconds,

and dropping to nearly one half in the course of 20 seconds. Although the general truth of this law was established beyond question, the tests to demonstrate the law of the variation of friction with time of application were too few to determine accurately the numerical constants.

(*c*) The friction of skidded wheels on rails is always very much less than the adhesion when the wheel is rolling on the rail—sometimes less than one third as much.

(*d*) An analysis of the tests all pointed to a law that the friction developed does *not* increase as rapidly as the *intensity* of pressure increases, but this may hardly be considered as an established law.

(*e*) The adhesion between the wheel and the rail appears to be independent of velocity. The adhesion here means the force that must be developed before the wheel will slip on the rail.

The practical effect of these laws is shown by the following observed phenomena:

(*a*) When the brakes are first applied (the velocity being very high), a brake pressure far in excess of the weight on the wheel (even three or four times as much) may be applied without skidding the wheel. This is partly due to the fact that the wheel has a very high rotative kinetic energy (which varies as the square of the velocity, and which must be overcome first), but it is chiefly due to the fact that the coefficient of friction at the higher velocity is very small (at 60 miles per hour it is about .07), while the adhesion between the wheel and the rail is independent of the velocity.

(*b*) As the velocity decreases the brake pressure must be decreased or the wheels will skid. Although the friction decreases with the time required to stop and increases with the reduction of speed, and these two effects tend to neutralise each other, yet unless the stop is very slow, the increase in friction due to reduction of speed is much greater than the decrease due to time, and therefore the brake pressure must not be greater than the weight on the wheel, unless momentarily while the speed is still very high.

(*c*) The adhesion between wheels and rails varies from .20 to .25 and over when the rail is dry. When wet and slippery it may fall to .18 or even .15. The use of sand will always raise it above .20, and on a dry rail, when the sand is not blown away by wind, it may raise it to .35 or even .40.

(*d*) Experiments were made with an automatic valve by which the brake-shoe pressure against the wheel should be reduced as the friction increased, but since (1) the essential requirement is that the friction produced by the brake-shoes shall not exceed the adhesion between rail and wheel, and since (2) the rail-wheel adhesion is a very variable quantity, depending on whether the rail is wet or dry, it has been found impracticable to use such a valve, and that the best plan is to leave it to the engineer to vary the pressure, if necessary, by the use of the brake-valve.

MECHANISM OF BRAKES.

423. Hand-brakes. The old style of brakes consists of brake-shoes of some type which are pressed against the wheel-treads by means of a brake-beam, which is operated by means of a hand-windlass and chain operating a set of levers. It is desirable that brakes shall not be set so tightly that the wheels shall be locked, and then slide over the track, producing flat places on them, which are very destructive to the rolling-stock and track afterward, on account of the impact occasioned at each revolution. With air-brakes the maximum pressure of the brake-shoes can be quite carefully regulated, and they are so designed that the maximum pressure exerted by any pair of brake-shoes on the wheels of any axle shall not exceed a certain per cent. of the weight carried by that axle when the car is *empty*, 90% being the figure usually adopted for passenger-cars and 70% for freight-cars. Consider the case of a freight-car of 100000 lbs. capacity, weighing 33100 lbs., or 8275 lbs. on an axle, and equipped with a hand-brake which operates the levers and brake-beams, which are sketched in Fig. 205. The dead weight on an axle is 8275 lbs.; 70% of this is 5792 lbs., which is the maximum allowable pressure per brake-beam, or 2896 lbs. per brake-shoe. With the dimensions shown, such a pressure will be produced by a pull of about 1158 lbs. on the brake-chain. The power gained by the brake-wheel is not equal to the ratio of the brake-wheel diameter to the diameter of the shaft, about which the brake-chain winds, which is about 16 to 1½. The ratio of the circumference of the brake-wheel to the length of chain wound up by one complete turn would be a closer figure. The loss of effi-

ciency in such a clumsy mechanism also reduces the effective ratio. Assuming the *effective* ratio as 6:1 it would require a pull of 193 lbs. at the circumference of the brake-wheel to exert 1158 lbs. pull on the brake-chain, or 5792 lbs. pressure on the wheels at *B*, and even this will not lock the wheels when the car is empty, much less when it is loaded. Note that the pressures at *A* and *B* are unequal. This is somewhat objectionable, but it is unavoidable with this simple form of brake-beam. More complicated forms to avoid this are sometimes used. Hand-brakes are, of course, cheapest in first cost, and even with the best of automatic brakes, additional mechanism to operate the brakes by hand in an emergency is always provided, but their slow operation when a quick stop is desired makes it exceedingly dangerous to attempt to run a train at high speed unless some automatic brake directly under the control of the engineer is at hand. The great increase in the

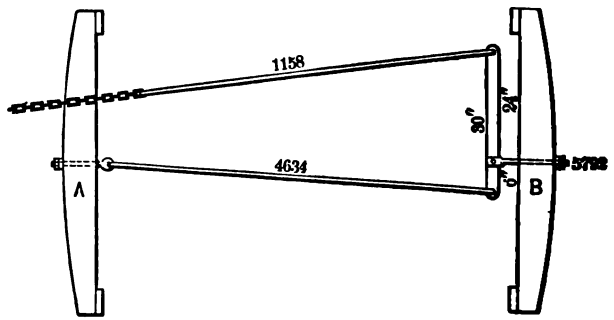


FIG. 205.—SKETCH OF MECHANISM OF HAND-BRAKE.

average velocity of trains during recent years has only been rendered possible by the invention of automatic brakes.

424. "Straight" air-brakes. The essential constructive features of this form of brake are (1) an air-pump on the engine, operated by steam, which compresses air into a reservoir on the engine; (2) a "brake-pipe" running from the reservoir to the rear of the engine and pipes running under each car, the pipes having flexible connections at the ends of the cars and engine; (3) a cylinder and piston under each car which

operates the brakes by a system of levers, the cylinder being connected to the brake-pipe. The reservoir on the engine holds compressed air at about 45 lbs. pressure. To operate the brakes, a valve on the engine is opened which allows the compressed air to flow from the reservoir through the brake-pipe to each cylinder, moving the piston, which thereby moves the levers and applies the brakes. The *defects* of this system are many: (1) With a long train, considerable time is required for the air to flow from the reservoir on the engine to the rear cars, and for an emergency-stop even this delay would often be fatal; (2) if the train breaks in two, the rear portion is not provided with power for operating the brakes, and a dangerous collision would often be the result; (3) if an air-pipe coupling bursts under any car, the whole system becomes absolutely helpless, and as such a thing might happen during some emergency, the accident would then be especially fatal.

This form of brake has almost, if not entirely, passed out of use. It is here briefly described in order to show the logical development of the form which is now in almost universal use, the automatic.

425. Automatic air-brakes. The above defects have been overcome by a method which may be briefly stated as follows: A reservoir for compressed air is placed under each car and the tender; whenever the pressure in these reservoirs is reduced for any reason, it is automatically replenished from the main reservoir on the engine; whenever the pressure in the brake-pipe is reduced for **any** cause (opening a valve at any point of its length, parting of the train, or bursting of a pipe or coupler), valves are automatically moved under each car to operate the piston and put on the brakes. *All* the brakes on the train are thus applied almost simultaneously. If the train breaks in two, *both* sections will at once have *all* the brakes applied automatically; if a coupling or pipe bursts, the brakes are at once applied and attention is thereby attracted to the defect; if an emergency should arise, such that the conductor desires to stop the train instantly without even taking time to signal to the engineer, he can do so by opening a valve placed on each car, which admits air to the train-pipe, which will set the brakes on the whole train, and the engineer, being able to discover instantly what had occurred, would shut off steam and do whatever else was necessary to stop the train as quickly as pos-

sible. The most important and essential detail of this system is the "automatic triple valve" placed under each car. Quoting from the Westinghouse Air-brake Company's Instruction Book, "A moderate reduction of air pressure in the train-pipe causes the greater pressure remaining stored in the auxiliary reservoir to force the piston of the triple valve and its slide-valve to a position which will allow the air in the auxiliary reservoir to pass directly into the brake-cylinder and apply the brake. A sudden or violent reduction of the air in the train-pipe produces the same effect, and in addition causes supplemental valves in the triple valve to be opened, permitting the pressure from the train-pipe to also enter the brake-cylinder, augmenting the pressure derived from the auxiliary reservoir about 20%, producing practically instantaneous action of the brakes to their highest efficiency throughout the entire train. When the pressure in the brake-pipe is again restored to an amount in excess of that remaining in the auxiliary reservoir, the piston- and slide-valves are forced in the opposite direction to their normal position, opening communication from the train-pipe to the auxiliary reservoir, and permitting the air in the brake-cylinder to escape to the atmosphere, thus releasing the brakes. If the engineer wishes to apply the brake, he moves the handle of the engineer's brake-valve to the right, which first closes a port, retaining the pressure in the main reservoir, and then permits a portion of the air in the train-pipe to escape. To release the brakes, he moves the handle to the extreme left, which allows the air in the main reservoir to flow freely into the brake-pipe, restoring the pressure therein."

426. Tests to measure the efficiency of brakes. Let v represent the velocity of a train in feet per second; W, its weight; F, the retarding force due to the brakes; d, the distance in feet required to make a stop; and g, the acceleration of gravity (32.16 feet per square second); then the kinetic energy possessed by the train (disregarding for the present the rotative kinetic energy of the wheels) $= \frac{Wv^2}{2g}$. The work done in stopping the train $= Fd$. $\therefore Fd = \frac{Wv^2}{2g}$. The ratio of the retarding force to the weight,

$$\frac{F}{W} = \frac{v^2}{2gd} = .0155\frac{v^2}{d}.$$

In order to compare tests made under varying conditions, the ratio $F \div W$ should be corrected for the effect of grade (+ or −), if any, and also for the proportion of the weight of the train which is on *braked* wheels. For example, a train weighed 46076 lbs., the proportion on braked wheels was 67%, speed 30 feet per second, length of stop 450 feet, track level. Substituting these values in the above formula, we find $(F \div W) = .124$. This value is really unduly favorable, since the ordinary track resistance helps to stop the train. This has a value of from 6 to 20 lbs. per ton, averaging say 10 lbs. per ton during the stop, or .005 of the weight. Since the effect of this is small and is nearly constant for all trains, it may be ignored in comparative tests. The grade in this case was level, and therefore grade had no effect. But since only 67% of the weight was on braked wheels, the ratio, on the basis of *all* the wheels braked, or of the weight reduced to that actually on the braked wheels, is $0.124 \div .67 = 0.185$. This was called a "good" stop, although as high a ratio as 0.200 has been obtained.

427. Brake-shoes. Brake-shoes were formerly made of wrought iron, but when it was discovered that cast-iron shoes would answer the purpose, the use of wrought-iron shoes was abandoned, since the cast-iron shoes are so much cheaper. A cheap practice is to form the brake-shoe and its head in one piece, which is cheaper in first cost, but when the wearing-surface is too far gone for further use, the whole casting must be renewed. The "Christie" shoe, adopted by the Master Car Builders' Association as standard, has a separate shoe which is fastened to the head by means of a wrought-iron key. The shoe is beveled $\frac{1}{4}''$ in a width of $3\frac{3}{8}''$ to fit the coned wheel. This is a greater bevel than the standard coning of a car-wheel. It is perhaps done to allow for some bending of the brake-beam and also so that the maximum pressure (and wear) should come on the outside of the tread, rather than next to the flange, where it might tend to produce sharp flanges. By concentrating the brake-shoe wear on the outer side of the tread, the wear on the tread is more nearly equalized, since the rail wears the wheel-tread chiefly near the flange. This same idea is developed still further in the "flange-shoes," which have a curved form to fit the wheel-flange and which bear on the wheel on the flange and on the outside of the tread. It is

claimed that by this means the standard form of the trea
better preserved than when the wear is entirely on the tr
The Congdon brake-shoe is one of a type in which wrou
iron pieces are inserted in the face of a cast-iron shoe. I
claimed that these increase the life of the shoe.

CHAPTER XVI.

TRAIN RESISTANCE.

428. Classification of the various forms. The various resistances which must be overcome by the power of the locomotive may be classified as follows:

(*a*) *Resistances internal to the locomotive*, which include friction of the valve-gear, piston- and connecting-rods, journal friction of the drivers; also all the loss due to radiation, condensation, friction of the steam in the passages, etc. In short, these resistances are the sum-total of the losses by which the power at the circumference of the drivers is less than the power developed by the boiler.

(*b*) *Velocity resistances*, which include the atmospheric resistances on the ends and sides; oscillation and concussion resistances, due to uneven track, etc.

(*c*) *Wheel resistances*, which include the rolling friction between the wheels and the rails of *all* the wheels (including the drivers); also the journal friction of all the axles, except those of the drivers.

(*d*) *Grade and curve resistances*, which include those resistances which are due to grade and to curves, and which are not found on a straight and level track.

(*e*) *Brake resistances*. As shown later, brakes consume power and to the extent of their use increase the energy to be developed by the locomotive.

(*f*) *Inertia resistances*. The resistance due to inertia is not generally considered as a train resistance because the energy which is stored up in the train as kinetic energy may be utilized in overcoming future resistances. But in a discussion of the demands on the tractive power of the engine, one of the chief items is the energy required to *rapidly* give to a starting train its normal velocity. This is especially true of suburban trains, which must acquire speed very quickly in order that

their general average speed between termini may be even reasonably fast.

429. Resistance internal to the locomotive. These are resistances which do not tax the adhesion of the drivers to the rails, and hence are frequently considered as not being a part of the train resistance properly so called. If the engine were considered as lifted from the rails and made to drive a belt placed around the drivers, then all the power that reached the belt would be the power that is ordinarily available for adhesion, while the remainder would be that consumed internally by the engine. The power developed by an engine may be obtained by taking indicator diagrams which show the actual steam pressure in a cylinder at any part of a stroke. From such a diagram the average steam pressure is easily obtained, and this average pressure, multiplied by the length of the stroke and by the net area of the piston, gives the energy developed by one half-stroke of one piston. Four times this product divided by 550 times the time in seconds required for one stroke gives the "indicated horse-power" Even this calculation gives merely the power behind the piston, which is several per cent. greater than the power which reaches the circumference of the drivers, owing to the friction of the piston, piston-rod, cross-head, connecting-rod bearings, and driving-wheel journals. (See § 411, Chapter XV.) By measuring the amount of water used and turned into steam, and by noting the boiler pressure, the energy possessed by the steam used is readily computed. The indicator diagrams will show the amount of steam that has been effective in producing power at the cylinders. The steam accounted for by the diagrams will ordinarily amount to 80 or 85% of the steam developed by the boiler, and the other 15 or 20% represents the loss of energy due to radiation, condensation, etc.

Locomotive resistance has been estimated and tabulated by a Committee of the Amer. Rwy. Eng. Assoc. and the results are given in Table XXIX, which is taken from the Manual of that Association. As a numerical illustration, what is the computed resistance for a Mikado locomotive of which the total weight of engine and tender is 315,000 lbs. of which 153,200 lbs. is carried on the drivers, at a velocity of 6 miles per hour? In this case, Item $A = (18.7 \times 76.6) + (80 \times 4) = 1432$ lbs. The weight carried on the engine and tender trucks $= 315{,}000 - 153{,}200 = 161{,}800$

= 80.9 tons. Item $B = (2.6 \times 80.9) + (20 \times 6) = 330$ lbs. Item C is comparatively insignificant at this low velocity. From the table, we read 9 lbs. Then the sum of A, B, and C = 1771 lbs., which must be subtracted from a computed tractive effort to obtain the estimated draw-bar pull.

TABLE XXIX. LOCOMOTIVE RESISTANCES.

Total Locomotive Resistance $= A + B + C$, in which

A = resistance between cylinder and rim of drivers, and in pounds
$= 18.7T + 80N$

in which T = tons weight on drivers, and
N = number of driving axles;

B = resistance of engine and tender trucks, and in pounds
$= 2.6T + 20N$

in which T = tons weight on engine and tender trucks
and N = number of truck axles;

C = head end or "air" resistance, and in pounds
$= .002V^2A$

in which V = velocity in miles per hour, and
A = end area of locomotive.

On the basis that the end area averages 125 square feet, the formula becomes $C = 0.25V^2$. The number of pounds air resistance for various velocities is as given below.

Vel.	Res.	Vel.	Res.	Vel.	Res.	Vel.	Res.	Vel.	Res.	Vel.	Res.
1	0.25	8	16.00	15	56	22	121	29	210	36	324
2	1.00	9	20.25	16	64	23	132	30	225	37	342
3	2.25	10	25.00	17	72	24	144	31	240	38	361
4	4.00	11	30	18	81	25	156	32	256	39	380
5	6.25	12	36	19	90	26	169	33	272	40	400
6	9.00	13	42	20	100	27	182	34	289	50	625
7	12.25	14	49	21	110	28	196	35	306	60	900

Draw-bar pull on level tangent equals the cylinder tractive power less the sum of the engine resistances.

At low speeds, the adhesion of the drivers should be considered and available draw-bar pull should never be estimated greater than 30% of weight on drivers at starting with use of sand, 25% of weight on drivers at running speeds.

Taken from Table 7 in "Economics" section of Manual of the Amer. Rwy. Eng. Assoc., 1915 edition.

430. Velocity resistance. (*a*) *Atmospheric.* This consists of the head and tail resistances and the side resistance. The head

and tail resistances are nearly constant for all trains of given velocity, varying but slightly with the varying cross-sections of engines and cars. The side resistance varies with the length of the train and the character of the cars, box-cars or flats, etc. Vestibuling cars has a considerable effect in reducing this side resistance by preventing much of the eddying of air-currents between the cars, although this is one of the least of the advantages of vestibuling. Atmospheric resistance is generally assumed to vary as the square of the velocity, and although this may be nearly true, it has been experimentally demonstrated to be at least inaccurate. Values for head resistance are given in Table XXIX, which are probably accurate enough for all practical purposes, especially at ordinary freight train velocities. A freight-train composed partly of flat-cars and partly of box-cars will encounter considerably more atmospheric resistance than one made exclusively of either kind, other things being equal. The definite information on this subject is very unsatisfactory, but this is possibly due to the fact that it is of little practical importance to know just how much such resistance amounts to.

(*b*) *Oscillatory and concussive.* These resistances are considered to vary as the square of the velocity. Probably this is nearly, if not quite, correct on the general principle that such resistances are a succession of impacts and the force of impacts varies as the square of the velocity. These impacts are due to the defects of the track, and even though it were possible to make a precise determination of the amount of this resistance in any particular case, the value obtained would only be true for that particular piece of track and for the particular degree of excellence or defect which the track *then* possessed. The general improvement of track maintenance during late years has had a large influence in increasing the possible train-load by decreasing the train resistance. The expenditure of money to improve track will give a road a large advantage over a competing road with a poorer track, by reducing train resistance, and thus reducing the cost of handling traffic.

431. **Wheel resistances.** (*a*) *Rolling friction of the wheels.* To determine experimentally the rolling friction of wheels, apart from all journal friction, is a very difficult matter and has never been satisfactorily accomplished. Theory as well as practice shows that the higher and the more perfect the

elasticity of the wheel and the surface, the less will be the rolling friction. But the determination, if made, would be of theoretical interest only.

The combined effect of rolling friction and journal friction is determinable with comparative ease. From the nature of the case no great reduction of the rolling friction by any device is possible. It is only a very insignificant part of the total train resistance.

(*b*) *Journal friction of the axles.* This form of resistance has been studied quite extensively by means of the measurement of the force required to turn an axle in its bearings under various conditions of pressure, speed, extent of lubrication, and temperature. The following laws have been fairly well established: (1) The coefficient of friction increases as the pressure diminishes; (2) it is higher at very slow speeds, gradually diminishing to a minimum at a speed corresponding to a train velocity of about 10 miles per hour, then slowly increasing with the speed; it is very dependent on the perfection of the lubrication, it being reduced to one sixth or one tenth, when the axle is lubricated by a bath of oil rather than by a mere pad or wad of waste on one side of the journal; (3) it is much lower at higher temperature, and *vice versa*. The practical effect of these laws is shown by the observed facts that (1) loaded cars have a less resistance per ton than unloaded cars, the figures being, for speeds of about 10 miles per hour, approximately:

For passenger- and loaded freight-cars..	4	lbs.	per	ton
" empty freight-cars.....	8	"	"	"
" street-cars.....	10	"	"	"
" freight-trucks without load.....	14	"	"	"

(2) When starting a train, the resistances are about 20 lbs. per ton, notwithstanding the fact that the velocity resistances are practically zero; at about 2 miles per hour it will drop to 10 lbs. per ton and above 10 miles per hour it may drop to 4 lbs. per ton if the cars are in good condition. (3) The resistance could probably be materially lowered if some practicable form of journal-box could be devised which would give a more perfect lubrication. (4) It is observed that freight-train loads must be cut down in winter by about 10 or 15% of the loads that the same engine can haul over the same track in summer. This is due partly to the extra roughness and inelasticity of the

track in winter, and partly to increased radiation from the engine wasting some energy, but this will not account for all of the loss, and the effect, which is probably due largely to the lower temperature of the journal-boxes, is very marked and costly. It has been suggested that a jacketing of the journal-boxes, which would prevent rapid radiation of heat and enable them to retain some of the heat developed by friction, would result in a saving amply repaying the cost of the device.

Roller journals for cars have been frequently suggested, and experiments have been made with them. It is found that they are very effective at low velocities, greatly reducing the starting resistance, which is very high with the ordinary forms of journals. But the advantages disappear as the velocity increases. The advantages also decrease as the load is increased, so that with heavily loaded cars the gain is small. The excess of cost for construction and maintenance has been found to be more than the gain from power saved.

432. Grade resistance. The amount of this may be computed with mathematical exactness. Assume that the ball or cylinder (see Fig. 206) is being drawn up the plane. If W

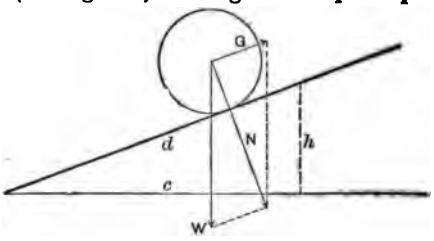


FIG. 206.

is the weight, N the normal pressure against the rail, and G the force required to hold it or to draw it up the plane with uniform velocity, the rolling resistances being considered zero or considered as provided for by other forces, then

$$G : W :: h : d, \quad \text{or} \quad G = \frac{Wh}{d};$$

but for all ordinary railroad grades, $d = c$ to within a tenth of 1%, i.e., $G = \frac{Wh}{c} = W \times$ rate of grade. In order that the student may appreciate the exact amount of this approximation the percentage of slope distance to its horizontal projection is given in the following tabular form:

Grade in per cent.	1	2	3	4	5
$\frac{\text{Slope dist.}}{\text{hor. dist.}} \times 100$.....	100.005	100.020	100.045	100.080	100.125

Grade in per cent.	6	7	8	9	10
$\frac{\text{Slope dist.}}{\text{hor. dist.}} \times 100$.....	100.180	100.245	100.319	100.404	100.499

This shows also the error on various grades of measuring with the tape on the ground rather than held horizontally. Since almost all railroad grades are less than 2% (where the error is but .02 of 1%), and anything in excess of 4% is unheard of for normal construction, the error in the approximation is generally too small for practical consideration.

If the rate of grade is 1 : 100, $G = W \times \frac{1}{100}$, i.e., $G = 20$ lbs. per ton; $\therefore$ for *any* per cent. of grade, $G = (20 \times$ per cent. of grade) pounds per ton. When moving up a grade this force G is to be overcome in addition to all the other resistances. When moving down a grade, the force G assists the motion and may be more than sufficient to move the train at its highest allowable velocity. The force required to move a train on a level track at ordinary freight-train speeds (say 20 miles per hour) is about 7 lbs. per ton. A down grade of $\frac{7}{20}$ of 1% will furnish the same power; therefore on a down grade of 0.35%, a freight-train would move indefinitely at about 20 miles per hour. If the grade were higher and the train were allowed to gain speed freely, the speed would increase until the resistance at that speed would equal W times the rate of grade, when the velocity would become uniform and remain so as long as the conditions were constant. If this speed was higher than a safe permissible speed, brakes must be applied and power wasted. The fact that one terminal of a road is considerably higher than the other does not necessarily imply that the extra power needed to overcome the difference of elevation is a total waste of energy, especially if the maximum grades are so low that brakes will never need to be applied to reduce a dangerously high velocity, for although more power must be

used in ascending the grades, there is a considerable saving of power in descending the grades. The amount of this saving will be discussed more fully in Chapter XXIII.

433. Curve resistance. Some of the principal laws will be here given without elaboration. A more detailed discussion will be given in Chapter XXII.

(*a*) While the total curve resistance increases as the degree of curve increases, the resistance *per degree of curve* is much greater for easy curves than for sharp curves; *e.g.*, the resistance on the excessively sharp curves (radius 90 feet) of the elevated roads of New York City is very much less *per degree of curve* than that on curves of 1° to 5°. (*b*) Curve resistance increases with the velocity. (*c*) The total resistance on a curve depends on the central angle rather than on the radius; i.e., two curves of the same central angle but of different radius would cause about the same total curve resistance. This is partly explained by the fact that the longitudinal slipping will be the same in each case. (See § 395, Chapter XV.) In each case also the trucks must be twisted around and the wheels slipped laterally on the rails by the same amount $\Delta°$. (See § 396, Chapter XV.)

434. Brake resistances. If a down grade is excessively steep so that brakes must be applied to prevent the train acquiring a dangerous velocity, the energy consumed is hopelessly lost without any compensation. When trains are required to make frequent stops and yet maintain a high average speed, considerable power is consumed by the application of brakes in stopping. All the energy which is thus turned into heat is hopelessly lost, and in addition a very considerable amount of steam is drawn from the boiler to operate the air-brakes, which consume the power already developed. It can be easily demonstrated that engines drawing trains in suburban service, making frequent stops, and yet developing high speed between stops, will consume a very large proportion of the total power developed by the use of brakes. Note the double loss. The brakes consume power already developed and stored in the train as kinetic or potential energy, while the operation of the brakes requires additional steam power from the engine.

435. Inertia resistance. The two forms of train resistance ch under some circumstances are the greatest resistances e overcome by the engine are the grade and inertia resist-

ances, and fortunately both of these resistances may be computed with mathematical precision. The problem may be stated as follows: What constant force P (in addition to the forces required to overcome the various frictional resistances, etc.) will be required to impart to a body a velocity of v feet per second in a distance of s feet? The required number of foot-pounds of energy is evidently Ps. But this work imparts a kinetic energy which may be expressed by $\frac{Wv^2}{2g}$. Equating these values, we have $Ps=\frac{Wv^2}{2g}$, or

$$P=\frac{Wv^2}{2gs}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (104)$$

The force required to increase the velocity from v_1 to v_2 may likewise be stated as $P=\frac{W}{2gs}(v_2^2-v_1^2)$. Substituting in the formula the values $W=2000$ lbs. (one ton), $g=32.16$, and $s=5280$ feet (one mile), we have

$$P=.00588(v_2^2-v_1^2).$$

Multiplying by $(5280\div3600)^2$ to change the unit of velocity to miles per hour, we have

$$P=.01267(V_2^2-V_1^2).$$

But this formula must be modified on account of the rotative kinetic energy which must be imparted to the wheels of the cars. The precise additional percentage depends on the particular design of the cars and their loading and also on the design of the locomotive. Consider as an example a box-car, 60000 lbs. capacity, weighing 33000 lbs. The wheels have a diameter of 36″ and their radius of gyration is about 13″. Each wheel weighs 700 lbs. The rotative kinetic energy of each wheel is 4877 ft.-lbs. when the velocity is 20 miles per hour, and for the eight wheels it is 39016 ft.-lbs. For greater precision (really needless) we may add 192 ft.-lbs. as the rotative kinetic energy of the axles. When the car is fully loaded (weight 93000 lbs.) the kinetic energy of translation is 1,244,340 ft.-lbs.; when empty (weight 33000 lbs.) the energy is 441540 ft.-lbs. The rotative kinetic energy thus adds (for this particular car) 3.15% (when the car is loaded) and 8.9% (when the car is empty) to the kinetic energy of translation. The kinetic

energy which is similarly added, owing to the rotation of the wheels and axles of the locomotive, might be similarly computed. For one type of locomotive it has been figured at about 8%. The variations in design, and particularly the fluctuations of loading, render useless any great precision in these computations. For a train of "empties" the figure would be high, probably 8 to 9%; for a fully loaded train it will not much exceed 3%. Wellington considered that 6% is a good average value to use (actually used 6.14% for "ease of computation"), but considering (*a*) the increasing proportion of live load to dead load in modern car design, (*b*) the greater care now used to make up *full* train-loads, and (*c*) the fact that *full* train-loads are the critical loads, it would appear that 5% is a better average for the conditions of modern practice. Even this figure allows something for the higher percentage for the locomotive and something for a few empties in the train. Therefore, adding 5% to the coefficient in the above equation, we have the true equation

$$P = .0133(V_2^2 - V_1^2), \quad . \quad . \quad . \quad . \quad . \quad (105)$$

in which V_2 and V_1 are the higher and lower velocities respectively, in *miles per hour*, and P is the force required *per ton* to impart that difference of velocity in a distance of *one mile*. If more convenient, the formula may be used thus: *

$$P_1 = \frac{70}{s}(V_2^2 - V_1^2), \quad . \quad . \quad . \quad . \quad . \quad (106)$$

in which s is the distance in feet and P_1 is the corresponding force.

As a numerical illustration, the force required per ton to impart a kinetic energy due to a velocity of 20 miles per hour in a distance of 1000 feet will equal

$$P_1 = \frac{70(400-0)}{1000} = 28 \text{ lbs.},$$

which is the equivalent (see § 432) of a 1.4% grade. Since the velocity enters the formula as V^2, while the distance enters only in the first power, it follows that it will require *four* times

* The slight approximation involved in the transformation from Eq. 105 to 106, by using the even number 70, is covered by allowing 4.6%, instead 5% for rotary kinetic energy.

the force to produce twice the velocity in the same distance, or that with the *same* force it will require four times the distance to attain twice the velocity.

As another numerical illustration, if a train is to increase its speed from 15 miles per hour to 60 miles per hour in a distance of 2000 feet, the force required (in addition to all the other resistances) will be

$$P_1 = \frac{70(3600-225)}{2000} = 118 \text{ lbs. per ton.}$$

This is equivalent to a 5.9% grade and shows at once that it would be impossible unless there were a very heavy down grade, or that the train was very light and the engine very powerful.

436. **Dynamometer tests.** These are made by putting a "dynamometer-car" between the engine and the cars to be tested. Suitable mechanism makes an automatic record of the force which is transmitted through the dynamometer at any instant, and also a record of the velocity at any instant. One of the practical difficulties is the *accurate* determination of the velocity at any instant when the velocity is fluctuating. When the velocity is decreasing, the kinetic energy of the train is being turned into work and the force transmitted through the dynamometer is less than the amount of the resistance which is actually being overcome. On the other hand, when the velocity is increasing, the dynamometer indicates a larger force than that required to overcome the resistances, but the excess force is being stored up in the train as kinetic energy. Grade has a similar effect, and the force indicated by the dynamometer may be greater or less than that required at the given velocity on a level by the force which is derived from, or is turned into, potential energy. The effect of curvature should be eliminated by subtracting from the dynamometer record 0.6 to 0.8 pound per ton per degree of curve, according to the rules for compensation of curvature as developed in § 511. Correct for grade by subtracting from the dynamometer record twenty pounds per ton for each percent of grade, assuming that the test train is moving *up* a grade; if the train is moving down grade, *add* a similar amount. Add (or subtract) the effect of *change* in velocity, as computed in § 435. Usually each dynamometer observation will need to be corrected by one or all of these corrections in order to determine what would have been the resistance on a *straight, level* track, at some definite *uniform* velocity.

In 1908–09 the Railway Eng. Dep't of the Univ. of Illinois conducted a series of tests, under the direction of Prof. E. C. Schmidt,* which were so elaborate and thorough that they definitely demonstrated that (*a*) the resistance per ton of any car depends very considerably upon the weight of the car, which is graphically shown in Fig. 206 *a*, and (*b*) the actual resistance per ton is variable and uncertain, and therefore no formula or resistance curve can assume to represent such resistance with a close percentage of accuracy. This uncertainty is illustrated by the fact that, in spite of the most elaborate care to eliminate all observational error and obtain uniform results, one typical group of plotted points had an average deviation of about 8% from the curve of average resistance and there was one instance of a 23% deviation. The variation in results is probably due to variable condition of the track (see § 430*b*) and shows that no one formula or curve, or set of them, is *closely* applicable to the variable track conditions found in the country or even to the variations found on any one road. The chief object in observing train resistance is to determine the tractive power required to haul a definite amount of traffic under certain known conditions, but these tests have confirmed what operating experience had already pointed out, that actual train resistance is so variable that there must be a considerable margin of tractive power in the locomotive or trains will be frequently stalled. Nevertheless, resistance formulae can be and are utilized for comparing proposed track locations and for computing, with a proper margin, the train load which may be attached to a locomotive of known tractive power.

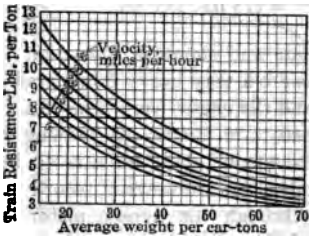
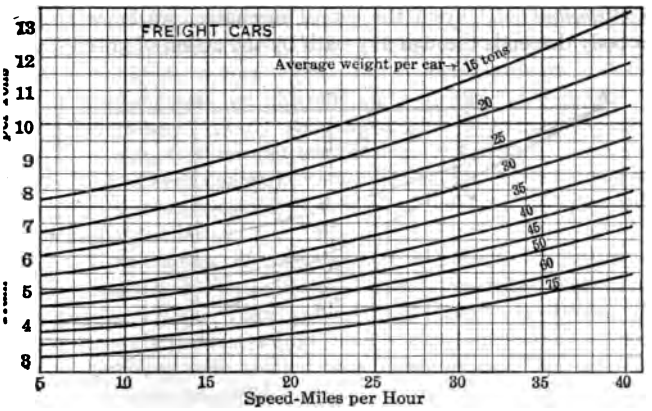


FIG. 206a.—RELATION BETWEEN RESISTANCE AND AVERAGE CAR WEIGHT, AT VARIOUS SPEEDS.
(Reduced from Fig. 10, Schmidt, Freight Car Resistance.)

The net result of these tests on 32 freight trains of various weights have been plotted in Fig. 206*b*, which shows ten curves, each for a different average car weight. For each curve, the resistance per ton increases with the velocity, being about 80% more for a velocity of 40 miles per hour than for a velocity of

* Univ. of Ill. Bull. 43, Freight Train Resistance, by Edward C. Schmidt.

5 miles per hour. Note that the upper curve (15 tons per car) s only applicable to a train of empties and the lower curve (75 ons per car) would mean a train of fully loaded cars. It should e fully realized that, in order to practically utilize these or other imilar curves as a measure of the tractive power demanded of , locomotive, due allowance should be made for grade, for curv- ture, and for the inertia effect of change in velocity; also that uch figures only claim to measure the resistance behind the ender, and that it does not apply if brakes have been used.



IG. 206b.—RELATION BETWEEN RESISTANCE AND SPEED, FOR VARIOUS AVERAGE WEIGHTS PER FREIGHT CAR.
(Reduced from Fig. 11, Schmidt, Freight Car Resistance.)

437. Gravity or " drop " tests. A drop test utilizes the force f gravity which may be measured with mathematical accuracy. he general method is to select a stretch of track which has a niform grade of about 0.7% and which is preferably straight r 2 or 3 miles. On such a grade cars with running gear a good condition may be started by a push. The velocity ill gradually increase until at some velocity, depending on ne resistances encountered, the cars will move uniformly. The nly work requiring extreme care with this method is the deter- ination of the velocity. If the velocity is fluctuating, as it during the time when it is of the greatest importance to know ne velocity, it is not sufficient to determine the time required run some long measured distance, for the average velocity

thus obtained would probably differ considerably from the velocity at the beginning and end of that space. If the train consists of five cars or more, the velocity may be determined electrically (as described by Wellington in his "Economic Location," etc., p. 793 *et seq.*) from the automatic record made on a chronograph of the passage of the first wheel and the last, the chronograph also recording automatically the ticks of a clock beating seconds. From this the exact time of the passage of the first and last wheels of the train of cars may be determined to the tenth or twentieth of a second.

Velocity-head. From theoretical mechanics we know that if a body descends through any path by the action of gravity, and

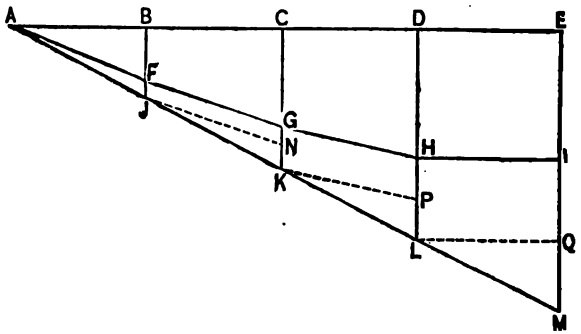


FIG. 207.—LOSS IN VELOCITY-HEAD.

is unaffected by friction, its velocity at any point in the direction of the path of motion is $V=\sqrt{2gh}$. If the body is retarded by resistances, its velocity at any point will be less than this. If AM, Fig. 207, represents any grade (exaggerated of course), then BJ, CK, etc., represent the actual fall at any point. Let BF represent the fall h_1, determined from $h_1=\frac{v_1^2}{2g}$, in which v_1 is the actual observed velocity at J. Then JF = the velocity-head consumed by the resistances between A and J. If the train continues to K, the corresponding h_2 is CG; the remaining fall GK consists of GN ($=JF$, which is the velocity-head lost back of J) and NK, the velocity-head lost between J and K. At some velocity (V_n) on any grade, the velocity will not further increase and the line $AFGHI$ will then be horizontal and at

a distance $(h_n) = EI$ below $A \ldots E$. The grade AM is the "grade of repose" for that velocity (V_n); i.e., it is the grade that would just permit the train to move indefinitely at the velocity V_n. The broken line $AFGHI$ should really be a curve, and the grade of repose at any point is the angle between AM and the tangent to that curve at the given point. The "grade of repose" by its definition gives the total resistance of the train at the particular velocity, or multiplying the grade of repose in per cent by 20 gives the pounds per ton of resistance. Thus being able to determine the total resistance in pounds per ton at any velocity, the variation of total resistance with velocity may be determined, and then by varying the resistances, using different kinds of cars, empty and loaded, box-cars and flats, the resistances of the different kinds at various velocities may be determined. Many tests have been made, on the above general plan, to determine track resistance, but, since it is impracticable and even dangerous to use this method for high velocities, the dynamometer-car method has been used for the most recent and reliable tests.

438. **Resistance of cars through switches.** It has always been realized that cars encounter greater resistance while passing through switches than on a straight unbroken track. This additional resistance would have a vital importance in case a passing siding were located on a ruling grade. The additional tractive force required to haul a train from a siding through a switch on to a main track would limit the length of train which might otherwise be hauled. Whenever a passing siding is essential on a ruling grade, the grade should be compensated, but the rate of compensation is still an uncertain quantity. An analogous problem is the rate of grade of a ladder track in a classification yard (see Chapter XIII, § 379) in order that, when switching cars by gravity from a hump, the added resistance, due to passing over the various frogs and switch rails on the ladder track, will not so exhaust the inertia due to the initial velocity that the cars cannot reach the desired locations on the classification tracks. Tests to determine such resistance were made in 1913–14, under the direction of Prof. C. L. Eddy, of the Case School of Applied Science.* The cars, usually singly but occasionally two, three or four together, were dropped from the top of a hump down a short 4% grade, by which they

* Bull, 175, Amer. Rwy. Eng. Assoc., March, 1915.

acquired a velocity varying from 14 to 21 miles per hour at the beginning of the ladder track, which had a downward grade of 1.175%. Velocities were observed at two places on the ladder track, by setting up at each place a pair of "contact points," usually 60 feet apart, by which the time of travelling the 60 feet was automatically recorded on a chronograph, which also recorded half seconds. The mean distance apart of the two pairs of contact points was at first 375 feet; then for other tests 400 feet and then 421.5 feet. Sometimes the velocity of the cars decreased while passing over this measured distance, and sometimes it increased. In any case the impelling force was the constant gravity force of $20 \times 1.175\% = 23$ pounds per ton, plus the inertia force due to the initial velocity. This net force, less the inertia force represented by the final velocity, equals the resisting force, in pounds per ton. As usual in such tests, the results were very variable, varying in 163 observations from a minimum of 4.5 to a maximum of 41.8 pounds per ton. The general average was about 22 pounds per ton, which is very nearly the gravity force (23.5 lbs.) of the ladder track used in this test. Note the increase in the average figure (22) above the average resistance per ton for whole trains of cars on a straight unbroken track, at the same average velocity of 15 to 20 m.p.h., which would vary from 3.5 to 9.5 pounds per ton—see Fig. 206*b*. A very small part of the increase is due to the extra atmospheric resistance *per ton* of one car over that of a train of cars, but the largest part of the excess resistance is that due to the frogs and switch points in the track, which, by their variable surface, variable elasticity and uneven support, cause shock resistances which average three or four times the normal resistance on an unbroken track. The above tests demonstrate (*a*) the very great increase of resistance on switches, and (*b*) that the resistance varies so greatly that no *precise* calculations can be made with respect to it. Although the average resistance was about 22 lbs. per ton, an allowance of 30 lbs. per ton would only cover 91% of the trials in the above test. It should also be noted that the switch work, made up of No. 8 frogs and split switches, was on the New York Central system, and was declared to be "in good order." It cannot therefore be claimed that this switch resistance was abnormally high.

439. American Railway Engineering Association Formula. In 1910, the Association Committee on Economics of Location

developed a formula with the special idea of its utilization in the comparative study of alternate locations of a railroad line, or in the operation of trains. An elaborate study of the very numerous formulae which had been published convinced the committee that all such formulae were either intrinsically worthless or that they were inapplicable to present conditions of track and rolling stock. After an exhaustive study of the results of recent dynamometer tests on the resistance of *freight* trains, with velocities varying from 5 to 35 m.p.h., it was declared that a formula which is sufficiently accurate for practical purposes can be put into the form

$$R = at + bn$$

in which t is the total weight of the train, in tons of 2000 lbs. and n is the number of cars; a and b are constants to be determined by tests. The values 2.78 and 113.9 for a and b respectively were first used on the basis of certain tests. Later, on the basis of an accumulation of additional tests, these constants were modified so as to have varying values according to the temperature and the following group of four formulae was recommended.

$$\left.\begin{array}{ll} A \text{ rating, temp.} = 35° \text{ F. or above;} & R = 2.2\,t + 122\,n \\ B \text{ rating, temp.} = 20° \text{ to } 35° \text{ F.;} & R = 3.0\,t + 137\,n \\ C \text{ rating, temp.} = 0° \text{ to } 20° \text{ F.;} & R = 4.0\,t + 153\,n \\ D \text{ rating, temp.} = \text{below } 0° \text{ F.;} & R = 5.4\,t + 171\,n \end{array}\right\} \quad . \quad (107)$$

These formulae apply only to level grade. When using them, suitable corrections for actual rate of grade and curvature, and a proper allowance for inertia, in accordance with the assumed method of operation, should be added to the resistance computed from Eq. 107.

Comparing these formulae with the results of the tests by Schmidt, we should use only the formula for A rating, since Schmidt's tests were all made at temperatures above 35° F. Assume a train of 53 empties, each weighing 18 tons, or a total of 954 tons, which is the value of t; $n = 53$; then the draw bar pull behind the tender equals

$$R = 2.2 \times 954 + 122 \times 53 = 2099 + 6466 = 8565 \text{ pounds.}$$

The mean resistance per ton would be $8565 \div 954 = 8.97$ pounds per ton. By Schmidt's curves (Fig. 206*b*) the resistance would

vary from about 7 lbs. per ton for a velocity of 5 m.p.h. to 11.4 lbs. per ton at 35 m.p.h., or a total of 6678 to 10876 lbs. resistance, depending on velocity. At a velocity of slightly over 20 m.p.h. the Schmidt curves show the same average resistance (8.97 lbs. per ton) for 18-ton cars.

A similar computation for a train of 30 cars weighing 70 tons each, or a total of 2100 tons, indicates a total resistance, by Eq. 107, of 8280 lbs. or 3.94 lbs. per ton. This again is the resistance per ton indicated by the Schmidt curves for 70-ton cars when the velocity is a little over 20 m.p.h.

The student should note that although the A.R.E.A. formula is independent of velocity, while the Schmidt curves indicate resistances varying as a function of the first power and also of the square of the velocity, the results at a velocity of about 20 m.p.h. are identical. Secondly both agree (up to 25 m.p.h.) that, although the loaded train weighs considerably more than twice as much as the train of empties, the pull on the draw bar is actually less, which forcibly illustrates the economy of operating full and heavily loaded cars.

The application of Eq. 107 to the operation of trains, or to train rating, is explained in Chapter XVIII, § 467.

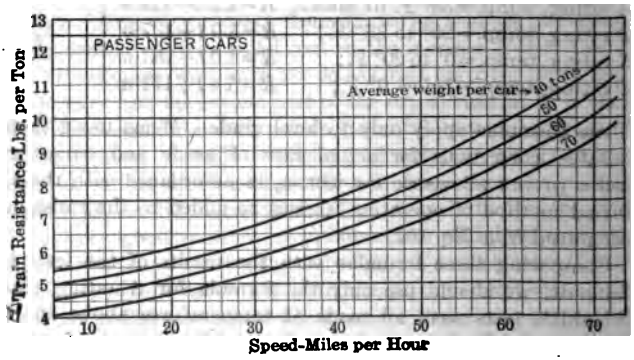


FIG. 207a.—RELATION BETWEEN RESISTANCE AND SPEED, FOR VARIOUS AVERAGE WEIGHTS PER PASSENGER CAR.

(Reduced from Fig. 6, Schmidt, Passenger Train Resistance.)

439a. **Passenger-car resistance.** In 1916, Prof. E. C. Schmidt made some tests on passenger-car resistance by the same general

methods used in freight-car tests, as described in § 436.* Tests were made on eighteen trains, of which the average car weight varied from 48.7 to 71.1 tons. 83% of the cars had six-wheeled trucks.

The curves plotted from these tests are shown on a reduced scale in Fig. 207*a*, which shows the same general form of curves as those of Fig. 206*b*. It should also be noted that the tests showed that the heavier cars have less resistance per ton than lighter cars, the same as for freight cars. Comparing the curves, where identical conditions make such comparisons possible, it may be noted that, in general, freight cars showed a *less* resistance per ton than passenger cars for the same velocity and weight of car. Many years ago a committee of the Am. Rwy. Master Mechanics Assoc. reported that "six-wheel trucks are found to produce greater resistance, and as a consequence absorb more hauling power than four-wheel trucks carrying the same weight of car." Six-wheeled trucks are considered essential for carrying especially heavy cars at high passenger-train speed, in spite of the proved added per-ton resistance. Since nearly all trains in the above tests included cars with both six-wheeled and four-wheeled trucks, it was impracticable to differentiate the results on this basis, but the fact that about 83% of the cars had six-wheeled trucks probably explains the higher per-ton results. When the passenger-car results are reduced to per-ton-per-axle, the freight-car and passenger-car results are more nearly uniform. Whenever these curves are used, it should be kept in mind that the effect of grade, curvature and inertia resistance have all been eliminated from these results. The tests were made in pleasant weather, during the summer. It should therefore be expected that the resistance in cold and windy weather would be materially greater.

It is interesting to note that the careful calculations made of the weight of the live load (passengers, baggage, mail and express) showed that the maximum load weighed only 5.2% of the gross train load, and therefore the cost of running a passenger train is measurably the same whether it runs full or absolutely empty.

* Proceedings, Amer. Rwy. Eng. Assoc., Vol. 18, p. 689.

CHAPTER XVII

COST OF RAILROADS.

440. General considerations. Although there are many elements in the cost of railroads which are roughly constant per mile of road, yet the published reports of the cost of railroads differ very widely. The variation in the figures is due to several causes. (*a*) Economy requires that a road shall be operated and placed on an earning basis as soon as possible. Therefore the reported cost of a road during the first few years of its existence is somewhat less than that reported later. This is well illustrated when a long series of consecutive reports from an old-established road is available; nearly every year there will be shown an addition to the previous figures. And this is as it should be. The magnificent road-beds of some old roads cannot be the creation of a single season. It takes many years to produce such settled perfect structures. (*b*) A large part of the variation is due to a neglect to charge up "permanent improvements" as additions to the cost of the road. For the first few years of the life of a road a great deal of work is done which is in reality a completion of the work of construction, and yet the cost of it is buried under the item "maintenance of way." For example, a long wooden trestle is replaced by an earth embankment and a culvert. Since the original trestle is to be considered a temporary structure, the *excess* of the cost of the permanent structure over that of the temporary structure should evidently be considered as an addition to the cost of the road. But if the filling-in was done slowly, a few train-loads at a time, and the work scattered over many years, the cost of operating the "mud-train" has perhaps been buried under "maintenance" charges. (*c*) The reports from which many of the following figures were taken have not always analyzed the items of cost with the same detail as has been here attempted, and to that is probably due many of the variations and apparent discrepancies.

The various items of cost will be classified as follows:

1. Preliminary financiering.
2. Surveys and engineering expenses.
3. Land and land damages.
4. Clearing and grubbing.
5. Earthwork, including rockwork; tunneling.
6. Bridges, trestles, and culverts.
7. Trackwork, material and track labor.
8. Buildings and miscellaneous structures.
9. Interest on construction.
10. Rolling stock.

441. Item 1. **PRELIMINARY FINANCIERING.** The cost of this preliminary work is exceedingly variable. The work includes the clerical and legal work of organization, printing, engraving of stocks and bonds, and (sometimes the most expensive of all) the securing of a charter. This sometimes requires special legislative enactments, or may sometimes be secured from a State railroad commission. It has been estimated that about 2% of the railway capital of Great Britain has been spent in Parliamentary expenses over the charters. These expenses are usually but a small percentage of the total cost of the enterprise, but for important lines the gross cost is large, while the amount of money thus spent by organizations which have never succeeded in constructing their roads is, in the aggregate, an enormous amount, although it is of course not ascertainable by any investigator.

Another occasional feature of the financing of a road must be kept in mind. The promoters of a railroad enterprise frequently endeavor to limit their own personal expenditures to the purely preliminary expenses as mentioned above. The project, after having been surveyed, mapped, and written up in a glowing "prospectus," is submitted to capitalists, in the endeavor to have them furnish money for construction, the money to be secured by bonds. If the project will stand it, the amount of the bond issue is made sufficient to pay the entire cost of the road, even with a discount of perhaps 15%. The bond issue may also provide for a very generous commission to the broker who is the intermediary between the promoters and the capitalists. The bond issue may even provide for repaying the promoters for their preliminary expenses. Frequently a considerable proportion of the capital stock goes to the capitalists

who take the bonds, the promoters retaining only such proportion as may be agreed upon. In such a case, the capital stock is "pure velvet," and costs nothing. Its future value, whatever it may be, is so much clear profit. The effect of such a financial policy is to burden the project with a capitalization which is far in excess of the actual cost of constructing the road. Comparatively few projects will stand such over-capitalization. The apparent financial failure of many railroads, which have gone into the hands of receivers is due to their inability to make returns on an over-capitalization rather than because they could not earn enough to pay the legitimate cost of their construction. These features of financiering are really foreign to the engineer's work, but he should know that many projects which would return a handsome profit on an investment amounting only to the legitimate cost, will be rejected by capitalists because it is apparent that there is not enough "velvet" in it.

442. Item 2. SURVEYS AND ENGINEERING EXPENSES. The comparison of a large number of itemized reports on the cost of construction shows that the cost of the "engineering" will average about 2% of the total cost of construction. This includes the cost of surveys and the cost of laying out and superintending the constructive work. The cost of mere surveying up to the time when construction actually commences has been variously quoted at $60, $75, and even $300 per mile. The lower figures generally refer to the hasty, ill-considered work which was formerly common and which has resulted in so much badly located road, much of which has been reconstructed, when improvements are practicable. See the introductory paragraphs of Chapter I. Except when the topography limits the location to one very obvious route, a thorough survey may cost about $300 per mile. In the estimate given at the end of this chapter the cost of "engineering and office expenses" is given at 5% of the cost of the construction work. The item thus includes the cost of the very considerable amount of clerical work and superintendence incident to the expenditure of such a large sum of money.

443. Item 3. Land and Land Damages. The cost of this item varies from the extreme, in which not only the land for right-of-way but also grants of public land adjoining the road are given to the corporation as a subsidy, to the other extreme

where the right-of-way can only be obtained at exorbitant prices. The width required is variable, depending on the width that may be needed for deep cuts or high fills, or the extra land required for yards, stations, etc. A strip of land 1 mile long and 8.25 feet wide contains precisely 1 acre. An average width of 4 rods (66 feet), therefore, requires 8 acres per mile. On the Boston & Albany Railroad the expenditure assigned to "land and land damages" averages over $25000 per mile. Of course this includes some especially expensive land for terminals and stations in large cities. Less than $300 per mile was assigned to this item by an unimportant 18-mile road.

444. **Item 4. CLEARING AND GRUBBING.** The cost of this may vary from zero to 100% for miles at a time, but as an average figure it may be taken as about 3 acres per mile at a cost of say $50 per acre. The possibility of obtaining valuable timber, which may be utilized for trestles, ties, or otherwise, and the value of which may not only repay the cost of clearing and grubbing, but also some of the cost of the land, should not be forgotten.

445. **Item 5. EARTHWORK.** This item also includes rock-work. The methods of estimating the cost of earthwork and rockwork have been discussed in Chapter III. The percentage of this item to the total cost is very variable. On a western prairie it might not be more than 5 to 10%. On a road through the mountains it will run up to 20 or 25%, and even more. The item also includes tunneling, which on some roads is a heavy item.

446. **Item 6. BRIDGES, TRESTLES, AND CULVERTS.** This item will usually amount to 5 or 6% of the total cost of the road. In special cases, where extensive trestling is necessary, or several large bridges are required, the percentage will be much higher. On the other hand, a road whose route avoids the watercourses may have very little except minor culverts. On the Boston & Albany the cost is given as $5860 per mile; on the Adirondack Railroad, $2845 per mile. Considering their relative character (double and single track), these figures are relatively what we might expect.

447. **Item 7. Trackwork.** This item will be considered as including everything above subgrade, except as otherwise itemized.

(a) **Ballast.** As already elaborated in Chapter VII, Ball the standards for depth of ballast, in order to produce a unifo pressure on sub-grade, have so increased that former estim are inapplicable. The increased depth now called for is usu provided by using a layer of sub-ballast made of comparativ inexpensive material, such as cinders, which, being a by-prod has only a nominal cost. The unit cost of ballast per cu yard varies from merely nominal to the cost of broken sto which may cost $1.50 or even $2.00 per cubic yard.

(b) **Ties.** Ties cost anywhere from $1.40 down to 50 c. a even less. At an average figure of 80c., 2640 ties per mile cost $2112 per mile of single track. The cheaper ties are usu smaller and more must be used per mile, and this tends to co pensate the difference in cost.

The following tabular form is convenient for reference:

TABLE XXX.—NUMBER OF CROSS-TIES PER MILE OF TRACK.

Number per 33′ rail.	Average spacing center to center.	Number per mile.
22	18.0 inches	3520
21	18.9 "	3360
20	19.8 "	3200
19	20.9 "	3040
18	22.0 "	2880
17	23.3 "	2720
16	24.75 "	2560
15	26.4 "	2400
14	28.3 "	2240
13	30.5 "	2080

(c) **Rails.** The total weight of the rails used per mile n best be seen by the tabular form.

A convenient and useful rule to remember is that the num of *long* tons (2240 lbs.) per mile of single track equals the wei of the rail per yard times $\frac{11}{7}$. The rule is exact. For exam there are 3520 yards of rail in a mile of single track; at 70 per yard this equals 246,400 lbs., or 110 long tons (exact but $70\times\frac{11}{7}=110$.

Any calculation of the required weight of rail for a gi weight of rolling-stock necessarily depends on the assumpti which are made regarding the support which the rails rec from the ties. This depends not only on the width and spac of the ties (which are determinable), but also on the supp

TABLE XXXI.—TONS PER MILE OF RAILS OF VARIOUS WEIGHTS.

Weight in lbs. per yd.	Tons (2240 lb.) per mile of single track.	Weight in lbs. per yd.	Tons (2240 lb.) per mile of single track.	Weight in lbs. per yd.	Tons (2240 lb.) per mile of single track.	Weight in lbs. per yd.	Tons (2240 lb.) per mile of single track.
8	12.571	25	39.286	55	86.429	85	133.571
10	15.714	30	47.143	60	94.286	90	141.429
12	18.857	35	55.000	65	102.143	95	149.286
14	22.000	40	62.857	70	110.000	100	157.143
16	25.143	45	70.714	75	117.857	110	172.857
20	31.429	50	78.571	80	125.714	120	188.571

About two per cent (2%) extra should be allowed for waste in cutting.

which the ties receive from the ballast, which is not only very uncertain but variable. No general rule can therefore claim any degree of precision, but the following is given by the Baldwin Locomotive Works: The weight per wheel which can be safely carried for each pound weight of rail per yard is approximately as follows:

Light rails; 60 lbs. and less per yard; 250 lbs.;
Medium rails; 60 lbs. to 90 lbs. per yard; 300 lbs.;
Heavy rails; 90 lbs. and over per yard; 350 lbs.

This assumes that the rails are properly supported by cross ties, not less than 14 per 30-ft. rail. For example, a Mikado locomotive with 153,200 lbs. on 8 drivers has a load of 19,150 lbs. per wheel. This divided by 300 gives 63.8. According to the rule, the rails for such a locomotive should weigh at least 63.8 lbs. per yard. But it should be noted that railroads which use Mikado locomotives will also have their track laid with heavier than 63.8 (or 65) pound rails. The rule should therefore be considered as the *minimum* permissible. A road with even one high-speed train, or a Class A road (§ 234), should use 80 to 90 lb. rails, even if not required by the above rule.

On the basis of 33-foot lengths, and 10% shorter lengths, varying by even feet down to 25 feet (see § 273 *e*), the average length, assuming an equal number each of the shorter length rails, would be 32.55 feet. Calculating similarly for 30-ft. rails, with 10% shorts to 24 feet, the average length would be 29.65 feet. 60-ft. rails, used extensively for electric roads, with 10% shorts to 40 feet, will have average length of 58.95 feet.

(d) **Splice-bars, track-bolts, and spikes.** These are usually sold by the pound, except the patented forms of rail-joints,

which are sold by the pair. In any case they are subject market fluctuations in price. As an approximate value following prices are quoted: Splice-bars, 2.50 cents per pou track-bolts, 4.0 cents; spikes, 3.25 cents. The weight of splice-bars will depend on the precise pattern adopted cross-section and length.

In Table XXXII are quoted, from a catalogue of the Illin Steel Co., the weights per foot of sections of angle-bars wh they recommend for various weights of rail and which are signed to fit standard A. S. C. E. rail sections of those weig The net weight of the angle-bars may be approximated subtracting about 2.5% to 4% from the gross weight to all for the bolt-holes. A deduction of 2.5% is usually about ri for the heavier sections. Their recommendations regard lengths of angle-bars do not include those for rails heavier th 50 pounds per yard. On the basis of a length of 24 inches four-hole splices and of 32 inches for six-hole splices, the weig of splice-bars have been computed for the several styles of spli for heavier rails, allowing 2.5% for the holes. The leng recommended for track bolts are those which will allow ab ½ inch for the nutlock and for margin, except for the lighter rail

TABLE XXXII.—SPLICE-BARS FOR VARIOUS WEIGHTS OF RAILS.

Weight of rail.	Length of angle-bar.	Weight per foot.	Weight of pair.	Proper size of track-bolt.	Proper siz of spikes
30	21″	4.49	15.1	2½″ × ⅞″	4 ″ × ½″
35	21″	4.7	15.9	2½″ × ⅞″	4½″ × ½″
40	21″	5.54	18.8	3 ″ × ⅞″	5 ″ × ½″
45	21″	6.3	21.5	3 ″ × ⅞″	5½″ × ½″
50	21″	6.97	23.4	3½″ × ⅞″	5½″ × ½″
55	24″	7.5	29.2	3½″ × ⅞″	5½″ × ½″
60	24″	8.4	32.8	3½″ × ⅞″	5½″ × ½″
65	24″	9.2	35.9	4 ″ × ⅞″	5½″ × ½″
	32″	9.6	49.9	4½″ × ⅞″	5½″ × ½″
70	24″	9.0	35.1	4 ″ × ⅞″	5½″ × ½″
	32″	10.0	52.0	4 ″ × ⅞″	5½″ × ½″
75	24″	10.68	42.6	4½″ × ⅞″	5½″ × ½″
	32″	11.9	61.9	4½″ × ⅞″	5½″ × ½″
80	24″	10.61	42.3	4½″ × ⅞″	5½″ × ½″
	32″	11.65	76.2	4½″ × ⅞″	5½″ × ½″
85	32″	12.4	64.5	4½″ × ⅞″	5½″ × ½″ or
90	32″	13.5	70.2	4½″ × ⅞″	5½″ × ½″ or
95	32″	14.7	76.4	4½″ × ⅞″	5½″ × ½″ or
100	32″	15.78	82.1	4½″ × ⅞″	5½″ × ½″ or

(e) Track-laying. Much depends on the force of men e ployed and the use of systematic methods; $528 per mile was t

TABLE XXXIII.—RAILROAD SPIKES.

Size measured under head.	Average number per keg of 200 pounds	Ties 24″ between centers, 4 spikes per tie, number per mile.		Suitable weight of rail.
		Pounds.	Kegs.	
5½″ × ⅝″	275	7680	38.40	90 to 100
5½″ × ½″	375	5632	28.16	45 " 100
5″ × ⅝″	400	5280	26.40	40 " 56
5″ × ½″	450	4692	23.46	40
4½″ × ⅝″	530	3984	19.92	35
4″ × ½″	600	3520	17.60	30
4½″ × 7/16″	680	3104	15.52	25 to 30

TABLE XXXIV.—TRACK-BOLTS.

Average number in a keg of 200 pounds.

Size of bolt.	Square nut.	Hexagonal nut.	Suitable rail.
3″ × ⅝″	366	395	40 pound
3″ × ½″	250	270	
3½″ × ⅝″	243	261	
3½″ × ½″	236	253	50
3½″ × ⅝″	229	244	55 to 60
4″ × ⅝″	222	236	65 " 70
4½″ × ⅝″	215	228	75
3½″ × ⅝″	170	180	
3½″ × ½″	165	175	
4″ × ⅝″	161	170	
4½″ × ⅝″	157	165	80
4½″ × ½″	153	160	85
4½″ × ⅝″	149	156	90

TABLE XXXV.—RAIL-JOINTS AND TRACK-BOLTS. NUMBER PER MILE OF TRACK.

Length of rail. Feet.	Average length of rail. Feet.	Number of rails or complete joints.	Number of bolts.	
			4-bolt.	6-bolt.
All 30	30	352	1408	2112
30–24	29.65	356.2	1425	2137
All 33	33	320	1280	1920
33–27	32.65	323.4	1294	1941
All 60	60	176	704	1056
60–40	58.95	179.1	717	1075

estimate formerly employed by the Pennsylvania Railroad. $500 per mile is the estimate given in § 451. See note at bottom of p. 536.

448. **Item 8. Buildings and Miscellaneous Structures.** Except for rough and preliminary estimates, these items must be individually estimated according to the circumstances. The subitems include depots, engine-houses, repair-shops, water-stations, section- and tool-houses, besides a large variety of smaller buildings. The structures include turn-tables, cattle-guards, fencing, road-crossings, overhead bridges, telegraph line, etc. The detailed estimate, given in § 451, illustrates the cost of these smaller items.

449. **Item 9. Interest on Construction.** The amount of capital that must be spent on a railroad before it has begun to earn anything is so very large that the interest on the cost during the period of construction is a very considerable item. The amount that must be charged to this head depends on the current rate of money on the time required for construction and on the ability of the capitalists to retain their capital where it will be earning something until it is actually needed to pay the company's obligations. Of course, it is not necessary to have the entire capital needed for construction on hand when construction commences. Assuming money to be worth 6%, that the work of construction will require one year, that the money may be retained where it will earn something for an average period of six months after construction commences, or, in other words, it will be out of circulation six months before the road is opened for traffic and begins to earn its way, then we may charge 3% on the total cost of construction.

450. **Item 10. Rolling Stock.** The cost depends on the traffic to be handled and bears very little relation to the total or the mileage cost of the roadbed and track. In each case the cost, at proper unit prices, of the locomotives and cars necessary to handle the estimated traffic must be computed.

451. **Detailed estimate of the cost of a line of road.** The following estimate was given in the *Engineering News* of Dec. 27, 1900, of the cost of the Duluth, St. Cloud, Glencoe & Mankato Railroad, 157.2 miles long.

The estimate is exactly as copied from the *Engineering News*. There are some numerical discrepancies. Item 26 should evidently be based on the sum of the first 25 items, and item 27

on the sum of the first 26. The figures in parentheses ( ) are deduced from the figures given.

1.	Right-of-way: 1905.3 acres (12.12 acres per mile) @ $100 per acre		$190530
2.	Clearing and grubbing. 144 acres (0.916 acre per mile) @ $50 per acre		7200
3.	Earth excavation. 1907590 cu. yds. (12135 cu. yds. per mile) @ 15 c		286138
4.	Rock excavation. 5100 cu. yds. (32.44 cu. yds. per mile) @ 80 c.		4080
5.	Wooden-box culverts. 508300 ft. B.M. @ $30 per M.	$15249	
	Iron-pipe culverts. 879840 lbs. @ 3c. per lb	26395	41644
6.	Pile trestling. 4600 lin. ft. @ 35 c. per lin. ft.	1610	
	Timber trestling. 509300 ft. B.M. @ $30 per M	15279	16889
7.	Bridge masonry: 5520 cu. yds. @ $8 per cu. yd	44160	
	Bridges, iron, 100 spans. 2000000 lbs. @ 4 c. per lb	80000	124160
8.	Cattle-guards		8750
9.	Ties (2640 per mile). 419813 (159.02 miles) @ 35 c.		146935
10.	Rails (70 lbs. per yd.): 110 tons per mile, 17492.2 tons (159.02 miles @ $26		384797
11.	Rail sidings (70 lbs. per yd.): 110 tons per mile, 3300 tons (30 miles @ $26		85800
12.	Switch timbers and ties		3300
13.	Spikes: 5920 lbs. per mile, 1107040 (187 m.) @ 1.75. c. per lb.		19373
14.	Splice-bars. 2635776 lbs. @ 1.35 c. per lb		35583
15.	Track-bolts (2 to joint (?)): 188458.3 lbs. @ 2.4 c. per lb		4520
16.	Track-laying 187.2 miles @ $500 per mile		93600
17.	Ballasting: 2152 cu. yds. per mile, 402854 (187.2 m.) @ 60 c.		241712
18.	Turn-out and switch furnishings		6450
19.	Road-crossings, 68040 ft. B.M. @ $30 per M		2041
20.	Section and tool-houses, 16 @ $800		12800
21.	Water-stations		15000
22.	Turn-tables, 6 @ $800		4800
23.	Depots, grounds, and repair-shops		78000
24.	Terminal grounds and special land damages		150000
25.	Fencing, 314 miles ($150 per mile)		47100
26.	Engineering and office expenses (5% of $1984458)		99222
27.	Interest on construction (3% of $2083680)		62510
28.	Rolling-stock ($5000 per mile)		786000
29.	Telegraph line: 157 miles @ $200 per mile		31400
			$3060340

Average cost per mile ready for operation, $19467.

Approximate cost of 130 miles from St. Cloud to Duluth, estimated at $23000 per mile.

Approximate cost of entire line from Albert Lea to Duluth, 287.2 miles, $6050340 ($21060 per mile).

Although the above estimate is now (1921) so old that the prices are obsolete, the list is retained since it is a typical analysis and may be utilized by making the proper changes in unit prices, which is always more or less necessary.

CHAPTER XVIII.

THE POWER OF A LOCOMOTIVE.

452. Pounds of steam produced. The power that can be developed by a locomotive depends very greatly on the quality of the coal burned and the design of the locomotive must correspond to the general kind or quality of coal to be used. A British thermal unit (symbolized as B.t.u.), is the quantity of heat required to raise the temperature of 1 lb. of pure water 1° F., when the water is at or near its maximum density at 39.1° F. When it is said that a certain grade of coal has 14000 B.t.u. it means that the heat in 1 lb. of that coal will raise the temperature of 14000 lbs. of water 1°, or, approximately, 100 lbs. of water 140°. But, although it only requires 180.9 heat units to heat water from 32° to 212°, it requires 965.7 more heat units to change it from water at 212° to steam at 212°. It requires only 53.6 more heat units to change it from steam at 212° to steam at 387.6° or with a pressure of 200 lbs. per square inch.

A study of locomotive tests made at the St. Louis Exposition resulted in the compilation of Table XXXVI, which is copied from the Proceedings of the American Railway Engineering Association, and is now included as Table I, in the "Economics" section of their Manual. It was found that the steam produced per square foot of heating surface is very nearly proportional to the coal burned per square foot of heating surface. The results are purposely made about 5% below the results obtained in the St. Louis tests to allow for ordinary working conditions.

453. Numerical example. The theory developed in this chapter will be illustrated numerically by applying it to a Mikado type of locomotive whose dimensions are as follows:

Cylinder	diam. 22″	Weight, driving wheels.	153,200 lbs.
Cylinder	stroke 28″	engine alone	196,100 lbs.
Driving wheel	diam. 57″	engine and tender	315,000 lbs.
Boiler pressure	185 lbs.	Heating surface, fire-box and tubes	2565 sq. ft.
Fire-box	length 102¾″	superheating surface.	580 sq. ft.
Fire-box grate area	width 65½″ 46.8 sq. ft.		

TABLE XXXVI.—AVERAGE EVAPORATION IN LOCOMOTIVE BOILERS BURNING BITUMINOUS AND SIMILAR COALS OF VARIOUS QUALITIES, AND FOR VARIOUS QUANTITIES CONSUMED PER SQUARE FOOT OF HEATING SURFACE PER HOUR.

(Based on feed water at 60° Fahrenheit, and boiler pressure 200 pounds)

Coal per square foot of heating surface per hour (lb.)	Steam per pound of coal of given thermal value (lb.)					
	15,000 B.t.u.	14,000 B.t.u.	13,000 B.t.u.	12,000 B.t.u.	11,000 B.t.u.	10,000 B.t.u.
0.8	7.86	7.34	6.81	6.29	5.76	5.24
0.9	7.58	7.07	6.57	6.06	5.56	5.05
1.0	7.31	6.82	6.34	5.85	5.36	4.87
1.1	7.06	6.59	6.12	5.65	5.18	4.71
1.2	6.82	6.37	5.91	5.46	5.00	4.55
1.3	6.59	6.15	5.71	5.27	4.83	4.39
1.4	6.37	5.95	5.52	5.10	4.67	4.25
1.5	6.17	5.76	5.35	4.94	4.52	4.11
1.6	5.97	5.57	5.18	4.78	4.38	3.98
1.7	5.79	5.40	5.02	4.63	4.25	3.86
1.8	5.61	5.24	4.86	4.49	4.12	3.74
1.9	5.44	5.08	4.71	4.35	3.99	3.63
2.0	5.27	4.92	4.57	4.22	3.86	3.51
2.1	5.12	4.78	4.44	4.10	3.75	3.41
2.2	4.97	4.64	4.31	3.98	3.64	3.31
2.3	4.83	4.51	4.19	3.86	3.54	3.22
2.4	4.69	4.38	4.07	3.75	3.44	3.13
2.5	4.56	4.26	3.95	3.65	3.34	3.04
2.6	4.44	4.14	3.84	3.55	3.25	2.96
2.7	4.32	4.03	3.74	3.46	3.17	2.88
2.8	4.21	3.93	3.64	3.37	3.09	2.80
2.9	4.10	3.83	3.55	3.28	3.01	2.73
3.0	3.99	3.73	3.46	3.19	2.93	2.66

The quantity of steam evaporated for intermediate quantities or qualities of coal can be found by interpolation.

On bad-water districts deduct the following from tabular quantities:

For each $\frac{1}{8}$ inch of accumulated scale..... 10 per cent
For each grain per U. S. gallon of foaming salts in the average feed water..... 1 per cent

Assume that this locomotive is using coal whose air-dried mine samples tested 13000 B.t.u.; then the average run-of-car coal would have about 90% of this or 11700 B.t.u. On the basis that a fireman can handle 4000 lbs. of coal per hour and maintain such work throughout his run, the coal may be fed at the rate of $(4000 \div 2565) = 1.56$ lbs. per hour per square foot of heating surface. Interpolating in Table XXXVI for 1.56 and 11700 we find that the pounds of steam per pound of coal would be 4.72. The tests at St. Louis showed that a reduction in

boiler pressure increased very slightly the amount of steam produced, but that this amount was only 0.5% greater when the pressure was 160 lbs. instead of 200 lbs. The effect of variation of pressure can therefore be ordinarily ignored. In this case it might add 0.2% or make the figure 4.73. Considering that a superheater adds from 15 to 25% to the efficiency, we will assume the average of 20% and say that 0.80 lb. of the superheated steam produced may be considered as having the same volume and pressure as 1 lb. of saturated steam. Then the amount of steam developed by 1 lb. of coal would be the equivalent of $4.73 \div 0.80 = 5.91$ lbs. Then the equivalent amount of steam developed per hour equals $5.91 \times 4000 = 23640$ lbs.

454. **Weight of steam per stroke at full cut-off.** This may be computed most easily by utilizing Table XXXVII, which is also taken (but somewhat amplified), from the Proceedings of the American Railway Engineering Association, and is now included as Table 2 in the "Economics" section of their Manual. The weight of steam per foot of stroke for 22 ins. diameter and 185 lbs. gauge pressure is 1.161 lbs. and for a stroke of 28 ins. ($2\frac{1}{3}$ ft.) it is 2.709 lbs. For a complete revolution of the drivers it is $4 \times 2.709 = 10.836$ lbs. Since the engine can develop the equivalent of 23640 lbs. of steam per hour and will use 10.836 lbs. at one revolution, it can run at a speed of $23640 \div 10.836 = 2182$ revolutions per hour, or 36.36 revolutions per minute, at full stroke and maintain full boiler pressure. The drivers are 57 ins. in diameter and, therefore, have a circumference of $(57 \div 12) \times 3.1416 = 14.923$ ft. The maximum engine speed for full stroke is $36.36 \times 14.923 = 542.6$ ft. per minute. Multiplying by 60 and dividing by 5280, or dividing by 88, we have 6.167 miles per hour as the maximum speed at which full stroke can be maintained, which is the value M for these conditions.

455. **Pounds of steam and per cent. of cut-off for multiples of M velocity.** In Table XXXVIII, also taken from the Proceedings of the American Railway Engineering Association and now included at Table 4 in the "Economics" section of the Manual, are given the pounds of steam per indicated horse-power hour for simple and for compound locomotives for various velocities, which are multiples of M, the maximum velocity at which the locomotive can use steam at full stroke and yet the
ır can maintain steam at full pressure. The table is com-
on the basis of 200 lbs. gauge pressure, but factors are

TABLE XXXVII.—WEIGHT OF STEAM USED IN ONE FOOT OF STROKE IN LOCOMOTIVE CYLINDERS.

(Cylinder diameter is for high-pressure cylinders in compound locomotives)

Diameter of cylinder (inches)	Weight of steam per foot of stroke for various gauge pressures.						
	220 lbs. per sq. in. (lb.)	210 lbs. per sq. in. (lb.)	200 lbs. per sq. in. (lb.)	190 lbs. per sq. in. (lb.)	180 lbs. per sq. in. (lb.)	170 lbs. per sq. in. (lb.)	160 lbs. per sq. in. (lb.)
12	0.405	0.389	0.370	0.354	0.337	0.321	0.304
13	0.475	0.456	0.435	0.415	0.396	0.376	0.357
14	0.551	0.529	0.504	0.482	0.459	0.436	0.414
15	0.633	0.607	0.579	0.553	0.527	0.501	0.476
15½	0.675	0.649	0.618	0.590	0.562	0.535	0.508
16	0.720	0.691	0.658	0.629	0.599	0.570	0.541
17	0.812	0.780	0.744	0.710	0.676	0.643	0.611
18	0.911	0.875	0.834	0.796	0.759	0.722	0.685
18½	0.962	0.924	0.881	0.841	0.801	0.762	0.724
19	1.015	0.975	0.928	0.887	0.845	0.804	0.763
19½	1.069	1.027	0.978	0.934	0.890	0.847	0.804
20	1.125	1.080	1.029	0.983	0.936	0.891	0.836
20½	1.181	1.134	1.081	1.032	0.984	0.936	0.888
21	1.240	1.191	1.134	1.083	1.032	0.982	0.932
22	1.361	1.307	1.245	1.189	1.133	1.078	1 023
23	1.487	1.428	1.361	1.300	1.238	1.178	1.118
24	1.620	1.555	1.482	1.416	1.348	1.283	1.218
25	1.758	1.688	1.608	1.536	1.462	1.392	1.322
26	1.901	1.825	1.739	1.661	1.582	1.506	1.430
27	2.050	1.968	1.875	1.792	1.706	1.624	1.542
28	2.204	2.117	2.017	1.926	1.835	1.745	1.657

For weight of steam used per revolution of drivers at full cut-off:
Multiply the tabular quantity by four times the length of stroke in feet for simple and four-cylinder compounds. For two-cylinder compounds multiply by two times the length of stroke.

given for other pressures. For example, continuing the above numerical problem, the pounds of steam per i.h.p.-hour, for a simple locomotive, at *M* velocity, and at 200 lbs. pressure, taken from Table XXXVIII, is 38.30; for 185 lbs. pressure we must multiply by the factor 1.0095, which makes the quantity 38.66. Dividing this into 23640, the steam produced per hour, we have 611.5, the i.h.p. at *M* velocity. Multiplying this by 33000, the foot-pounds per minute in one horse-power, and dividing by 542.6, the velocity in feet per minute, we have 37190, the cylinder tractive power in pounds, when burning 4000 lbs. of coal per hour and running at 6.167 m.p.h.

TABLE XXXVIII.—MAXIMUM CUT-OFF AND POUNDS OF STEAM PER I.H.P.-HOUR FOR VARIOUS MULTIPLES OF *M*.

(*M* is maximum velocity in miles per hour at full cut-off, with boiler pressure at 200 pounds per square inch)

Velocity	Cut-off per cent	Pounds steam per I.H.P.-hour		Velocity	Cut-off per cent	Pounds steam per I.H.P.-hour	
		Simple	Compound			Simple	Compound
1.0 *M*	Full	38.30	25.80	2.9 *M*	38.5	24.37	21.04
1.1 "	94.4	36.46	24.36	3.0 "	37.0	24.22	21.21
1.2 "	89.1	34.89	23.24	3.2 "	34.2	24.00	21.57
1.3 "	84.3	33.56	22.35	3.4 "	31.8	23.85	21.93
1.4 "	79.7	32.41	21.65	3.6 "	29.8	23.80	22.27
1.5 "	75.4	31.40	21.14	3.8 "	28.0	23.80	22.57
1.6 "	71.4	30.49	20.77	4.0 "	26.4	23.87	22.85
1.7 "	67.7	29.67	20.52	4.25 "	24.7	24.05	23.22
1.8 "	64.3	28.93	20.40	4.50 "	23.3	24.24	23.56
1.9 "	61.0	28.25	20.40	4.75 "	22.1	24.44	23.85
2.0 "	58.0	27.62	20.40	5.0 "	21.1	24.64	24.15
2.1 "	55.2	27.05	20.40	5.5 "	19.5	24.98	24.70
2.2 "	52.6	26.52	20.40	6.0 "	18.4	25.20	
2.3 "	50.1	26.06	20.40	6.5 "	17.6	25.45	
2.4 "	47.8	25.67	20.40	7.0 "	17.1	25.60	
2.5 "	45.7	25.32	20.47	7.5 "	16.7	25.70	
2.6 "	43.7	25.02	20.60	8.0 "	16.4	25.80	
2.7 "	41.8	24.76	20.73	9.0 "	16.1	25.90	
2.8 "	40.1	24.54	20.88				

For steam per i.h.p.-hour for other boiler pressure take the following percentages of values given in table:

160 lb., 103.0%.	180 lb., 101.3%	210 lb., 99.5%
170 lb., 102.1%	190 lb., 100.6%	200 lb., 99.2%

456. Draw-bar Pull. To obtain the draw-bar pull we must deduct the engine resistance. These have already been discussed in § 429 and the numerical value of the resistance of this same locomotive has been there computed to be about 1771 lbs. Subtracting this from 37190 we have 35419 lbs., the estimated draw-bar pull for that speed and coal consumption.

457. Effect of increasing the rate of coal consumption. To note the effect of increasing the rate of coal consumption, the problem may be again worked through on the basis that the rate of coal consumption is increased, even temporarily, from 4000 lbs. to 5000 lbs. per hour. The steam developed per pound of coal is reduced from 5.91 to 5.23, but the total steam produced per hour is increased from 23640 to 26150. The in eased capacity comes through a loss of efficiency. The inc ed steam

production raises the velocity at which full stroke may be maintained from 6.167 m.p.h to 6.820 m.p.h and the i.h.p. from 611.5 to 676.4. But the computed cylinder tractive power is practically identical, the numerical computation of 37190 being only changed to 37189. But these cylinder tractive powers are each computed for the "*M*" velocities, the maximum velocities at which full stroke can be maintained, and "*M*" is higher with increased coal consumption. For a real comparison, the figures must be reduced to the same velocity, e.g., the working velocity of 10 m.p.h. $10 \div 6.167 = 1.621$, the multiple for the original problem. For 5000 lbs. of coal per hour, *M* velocity is

TABLE XXXIX*.—PER CENT CYLINDER TRACTIVE POWER FOR VARIOUS MULTIPLES OF *M*.

(*M* is maximum velocity in miles per hour at which boiler pressure can be maintained with full cut-off)

Velocity	Per cent (Compound)	Per cent (Simple)	Velocity	Per cent (Compound)	Per cent (Simple)	Velocity	Per cent (Compound)	Per cent (Simple)
Start	135.00	106.00	3.6 *M*	32.40	44.75	6.4 *M*		23.59
0.5 *M*	103.00	103.00	3.7 "	31.25	43.56	6.5 "		23.18
1.0 "	100.00	100.00	3.8 "	30.10	42.39	6.6 "		22.79
1.1 "	96.28	95.57	3.9 "	29.14	41.24	6.7 "		22.42
1.2 "	92.55	91.53	4.0 "	28.24	40.10	6.8 "		22.06
1.3 "	88.83	87.83	4.1 "	27.38	39.00	6.9 "		21.71
1.4 "	85.12	84.46	4.2 "	26.56	37.96	7.0 "		21.38
1.5 "	81.40	81.37	4.3 "	25.77	36.97	7.1 "		21.06
1.6 "	77.68	78.55	4.4 "	25.03	36.03	7.2 "		20.75
1.7 "	73.96	75.97	4.5 "	24.34	35.13	7.3 "		20.45
1.8 "	70.25	73.60	4.6 "	23.69	34.26	7.4 "		20.16
1.9 "	66.54	71.41	4.7 "	23.07	33.41	7.5 "		19.88
2.0 "	63.21	69.37	4.8 "	22.48	32.59	7.6 "		19.61
2.1 "	60.20	67.47	4.9 "	21.92	31.82	7.7 "		19.34
2.2 "	57.48	65.67	5.0 "	21.38	31.11	7.8 "		19.08
2.3 "	54.97	63.94	5.1 "	20.87	30.42	7.9 "		18.82
2.4 "	52.68	62.22	5.2 "	20.37	29.75	8.0 "		18.57
2.5 "	50.42	60.55	5.3 "	19.89	29.10	8.1 "		18.33
2.6 "	48.16	58.92	5.4 "	19.43	28.48	8.2 "		18.09
2.7 "	46.08	57.33	5.5 "	18.99	27.87	8.3 "		17.86
2.8 "	44.10	55.78	5.6 "		27.33	8.4 "		17.64
2.9 "	42.29	54.26	5.7 "		26.81	8.5 "		17.43
3.0 "	40.57	52.78	5.8 "		26.30	8.6 "		17.22
3.1 "	38.95	51.33	5.9 "		25.81	8.7 "		17.01
3.2 "	37.42	49.91	6.0 "		25.34	8.8 "		16.82
3.3 "	35.98	48.55	6.1 "		24.88	8.9 "		16.63
3.4 "	34.66	47.24	6.2 "		24.44	9.0 "		16.45
3.5 "	33.53	45.97	6.3 "		24.01			

* Table 5 in "Economics" Section of Manual of American Railway Engineering Association.

6.820 m.p.h., and the multiple is 1.466. From Table XXXIX we find that the percentages of cylinder tractive power for simple engines for these two multiples of M are 78.01 and 82.42, respectively. The higher value is 105.7% of the lower, which shows that, in this case, adding 25% to the rate of coal consumption adds only 5.7 to the cylinder tractive power at 10 m.p.h.

458. **Effect of using a better quality of coal.** As another instructive variation of the same problem, assume that the coal has effective B.t.u. of 13000, instead of only 11700. It will be found that steam will be produced more rapidly, the M velocity is 6.867 m.p.h. and the horsepower at that velocity is 680.3, but the cylinder power is computed to be 37191 lbs., which is again almost identical with the previous values, although the M velocity is still higher. The multiple for 10 m.p.h. is 1.456 and by Table XXXIX the per cent. of cylinder tractive power is 82.73, which is an increase of 6% over 78.01%, showing that the increase in effective B.t.u. from 11700 to 13000 adds 6% to the cylinder tractive power at 10 m.p.h.

459. **Check with approximate rule.** Applying Eq. 103 to the above data on the basis that the "effective steam pressure" is 85% of the gauge pressure (185) or 157 lbs., we will have

$$\text{Tractive force} = \frac{22^2 \times 157 \times 28}{57} = 37327 \text{ lbs.}$$

This agrees with the more precise value (37190) computed above to within one-half of one per cent. This rule is more simple as a method of obtaining merely the maximum tractive power at slow velocities, but the previous method, although longer, is preferable, since it computes the critical velocity M, and also the tractive force at higher velocities.

460. **Tractive Force at Higher Velocities.** At higher velocities than M, the cylinder power falls off quite rapidly, since the steam is cut off at part stroke and is used expansively. The proper per cent of cut-off for any given velocity and the number of pounds of steam per i.h.p. are shown in Table XXXVIII, in which is give the per cent of cylinder tractive power for multiples of M. The table shows, for example, that, for simple engines, the cylinder tractive power is 69.37% of its value for full stroke when the velocity is $2M$ and that when the velocity is increased to $5M$ the tractive power is reduced to 31.11%.

Applying this to the above numerical problem, when $M = 6.167$ m.p.h., the cylinder tractive power is reduced to 31.11% of 37190, or 11570 lbs., but, since the velocity is five times as great, the horse-power developed is $31.11\% \times 5 = 1.55$ times as great. It should be noted that Table XXXIX shows a slight excess of tractive power (6% when starting), for the simple engine. This is due to the fact that with very low velocities the cylinder pressure more nearly equals the full boiler pressure and there is not the usual reduction of about 15%. Also, compound locomotives are operated with all the cylinders using full-pressure steam, which increases their effectiveness at starting about 35%, although at some loss in economy of steam due to compounding. But since the starting resistances are so much greater than the resistances above 5 miles per hour, the extra assistance is very timely.

Any competent locomotive designer will, of course, make a design such that there is a proper relation between cylinder power and tractive adhesion. In the above case, 106% of $37190 = 39421$ lbs., which is 25.7% of the weight on the drivers, and this is just about the ratio of adhesion which may be expected.

Velocity.		Cylinder tractive. power		Locomotive resistance pounds.	Draw-bar pull. pounds
Multiples of M.	Miles per hour.	Per cent.	Pounds.		
0.0	0.000	106.00	39421	1762	37659
1.0	6.167	100.00	37190	1771	35419
1.2	7.400	91.53	34040	1776	32264
1.5	9.250	81.37	30261	1783	28478
2.0	12.334	69.37	25799	1800	23999
3.0	18.501	52.78	19629	1847	17782
4.0	24.668	40.10	14913	1913	13000
5.0	30.835	31.11	11570	1999	9571
6.0	37.002	25.34	9424	2104	7320

A graphical illustration of the variation in tractive power and velocity may be obtained by computing first and setting down in tabular form the multiple values of M (6.167); the percentages taken from Table XXXIX, for each multiple of M; the products of each percentage times the tractive force (37190), for M velocity; the locomotive resistance, from Table XXIX, for each velocity; and the net draw-bar pull for each velocity. These several values for cylinder tractive power and for draw-bar pull may be plotted as shown in Fig. 208.

The student should realize that the above values represent the maximum draw-bar pull which the locomotive *can* produce, provided the fire-box is fed with 4000 lbs. of coal per hour. These draw-bar pulls as given will overcome the resistance of a train of some definite weight, at uniform speed, along a straight level track, at the several velocities given. A less weight of train will be drawn somewhat faster; or, it will travel at the same speed by using less coal or by throttling the steam and, perhaps, wasting it at the blow-off. A heavier train could not maintain such speed. While the values given are approximately correct, a variation in the quality of the coal, or in the condition of the

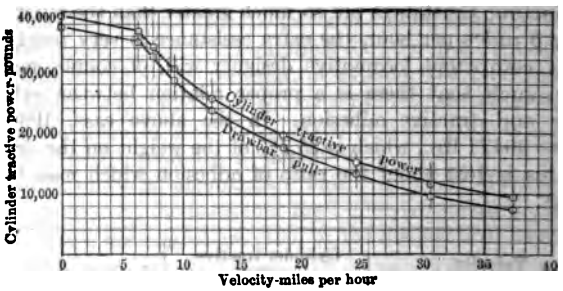


FIG. 208.—TRACTIVE POWER, MIKADO LOCOMOTIVE.

track, or in the firing, or in the management by the engineman, will alter the results materially, and they should not be relied on to give an accurate measure of what can and will be accomplished at all times. But the method is useful and dependable in comparing two types of engines, or, for comparing the operating results of light trains at faster speed or heavier trains at slower speed, using the same engine, or, as shown later, of comparing the operating results of using a certain type of engine on two grades and thus estimating the value of reducing the higher grade.

461. Effect of Grade on Tractive Power. The effect of grade on tractive power is best shown by some numerical computations whose results are plotted in Fig. 209. The cylinder tractive power was computed for three engines of greatly different total weight and power, but which had driving-axle loads nearly identical (about 50750 lbs.), and, therefore, by the Baldwin

Locomotive Works rule, given in § 268, could all be operated on the same kind of track. Using the rule, $\frac{1}{2} \times 50750 \div 300 = 84.5$, which means that the rails should weigh at least 85 lbs. per yard. Making computations for these locomotives, using 12000 B.t.u. coal, similar to those already detailed in §§ 453 *et seq.*, it was found that the cylinder tractive powers of the Pacific, Mikado, and Mallet locomotives were 29718, 33575, 49095 lbs., respectively, when the velocity was uniformly 10 m.p.h. and the locomotives each burned 4000 lbs. of coal per hour. The several engine resistances at 10 m.p.h. are easily computed from Table XXIX and are tabulated below.

Engine characteristics (At velocity $V = 10$ m.p.h.)	Pacific 4–6–2 (lb.)	Mikado 2–8–2 (lb.)	Mallet 2–8–8–2 (lb.)
Cylinder tractive power.....	29,718	33,575	49,095
Engine resistance on level.....	2,205	2,648	4,864
Draw-bar pull on level.....	27,513	30,927	44,231
Draw-bar pull on 3% grade....	15,213	18,207	25,631

The net values, or the draw-bar pulls, are plotted on the left-hand vertical line of Fig. 209, and in each case are the left-hand ends of the solid lines which show the tractive powers of the locomotives. On a 3% grade the grade resistances for the locomotives equal 60 lbs. per ton, and are 12300, 12720 and 18600 lbs., respectively. This reduces the effective draw-bar pull approximately 40% in each case. Since this reduction varies uniformly with the grade, we may plot the three values, 15213, 18207 and 25631, on the 3% vertical line and draw straight lines which represent in each case the tractive power of the locomotive at 10 m.p.h. and on any grade within that range.

Assume trains of cars, all averaging 50 tons per car and varying from 10 cars weighing 500 tons to 50 cars weighing 2500 tons. The resistances at 10 m.p.h on a level grade are given by Eq. 121, and may be plotted on the left-hand vertical line of Fig. 209. Grade adds resistance proportional to the grade. For example, on a 0.7% grade the grade resistance per ton is 14 lbs. and for 2500 tons is 35000 lbs. Adding this to 11580, the tractive resistance, we have 46580, which we plot on the 0.7% vertical line. It is indicated by a small circle. Joining the two points gives the resistance line for 2500 tons hauled at 10 m.p.h. The circles on the other lines indicate similar computations. The inter-

sections of these resistance lines with the lines of tractive power indicate the relative power of each locomotive. For example, the 1000-ton train can be hauled by the Pacific locomotive at 10 m.p.h. up a 0.96% grade, but a Mikado can do the same on a 1.1% grade, while the Mallet can do it on a 1.52% grade.

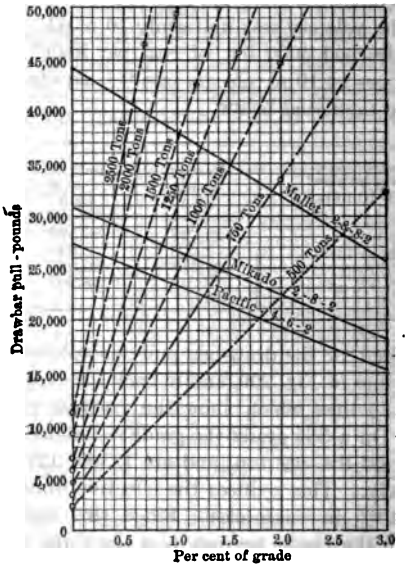


FIG. 209.—CURVES SHOWING EFFECT OF GRADE ON TRACTIVE POWER.

All of these calculations were made on the basis of burning 4000 lbs. of coal per hour, which, as before stated, is the practical limit of what an ordinary fireman can be expected to do for an extended run.

The description of the Mallet locomotive (built by the Baldwin Locomotive Works), stated that its tractive power is 91000 lbs. A computation of its cylinder tractive power at *M* velocity, using 12000 B.t.u. coal, shows it to be 95389 lbs. Subtracting the engine resistance (4843 lbs.), we would have 90546 lbs., which is a very fair check, especially as the Baldwin Locomotive Works method of calculation is different.

462. Acceleration-speed curves. The time required for an engine of given weight and power to haul a train of known weight and resistance over a track with known grades and curvature is an important and necessary matter for an engineer to compute, since the saving in time has such a value as to justify constructive or operating changes which will reduce that time. Fig. 208 shows that the draw-bar pull is very much greater at very low velocities than at the moderate speed of even 15 m.p.h. In spite of the increased resistance at these low velocities the margin of power left for acceleration is also greater and the "speed curve" is really a curve and not a straight line. Its general form may be most easily developed by a numerical example, especially as each case has its own special curve.

Illustrative Example. The Mikado locomotive, whose characteristics have already been investigated in §§ 453 *et seq.*, has draw-bar pulls at various velocities as shown in the tabular form in § 460, to which frequent reference must be made in this demonstration. Assume that this locomotive starts from rest on a 0.4% upgrade, hauling a train of 14 cars, each weighing 50 tons, and a caboose weighing 10 tons. Then the normal level tractive resistance, by Eq. 107, § 439, equals

$$R = (2.2 \times 710) + (122 \times 15) = 3392 \text{ lbs.}$$

The grade resistance of the cars will be $20 \times 0.4 \times 710 = 5680$ lbs. The extra starting resistance will be considered as 6 lbs. per ton, or 4260 lbs.. These three items total 13332 lbs. The average draw-bar pull of the locomotive at velocities between zero and M velocity, which is 6.167 m.p.h., is $\frac{1}{2}(37659+35419) = 36539$ lbs., but this must be diminished in this case by $20 \times 0.4 \times 157.5 = 1260$ lbs. for grade and by $157.5 \times 6 = 945$ lbs. for starting resistance, leaving a net draw-bar pull of 34334 lbs., excluding the force required for the acceleration of the locomotive. The net force available for acceleration of both the locomotive and the train is $34334 - 13332 = 21002$ lbs., or prorated, is $21002 \div (157.5 + 710) = 24.21$ lbs. per ton. Transposing Eq. 106, with $V_1 = O$, $V_2 = 6.167$, and $P = 24.21$ lbs., we have $s = 70(38.03 - 0) \div 24.21 = 110$ feet, the distance required to attain a velocity of 6.167 m.p.h.

While the velocity is increasing from 1.0 M to 1.2 M, the mean draw-bar pull is $\frac{1}{2}(35419+32264) - 1260 = 32582$ lbs., less the accelerative resistance of the locomotive. Subtracting the

tractive and grade resistances of the cars, we have 32582 − 3392 − 5680 = 23510 lbs. Note that there is no longer any starting resistance. The accelerative force in pounds per ton is 23510 ÷ 867.5 = 27.10. The distance s required to increase the velocity from 6.167 m.p.h. to 7.400 m.p.h., is 70(54.76 − 38.03) + 27.10 = 43 feet. Similarly the distances required to increase the velocity from 1.2 M to 1.5 M, from 1.5 M to 2M, etc., are computed as in the accompanying tabular form.

The corresponding distances and velocities have been plotted in Fig. 210. The velocity of 10 m.p.h. is acquired in a little over 300 feet, but it requires 500 feet to acquire a velocity of 12.33 m.p.h. and about 16000 feet to raise it to 29 m.p.h. The force, in pounds per ton, available for acceleration, is maximum at low velocities, after the extra starting resistance is overcome. As the margin per ton for acceleration becomes less and less, the greater is the distance required to increase the velocity 1 mile per hour—especially through the last increments—up to the velocity at which the net draw-bar pull exactly equals the total car resistance and the velocity becomes uniform, which is later computed to be 4.78 M. There is an approximation in using *average* draw-bar pulls between the different velocities at which the draw-bar pull has been definitely computed, but the computed distances are practically correct up to 4 M velocity or 24.67 m.p.h. But the computation for the distance required to increase the velocity from 4 M up to 4.78 M is far less accurate if the average draw-bar pull is used. The effective pull at 4 M velocity equals 13000 − 1260 = 11740, less the accelerative resistance of the locomotive. The tractive and grade resistance of the cars at this velocity is 3392 + 5680 = 9072. This leaves 11740 − 9072 = 2668 lbs. available for acceleration of both locomotive and cars. The reduction in tractive force between 4 M velocity and 5 M velocity (see § 460), is 13000 − 9571 = 3429 lbs. By proportionate interpolation we would then say that the excess force available for acceleration would be exhausted at (2668 ÷ 3429) = .78 of the interval, or at a velocity of 4.78 M, or 29.48 m.p.h. The mean accelerative force is one-half of 2668, or 1334 lbs., which is 1.53 lbs. per ton of train. The distance, by an inversion of Eq. 106, is computed to be 11925 feet. Owing to the approximate equality of working force and resistance and the momentary variations in both, the precise point where the acceleration would cease and the velocity would

DATA AND COMPUTATIONS FOR ACCELERATION AND RETARDATON CURVES.

	Velocities.				Tractive Forces.						Distances.		Time.
	Feet per sec.	Range, miles per hour.		Mean, feet per sec.	Mean draw-bar pull, level, lbs.	Locomotive resistance, grade plus start* lbs.	Actual draw-bar pull, average, lbs.	Car resistance tractive grade, plus start* lbs.	Difference effective for acceleration or retardation. lbs.	Net force per ton, lbs.	Acceleration, or retardation, feet	Total from start, feet.	sec.
	0.00	0.00	6.167	4.52	36539	*2205	34334	*13332	21002	24.21	110	110	24
	9.04	6.167	7.40	9.95	33842	1260	32582	9072	23510	27.10	43	153	4
	10.86	7.40	9.25	12.22	30371	1260	29111	9072	20039	23.10	93	246	8
Acceleration.....	13.57	9.25	12.33	15.83	26239	1260	24979	9072	15907	18.34	254	500	16
	18.09	12.33	18.50	22.61	20891	1260	19631	9072	10559	12.17	1094	1594	48
	27.13	18.50	24.67	31.66	15391	1260	14131	9072	5059	5.83	3196	4790	101
	36.18	24.67	29.48	39.71	11666	1260	10406	9072	1334	1.53	11925	16715	300
	43.24	29.48	24.67	39.71	11662	3780	7882	20432	12550	14.46	1262	1262	32
Retardation.....	36.18	24.67	18.50	31.66	15391	3780	11611	20432	8821	10.17	1832	3094	58
	27.13	18.50	12.33	22.61	20891	3780	17111	20432	3321	3.83	3477	6571	154
	18.09	12.33	12.21	17.99	24106	3780	20326	20432	106	0.122	1681	8252	93

* The extra starting resistance only applies to the first item.

actually become uniform would be be very uncertain. Fortunately the inaccuracy is of little or no practical importance and for the purposes of our calculations we may call this last interval 11925 feet, assuming that the grade is as long as 16715 feet or 3.1 miles. If the 0.4% grade continued indefinitely the train would travel at this uniform velocity as long as the locomotive operated on the basis assumed for this problem. Note that Fig. 210 would have to be extended to nearly three times its

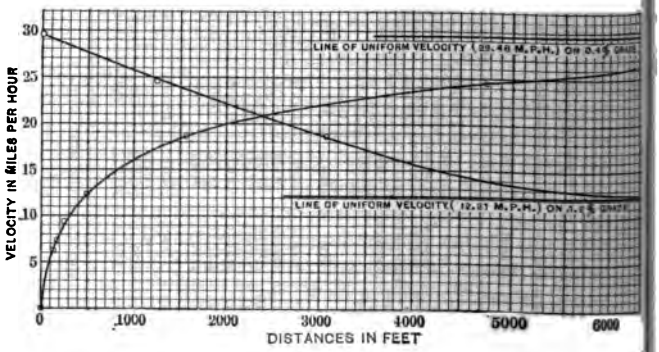


FIG. 210.

present length before the time curve would reach and become tangent to the "line of uniform velocity."

463. Retardation-speed curves. When, on account of grade resistance, the total of tractive and grade resistance is greater than the draw-bar pull, there is retardation.

Illustrative Example. Continuing the numerical problem of § 462, assume that, while moving up the 0.4% grade at a velocity of 4.78 M, or 29.48 m.p.h., the train reaches a grade of +1.2%. The grade resistance of the cars will be $20 \times 1.2 \times 710 = 17040$ lbs. The tractive resistance will be 3392 lbs., as before, making a total of 20432 lbs. Interpolating in the tabular form in § 460 for the draw-bar pull at 4.78 M velocity, we find 10325; at 4 M it is 13000 and the mean is 11662; but from this must be subtracted $20 \times 1.2 \times 157.5 = 3780$ for grade resistance of the locomotive, leaving 7882 lbs. for the net draw-bar pull. The retarding force is $20432 - 7882 = 12550$; or in pounds per ton of train, is $12{,}550 \div 867.5 = 14.46$. As before, using an inversion of

Eq. 106, $s=(29.48^2-24.67^2)70\div 14.46=1262$ feet, the distance at which the velocity would reduce to 4 M. As before, the other quantities may be computed and recorded, with less danger of confusion and error, by tabulating them, as given in § 462.

The mean velocity, when retarding from 4.78 M to 4.0 M, reduced to feet per second, is as before 39.71 feet per second, and dividing this into the distance, 1262 feet, gives 32, the time in seconds. The quantities for the reduction in velocity from 4 M to 3 M and from 3 M to 2 M are computed similarly. The level draw-bar pull for 1.5 M is 28478 (see § 460), and by subtracting 3780, we get 24698 lbs. the actual net pull on the grade. Similarly, the actual pull at 2 M is 20219 lbs. The increase from 20219 to 20432 is $\frac{213}{4479}=4.7\%$ of the interval from 20219 to 24698 and $4.7\%\times.5=.02$; therefore, the actual draw-bar pull just equals the resistance at $2.00-.02=1.98M$, or 12.21 m.p.h. The deficiency of draw-bar pull at $2.0\,M=20{,}432-20219=213$ lbs. At 1.98 M the deficiency is zero and, therefore, the mean deficiency is one-half of 213, or 106. Dividing this by 867.5, we have 0.122, which is the value of P in Eq. 106. Then

$$s=(152.01-149.08)70\div 0.122=1681 \text{ ft.}$$

Velocities in miles per hour can be readily converted into velocities in feet per second by multiplying by 1.4667. Averaging the two velocities at the beginning and the end of each period gives the mean velocity; and dividing each of these into the distance for that period gives the time in seconds.

464. Drifting. The tractive resistance of the cars of the problem just worked out is 3392 lbs.; the locomotive resistance at 20 m.p.h. is 1862 lbs., or a total of 5254 lbs. Variation in velocity will affect this but little. Dividing by 867.5, the total weight in tons, we have 6.06 lbs., the resistance per ton, from which the equivalent rate of grade is $6.06\div 20=.303\%$. This means practically that when this train is running *down* a grade which is over .303% it will run by gravity and steam may be shut off. If the grade is much greater than .303% the acceleration on the downgrade may become so great, if the grade is very long, that the velocity may become objectionably high.

Illustrative Example. Assume that the limiting safe velocity for freight trains, considering the condition of track and rolling

stock, is 35 m.p.h.; assume that the train we have been consider reaches a 0.4% downgrade at a velocity of 15 m.p.h. How fa down the grade will it run with steam shut off, before the speed reaches 35 m.p.h. and brakes must be applied? There is no question here of variable tractive power since the only motive power is gravity. The resistance is nearly independent of velocity and we will here assume it to be so and utilise Table XLII. At 15 m.p.h. the train has a velocity head of 7.90 feet. At 35 m.p.h. the velocity head is 43.01 feet. The train can, therefore, drop down the grade a vertical height of $43.01-7.90=35.11$ feet before the velocity reaches 35 m.p.h. On a 0.4% grade the distance required for such a fall is $35.11\div.004=8777$ feet. The problem in § 462 assumed that the 0.4% grade is 16715 feet or more, and this shows what will happen to the trains moving in the opposite direction.

But it must not be thought that there is no loss of energy during drifting. Even though no steam is used in the cylinders, some is frequently wasted at the safety valve and more is used in operating brakes and in maintaining the brake air-reservoir at full pressure. But the greatest loss of heat is that due to radiation, especially in winter, in spite of all the jacketing devices to retain heat. Although the results of the numerous tests which have been made are quite variable, the following approximate averages may be used: The loss due to radiation while standing may be figured at 120 lbs. of coal per hour per 1000 square feet of heating surface; while drifting the loss will increase to 220 lbs. per hour. The amount of coal used for firing up will be about 510. This is based on the use of 12000 B.t.u. coal. The better the coal, the less will be used.

Illustrative Example. The Mikado locomotive we have been considering has 2565 square feet of heating surface. It will then require about $2.565\times510=1308$ lbs. of coal to fire up. While drifting down the grade, referred to above, a distance of 8777 feet, the average velocity is $\frac{1}{2}(15+35)=25$ m.p.h. $=36.67$ ft. per sec. and the required time is $8777\div36.67=239$ seconds $=3$ min. 59 sec. $=.066$ hour. The coal used while drifting down this short run would be

$$220\times2.565\times.066=37 \text{ lbs.}$$

At this point brakes would need to be applied and the coal spent in drifting beyond this point must be compensated for

in the total time spent on the run and also to compute the total amount of coal consumed while drifting. Although this item of 37 lbs. is relatively very small, its method of computation is typical of the computation of the several items to make up the total of coal consumed during a trip.

465. Review of computed power of one locomotive. It was assumed that it started on a +0.4% grade with a load of 15 cars weighing 710 tons. After moving 16715 feet (assuming that the grade was that long), and doing it in 493 seconds, or 8 minutes 13 seconds, the train acquired a velocity of 29.48 m.p.h. and the power of the locomotive would then be sufficient, when burning 4000 lbs. of coal per hour, to keep it moving up such a grade indefinitely at that velocity. In case the grade were not as long as 16715 feet, it would be necessary to compute the velocity where the rate of grade changed and make that the basis for the computation on the succeeding grade. But, assuming that the grade were as long as 16715 feet, or more, and that the velocity of 29.48 m.p.h. had been acquired, and that the train had run at that speed for some distance—although this does not modify the problem—the train is assumed to reach a still steeper grade +1.2%. The velocity then begins to decrease and in a total distance of 8252 feet and a total time of 337 seconds, or 5 minutes 37 seconds, the velocity is reduced to 12.21 m.p.h., at which velocity the locomotive is able to make steam fast enough to overcome the higher resistance on the steeper grade. From that point on, assuming that the 1.2% grade is longer than 8252 feet, the train would continue for the remaining length of that grade at the velocity of 12.21 m.p.h.

As before stated, precision in the above results depends on many factors (such as B.t.u. of coal used, or the actual consumption in pounds per hour), which are somewhat variable. Sometimes the variation of these factors from the values used above is known; sometimes it is unknown and then the accuracy of the results is correspondingly uncertain. But whether accurately known or not, when this method is used, employing the best values for the factors which are obtainable, the method shows a valuable *comparison* of two proposed alinements or grades. In such a comparison, any error in the factors will affect both results nearly, if not quite, equally, and the comparative results will still be substantially correct.

466. Selection of route. The preceding articles may be utilized in comparing two routes. If one of the lines is already in operation, the engineer has the great advantage of being able to determine by test exactly what results may be obtained on that line and what factors should be used in computations. It is then only necessary to compute the quantities for the proposed new line. When both lines are "on paper" there is less certainty as to the accuracy of the results, except that the line which is shown to be most advantageous will probably continue to be most advantageous even if the uncertain factors used in the comparison are somewhat changed. Using the methods outlined in §§ 462 to 464, there will be computed the behavior of an assumed type of locomotive, hauling one or more types of train load, and passing over tracks having definite grades and lengths. The effect of curves may be disregarded provided that the grades were properly compensated during original construction, and then the rate of grade for the entire length of straight and curved track may be taken as the rate on the straight track. If the rate of grade is actually uniform, even through the curves, then the lengths of curved track must be computed separately and on the basis of a rate of grade equal to the actual rate plus an allowance of .035% for each degree of curve. The behavior of a train from starting to stopping must be computed, making due allowance for each change in condition which will affect the hauling power of the locomotive. The locomotive is assumed to be working at the limit of its steaming capacity, except when drifting with steam shut off on a down grade, or when brakes are applied, either to prevent objectionably high velocity on a down grade or to make a stop. The action of brakes during a service stop (as distinguished from an emergency stop), may be considered as a retarding force varying from 10% to 20% of the train weight. Unfortunately brake action is so variable, being directly under the control of the locomotive engineer and varying from zero to the full braking power, that any computation of energy used in operating them or of the effect of the brakes is impracticable except on the basis of arbitrary assumptions such as the requirement that the brakes are used in such a way that a train will be retarded at a specified rate. The performance of the locomotive over the entire division, the total time required, its velocity in critical places, etc., can be computed. In §§ 462 and 463 it

was shown that the locomotive considered could haul the particular train considered up a 0.4% grade at a velocity of 29.48 m.p.h. and maintain such speed indefinitely; also that it could haul the same train up a 1.2% grade at 12.21 m.p.h. and maintain its velocity indefinitely. This of course,, means that a much heavier train could be hauled up the 0.4% grade and that a somewhat heavier train could be hauled up the 1.2% grade without being stalled, although the velocities in each case would be reduced. There are an infinite number of combinations, but there are usually some considerations which narrow the choice. Even after construction is complete these tables may be utilized in a study of the most economical combination of type of locomotive and amount of train load for the track conditions as they may exist.

467. Rating of locomotives. The maximum power of a locomotive on any grade at M velocity is measured by its "rating."

Let P = the tractive power of the locomotive, measured at the rim of the drivers;
E = Weight of engine and tender, in pounds;
W = Weight of cars behind tender, in pounds;
r = rate of grade, or the ratio of vertical to horizontal;
a = a constant, which as determined by tests = 2.2 lbs. per ton or .0011 lb. per pound of train;
b = a constant, which as determined by tests = 122 lbs. per ton. a and b are the same constants as are used in § 439.
n = number of cars in train.

Then $P = (E+W)\ (r+a)+bn$.

Transforming,

$$\frac{P}{r+a} - E = W + n\frac{b}{r+a}. \quad \ldots\ldots \quad (122)$$

The right-hand side of this equation is called the "rating," A, and is the weight of the train behind the tender plus the number of cars times a quantity made up of two constants and the rate of grade. This quantity is independent of any special engine or train values and may be tabulated for various rates of grade, as given in Table XL.

Examples. The Mikado locomotive considered in §§ 453, *et seq.*, has a tractive power, measured at the rim of the drivers,

TABLE XL.—LOCOMOTIVE RATING DISCOUNTS.

VALUES OF $b \div (r \times a)$ FOR VARIOUS GRADES.

(In tons per car.)

Grade R (per cent).	Tons per car $b \div (r+a)$.	Grade R (per cent).	Tons per car $b \div (r+a)$.	Grade R (per cent).	Tons per car $b \div (r+a)$.	Grade R (per cent).	Tons per car $b \div (r+a)$.	Grade R (per cent).	Tons per car $b \div (r+a)$.
Level	55	0.5	10.0	1.0	5.5	1.5	3.8	2.0	2.8
0.1	29	0.6	8.5	1.1	5.0	1.6	3.6	2.1	2.7
0.2	20	0.7	7.5	1.2	4.6	1.7	3.4	2.2	2.6
0.3	14	0.8	6.7	1.3	4.3	1.8	3.2	2.3	2.5
0.4	12	0.9	6.0	1.4	4.0	1.9	3.0	2.4	2.4

at M velocity, or 6.167 m.p.h., of $37190 - 1432 = 35758$ lbs., which equals P; 1432 is the locomotive resistance between cylinder and rim of drivers, see § 429. The weight of engine and tender is 315000 lbs. What is its rating on a 1.2% grade? The value of r for a 1.2% grade $= .012$; $a = .0011$ lb. per pound. Then

$$A = \frac{P}{r+a} - E = \frac{35758}{.012+.0011} - 315000 = 2{,}414{,}000 \text{ lbs.} = 1207 \text{ tons},$$

which is the rating for that locomotive for a 1.2% grade. But this does not mean 1207 tons of cars. Placing this equal to the right-hand side of Eq. 122, we have

$$1207 = W + n\frac{b}{r+a}.$$

The value of $\frac{b}{r+a}$ for a 1.2% grade is given in Table XL as 4.6.

Then $$W = 1207 - 4.6n,$$

which shows that the weight of train depends on the number of cars. Assume that $n = 16$. Then $W = 1133.4$ and the average weight per car is 70.8 tons. Assume that the cars are all "empties," weighing 18 tons each; then $W = 18n$, and

$$n = 1207 \div (18 + 4.6) = 53.4,$$

which must be interpreted as 53 empty cars.

In the above examples the pulling power P is d[illegible]nined on the basis of the locomotive working at the maximum [illegible]ocity M at

which it can maintain full stroke. See § 455. This represents practically the maximum power of the locomotive. The velocity M is usually from 4 to 7 miles per hour and is as low as should be allowed on maximum grades, since an attempt to utilize a slightly higher tractive force at a somewhat lower velocity would probably result in stalling the train if an unexpected resistance in the track slightly increased the normal resistance.

CHAPTER XIX.

THE PROMOTION OF RAILROAD PROJECTS.

468. Method of formation of railroad corporations. Many business enterprises, especially the smaller ones, are financed entirely by the use of money which is put into them directly in the form of stock or mere partnership interest. A railroad enterprise is frequently floated with a comparatively small financial expenditure on the part of the original promoters. The promoters become convinced that a railroad between *A* and *B*, passing through the intermediate towns of *C* and *D*, with others of less importance, will be a paying investment. They organize a company, have surveys made, obtain a charter, and then, being still better able (on account of the additional information obtained) to exploit the financial advantages of their scheme, they issue a prospectus and invite subscriptions to bonds. Sometimes a portion of these bonds are guaranteed, principal and interest, or perhaps the principal alone, by townships or by the national government. The cost of this preliminary work, although large in gross amount if the road is extensive, is yet but an insignificant proportion of the total amount involved. The proportionate amount that can be raised by means of bonds varies with the circumstances. In the early history of railroad building, when a road was projected into a new country where the traffic possibilities were great and there was absolutely no competition, the financial success of the enterprise would seem so assured that no difficulty would be experienced in raising from the sale of bonds all the money necessary to construct and equip the road. But the promoters (or stockholders) must furnish all money for the preliminary expenses, and must make up all deficiencies between the proceeds of the sale of the bonds and the capital needed for construction.

"In theory, stocks represent the property of the responsible owners of the road, and bonds are an encumbrance on that

property. According to this theory, a railroad enterprise should begin with an issue of stock somewhere near the value of the property to be created and no more bonds should be issued than are absolutely necessary to complete the enterprise. Now it is not denied that there are instances in which this theory is followed out. In New England, for example, as well as in some of the Southern States, there are a few roads represented wholly by stock or very lightly mortgaged. But this theory does not conform to the general history of railway construction in the United States, nor is it supported by the figures that appear in the summary. The truth is, railroads are built on borrowed capital, and the amount of stock that is issued represents in the majority of cases the difference between the actual cost of the undertaking and the confidence of the public expressed by the amount of bonds it is willing to absorb in the ultimate success of the venture." *

"The same general law obtains and has always obtained throughout the world, that such properties (as railways) are always built on borrowed money up to the limit of what is regarded as the positive and certain minimum value. The risk only—the dubious margin which is dependent upon sagacity, skill, and good management—is assumed and held by the company proper who control and manage the property." †

469. The two classes of financial interests—the security and profits of each. From the above it may be seen that stocks, bonds, car-trust obligations, and even current liabilities represent railroad capital. The issue of the bonds "was one means of collecting the capital necessary to create the property against which the mortgage lies." The variation between these interests lies chiefly in the security and profits of each. The current liabilities are either discharged or, as frequently happens, they accumulate until they are funded and thus become a definite part of the railroad capital.

The growth of this tendency is shown in the following tabular form (see next page):

The bonded interest has greater security than the stock, but less profit. The interest on the bonds must be paid before any money can be disbursed as dividends. If the bond interest

* Henry C. Adams, Statistician, U. S. Int. Con. Commission.
† A. M. Wellington, Economic Theory of Railway Location.

Capitalization of Railroads in the United States.	June 30, 1898.		June 30, 1912.		Dec. 31, 1918.	
	Amount, millions.	Per cent.	Amount, millions.	Per cent.	Amount, millions.	Per cent.
Stocks	5311	44.6	8622	43.7	8678	43.2
Funded debt	5510	46.3	11130	56.3	11406	56.8
Current liabilities, etc.	1087	9.1				

is not paid, a receivership, and perhaps a foreclosure and sale of the road, is a probability, and in such case the stockholder's interests are frequently wiped out altogether. The bondholder's real profit is frequently very different from his nominal profit. He sometimes buys the bonds at a very considerable discount, which modifies the rate which the interest received bears to the amount really invested. Even the bondholder's security may suffer if his mortgage is a second (or fifth) mortgage, and the foreclosure sale fails to net sufficient to satisfy all previous claims.

On the other hand, the stockholder, who may have paid in but a small proportion of his subscription, *may*, if the venture is successful, receive a dividend which equals 50 or 100% of the money actually paid in, or, as before stated, his entire holdings may be entirely wiped out by a foreclosure sale. When the road is a great success and the dividends very large, additional issues of stock are generally made, which are distributed to the stockholders in proportion to their holdings, either gratuitously or at rates which give the stockholders a large advantage over outsiders. This is the process known as "watering." While it may sometimes be considered as a legitimate "salting down" of profits, it is frequently a cover for dishonest manipulation of the money market.

For the twelve years between 1887 and 1899 about *two thirds* of all the railroad stock in the United States paid *no* dividends, while of those that paid dividends the average rate varied from 4.96 to 5.74%. The year from June 30, 1898, to June 30, 1899, was the most prosperous year of the group, and yet nearly 60% of all railroad stock paid *no* dividend, and the average rate paid by those which paid at all was 4.96%. The total amount distributed in dividends was greater than ever before, but the average rate is the least of the above group because many roads, which had passed their dividends for many previous

years, distinguished themselves by declaring a dividend, even though small. During that same period but 13.35% of the stock paid over 6% interest. The total dividends paid amounted to but 2.01% of all the capital stock, while investments ordinarily are expected to yield from 4 to 6% (or more) according to the risk. Of course the effect of "watering" stock is to decrease the nominal rate of dividends, but there is no dodging the fact that, watered or not, even in that year of "good times," about 60% of all the stock paid *no* dividends. Unfortunately there are no accurate statistics showing how much of the stock of railroads represents actual paid-in capital and how much is "water." The great complication of railroad finances and the dishonest manipulation to which the finances of some railroads have been subjected would render such a computation practically worthless and hopelessly unreliable now.

During the year ending June 30, 1898 (which may in general be considered as a sample), 15.82% of the funded debt paid no interest. About one third of the funded debt paid between 4 and 5% interest, which is about the average which is paid.

The income from railroads (both interest on bonds and dividends on stock) may be shown graphically by diagrams, such as are given in the annual reports of the Interstate Commerce Commission. They show that while railroad investments are occasionally very profitable, the average return is less than that of ordinary investments *to the investors*. The *indirect* value of railroads in building up a section of country is almost incalculable and is worth many times the cost of the roads. It is a discouraging fact that very few railroads (old enough to have a history) have escaped the experience of a receivership, with the usual financial loss to the then stockholders. But there is probably not a railroad in existence which, however much a financial failure in itself, has not profited the community more than its cost.

470. The small margin between profit and loss to projectors. When a railroad is built entirely from the funds furnished by its promoters (or from the sale of stock) it will generally be a paying investment, although the rate of payment may be very small. The percentage of receipts that is demanded for actual operating expenses is usually about 67%. The remainder will usually pay a reasonable interest on the total capital involved. But the operating expenses are frequently 90 and even 100% of

the gross receipts. In such cases even the bondholders do not get their due and the stockholders have absolutely nothing. Therefore the stockholder's interest is very speculative. A comparatively small change in the business done (as is illustrated numerically in § 472) will not only wipe out altogether the dividend—taken from the last small percentage of the total receipts and which may equal 50% or more of the capital stock *actually paid in*—but it may even endanger the bondholders' security and cause them to foreclose their mortgage. In such a case the stockholders' interest is usually entirely lost. It does not alter the essential character of the above-stated relations that the stockholders sometimes protect themselves somewhat by buying bonds. By so doing they simply decrease their risk and also decrease the possible profit that might result from the investment of a given total amount of capital.

471. **Extent to which a railroad is a monopoly.** It is a popular fallacy that a railroad, when not subject to the direct competition of another road, has an absolute monopoly—that it controls "all the traffic there is" and that its income will be practically independent of the facilities afforded to the public. The growth of railroad traffic, like the use of the so-called necessities or luxuries of life, depends entirely on the supply and the cost (in money or effort) to obtain it. A large part of railroad traffic belongs to the unnecessary class—such as traveling for pleasure. Such traffic is very largely affected by mere matters of convenience, such as well-built stations, convenient terminals, smooth track, etc. The freight traffic is very largely dependent on the possibility of delivering manufactured articles or produce at the markets so that the *total* cost of production and transportation shall not exceed the total cost in that same market of similar articles obtained elsewhere. The creation of facilities so that a factory or mine may successfully compete with other factories or mines will develop such traffic. The receipts from such a traffic may render it possible to still further develop facilities which will in return encourage further business. On the other hand, even the partial withdrawal of such facilities may render it impossible for the factory or mine to compete successfully with rivals; the traffic furnished by them is completely cut off and the railroad (and indirectly the whole community) suffers correspondingly. The "strictly necessary" traffic is thus so small that few railroads could pay

their operating expenses from it. The dividends of a road come from the last comparatively small percentage of its revenue, and such revenue comes from the "unnecessary" traffic which must be coaxed and which is so easily affected by apparently insignificant "conveniences."

472. Profit resulting from an increase in business done; loss resulting from a decrease. In a subsequent chapter it will be shown that a large portion of the operating expenses are independent of small fluctuations in the business done and that the operating expenses are roughly two thirds of the gross revenue. Assume that by changes in the alinement the business obtained has been increased (or diminished) 10%. Assume for simplicity that the operating expenses on the revised track are the same as on the route originally planned; also that the cost of the track is the same and hence the fixed charges are assumed to be constant for all the cases considered. Assume the fixed charges to be 28%. The additional business, when carried in cars otherwise but partly filled will hardly increase the operating expenses by a measurable amount. When extra cars or extra trains are required, the cost will increase up to about 60% of the average cost per train mile. We may say that 10% increase may in general be carried at a rate of 40% of the average cost of the traffic. A reduction of 10% in traffic may be assumed to reduce expenses a similar amount. The effect of the change in business will therefore be as follows:

	Business increased 10%.		Business decreased 10%.	
Operating exp. = 67	67(1 + 10% × 40%) =	69.68	67(1 − 10% × 40%) =	64.32
Fixed charges = 28	.....	28.00	.....	28.00
95		97.68		92.32
Total income... 100	Income.....	110.00	Income.....	90.00
Available for dividends..... 5	Available for dividends.....	12.32	Deficit.....	2.32

In the one case the increase in business, which may often be obtained by judicious changes in the alinement or even by better management without changing the alinement, more than doubles the amount available for dividends. In the other case the profits are gone, and there is an absolute deficit. The above is a numerical illustration of the argument, previously

stated, of the small margin between profit and loss to the original projectors.

473. Estimation of probable volume of traffic and of probable growth. Since traffic and traffic facilities are mutually interdependent and since a large part of the normal traffic is merely potential until the road is built, it follows that the traffic of a road will not attain its normal volume until a considerable time after it is opened for operation. But the estimation even of this normal volume is a very uncertain problem. The estimate may be approached in three ways:

1st. The actual gross revenue derived by all the railroads in that section of the country (as determined by State or U. S. Gov. reports) may be divided by the total population of the section and thus the average annual expenditure per head of population may be determined. A determination of this value for each one of a series of years will give an idea of the normal rate of growth of the traffic. Multiplying this annual contribution by the population which may be considered as tributary gives *a valuation* of the possible traffic. Such an estimate is unreliable (*a*) because the *average* annual contribution may not fit that particular locality, (*b*) because it is very difficult to correctly estimate the number of the true tributary population, especially when other railroads encroach more or less into the territory. Since a rough value of this sort may be readily determined, it has its value as a check, if for nothing else.

2d. The actual revenue obtained by some road whose circumstances are as nearly as possible identical with the road to be considered may be computed. The weak point consists in the assumption that the character of the two roads is identical, or in incorrectly estimating the allowance to be made for observed differences. The method of course has its value as a check.

3d. A laborious calculation may be made from an actual study of the route—determining the possible output of all factories, mines, etc., the amount of farm produce and of lumber that might be shipped, with an estimate of probable passenger traffic based on that of like towns similarly situated. This method is the best when it is properly done, but there is always the danger of leaving out sources of income—both existent and that to be developed by traffic facilities, or, on the other hand, of overestimating the value of expected traffic. In the

ollowing tabular form are shown the population, gross receipts, receipts per head of population, mileage, earnings per nile of line operated, and mileage per 10,000 of population for he whole United States. It should be noted that the values re only *averages*, that individual variations are large, and that nly a very rough dependence may be placed on them as applied o any particular case.

Year.	Population (estimated).	Gross receipts.	Receipts per head of population.	Mileage†	Earnings per mile of line operated.	Mileage per 10,000 population.‡
888...	60,100,000	$910,621,220	$15.15	136,884	$6653	24.94
889...	61,450,000	964,816,129	15.81	153,385	6290	25.67
890...	*62,801,571	1051,877,632	16.75	156,404	6725	26.05
891...	64,150,000	1096,761,395	17.10	161,275	6801	26.28
892...	65,500,000	1171,407,343	17.89	162,397	7213	26.19
893...	68,850,000	1220,751,874	18.26	169,780	7190	26.40
894...	68,200,000	1073,361,797	15.74	175,691	6109	26.20
895...	69,550,000	1075,371,462	15.46	177,746	6050	25.97
896...	70,900,000	1150,169,376	16.22	181,983	6320	25.78
897...	72,350,000	1122,089,773	15.53	183,284	6122	25.53
898...	73,600,000	1247,325,621	16.95	184,648	6755	25.32
899...	74,950,000	1313,610,118	17.53	187,535	7005	25.25
900...	*76,295,220	1487,044,814	19.49	192,556	7722	25.44
901...	77,863,000	1588,526,037	20.47	195,562	8123	25.52
902...	79,431,000	1726,380,267	21.88	200,155	8625	25.76
903...	80,998,000	1900,846,907	23.70	205,314	9258	26.03
904...	82,566,000	1975,174,091	24.23	212,243	9306	26.34
905...	84,134,000	2082,482,406	25.15	216,974	9508	26.44
906...	85,701,000	2325,765,167	27.65	222,340	10460	26.78
907...	87,279,000	2589,105,578	29.63	227,455	11383	26.38
908...	88,837,000	2393,805,989	26.95	231,540	10338	26.30
909...	90,405,000	2418,677,538	26.71	234,800	10301	26.20
910...	*91,972,266	2750,667,435	29.91	238,609	11528	26.14
911...	93,572,266	2789,761,669	29.81	244,476	11411	26.10
912...	95,172,266	2842,695,382	29.87	247,981	11463	25.93

Actual. †Excludes a small percentage not reporting "gross receipts." ‡Actual mileage.

The probable growth in traffic, after the traffic has once ttained its normal volume, is a small but almost certain quantity. n the above tabular form this is indicated by the gradual rowth in "receipts per head of population" from 1897 to 907. Then the sudden drop due to the panic of 1907 is clearly ndicated, and also the gradual growth in the last few years. Even in England, where the population has been nearly station- ry for many years, the growth though small is unmistakable. On the other hand the growth in some of the Western States

has been very large. For example, the gross earnings p of population in the State of Iowa increased from $1.42 to $10.00 in 1870, and to $19.46 in 1884.

There will seldom be any justification in building to ac date a larger business than what is "in sight." Ev could be anticipated with certainty that a large inc business would come in ten years, there are many reas it would be unwise to build on a scale larger than that r for the business to be immediately handled. Even th may cost more in the future to provide the added ac dations (*e.g.* larger terminals, engine-houses, etc.), th expense will be nearly if not quite offset by the interes by avoiding the larger outlay for a period of years whi often prove much longer than was expected. A still m portant reason is the avoidance of uselessly sinking m a time when every cent may be needed to insure the of the enterprise as a whole.

474. Probable number of trains per day. Increas growth of traffic. The number of passenger trains p cannot be determined by dividing the total number of pas estimated to be carried per day by the capacity of t that can be hauled by one engine. There are many railroads, running three or four passenger trains per da way, which do not carry as many passengers all told carried on one heavy train of a trunk line. But beca bulk of the passenger traffic, especially on such ligh roads, is "unnecessary" traffic (see § 471) and must be enc and coaxed, the trains must be run much more fre than mere capacity requires. The minimum number of ger trains per day on even the lightest-traffic road sh two. These need not necessarily be passenger trains exch They may be mixed trains.

The number required for freight service may be kep nearly according to the actual tonnage to be moved. A one local freight will be required, and this is apt to be consi within the capacity of the engine. Some very ligh roads have little else than local freight to handle, and o there is less chance of economical management. Road heavy traffic can load up each engine quite accurately ac to its hauling capacity and the resulting economy is great. tuations in traffic are readily allowed for by adding on o

ping off one or more trains. Passenger trains must be run on regular schedule, full or empty. Freight trains are run by train-despatcher's orders. A few freight trains per day may be run on a nominal schedule, but all others will be run as extras. The criterion for an increase in the number of passenger trains is impossible to define by set rules. Since it should always come before it is absolutely demanded by the train capacity being overtaxed, it may be said in general terms that a train should be added when it is believed that the consequent increase in facilities will cause an increase in traffic the value of which will equal or exceed the added expense of the extra train.

475. Effect on traffic of an increase in facilities. The term facilities here includes everything which facilitates the transport of articles from the door of the producer to the door of the consumer. As pointed out before, in many cases of freight transport, the reduction of facilities below a certain point will mean the entire loss of such traffic owing to local inability to successfully compete with more favored localities. Sometimes owing to a lack of facilities a railroad company feels compelled to make some concession which is a virtual reduction on what would normally be the freight rate. In competitive freight business such a method of procedure is a virtual necessity in order to retain even a respectable share of the business. Even though the railroad has no direct competitor, it must if possible enable its customers to meet their competitors on even terms. In passenger business the effect of facilities is perhaps even more marked. The pleasure travel will be largely cut down if not destroyed.

476. Loss caused by inconvenient terminals and by stations far removed from business centers. This is but a special case of the subject discussed just in the preceding paragraph. The competition once existing between the West Shore and the New York Central was hopeless for the West Shore from the start. The possession of a terminal at the Grand Central Station gave the New York Central an advantage over the West Shore, with its inconvenient terminal at Weehawken, which could not be compensated by any obtainable advantage by the West Shore. This is especially true of the passenger business. The through freight business passing through or terminating at New York is handled so generally by means of floats that the disadvantage in this respect is not so great. The

enormous expenditure (roughly $10,000,000) made
Pennsylvania R. R., on the Broad Street Station (and
proaches) in Philadelphia, a large part of which was
crossing the Schuylkill River and running to City Hall
rather than retain their terminal in West Philadelphi
illustration of the policy of a great road on such a c
The fact that the original plan and expenditure has be
largely increased since the first construction proves t
management has not only approved the original large
but saw the wisdom of making a very large increase in
penditure.

The construction of great terminals is comparatively in
and seldom concerns the majority of engineers. But an
has frequently to consider the question of the locati
way station with reference to the business center of th
The following points may (or may not) have to be con
and the real question consists in striking a proper
between conflicting considerations.

(1) During the early history of a railroad enterpri
especially needful to avoid or at least postpone all expe
which are not demonstrably justifiable.

(2) The ideal place for a railroad station is a local
mediately contiguous to the business center of the tow
location of the station even one fourth of a mile from t
result in a loss of business. Increase this distance to
and the loss is very serious. Increase it to five miles
loss approaches 100%.

(3) The cost of the ideal location and the necessa
of way may be a very large sum of money for the new en
On the other hand the increase in property values an
general prosperity of the town, caused by the railroa
will so enhance the value of a more convenient location
cost at some future time will generally be extravagan
absolutely prohibitory. The original location is therefo
ordinary conditions a finality.

(4) To some extent the railroad will cause a move
the business center toward it, especially in the establ
of new business, factories, etc., but the disadvantages
to business already established is permanent.

(5) In any attempt to compute the loss resulting
location at a given distance from the business center it

recognized that each problem is distinct in itself and that any change or growth in the business of the town changes the amount of this loss.

The argument for locating the station at some distance from the center of the town may be based on (*a*) the cost of right of way, thus involving the question of a large initial outlay, (*b*) the cost of very expensive construction (*e.g.* bridges), again involving a large initial outlay, (*c*) the avoidance of excessive grade into and out of the town. It sometimes happens that a railroad is following a line which would naturally cause it to pass at a considerable elevation above (rarely below) the town. In this case there is to be considered not only the possible greater initial cost, but the even more important increase in operating cost due to the introduction of a very heavy grade. The loss of business due to inconvenient location can only be guessed at. Wellington says that at a distance of one mile the loss would average 25%, with upper and lower limits of 10 and 40%, depending on the keenness of the competition and other modifying circumstances. For each additional mile reduce 25% of the preceding value. While such estimates are grossly approximate, yet with the aid of sound judgment they are better than nothing and may be used to check gross errors.

477. General principles which should govern the expenditure of money for railroad purposes. It will be shown later that the elimination of grade, curvature, and distance have a positive money value; that the reduction of ruling grade is of far greater value; that the creation of facilities for the handling of a large traffic is of the highest importance and yet the added cost of these improvements is sometimes a large percentage of the cost of *some road* over which it would be physically possible to run trains between the termini.

The subsequent chapters will be largely devoted to a discussion of the value of these details, but the general principles governing the expenditure of money for such purposes may be stated as follows:

1. No money should be spent (beyond the unavoidable minimum) unless it may be shown that the addition is in itself a profitable investment. The additional sum may not wreck the enterprise and it may add something to the value of the road, but unless it adds more than the improvement costs it is not justifiable.

2. If it may be positively demonstrated that an improvement will be more valuable to the road than its cost, it should certainly be made even if the required capital is obtained with difficulty. This is all the more necessary if the neglect to do so will permanently hamper the road with an operating disadvantage which will only grow worse as the traffic increases.

3. This last principle has two exceptions: (*a*) the cost of the improvement may wreck the whole enterprise and cause a total loss to the original investors. For, unless the original promoters can build the road and operate it until its stock has a market value and the road is beyond immediate danger of a receivership, they are apt to lose the most if not all of their investment; (*b*) an improvement which is very costly although unquestionably wise may often be postponed by means of a cheap temporary construction. Cases in point are found at many of the changes of alinement of the Pennsylvania R. R., the N. Y., N. H. & H. R. R., and many others. While some of the cases indicate faulty original construction, at many of the places the original construction was wise, considering the then scanty traffic, and now the improvement is wise considering the great traffic.

478. Study of railroad economics—its nature and limitations. The multiplicity of the elements involved in most problems in railroad construction preclude the possibility of a solution which is demonstrably perfect. Barring out the comparatively few cases in this country where it is difficult to obtain any practicable location, it may be said that a comparatively low order of talent will suffice to locate anywhere a railroad over which it is physically possible to run trains. It may be very badly located for obtaining business, the ruling grades may be excessive, the alinement may be very bad, and the road may be a hopeless financial failure, and yet trains can be run. Among the infinite number of possible locations of the road, the engineer must determine the route which will give the best railroad property for the least expenditure of money—the road whose earning capacity is so great that after paying the operating expenses and interest on the bonds the surplus available for dividends or improvements is a maximum.

An unfortunate part of the problem is that e[rro]rs blunders are not always readily apparent nor their magnitu[de]. A defective dam or bridge will give way and every o[ne] realizes the

failure, but a badly located railroad affects chiefly the finances of the enterprise by a series of leaks which are only perceptible and demonstrable by an expert, and even he can only say that certain changes would probably have a certain financial value.

479. Outline of the engineer's duties. The engineer must realize at the outset the nature and value of the conflicting interests which are involved in variable amount in each possible route.

(a) **The maximum of business must be obtained,** and yet it *may* happen that some of the business may only be obtained by an extravagant expenditure in building the line or by building a line very expensive to operate.

(b) **The ruling grades should be kept low,** and yet this *may* require a sacrifice in business obtained and also *may* cost more than it is worth.

(c) **The alinement should be made as favorable as possible;** favorable alinement reduces the future operating expenses, but it may require a very large immediate outlay.

(d) **The total cost** must be kept within the amount at which the earnings will make it a profitable investment.

(e) **The road must be completed and operated** until the "normal" traffic is obtained and the road is self-supporting without exhausting the capital obtainable by the projectors; for no matter how valuable the property may ultimately become, the projectors will lose nearly, if not quite, all they have invested if they lose control of the enterprise before it becomes a paying investment.

Each new route suggested makes a new combination of the above conflicting elements. The engineer must select a route by first eliminating all lines which are manifestly impracticable and then gradually narrowing the choice to the best routes whose advantages are so nearly equal that a closer detailed comparison is necessary.

The ruling grade and the details of alinement have a large influence on the operating expenses. A large part of this course of instruction therefore consists of a study of operating expenses under average normal conditions, and then a study of the effect on operating expenses of given changes in the alinement.

CHAPTER XX.

OPERATING EXPENSES.*

480. Distribution of gross revenue. When a railroad comprises but one single property, owned and operated by itself, the distribution of the gross revenue is a comparatively simple matter. The operating expenses then absorb about two thirds of the gross revenue; the fixed charges (chiefly the interest on the bonds) require about 25 or 30% more, leaving perhaps 3 to 8% (more or less) available for dividends. The report on the Fitchburg R. R. for 1898 shows the following:

Operating expenses	$5,083,571	69.1%
Fixed charges	1,567,640	21.3%
Available for dividends, surplus, or permanent improvements	708,259	9.6%
Total revenue	$7,359,470	100.0%

But the financial statements of a large majority of the railroad corporations are by no means so simple. The great consolidations and reorganizations of recent years have been effected by an exceedingly complicated system of leases and sub-leases, purchases, "mergers," etc., whose forms are various. Railroads in their corporate capacity frequently own stocks and bonds of other corporations (railroad properties and otherwise) and receive, as part of their income, the dividends (or bond interest) from the investments.

The Interstate Commerce Commission annually makes a report of the income and profit-and-loss account of all the railroads of the United States, considered as one system. For example, the statement for the year 1912 includes the following items. Operating revenues from rail operations $2,842,606,382; operating expenses due to rail operations $1,972,415,776, which is 69.4%. Interest on funded debt used up 13.9% of the rev-

* The operating expenses of railroads have been utterly abnormal during and since the Great War. The figures of this chapter are not now (1921) applicable to present conditions, but corresponding figures, revised to date, would not be typical. The chapter therefore stands untouched until new figures, representing normal conditions, are available.

enues, and taxes 4.2%. There were other miscellaneous incomes and expenditures which caused a net loss of another 2.0% of revenue, leaving 10.5% or $299,361,208 which were issued as dividends. These dividends are about 3.4% of the outstanding stock. The percentage to the amount of money actually paid for the stock is unknown and unknowable.

481. Operating expenses per train-mile. The uniformity in the average operating expenses per train mile for light-traffic and heavy-traffic roads and for long and short roads is very remarkable. This is illustrated by a comparison of figures for ten heavy traffic roads and ten small roads selected *at random*, except that each had a mileage of less than 100 miles.

OPERATING EXPENSES PER TRAIN-MILE ON LARGE AND SMALL ROADS (1904 AND 1910).

	Mileage.		Operating expenses per train-mile.		Ratio expenses to earnings per cent.	
	1904.	1910.	1904.	1910.	1904.	1910.
Whole United States	220,112	240,439	1.314	1.489	67.79	66.29
Canadian Pacific	8,332	10,271	1.320	1.504	68.72	65.41
C., B. & Q.	8,326	9,040	1.313	1.710	64.35	71.71
Chicago & Northwestern	7,412	7,629	1.136	1.306	66.61	70.31
Southern Railway	7,197	7,050	1.048	1.234	70.30	67.43
C., R. I. & P.	6,761	7,396	1.199	1.344	72.90	73.07
Northern Pacific	5,619	6,189	1.392	1.824	52.26	61.71
A., T. & S. F.	5,031	7,460	1.305	1.626	60.05	64.33
Great Northern	4,489	7,147	1.464	1.808	49.72	60.53
Illinois Central	4,374	4,551	1.107	1.409	70.02	74.84
Atlantic Coast Line	4,229	4,491	0.984	1.213	58.95	62.44
Average of ten	.....	.....	1.227	1.498	63.39	67.18
Montpelier & Wells River	44	50	1.169	1.430	80.73	75.08
Somerset Railway Co.*	42	94	0.802	1.314	59.37	76.65
Huntingdon & Broadtop Mountain	66	70	0.950	2.052	52.10	96.40
Lehigh & New England	96	170	0.793	2.045	69.80	62.84
Ligonier Valley	11	16	1.427	1.480	69.33	49.15
Newburgh, Dutchess & Connecticut †	59	.....	0.922	.....	85.09	.....
Susquehanna & New York	55	80	1.368	1.028	78.47	77.81
Detroit & Charlevoix	51	51	1.424	1.010	67.52	99.53
Harriman & Northeastern *	20	20	2.162	1.733	79.26	63.70
Galveston, Houston & Henderson	50	50	1.556	1.759	47.27	70.37
Average of ten (or nine)	.....	.....	1.257	1.539	68.89	74.61

* Subsidiary road since 1904.

† Merged since 1904; separate figures not available.

The fluctuations of the average cost per train-mile for sev years past may be noted from the following tabular form:

AVERAGE COST PER TRAIN-MILE (FOR WHOLE U. S.) IN CEN

Year.	Cents.	Year.	Cents.	Year.	Cents.	Year.	Cent
1890	96.006	1896	93.838	1902	117.960	1908	147.
1891	95.707	1897	92.918	1903	126.604	1909	143.
1892	96.580	1898	95.635	1904	131.375	1910	148.
1893	97.272	1899	98.390	1905	132.140	1911	154.
1894	93.478	1900	107.288	1906	137.060	1912	159.
1895	91.829	1901	112.292	1907	146.993		

The enforced economies after the panic of 1893 are shown. The reduction generally took the form of a lowe of the standards of maintenance of way and of maintenanc equipment. The marked advance since 1895 is partly due the necessity for restoring the roads to proper conditions, rep ishing worn-out equipment and providing additional eq ment to handle the greatly increased volume of business. recent advance is chiefly due to the increase in wages and generally increased cost of supplies.

It may be noted from the I. C. C. reports that the cases wh the operating expenses per train-mile and the ratio of expen to earnings vary very greatly from the average are alm invariably those of the very small roads or of "junction road where the operating conditions are abnormal. For example, c little road, with a total length of 13 miles and total annual ope ting expenses of $5342, spent but 22½c. per train-mile, which p cisely exhausted its earnings. This precise equality of earni and expenses suggests jugglery in the bookkeeping. As anot abnormal case, a road 44 miles long spent $3.81 per train-mi which was nearly *fourteen* times its earnings. In another cas road 13 miles long earned $7.76 per train-mile and spent $6. (78%) on operating expenses, but the fixed charges were abn mal and the earnings were less than half the sum of the operati expenses and fixed charges. The *normal* case, even for t small road, is that the cost per train-mile and the ratio of oper ing expenses to earnings will agree fairly well with the aver and when there is a marked difference it is generally due some abnormal conditions of expenses or of earning capacity.

482. Reasons for uniformity in expenses per train-mi The chief reason is that, although on the heavy-traffic ro everything is kept up on a finer scale, better roadbed, heavi

rails, better rolling stock, more employees, better buildings, stations, and terminals, etc., yet the number of trains is so much greater that the divisor is just enough larger to make the average cost about constant. This is but a general statement of a fact which will be discussed in detail under the different items of expense.

483. Detailed classification of expenses with ratios to the total expense. The Interstate Commerce Commission now publishes each year a classification with detailed summation for the cost of each item. These summations are made up from reports furnished by railroads which have (in the reports recently made) represented over 99% of the total traffic handled. In the annexed tabular form (Table XLI) are shown the percentages which each item bears to the total. The railroads have been divided into two classes, "large" and "small," as indicated below. Large roads report on 116 items which are combined and condensed with 44 items for small roads.

"Large roads" are those with mileage greater than 250 miles, or those with operating revenues greater than $1,000,000. Roads subsidiary to "large roads" are also included in this class.

"Small roads" are those with mileage less than 250 miles and also with operating revenues less than $1,000,000.

484. Amounts and percentages of the various items. The I. C. C. report for the year ending June 30, 1909, was the first to include the distribution of expenses according to the present classification. The items as given are reliable and may be utilized, as far as any such computations are to be depended on, in estimating future expenses. The chief purpose of this discussion is to point out those elements of the cost of operating trains which may be affected by such changes of location as an engineer is able to make. There are some items of expense with which the engineer has not the slightest concern, nor will they be altered by any change in alinement or constructive detail which he may make. In the following discussion such items will be passed over with a brief discussion of the sub-items included.

MAINTENANCE OF WAY AND STRUCTURES.

485. Items 2 to 5. Track material. The relative cost of ballast, ties, rails and other track material, as shown by com-

TABLE XLI.—ANALYSIS OF OPERATING EXPENSES OF ALL "LARGE"* RAILROADS IN THE UNITED STATES FOR YEAR ENDING JUNE 30, 1912, SHOWING PERCENTAGE OF EACH ITEM TO TOTAL AND COST IN CENTS PER TRAIN-MILE.

Item. No.	Account.	Total Amount (thousands)	Per cent of total Expenses	Cents per Train-Mile.
	Maintenance of Way and Structures.			
1	Superintendence	$18,789	0.990	1.58
2	Ballast	7,157	0.377	.60
3	Ties	55,463	2.921	4.65
4	Rails	16,438	.866	1.38
5	Other track material	17,346	.914	1.45
6	Roadway and track	129,397	6.815	10.84
7	Removal of snow, sand, and ice	6,920	.364	.58
8	Tunnels	1,141	.060	.10
9	Bridges, trestles, and culverts	27,712	1.460	2.32
10–12	Crossings, all; fences; snow structures	8,066	.425	.68
13–15	Signals, telegraph, electrical power transmission	13,681	.720	1.14
16, 17	Buildings, grounds, docks, wharves	35,389	1.864	2.96
18	Roadway tools and supplies	4,480	.236	.38
19	Injuries to persons	1,989	.105	.17
20, 21	Stationery, printing and other expenses	1,038	.054	.09
22, 23	Joint tracks, etc. (net balance)	3,463	.182	.29
		$348,471	18.353	29.20
	Maintenance of Equipment.			
24	Superintendence	$13,175	.694	1.10
	Repairs, renewals and depreciation:			
25–30	Locomotives, steam and electric	175,889	9.263	14.74
31–33	Cars, passenger	38,968	2.052	3.26
34–36	Cars, freight	183,968	9.690	15.41
37–39	Equipment, electrical, car	318	.017	.03
40–42	Equipment, floating	1,333	.071	.11
43–45	Equipment, work	6,128	.322	.51
46	Equipment, shop (machinery and tools)	10,418	.548	.87
47	Equipment, power plant	268	.014	.02
48	Injuries to persons	1,818	.096	.15
49, 50	Stationery, printing and other expenses	4,036	.213	.34
51, 52	Joint equipment, at terminals (net balance)	676	.036	.06
		$436,995	23.016	36.61
	Traffic Expenses.			
53–60	Agencies; advertising; fast freight lines; etc.	$59,047	3.110	4.95

* The "large" roads here reported represent 88% of the total mileage.

paring either the gross amounts or the percentages in Table XLI, is suggestive and instructive. The fact that ties cost considerably more than all other track material combined shows

TABLE XLI. (*Continued*).—ANALYSIS OF OPERATING EXPENSES OF ALL "LARGE" RAILROADS IN THE UNITED STATES FOR YEAR ENDING JUNE 30, 1912, SHOWING PERCENTAGE OF EACH ITEM TO TOTAL AND COST IN CENTS PER TRAIN-MILE.

Item No.	Account.	Total Amount (thousands).	Per cent of total expenses.	Cents per train-mile.
	Transportation Expenses.			
61, 62	Superintendence and train dispatching	$40,743	2.146	3.41
63	Station employees	133,877	7.051	11.22
64–66	Weighing; car service association; coal and ore docks	15,949	.839	1.33
67–70	Yards (wages, expenses, supplies)	76,069	4.007	6.37
71–76	Yard locomotives (enginemen, fuel, water, lubricants, supplies)	74,370	3.917	6.23
77, 78 104, 105	Operating joint tracks, terminals, yards, and facilities (net balance)	10,430	.550	.88
79, 80	Motormen and road enginemen	120,966	6.371	10.14
81	Road locomotives, engine-house expenses	33,951	1.788	2.84
82	Road locomotives, fuel	194,142	10.225	16.27
83	Road locomotives, water	12,482	.657	1.04
84, 85	Road locomotives, lubricants and other supplies	7,430	.392	.62
86, 87	Operating power plants, purchased power	1,797	.095	.15
88	Road trainmen	128,339	6.759	10.75
89	Train supplies and expenses	34,462	1.815	2.89
90–92	Interlockers, signals, flagmen, draw-bridges	17,831	.939	1.49
93	Clearing wrecks	5,167	.272	.43
94–98	Telegraph, floating equipment, stationery, miscellaneous	20,009	1.054	1.68
99–103	Loss and damage to property, personal injuries	56,838	2.994	4.76
		$984,852	51.871	82.51
	General Expenses.			
106–116	Salaries of general officers, clerks, etc.; law, insurance, pensions, miscellaneous	69,297	3.650	5.81
	Total operating expenses	$1,898,662	100.000	159.08

the importance of any possible saving in tie renewals. It is also significant that the relative importance of ties has increased in the last few years, and that the relative increase has not been due to a reduction in the cost of other track material. Apparently the lengthening of the average life of ties, due to preservative processes, the use of tie-plates, and greater care to avoid the premature withdrawal from the track of ties which

are still serviceable, has not kept pace with the increase in the average cost per tie. The cost of rails has advanced because of (*a*) the very general adoption of heavier rails; (*b*) the almost universal substitution of more expensive open-hearth steel for Bessemer, on account of greater reliability and durability, and (*c*) the increase in cost of all steel products.

486. Item 6. Roadway and track. This item is three-eighths of the total cost of maintenance of way and structures. It consists chiefly of the wages of trackmen. There has been an almost steady increase in the daily wages of section foremen and other trackmen since 1900, as shown below:

	1900	1901	1902	1903	1904	1905	1906
Section foremen.....	1.68	1.71	1.72	1.78	1.78	1.79	1.80
Other trackmen.....	1.22	1.23	1.25	1.31	1.33	1.32	1.36
No. of trackmen per 100 miles.....	118	122	140	147	136	143	155

	1907	1908	1909	1910	1911	1912
Section foremen.....	1.90	1.95	1.96	1.99	2.07	2.09
Other trackmen.....	1.46	1.45	1.38	1.47	1.50	1.50
No. of trackmen per 100 miles.....	162	130	136	157	147	142

The average number of section foremen per 100 miles of line has remained almost constant at 18. Although there have been fluctuations in the number of "other trackmen" required per 100 miles of line, there has been in general a very substantial increase. These two causes combined (increased number and increased wages) have had a great influence in producing the regular and steady increase in the average cost of a train-mile, as shown in § 481.

487. Items 8 to 15. Maintenance of track structures. As a matter of economics, the locating engineer has little or no concern with the cost of maintaining track structures. If he is comparing two proposed routes it would be seldom that they would be so different that he would be justified in attempting to compute a train-mile difference in cost of operation, based on differences in these items. Of course, one proposed line might call for one or more tunnels which the alternate line does not have, and the annual cost of maintaining the tunnels would increase the cost of operation. Such a case would justify special consider-

tion. So far as the maintenance of small bridges and culverts are concerned it would usually be sufficiently accurate to consider that a proposed change of line, involving perhaps several miles of road, would require substantially the same number of bridges and culverts, and therefore that the cost of maintaining them would be the same by either line. The error involved in such an assumption would usually be insignificant, unless there was a very large and material difference in the two lines in this respect. Under such conditions special computations should be made. The items total less than 3% for small roads and still less for large roads.

MAINTENANCE OF EQUIPMENT.

488. Items 25 to 27. Repairs, renewals and depreciation of steam and electric locomotives. The item is of interest to the locating engineer because he must appreciate the effect on locomotive repairs and renewals of an addition to distance. A large part of the repairs of locomotives are due to the wear of wheels, which is largely caused by curvature. Therefore the value of any reduction of curvature is a matter of importance, and this will be considered in Chapter XXII. A considerable portion of the deterioration of a locomotive is due to grade, and the economic advantages of reductions of grade will be considered in Chapter XXIII.

This item includes the expenses of work whose effect is supposed to last for an indefinite period. It does *not* include the expense of cleaning out boilers, packing cylinders, etc., which occurs regularly and which is charged to items 72 or 81. It does include all current repairs, general overhauling, and even the replacement of old and worn-out locomotives by new ones to the extent of keeping up the original standard and number. Of course additions beyond this should be considered as so much increase in the original capital investment. As a locomotive becomes older the *annual* repair charge becomes a larger percentage on the first cost, and it may become as much as one-fourth and even one-third of the first cost. When a locomotive is in this condition it is usually consigned to the scrap-pile; the annual cost for maintenance becomes too large an item for its annual mileage. The effect on expenses of increasing the weight of engines is too complicated a problem to be solved accurately, but

certain elements of it may be readily computed. While the cost of repairs is greater for the heavier engines, the increase is only about one-half as fast as the increase in weight—some of the subitems not being increased at all.

TRANSPORTATION.

489. Items 71 to 76. Yard-engine expenses. By comparing these items with the corresponding items (80 to 85) for road engines, it may be seen that the total expenses assignable to yard engines are about 20% of those of road engines; the relative fuel charge for 1912 was 15.6%. The number of switching locomotives in the United States in 1912 was 9529 or 15.3% of the total number, 62,262. The relative charge for wages of enginemen was 26.2%. This higher proportionate charge is probably due to the fact that the wages for yard enginemen must necessarily be on a per diem basis, but the wages of road enginemen are generally on a mileage basis, as explained later. On the other hand the mileage of a yard engine is usually comparatively low, and the coal consumed will be correspondingly, although not proportionately, low. It must also be remembered that these figures are exclusive of the work and equipment of switching and terminal companies.

490. Item 80. Road enginemen. This item requires 6% of the total operating expenses. The enginemen are usually paid on a mileage basis, or by the trip, except on very small railroads. On very short roads, where a train crew may make two, three, or even four complete round trips per day, they may readily be paid by the day, so many round trips being considered as a day's work, but on roads of great length, where all trains, and especially freight-trains, are run day and night, weekday and Sunday, all trainmen are necessarily paid by the trip. The pay for a trip is figured on a mileage basis except that a trip is usually considered to have a minimum length of 100 miles or 10 hours of time. Eight hours was fixed as standard by the "Adamson" law, in 1916. All extra time is called "overtime" and is paid for at an extra rate. The basis of train wages is too complicated for any brief discussion. Even the basis is constantly changing, the only uniform feature being a steady increase.

The increase in the average wages paid to enginemen and firemen since 1900 is plainly shown by the following figures:

INCREASE IN DAILY WAGES, FROM 1900 TO 1912.

	1900	1901	1902	1903	1904	1905	1906
	$	$	$	$	$	$	$
Enginemen	3.75	3.78	3.84	4.01	4.10	4.12	4.12
Firemen	2.14	2.16	2.20	2.28	2.35	2.38	2.42

	1907	1908	1909	1910	1911	1912
	$	$	$	$	$	$
Enginemen	4.30	4.45	4.44	4.55	4.79	5.00
Firemen	2.54	2.64	2.67	2.74	2.94	3.02

491. **Item 82. Fuel for road locomotives.** This item includes every subitem of the entire cost of the fuel until it is placed in the engine-tender. The cost therefore includes not only the first cost at the point of delivery to the road, but also the expense of hauling it over the road from the point of delivery to the various coaling-stations and the cost of operating the coal-pockets from which it is loaded on to the tenders. Even though the cost may be fairly regular for any one road, the cost for different roads is exceedingly variable. There has been an almost steady increase in the *percentage* of the cost of this item per train-mile since 1897. Items 73 and 82 amounted to nearly 12% of the total operating expenses in 1912, and required an actual expenditure of nearly $225,000,000. It is the largest item in the whole cost of railroad operation. Although some roads, which traverse coal-regions and perhaps actually own the coal-mines, are able to obtain their coal for a cost which may be charged up as $1 per ton or less, there are many roads which are far removed from coal-fields which have to pay $3 or $4 per ton, on account of the excessive distance over which the coal must be hauled. Unfortunately the figures published by the Interstate Commerce Commission do not show the variations in the percentage of this item in different localities. A surprisingly large percentage of the fuel consumed is not utilized in drawing a train along the road. A portion of this percentage is used in firing-up. A portion is wasted when the engine is standing still, which is a considerable proportion of the whole time. The policy of banking fires instead of drawing them reduces the injury resulting from great fluctuations in temperature, but in a general way we may say that there is but little, if any, saving in fuel by banking the fires, and therefore we may consider that

almost a fire-box full of coal is wasted whether the fires are banked or drawn. As given in § 464, the fuel used by a locomotive in firing-up may be estimated as 510 lbs. per 1000 square feet of heating surface, based on using 12000 B.t.u. coal. But even the amount of coal required to produce the required steam-pressure in the boiler from cold water does not represent the total loss. The train-dispatcher, in his anxiety that engines shall be ready when needed, will sometimes order out the locomotives which remain somewhere in the yard, perhaps exposed to cold weather, and blow off steam for several hours before they make an actual start. This loss has been estimated as 120 lbs. per hour per 1000 square feet of heating surface, but it would evidently be far greater on a windy winter day than on a calm summer day. A freight-train, especially on a single-track road, will usually spend several hours during the day on sidings, and when a single-track road is being run to the limit of its capacity, or when the management is not good, the time will be still greater. It is estimated that the amount lost through a 2½-inch safety-valve in one minute would represent the consumption of 15 pounds of coal, which would be sufficient to haul 100 tons on a mile of track with easy grades. Again we see that the amount thus lost is exceedingly variable and almost non-computable, although as a rough estimate the amount has been placed at from 3 to 6% of the total. Another very large subitem of loss of useful energy is that occasioned by stopping and starting. A train running 30 miles per hour has enough kinetic energy to move it on a straight level track for more than two miles. Therefore, every time a train running at 30 miles per hour is stopped, enough energy is consumed by the brakes to run it about two miles. There is a double loss, not only due to the fact of the loss of energy, but also because the power of the locomotive has been consumed in operating the brakes. When the train is again started, this kinetic energy must be restored to the train in addition to the ordinary resistances which are even greater, on account of the greater resistance at very low velocities. Of course, the proportion of fuel thus consumed depends on the frequency of the stops. It was demonstrated by some tests on the Manhattan Elevated Road in New York City, where the stops average one in every three-eighths of a mile, that this cause alone would account for the consumption of nearly three-fourths of the fuel. On ordinary railroads

the proportion, of course, will not be nearly so great, but there is reason to believe that 10 to 20% is not excessive as an average figure.

492. Item 88. Road trainmen. This item includes the wages of conductors and "other trainmen." As in the case of all other employees, the average daily wages have advanced since 1900 as shown below:

AVERAGE DAILY WAGES OF CONDUCTORS AND OTHER TRAINMEN, 1900 TO 1912.

	1900	1901	1902	1903	1904	1905	1906
	$	$	$	$	$	$	$
Conductors	3.17	3.17	3.21	3.38	3.50	3.50	3.51
Other trainmen	1.96	2.00	2.04	2.17	2.27	2.31	2.35

	1907	1908	1909	1910	1911	1912
	$	$	$	$	$	$
Conductors	3.69	3.81	3.81	3.91	4.16	4.29
Other trainmen	2.54	2.60	2.59	2.69	2.88	2.96

These figures are of vital importance from an economic standpoint, since they show a constant tendency to increase and thereby raise the average cost of a train-mile. And as there is no present indication of any limit to this increase, all economic calculations which attempt to predict future expenses, even for a few years in advance, must allow for these and other increased expenses.

493. Item 89. Train supplies and expenses. These items, which average about 1.8%, include the large list of consumable supplies such as lubricating oil, illuminating-oil or gas, ice, fuel for heating, cleaning materials, etc., which are used on the cars and not on the locomotives. The consumption of some of these articles is chiefly a matter of time. In other cases it is a function of mileage. The effect of changes which an engineer may make on this item will be considered when estimating the effect of the changes.

494. Items 93, 99 to 103. Clearing wrecks, loss, damage and injuries to persons and property. These expenses are fortuitous and bear no absolute relation either to the number of miles of road or the number of train-miles. While they depend largely on the standards of discipline on the road, even the best of roads have to pay some small proportion of their earnings to these

items. While we might expect that a road with heavy traffic would have a larger proportion of train accidents than a road of light traffic, it is usually true that on the heavy-traffic roads the precautions taken are such that they are usually freer from accidents than the light-traffic roads. During recent years there has been a very perceptible increase in the percentages of these items, particularly in the compensations paid for "injuries to persons." The increase in this item coincides with the increase already noted in the number of passengers killed during recent years. The possible relation between curvature and accidents is discussed in § 507, but otherwise the locating engineer has no concern with these items.

495. Items 104, 105. Operating joint tracks and facilities, Dr. and Cr. A large part of these debit and credit charges are those for car per diem and mileage charges. This is a charge paid by one road to another for the use of cars, which are chiefly freight-cars. To save the rehandling of freight at junctions, the policy of running freight-cars from one road to another is very extensively adopted. Since the foreign road receives its mileage proportion of the freight charge, it justly pays to the road owning the car at a rate which is supposed to represent the value of the use of the freight-car for the number of miles traveled. The foreign road then loads up the freight-car with freight consigned to some point on the home road and sends it back, paying mileage for the distance traveled on the foreign road, a proportional freight charge having been received for that service. All of these movements of freight-cars are reported to a car association, which, by a clearing-house arrangement, settles the debit and credit accounts of the various roads with each other. Such is the simple theory. In practice the cars are not sent back to the home road at once, but wander off according to the local demand. As long as a strict account is kept of the movements of every car, and as long as the home road is paid the charge which really covers the value of lost service, no harm is done to the home road, except that sometimes, when business has suddenly increased, the home road cannot get enough cars to handle its own business. The value of the car is then abnormally above its ordinary value, and the home road suffers for lack of the rolling stock which belongs to it. Formerly such charges were paid strictly according to the mileage. This developed the intolerable condition that loaded cars would be

run onto a siding and left there for several days, simply because it was not convenient to the consignee to unload the car immediately. On the mileage basis the car would be earning nothing, and, since the road on which the car then was had no particular interest in the car, the car was allowed to stand to suit the convenience of the consignee. To correct this evil a system of per diem charges has been developed, so that a railroad has to pay a per diem charge for every foreign car on its lines. To reduce this charge as much as possible the railroads compel consignees, under penalty of heavy demurrage charges, to unload cars promptly. The running of freight-cars on foreign lines is now settled almost exclusively on the per diem basis, but the running of passenger-cars over other lines, as is done on account of the advantages of through-car service, as well as the running of Pullmans and other special cars, is still paid for on the mileage basis. To the extent to which this charge is settled on the mileage basis, any change in distance which the engineer may be able to effect in the length of the road will have its influence on this item, but when the freight-car business, which comprises by far the larger part of the running of cars over foreign lines, is settled on the per diem basis no changes in alinement which the engineer may make will affect the item appreciably.

Switching Charges. Where two or more railroads intersect there will be a considerable amount of shifting of cars, chiefly freight-cars, from one road to the other. This shifting at any one junction may be done entirely by the engines of one road or perhaps by those of both roads. A portion of the expense of this work is charged up against the other road by the road which does the work. The total amount of this work is carefully accounted for by a clearing-house arrangement, and the balance is charged up against the road which has done the least work. The item is very small, is fairly uniform year by year, and is seldom, if ever, affected by changes of alinement.

Other Items. All of the remaining items, as stated in Table XLI, are of no concern to the locating engineer. They are either general expenses, such as the salaries of general officers, insurance or law expenses, or are special items, such as advertising or the operation of marine equipment which will not be changed by any variations in distance, curvature, or grades which a locating engineer may make. There is therefore no need for their further discussion here.

CHAPTER XXI.

DISTANCE.

496. Relation of distance to rates and expenses. Ra
are usually based on distance traveled, on the appa
hypotheses that each additional mile of distance adds its p
portional amount not only to the service rendered but als
the expense of rendering it. Neither hypothesis is true.
value of the service of transporting a passenger or a ton
freight from *A* to *B* is a more or less uncertain gross amo
depending on the necessities of the case and independent
the exact distance. Except for that very small part of pas
ger traffic which is undertaken for the mere pleasure of travel
the general object to be attained in either passenger or fre
traffic is the transportation from *A* to *B*, however it is attai
A mile greater distance does not improve the service rende
in fact, it consumes valuable time of the passengers and peri
deteriorates the freight. From the standpoint of service
dered, the railroad which adopts a more costly construction
thereby saves a mile or more in the route between two pl
is thereby fairly entitled to additional compensation ra
than have it cut down as it would be by a strict mileage r
The actual value of the service rendered may therefore v
from an insignificant amount which is less than any reason
charge (which therefore discourages such traffic) and its v
in cases of necessity—a value which can hardly be measure
money. If the passenger charge between New York and Ph
delphia were raised to $5, $10, or even $20, there would stil
some passengers who would pay it and go, because to t
it would be worth $5, $10, or $20, or even more. Third
when they pay $2.25 they are not paying what the servic
worth to them. The service rendered cannot therefore
made a measure of the charge, nor is the service rendered p
portional to the miles of distance.

The idea that the cost of transportation is proportiona

the distance is much more prevalent and is in some respects more justifiable, but it is still far from true. This is especially true of passenger service. The extra cost of transporting a single passenger is but little more than the cost of printing his ticket. Once aboard the train, it makes but little difference to the railroad whether he travels one mile or a hundred. Of course there are certain very large expenses due to the passenger traffic which must be paid for by a tariff which is rightfully demanded, but such expenses have but little relation to the cost of an additional mile or so of distance inserted between stations. The same is true to a slightly less degree of the freight traffic. As shown later, the items of expense in the total cost of a train-mile, which are directly affected by a small increase in distance, are but a small proportion of the total cost.

497. The conditions other than distance that affect the cost; reasons why rates are usually based on distance. Curvature and minor grades have a considerable influence on the cost of transportation, as will be shown in detail in succeeding chapters, but they are never considered in making rates. Ruling grades have a very large influence on the cost, but they are likewise disregarded in making rates. An accurate measure of the effect of these elements is difficult and complicated and would not be appreciated by the general public. Mere distance is easily calculated; the public is satisfied with such a method of calculation; and the railroads therefore adopt a tariff which pays expenses and profits even though the charges are not in accordance with the expenses or the service rendered

EFFECT OF DISTANCE ON RECEIPTS.

498. Classification of traffic. There are various methods of classifying traffic, according to the use it is intended to make of the classification. The method here adopted will have reference to its competitive or non-competitive character and also to the method of division of the receipts on through traffic. Traffic may be classified first as "through" and "local"—through traffic being that traveling over two (or more) lines, no matter how short or non-competitive it may be; "local" traffic is that confined entirely to one road. A fivefold classification is however necessary—which is:

A. Non-competitive local—on one road with no choice of routes

B. Non-competitive through—on two (or more) roads with no choice.

C. Competitive local—a choice of two (or more) routes the entire haul may be made on the home road.

D. Competitive through—direct competition between or more routes each passing over two or more lines.

E. Semi-competitive through—a non-competitive haul on home road and a competitive haul on foreign roads.

There are other possible combinations, but they all reduce one of the above forms so far as their essential effect is concerned.

499. Method of division of through rates between roads run over. Through rates are divided between roads run over in proportion to the mileage. There may be terminal charges and possibly other more or less arbitrary deductions to be taken from the total amount received before when the final division is made the remainder is divided according to the mileage. On account of this method of division and also because non-competitive rates are always fixed according to the distance, there results the unusual feature that, on curvature and grade, there is a compensating advantage in increased distance, which applies to all the above kinds of traffic except one (competitive local), and that the compensation is sometimes sufficient to make the added distance an actual source of profit. It has been estimated that the cost of hauling a train an additional mile is only 33 to 49% of the average cost. Therefore in all non-competitive business (local or through) where the rate is according to the distance, there is an actual profit in all such added distance. In competitive local business, in which the rate is fixed by competition and has practically no relation to distance, any additional distance is dead loss. In competitive through business the profit or loss depends on the distances involved. This may best be demonstrated by examples.

500. Effect of a change in the length of the home road on its receipts from through competitive traffic. Suppose the home road is 100 miles long and the foreign road is 150 miles long. Then the home road will receive $\frac{100}{100+150}=40\%$ of the through rate.

Suppose the home road is lengthened 5 miles; then

receive $\frac{105}{105+150}=41.176\%$ of the through rate. The traffic being competitive, the rate will be a fixed quantity regardless of this change of distance. By the first plan the rate received is 0.4% per mile; adding 5 miles, the rate for the original 100 miles may be considered the same as before; and that the additional 5 miles receive 1.176%, or 0.235% per mile. This is 59% of the original rate per mile, and since this is more than the cost per mile for the additional distance, the added distance is evidently in this case a source of distinct profit. On the other hand, if the line is shortened 5 miles, it may be similarly shown that not only are the receipts lessened, but that the saving in operating expenses by the shorter distance is less than the reduction in receipts.

A second example will be considered to illustrate another phase. Suppose the home road is 200 miles long and the foreign road is 50 miles long. In this case the home road will receive $\frac{200}{200+50}=80\%$ of the through rate. Suppose the home road is lengthened 5 miles; then it will receive $\frac{205}{205+50}=80.392\%$ of the through rate. By the first plan the rate received is 0.400% per mile; adding 5 miles, there is a surplus of 0.392, or 0.0784 per mile, which is but 19.6% of the original rate. At this rate the extra distance evidently is not profitable, although it is not a dead loss—there is some compensation.

501. The most advantageous conditions for roads forming part of a through competitive route. From the above it may be seen that when a road is but a short link in a long competitive through route, an addition to its length will increase its receipts and increase them more than the addition to the operating expenses.

As the proportionate length of the home road increases the less will this advantage become, until at some proportion an increase in distance will just pay for itself. As the proportionate length grows greater the advantage becomes a disadvantage until, when the competitive haul is entirely on the home road, any increase in distance becomes a net loss without any compensation. It is therefore advantageous for a road to be a short link in a long competitive route; an increase in that link

will be financially advantageous; if the total length is le
that of the competing line, the advantage is still great
then the rate received per mile will be greater.

**502. Effect of the variations in the length of haul a
classes of the business actually done.** The above di
refer to particular lengths of haul and are not necessari
total lengths of the road. Each station on the ro
traffic relations with an indefinite number of traffic
all over the country. The traffic between each stat
the road and any other station in the country between
traffic may pass therefore furnishes a new combinatic
effect of which will be an element in the total effe
change of distance. In consequence of this, any *exact* s
of such a problem becomes impracticable, but a suffi
accurate solution for all practical purposes is frequen
tainable. For it frequently happens that the great bu
road's business is non-competitive, or, on the other h
may be competitive-through, and that the proportion
or more definite kinds of traffic is so large as to over
the other miscellaneous traffic. In such cases an appro
but sufficiently accurate solution is possible.

503. General conclusions regarding a change in di
(a) In *all* non-competitive business (local and throug
added distance is actually profitable. Sometimes pra
all of the business of the road is non-competitive; a consi
proportion of it is always non-competitive.

(b) When the competitive local business is very large
competitive through business has a very large averag
haul compared with the foreign haul, the added dist
a source of loss. Such situations are unusual and are ge
confined to trunk lines.

(c) The above may be still further condensed to the
conclusion that there is always *some* compensation for the
cost of operating an added length of line and that it fre
is a source of actual profit.

(d) There is, however, a limitation which should not
sight of. The above argument may be carried to the
conclusion that, if added distance is profitable, the c
should purposely lengthen the line. But added distance
added operating expenses. A sufficient tariff to meet th
tax on the community—a tax which more or less disc

traffic. It is contrary to public policy to burden a community with an avoidable expense. But, on the other hand, a railroad is not a charitable organization, but a money-making enterprise, and cannot be expected to unduly load up its first cost in order that subsequent operating expenses may be unduly cheapened and the tariff unduly lowered. A common reason for increased distance is the saving of the first cost of a very expensive although shorter line.

(e) Finally, although there is a considerable and uncompensated loss resulting from curvature and grade which will justify a considerable expenditure to avoid them, there is by no means as much justification to incur additional expenditure to avoid distance. Of course needless lengthening should be avoided. A moderate expenditure to shorten the line may be justifiable, but large expenditures to decrease distance are never justifiable except when the great bulk of the traffic is exceedingly heavy and is competitive.

504. Justification of decreasing distance to save time. It should be recalled that the changes which an engineer may make which are physically or financially possible will ordinarily have but little effect on the time required for a trip. The time which can thus be saved will have practically no value for the freight business—at least any value which would justify changing the route. When there is a large directly competitive passenger traffic between two cities (*e.g.* New York to Philadelphia) a difference of even 10 minutes in the time required for a run might have considerable financial importance, but such cases are comparatively rare. It may therefore be concluded that the value of the time saved by shortening distance will not ordinarily be a justification for increased expense to accomplish it.

505. Effect of change of distance on the business done. The above discussion is based on the assumption that the business done is unaffected by any proposed change in distance. If a proposed reduction in distance involves a loss of business obtained, it is almost certainly unwise. But if by increasing the distance the original cost of the road is decreased (because the construction is of less expensive character), and if the receipts are greater, and are increased still more by an increase in business done, then the change is probably wise. While it is almost impossible in a subject of such complexity to give a general

rule, the following is generally safe: Adopt a route of such l
that the annual traffic per mile of line is a maximum.
statement may be improved by allowing the element of o
cost to enter and say, adopt a route of such length that the a
traffic per mile of line divided by the average cost per
a maximum. Even in the above the operating cost per
as affected by the curvature and grades on the various
does not enter, but any attempt to formulate a genera
which would allow for variable operating expenses woul
dently be too complicated for practical application.'

CHAPTER XXII.

CURVATURE.

506. General objections to curvature. In the popular mind curvature is one of the most objectionable features of railroad alinement. The cause of this is plain. The objectionable qualities are on the surface, and are apparent to the non-technical mind. They may be itemized as follows:

1. Curvature increases **operating expenses** by increasing (**a**) the required tractive force, (**b**) the wear and tear of roadbed and track, (**c**) the wear and tear of equipment, and (**d**) the required number of track-walkers and watchmen.

2. It may affect the **operation of trains** (**a**) by limiting the length of trains, and (**b**) by preventing the use of the heaviest types of engines.

3. It may affect **travel** (**a**) by the difficulty of making time, (**b**) on account of rough riding, and (**c**) on account of the apprehension of danger.

4. There is actually an increased **danger** of collision, derailment, or other form of accident.

Some of these objections are quite definite and their true value may be computed. Others are more general and vague and are usually exaggerated. These objections will be discussed in inverse order.

507. Financial value of the danger of accident due to curvature. At the outset it should be realized that in general the problem is *not* one of curvature *vs.* no curvature, but simply sharp curvature *vs.* easier curvature (the central angle remaining the same), or a greater or less percentage of elimination of the degrees of central angle. A straight road between termini is in general a financial (if not a physical) impossibility. The practical question is then, how much is the financial value of such diminution of danger that may result from such eliminations of curvature as an engineer is able to make?

In the year 1898 there were 2228 railroad accidents reported by the *Railroad Gazette*, whose lists of all accidents worth reporting are very complete. Of these a very large proportion clearly had no relation whatever to curvature. But suppose we assume that 50% (or 1114 accidents) were directly caused by curvature. Since there are approximately 200,000 curves on the railroads of the country, there was on the average an accident for every 179 curves during the year. Therefore we may say, according to the theory of probabilities, that the chances are even that an accident may happen on any particular curve in 179 years. This assumes all curves to be equally dangerous, which is not true, but we may temporarily consider it to be true. If, at the time of the construction of the road, $1.00 were placed at compound interest at 5% for 179 years, it would produce in that time $620.89 for each dollar saved, wherewith to pay all damages, while the amount necessary to eliminate that curvature, even if it were possible, would probably be several thousand dollars. The number of passengers carried one mile for one killed in 1898–99 was 61,051,580. If a passenger were to ride continuously at the rate of sixty miles per hour, day and night, year after year, he would need to ride for more than 116 years before he had covered such a mileage, and even then the probabilities of his death being due to curvature or to such a reduction of curvature as an engineer might accomplish are very small. Of course particular curves are often, for special reasons, a source of danger and justify the employment of special watchmen. They would also justify very large expenditures for their elimination if possible. But as a general proposition it is evidently impossible to assign a definite money value to the danger of a serious accident happening on a particular curve which has no special elements of danger.

Another element of safety on curved track is that trait of human nature to exercise greater care where the danger is more apparent. Many accidents are on record which have been caused by a carelessness of locomotive engineers on a *straight* track when the extra watchfulness usually observed on a curved track would have avoided them.

508. Effect of curvature on travel. (a) **Difficulty in making time.** The general use of transition curves has largely eliminated the necessity for reducing speed on curves, and even when the speed is reduced it is done so easily and quickly by means

of air-brakes that but little time is lost. If two parallel lines were competing sharply for passenger traffic, the handicap of sharp curvature on one road and easy curvature on the other might have a considerable financial value, but ordinarily the *mere reduction* of time due to sharp curvature will not have any computable financial value.

(b) **On account of rough riding.** Again, this is much reduced by the use of transition curves. Some roads suffer from a general reputation for crookedness, but in such cases the excessive curvature is practically unavoidable. This cause probably does have some effect in influencing competitive passenger traffic.

(c) **On account of the apprehension of danger.** This doubtless has its influence in deterring travel. The amount of its influence is hardly computable. When the track is in good condition and transition curves are used so that the riding is smooth, even the apprehension of danger will largely disappear.

Travel is doubtless more or less affected by curvature, but it is impossible to say how much. Nevertheless the engineer should not ordinarily give this item any financial weight whatsoever. Freight traffic (two thirds of the total) is unaffected by it. It chiefly affects that limited class of sharply competitive passenger traffic—a traffic of which most roads have not a trace.

509. Effect on operation of trains. (a) Limiting the length of trains. When curvature actually limits the length of trains, as is sometimes true, the objection is valid and serious. But this can generally be avoided. If a curve occurs on a ruling grade without a reduction of the grade sufficient to compensate for the curvature, then the resistance on that curve will be a maximum and that curve will limit the trains to even a less weight than that which may be hauled on the ruling grade. In such cases the unquestionably correct policy is to "compensate for curvature," as explained later (see §§510, 511), and not allow such an objection to exist. It is *possible* for curvature to limit the length of trains even without the effect of grade. On the Hudson River R. R. the total net fall from Albany to New York is so small that it has practically no influence in determining grade. On the other hand, a considerable portion of the route follows a steep rocky river bank which is so crooked that much curvature is unavoidable and very sharp curvature

can only be avoided by very large expenditure. As a co
sharp curvature has been used and the resistance on t
is far greater than that of any fluctuations of grade
was necessary to use. Or, at least, a comparativ
expenditure would suffice to cut down any grade s
resistance would be less than that of some curve wh
not be avoided except at an enormous cost. And s
since the length of trains is really limited by curvatu
grades of 0.3 to 0.5% have been freely introduc
might be removed at comparatively small expense T
case is very unusual. Low grades are usually associ
generally level country where curvature is easily
as in the Camden and Atlantic R. R. Even in the
case of the Hudson River road the maximum cur
only equivalent to a comparatively low ruling grade.

(b) Preventing the use of the heaviest types of engi
validity of this objection depends somewhat on the
curvature and the detailed construction of the engin
some types of engines might have difficulty on curv
tremely short radius, yet the objection is ordinarily
This will best be appreciated when it is recalled that t
solidation" type was originally designed for use on
curvature of the mountain divisions of the Lehigh Val
and that the type has been found so satisfactory th
been extensively employed elsewhere. It should al
membered that during the Civil War an immense tr
passed over a hastily constructed trestle near Peterst
the track having a radius of 50 feet. As a result of a
at Renovo on the Philadelphia and Erie R. R. by
Dripps, Gen. Mast. Mech., in 1875,* it was claime

COMPENSATION FOR CURVATURE.

510. Reasons for compensation. The effect of curvature on a grade is to increase the resistance by an amount which is equivalent to a material addition to that grade. On minor grades the addition is of little importance, but when the grade is nearly or quite the ruling grade of the road, then the additional resistance induced by a curve will make that curve a place of maximum resistance and the real maximum will be a "virtual grade" somewhat higher than the nominal maximum. If, in Fig. 211,

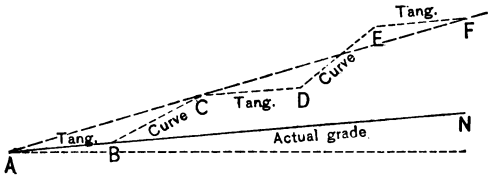


FIG. 211.

AN represents an actual *uniform* grade consisting of tangents and curves, the "virtual grade" on curves at BC and DE may be represented by BC and DE. If BC and DE are very long, or if a stop becomes necessary on the curve, then the full disadvantage of the curve becomes developed. If the whole grade may be operated without stoppage, then, as elaborated further in the next chapter, the whole grade may be operated as if equal to the average grade, AF, which is better than BC, although much worse than AN. The process of "compensation" consists in reducing the grade on every curve by such an amount that the actual resistance on each curve, due to both curvature and grade, shall precisely equal the resistance on the tangent. The practical effect of such reduction is that the "virtual" grade is kept constant, while the nominal grade fluctuates.

One effect of this is that (see Fig. 212) instead of accomplishing the vertical rise from A to G (i.e., HG) in the horizontal distance AH, it requires the horizontal distance AK. Such an addition to the horizontal distance can usually be obtained by proper development, and it should always be done on a ruling

grade. Of course it is possible that it will cost more to accomplish this than it is worth, but the engineer should be sure of this before allowing this virtual increase of the grade.

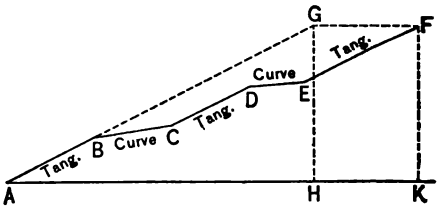


FIG. 212.

European engineers early realized the significance of unreduced curvature and the folly of laying out a uniform ruling grade regardless of the curvature encountered. Curve compensation is now quite generally allowed for in this country, but thousands of miles have been laid out without any compensation. A very common limitation of curvature and grade has been the alliterative figures 6° curvature and 60 feet per mile of grade, either singly or in combination. Assuming that the resistance on a 6° curve is equivalent to a 0.3% grade (15.84 feet per mile), then a 6° curve occurring on a 60-foot grade would develop more resistance than a 75-foot grade on a tangent. The "mountain cut-off" of the Lehigh Valley Railroad near Wilkesbarre is a fine example of a heavy grade compensated for curvature, and yet so laid out that the virtual grade is uniform from bottom to top, a distance of several miles.

511. **The proper rate of compensation.** This evidently is the rate of grade of which the resistance just equals the resistance due to the curve. But such resistance is variable. It is greater as the velocity is lower; it is generally about 2 lbs. per ton (equivalent to a 0.1% grade) per degree of curve when starting a train. On this account, the compensation for a curve which occurs at a known stopping-place for the heaviest trains should be 0.1% per degree of curve. The resistance is not even strictly proportional to the degree of curvature, although it is usually considered to be so. In fact most formulæ for curve resistance are based on such a relation. But if the experimentally determined resistances for low curvatures are applied to the excessive curvature of the New York Elevated road, for example, the

rules become ridiculous. On this account the compensation *per degree of curve* may be made less on a sharp curve than on an easy curve. The compensation actually required for very fast trains is less than for slow trains, say 0.02 or 0.03% per degree of curve; but since the comparatively slow and heavy freight trains are the trains which are chiefly limited by ruling grade, the compensation must be made with respect to those trains. From 0.04 to 0.05% per degree is the rate of compensation most usually employed for average conditions. Curves which occur *below* a known stopping-place for *all* trains need not be compensated, for the extra resistance of the curve will be simply utilized in place of brakes to stop the train. If a curve occurs just *above* a stopping-place, it is very serious and should be amply compensated. Of course the down-grade traffic need not be considered.

It sometimes happens that the ordinary rate of compensation will consume so much of the vertical height (especially if the curvature is excessive) that a steeper through grade must be adopted than was first computed, and then the trains might stall on the tangents rather than on the curves. In such cases a slight reduction in the rate of compensation might be justifiable.

The following rules have been approved by the Amer. Rwy. Eng. Assoc.

1. Compensate .03% per degree (*a*) when the length of curve is less than half the length of the longest train; (') when a curve occurs within the first 20 feet of rise of a grade; (*c*) when curvature is in no sense limiting.

2. Compensate .035% per degree (*a*) when curves are between one-half and three-quarters as long as the longest train; (*b*) when the curve occurs between 20 feet and 40 feet of rise from the bottom of the grade.

3. Compensate .04% per degree (*a*) where the curve is habitually operated at low speed; (*b*) where the length of the curve is longer than three-quarters of the length of the longest train; (*c*) where elevation is excessive for freight trains; (*d*) at all places where curvature is likely to be limiting.

4. Compensate .05% per degree wherever the loss of elevation can be spared.

512. The limitations of maximum curvature. What is the maximum degree of curvature which should be allowed on any

road? It has been shown that sharp curvature does not prevent the use of the heaviest types of engines, and although a sharp curve unquestionably increases operating expenses, the increase is but one of degree with hardly any definite limit. The general character of the country and the gross capital available (or the probable earnings) are generally the true criterions.

A portion of the road from Denver to Leadville, Col., is an example of the necessity of considering sharp curvature. The traffic that might be expected on the line was so meagre and yet the general character of the country was so forbidding that a road built according to the usual standards would have cost very much more than the traffic could possibly pay for. The line as adopted cost about $20,000 per mile, and yet in a stretch of 11.2 miles there are about 127 curves. One is a 25° 20′ curve, twenty-four are 24° curves, twenty-five are 20° curves, and seventy-two are sharper than 10°. If 10° had been made the limit (a rather high limit according to usual ideas), it is probable that the line would have been found impracticable (except with prohibitive grades) unless four or five times as much per mile had been spent on it, and this would have ruined the project financially.

For many years the main-line traffic of the Baltimore and Ohio R. R. has passed over a 300-foot curve (19° 10′) and a 400-foot curve (14° 22′) at Harper's Ferry. A few years ago some reduction was made in this by means of a tunnel, but the fact that such a road thought it wise to construct and operate such curves (and such illustrations on the heaviest-traffic roads are quite common) shows how foolish it is for an engineer to sacrifice money or (which is much more common) sacrifice gradients in order to reduce the *rate* of curvature on a road which at its best is but a second- or third-class road.

Of course such belittling of the effects of curvature may be (and sometimes is) carried to an extreme and cause an engineer to fail to give to curvature its due consideration. Degrees of central angle should always be reduced by all the ingenuity of the engineer, and should only be limited by the general relation between the financial and topographical conditions of the problem. Easy curvature is in general better than sharp curvature and should be adopted when it may be do e at a small financial sacrifice, especially since it reduces ce generally and may even cut down the initial cost of that ction of the

road. But large financial expenditures are rarely, if ever, justifiable where the net result is a mere increase in radius without a reduction in central angle. An analysis of the changes which have been so extensively made during late years on the Penn. R. R. and the N. Y., N. H. & H. R. R. will show invariably a reduction of distance, or of central angle, or both, and perhaps incidentally an increase in radius of curvature. There are but few, if any, cases where the sole object to be attained by the improvement is a mere increase in radius.

The requirements of standard M. C. B. car-couplers have virtually placed a limitation on the radius on account of the corners of adjacent cars striking each other on very sharp curves. This limitation has been crystallized into a rule on the P. R. R. that no curve, even that of a siding, can have a less radius than 175 feet, which is nearly the radius of a 33° curve. Of course only the most peremptory requirements of yard work would justify the employment of such a radius.

CHAPTER XXIII.

GRADE.

513. Two distinct effects of grade. The effects of grade on train expenses are of two distinct kinds; one possible effect is very costly and should be limited even at considerable expenditure; the other is of comparatively little importance, its cost being slight. As long as the length of the train is not limited, the occurrence of a grade on a road simply means that the engine is required to develop so many foot-pounds of work in raising the train so many feet of vertical height. For example, if a freight train weighing 600 tons (1,200,000 lbs.) climbs a hill 50 feet high, the engine performs an *additional* work of creating 60,000,000 foot-pounds of potential energy. If this height is surmounted in 2 miles and in 6 minutes of actual time (20 miles per hour), the *extra* work is 10,000,000 foot-pounds per minute, or about 303 horse-power. But the disadvantages of such a rise are always largely compensated. Except for the fact that one terminus of a road is generally higher than the other, every up grade is followed, more or less directly, by a down grade which is operated partly by the potential energy acquired during the previous climb. But when we consider the trains running in both directions even the difference of elevation of the termini is largely neutralized. If we could eliminate frictional resistances and particularly the use of brakes, the *net* effect of minor grades on the operation of minor grades in both directions would be zero. Whatever was lost on any up grade would be regained on a succeeding down grade, or at any rate on the return trip. On the very lowest grades (the limits of which are defined later) we may consider this to be literally true, viz., that nothing is lost by their presence; whatever is temporarily lost in climbing them is either immediately regained on a subsequent light down grade or is regained on the return trip. If a stop is required at the bottom of a sag, there is a net and uncompensated loss of energy.

On the other hand, if the length of trains is limited by the grade, it will require more trains to handle a given traffic. The receipts from the traffic are a definite sum. The cost of handling it will be nearly in proportion to the number of trains. Assume that by lowering the rate of ruling grade it becomes possible to handle such an increased number of cars with one engine that four engines can haul as many cars on the reduced grade as five engines could haul on the higher grade and at a cost but slightly more than four-fifths as much. The effect of this on dividends may readily be imagined.

514. Application to the movement of trains of the laws of accelerated motion. When a train starts from rest and acquires its normal velocity, it overcomes not only the usual tangent resistances (and perhaps curve and grade resistances), but it also performs work in storing into the train a vast fund of kinetic energy. This work is not lost, for every foot-pound of such energy may later be utilized in overcoming resistances, provided it is not wasted by the action of train-brakes. If for a moment we consider that a train runs without any friction, then, when running at a velocity of v feet per second, it possesses a kinetic energy which would raise it to a height h feet, when $h=\frac{v^2}{2g}$, in which g is the acceleration of gravity $=32.16$. Assuming that the engine is exerting just enough energy to overcome the frictional resistances, the train would climb a grade until the train was raised h feet above the point where its velocity was v. When it had climbed a height h' (less than h) it would have a velocity $v_1=\sqrt{2g(h-h')}$. As a numerical illustration, assume $v=30$ miles per hour $=44$ feet per second. Then $h=\frac{v^2}{2g}=30.1$ feet, and assuming that the engine was exerting just enough force to overcome the rolling resistances on a level, the kinetic energy in the train would carry it for two miles up a grade of 15 feet per mile, or half a mile up a grade of 60 feet per mile. When the train had climbed 20 feet, there would still be 10.1 feet left and its velocity would be $v_1=\sqrt{2g(10.1)}=25.49$ feet per second $=17.4$ miles per hour. These figures, however, must be slightly modified on account of the weight and the revolving action of the wheels, which form a considerable percentage of the total weight of the train. When train velocity is being

acquired, part of the work done is spent in imparting the energy of rotation to the driving-wheels and various truck-wheels of the train. Since these wheels run on the rails and must turn as the train moves, their rotative kinetic energy is just as effective—as far as it goes—in becoming transformed back into useful work. The proportion of this energy to the total kinetic energy has already been demonstrated (see Chapter XVI, § 435). The value of this correction is variable, but an average value of 5% has been adopted for use in the accompanying tabular form (Table XLII), in which is given the corrected "velocity head" corresponding to various velocities in miles per hour. The table is computed from the following formula:

$$\text{Velocity head} = \frac{v^2 \text{ in ft. per sec.}}{64.32} = \frac{2.151V^2 \text{ in m. per h.}}{64.32} = 0.03344V^2$$

adding 5% for the rotative kinetic energy of the wheels, $0.00167V^2$

The corrected velocity head therefore equals $0.03511V^2$

Part of the figures of Table XLII were obtained by interpolation and the final *hundredth* may be in error by one unit, but it may readily be shown that the final hundredth is of no practical importance. It is also true that the chief use made of this table is with velocities much less than 45 miles per hour. Corresponding figures may be obtained for higher velocities, if desired, by multiplying the figure for *half* the velocity by *four*.

515. Construction of a virtual profile. The following simple demonstration will be made on the basis that the ordinary tractive resistances and also the tractive force of the locomotive are independent of velocity. For a considerable range of velocity which includes the most common freight-train velocities the first assumption is practically true; the second assumption is so nearly true under certain possible operative conditions that it may serve as a preliminary to the more accurate solution. It may best be illustrated by considering a simple numerical example.

Assume that Fig. 213 shows the profile of a section of road and that the grade of AE is 0.40%, which is 21.12 feet per mile. Assume also that a freight engine is climbing up the grade at a uniform velocity of 20 miles per hour. But since the train is moving at 20 miles per hour it has a kinetic energy corresponding to a velocity of 14.05 feet (see Table XLII). At A it encounters a down-grade of 0.20 per cent, which is 1500 feet lo Although

AB has a down-grade of only 0.20%, its grade with respect the up-grade of AE (0.40%) is 0.60%. Therefore B is 9.00 feet below B'. Since the work done by the engine would have carried the train up to the point B' with a velocity of 20 miles per hour, the *virtual* drop of 9 feet will increase the velocity head from 14.05 feet to 23.05 feet, which corresponds to the velocity of 25.6 miles per hour, and this will actually be the velocity of the train at the point B. At B the grade changes to a 1.0% up-grade for a distance of 2300 feet.

The approach of the grade BC to the grade $B'C$ is at the rate of $1.0-0.4=0.6\%$ and therefore, the point C will be reached in 1500 feet. In the remaining 800 feet the line will climb to D, which is 4.8 feet above D'. Although at B the train is moving at the rate of 25.6 miles per hour and the engine is working at such a rate that it will carry the train up a 0.4% grade, yet when climbing up a 1.0% grade it consumes its kinetic energy in overcoming the additional grade. When it reaches C, it has lost the additional kinetic energy which it gained from A to B, and as it continues it loses even more. When it reaches D, it has lost 4.8 ft. more and its velocity head is reduced to $14.05-4.8=9.25$ ft., which corresponds to a velocity of 24.2 miles per hour. At D the grade changes to +0.1%.

FIG. 212.—TYPICAL PROFILE OF ROAD SECTION.

TABLE XLII—VELOCITY HEAD (REPRESENTING THE KINETIC ENERGY) OF TRAINS MOVING AT VARIOUS VELOCITIES.

Vel. mi. hr.	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
5	0.88	0.91	0.95	0.99	1.02	1.06	1.10	1.14	1.18	1.22
6	1.26	1.31	1.35	1.40	1.44	1.48	1.53	1.58	1.62	1.67
7	1.72	1.77	1.82	1.87	1.92	1.97	2.03	2.08	2.14	2.19
8	2.25	2.30	2.36	2.42	2.48	2.54	2.60	2.66	2.72	2.78
9	2.85	2.91	2.97	3.04	3.10	3.17	3.24	3.30	3.37	3.44
10	3.51	3.58	3.65	3.72	3.79	3.87	3.95	4.02	4.10	4.17
11	4.25	4.33	4.41	4.49	4.57	4.65	4.73	4.81	4.89	4.97
12	5.06	5.15	5.23	5.32	5.41	5.50	5.58	5.67	5.75	5.84
13	5.93	6.02	6.12	6.21	6.31	6.40	6.50	6.59	6.69	6.78
14	6.88	6.98	7.08	7.19	7.29	7.39	7.49	7.60	7.70	7.80
15	7.90	8.00	8.11	8.22	8.33	8.44	8.55	8.66	8.77	8.88
16	8.99	9.10	9.21	9.32	9.43	9.55	9.67	9.79	9.91	10.03
17	10.15	10.27	10.39	10.51	10.63	10.75	10.87	10.99	11.12	11.25
18	11.38	11.50	11.63	11.76	11.89	12.02	12.15	12.28	12.41	12.55
19	12.68	12.81	12.95	13.08	13.22	13.35	13.49	13.63	13.77	13.91
20	14.05	14.19	14.33	14.47	14.61	14.75	14.89	15.04	15.19	15.34
21	15.49	15.64	15.79	15.94	16.09	16.24	16.39	16.54	16.69	16.84
22	17.00	17.15	17.30	17.46	17.62	17.78	17.94	18.10	18.26	18.42
23	18.58	18.74	18.90	19.06	19.22	19.38	19.55	19.72	19.89	20.06
24	20.23	20.40	20.57	20.74	20.91	21.08	21.25	21.42	21.59	21.77
25	21.95	22.12	22.30	22.48	22.66	22.84	23.02	23.20	23.38	23.56
26	23.74	23.92	24.10	24.28	24.46	24.65	24.84	25.03	25.22	25.41
27	25.60	25.79	25.98	26.17	26.36	26.55	26.74	26.93	27.13	27.33
28	27.53	27.73	27.93	28.13	28.33	28.53	28.73	28.93	29.13	29.33
29	29.53	29.73	29.93	30.13	30.34	30.55	30.76	30.97	31.18	31.39
30	31.60	31.81	32.02	32.23	32.44	32.65	32.86	33.08	33.30	33.52
31	33.74	33.96	34.18	34.40	34.62	34.84	35.06	35.28	35.50	35.72
32	35.95	36.17	36.39	36.62	36.85	37.08	37.31	37.54	37.77	38.00
33	38.23	38.46	38.69	38.92	39.15	39.38	39.62	39.86	40.10	40.34
34	40.58	40.82	41.06	41.30	41.54	41.78	42.02	42.26	42.51	42.76
35	43.01	43.26	43.51	43.76	44.01	44.26	44.51	44.76	45.01	45.26
36	45.51	45.76	46.01	46.26	46.52	46.78	47.04	47.30	47.56	47.82
37	48.08	48.34	48.60	48.86	49.12	49.38	49.64	49.91	50.18	50.45
38	50.72	50.99	51.26	51.53	51.80	52.07	52.34	52.61	52.88	53.15
39	53.42	53.69	53.96	54.23	54.51	54.79	55.07	55.35	55.63	55.91
40	56.19	56.47	56.75	57.03	57.31	57.59	57.87	58.16	58.45	58.74
41	59.03	59.32	59.61	59.90	60.19	60.48	60.77	61.06	61.35	61.64
42	61.94	62.23	62.52	62.82	63.12	63.42	63.72	64.02	64.32	64.62
43	64.92	65.22	65.52	65.82	66.12	66.43	66.74	67.05	67.36	67.67
44	67.98	68.29	68.60	68.91	69.22	69.53	69.84	70.15	70.46	70.78

Here we have the rather surprising condition that, although the grade is actually rising, it is virtually a down-grade under the given conditions, for the engine is working harder than is required to run up merely a 0.1% grade and hence will gain in velocity. At *E*, a distance of 1600 feet from *D*, it reaches what

would have been a uniform 0.4% grade from A to E and the grade continues at that rate. Although the train has actually climbed 1.6 feet from D to E, it has virtually fallen the 4.8 feet between D and D', and the velocity head has increased from its value of 9.25 feet at D to 14.05 feet, and its velocity is again 20 miles per hour. The upper line represents the " virtual profile," which may always be drawn by measuring off to the proper scale at every point an ordinate which is the velocity head at that point. Since the engine is working uniformly, the virtual profile is in this case a straight line.

As another case, assume that a train is climbing the grade AE and exerting a pull just sufficient to maintain a constant velocity up that grade. Then $A'B'$ (parallel to AB) is the virtual profile, AA' representing the velocity head. A stop being required at C, steam is shut off and brakes are applied at B, and the velocity head BB' reduces to zero at C.

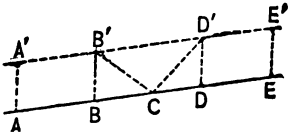


FIG. 214.

The train starts from C, and at D attains a velocity corresponding to the ordinate DD'. At D the throttle may be slightly closed so that the velocity will be uniform and the virtual grade is $D'E'$, parallel to DE.

From the above it may be seen that a virtual profile has the following properties:

(*a*) When the velocity is *uniform*, the virtual profile is parallel with the actual.

(*b*) When the velocity is increasing the profiles are separating; when decreasing the profiles are approaching.

(*c*) When the velocity is zero the profiles coincide.

(*d*) The virtual grade at any place is a measure of the work required of the engine beyond that required to overcome merely the tractive resistances. If it is horizontal it shows that the engine is doing nothing besides overcoming the tractive resistances. If it is upward and is uniform, as in Fig. 213, it shows that it is working uniformly and is storing in the train " potential " energy which may be utilized on the return trip if it is not utilized to overcome tractive resistance in moving down a succeeding down-grade. If it is downward, as from B' to C, Fig. 214, it shows that the train is giving up kinetic energy, probably consuming most of it in brakes, but utilizing some of it

to furnish the tractive power to run from B to C and also to overcome the grade from B to C.

516. Variation in draw-bar pull. The above demonstration has been made on the basis that the draw-bar pull is constant throughout. It is shown in Chapter XVIII that, when the engine is working at its full capacity the draw-bar pull decreases as the velocity increases, which is chiefly due to the fact that if we attempt to use full stroke at 2 M or 3 M velocity the steam will be so rapidly exhausted from the boiler that the pressure will fall. Therefore the valves are set to cut off so as to use the steam expansively but as this reduces the average pressure in the cylinder, then (see Eq. 103), the tractive power must be less. The reduction of tractive power for several multiples of M is shown in Table XXXIX. For example, in the numerical problem given above, and assuming the use of the Mikado engine whose characteristics have already been computed, the velocity at $A = 20 \div 6.167 = 3.25\ M$ and the tractive power at this velocity is 49.23% of its power at M velocity. From the tabular form in § 469 the draw-bar pull at 3.25 M-velocity may be found by interpolation to be 16587 lbs. Similarly at B the velocity is *expected* to be 25.6 m.p.h. $= 4.15\ M$, and then the tractive power is 38.48% and the draw-bar pull only 12484 lbs., about 75% of the pull at A. But since the draw-bar pull is so much reduced the velocity evidently would not be increased the theoretical amount due to the virtual drop BB'. On the other hand, when the train reaches D, where the velocity is *supposed* to be 16.2 m.p.h. $= 2.62\ M$, the draw-bar pull would be 20144, which is over 121% of the normal pull at 3.25 M velocity. The average pull between B and D is 16314 or within 2% of the normal 16587. The average between A and E, assuming that the theoretical velocities at B and D were actually realized, would be about 2% below the assumed pull at A. The 3000-foot sag ABC will be passed in 90 seconds and no very great reduction in boiler power could take place in that time, especially if the fireman used extra care to maintain the pressure. Investigators have declared that tests of trains, with a dynamometer car between the tender and cars, have shown a practically uniform draw-bar pull, with an unchanged throttle and with velocities varying substantially on the principles indicated above. If the sag ABC is excessively long or deep the reduction of tractive force with increased ·city would be so great that the error of the method would be

too great for practical use. But experience has proven that for ordinary cases the method can be used with substantial accuracy.

517. Use, value, and possible misuse. The essential feature respecting grades is the demand on the locomotive. From the foregoing it may readily be seen that the ruling grade of a road is not necessarily the steepest nominal grade. When a grade may be operated by momentum, i.e., when every train has an opportunity to take "a run at the hill," it may become a very harmless grade and not limit the length of trains, while another grade, actually much less, which occurs at a stopping-place for the heaviest trains, will require such extra exertion to get trains started that it may be the worst place on the road. Therefore the true way to consider the value of the grade at any critical place on the road is to construct a virtual profile for that section of the road. The required length of such a profile is variable, but in general may be said to be limited by points on each side of the critical section at which the velocity is definite, as at a stopping-place (velocity zero), or a long heavy grade where it is the minimum permissible, say M miles per hour.

Since the velocities of different trains vary, each train will have its own virtual profile at any particular place. Fast passenger trains are less affected than slow freight trains. The requirement of high average speed necessitates the use of powerful engines, and grades which would stall a heavy freight will only cause a momentary and harmless reduction of speed of the fast passenger train.

A possible misuse of virtual profiles lies in the chance that a station or railroad grade crossing may be subsequently located on a heavy grade that was designed to be operated by momentum. But this should not be used as an argument against the employment of a virtual profile. The virtual profile shows the *actual state of the case* and only points out the necessity, if an unexpected requirement for a full stoppage of trains at a critical point has developed, of changing the location (if a station), or of changing the grade by regrading or by using an overhead crossing.

518. Undulatory grades. Advantages. Money can generally be saved by adopting an actual profile which is not strictly uniform—the matter of compensation for curvature being here

ignored. Its effect on the operation of trains is harmless provided the sag or hump is not too great. In Fig. 215 the undulatory grade may actually be operated as a uniform grade *AG*. The sag at *C* must be considered as a sag, even though *BC* is actually an up grade. But the engine is supposed to be working hard enough to carry a train at uniform velocity up a grade *AG*. Therefore it *gains* in velocity from *B* to *C*, and from *C* to *D* loses an equal amount. It may even be proven that the *time* re-

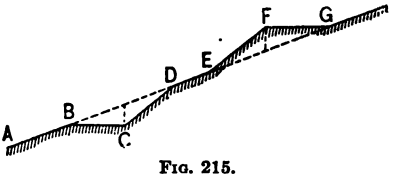


FIG. 215.

quired to pass the sag will be slightly less than the time required to run the uniform grade.

Disadvantages. The hump at *F* is dangerous in that, if the velocity at *E* is not equal to that corresponding to the extra velocity-head ordinate at *F*, the train will be stalled before reaching *F*. In practice there should be considerable margin. Any train should have a velocity of at least *M* (see § 455) in passing any summit. An extra heavy head wind, slippery rails, etc., may use up any smaller margin and stall the train. If the grade *AG* is a ruling grade, then *no* bump should be allowed under any circumstances. For the heaviest trains are supposed to be so made up that the engine will *just* haul them up the ruling grades—of course with some margin for safety. Any increase of this grade, however short, would probably stall the train.

Safe limits. Since over 99.4% of all freight cars are now equipped with train brakes and automatic couplers, there is not now the limitation which formerly existed about operating freight trains at high speeds, but it may frequently happen that it would be undesirable to run a freight train through a deep sag at such a velocity as would result from a free run and it would therefore become necessary to use brakes, which will add a distinct element of cost.

The term "safe limits" as used here, refers to the limits within

which a freight train may be safely operated without the application of brakes or varying the work of the engine. Of course much greater undulations are frequently necessary and are safely operated, but it should be remembered that they add a distinct element to the cost of operating trains and that they must not be considered as harmless or that they should be introduced unless really necessary.

RULING GRADES.

519. Definition. Ruling grades are those which limit the weight of the train of cars which may be hauled by one engine. The subject of "pusher grades" will be considered later. For the present it will suffice to say that on all well-designed roads the large majority of the grades on any one division are kept below some limit which is considered the ruling grade. If a heavier grade is absolutely necessary no special expense will be made to keep it below a rate where the resistance is twice (or possibly three times) the resistance on the ruling grade, and then the trains can be hauled unbroken up these few special grades with the help of one (or two) pusher engines. So far as limitation of train length is concerned, these pusher grades are no worse than the regular ruling grades and, except for the expense of operating the pusher engines (which is a separate matter), they are not appreciably more expensive than any ruling grade. As before stated, the engineer cannot alter very greatly the ruling grade of the road when the general route has been decided on. He may remove sags or humps, or he may lower the natural grade of the route by development in order to bring the grade within the adopted limit of ruling grade.

520. Choice of ruling grade. It is of course impracticable for an engine to drop off or pick up cars according to the grades which may be encountered along the line. A train load is made up at one terminus of a division and must run to the other terminus. Excluding from consideration any short but steep grades which may *always* be operated by momentum, and also all pusher grades, the maximum grade on that division is the ruling grade.

It will evidently be economy to reduce the few grades which naturally would be a little higher than the great majority of

others until such a large amount of grade is at some uniform limit that a reduction at all these places would cost more than it is worth. The precise determination of this limit is practically impossible, but an approximate value may be at once determined from a general survey of the route. The distance apart of consecutive control points (see § 18) into their difference of elevation is a first trial figure for the rate of the grade. If a grade even approximately uniform is impossible owing to the elevation of intervening ground, the worst place may be selected and the natural grade of that part of the route determined. If this grade is much steeper than the general run of the natural grades, it may be policy to reduce it by development or to boldly plan to operate that place as a pusher grade. The choice of possible grades thus has large limitations, and it justifies very close study to determine the best combination of grades and pusher grades. When the choice has narrowed down to two limits, the lower of which may be obtained by the expenditure of a definite extra sum, the choice may be readily computed, as will be developed.

521. Maximum train load on any grade. The Mikado locomotive, whose characteristics were analyzed in Chapter XVIII, has a net pulling power at the rim of the drivers, at *M* velocity, of 35758 lbs. which is 23.3% of 153,200, the weight on the drivers. This percentage is slightly over $\frac{2}{4}\frac{3}{0}$. Increasing the percentage 6% on account of increased power at starting we have 24.7% or nearly $\frac{1}{4}$. On the other hand, wet, slippery rails may render the adhesion as low as $\frac{1}{5}$ and thus limit the actual drawing power. Although the real power of a locomotive depends on the velocity at which it seems desirable to run, the maximum tractive power at "*M*" velocity can always be approximately estimated as $\frac{1}{4}$ of the weight on the drivers. In Table XLIII are given the weights of several types of locomotives together with their tractive powers at three ratios of adhesion. These values are useful when the more elaborate method detailed in Chapter XVIII is not considered necessary.

The maximum train load on any grade depends on the character and number of the cars, as well as on their gross weight. The approximate resistance of cars is given by Eq. 121 as $R = 2.2\,i + 122\,n$. Applying this to a steel box-car weighing 24 tons net and loaded with 100,000 lbs., the resistance would be 285 lbs. or 3.85 lbs. per ton. Empty, the resistance would be 7.28 lbs. per

TABLE XLIII.—TRACTIVE POWER OF VARIOUS TYPES OF STANDARD-GAUGE LOCOMOTIVE AT VARIOUS RATES OF ADHESION.

Type of locomotive.	Total weight of engine and tender.		Weight of engine only.	Weight on the drivers.	Tractive power when ratio of adhesion is		
	Lbs.	Tons.			$\frac{1}{4}$	$\frac{2}{9}$	$\frac{1}{5}$
Atlantic, 4–4–2.....	340,000	170.0	199,400	105,540	26,385	23,740	21,100
Atlantic, 4–4–2, four cylinder compound	368,800	184.4	206,000	115,000	28,750	25,875	23,000
Pacific, 4–6–2.....	343,600	171.8	218,000	142,000	35,500	31,950	28,400
Pacific, 4–6–2.....	403,780	201.9	226,700	151,900	37,975	34,180	30,380
Ten-wheel, 4–6–0...	321,000	160.5	201,000	154,000	38,500	34,650	30,800
Prairie, 2–6–2.....	366,500	183.2	212,500	154,000	38,500	34,650	30,800
Consolidation, 2–8–0	214,000	107.0	120,000	106,000	26,500	23,850	21,200
Consolidation, 2–8–0	366,700	183.3	221,500	197,500	49,375	44,440	39,500
Mikado, 2–8–2.....	405,500	202.7	259,000	196,000	49,000	44,100	39,200
Mikado, 2–8–2.....	315,000	157.5	196,100	153,200	38,300	34,470	30,640

ton. Applying the formula to a wooden box-car weighing 15 tons net and carrying 60,000 lbs., the resistances for the car full and empty would be 4.9 and 10.3 lbs. per ton, respectively. Three and 10 pounds per ton are the ordinary extremes. Although resistances of less than 3 lbs. per ton have been measured for whole trains of heavy-loaded coal cars, there are usually enough light-weight cars and empties in a train to increase the average per ton resistance to perhaps 6 lbs. per ton.

The Mikado locomotive, referred to above, had a draw-bar pull on a level at *M* velocity (6.167 m.p.h.) of 35,419 lbs. How much of a load could it draw up a 1.2% grade at *M* velocity? Assume that the cars have a weight and character such that the average resistance would be 6 lbs. per ton. The grade resistance of the locomotive is $315{,}000 \times .012 = 3780$, which subtracted from 35,419 leaves 31,639, the pull available for the cars. Then, calling T the tons weight of cars

$$31{,}639 = 6T + (20 \times 1.2 \times T) = 30T, \text{ and } T = 1054.$$

It should be noted that this computed tonnage is on the basis of an assumed tractive resistance of 6 lbs. per ton. In § 467 the tractive power of this same locomotive, on the same grade, is computed, by the regular rating formula, to be 16 fully loaded cars, weighing 70.8 tons per car, a total load of 1133 tons, *or* 53 empties, weighing 18 tons per car, a total load of 954 tons. The above value of T is approximately the mean of these two extremes. For general computations, when the character of

the train load is unknowable, some such average value, as used above, is probably as accurate as it is possible to utilize it.

522. Proportion of the traffic affected by the ruling grade. Some very light traffic roads are not so fortunate as to have a traffic which will be largely affected by the rate of the ruling grade. When passenger traffic is light, and when, for the sake of encouraging traffic, more frequent trains are run than are required from the standpoint of engine capacity, it may happen that no passenger trains are really limited by any grade on the road—i.e., an extra passenger car *could* be added if needed. The maximum grade then has no worse effect (for passenger trains) than to cause a harmless reduction of speed at a few points. The local freight business is frequently affected in practically the same way. All coal, mineral, or timber roads are affected by the rate of ruling grade as far as such traffic is concerned. Likewise the through business in general merchandise, especially of the heavy traffic roads, will generally be affected by the rate of ruling grade. Therefore in computing the effect of ruling grade, the total number of trains on the road should not ordinarily be considered, but only the trains to which cars are added, until the limit of the hauling power of the engine on the ruling grades is reached.

PUSHER GRADES.

523. General principles underlying the use of pusher engines. On nearly all roads there are some grades which are greatly in excess of the general average rate of grade, and these heavy grades cannot usually be materially reduced without an expenditure which is excessive and beyond the financial capacity of the road. If no pusher engines are used, the length of all heavy trains is limited by these grades. The financial value of the reduction of such ruling grades has already been shown. But in the operation of pusher grades there is incurred the additional cost of pusher-engine service, for a pusher engine must run *twice* over the grade for each train which is assisted. It is possible for this additional expense to equal or even exceed the advantage to be gained. In any case it means the adoption of the lesser of two evils, or the adoption of the more economical method. The work of overcoming the normal resistances of so many loaded cars over so many miles of track and of lifting so many tons up the gross differences of elevation of predetermined points of the line is approximately the same whatever the exact

e, and if the grades are so made that fewer engines working
e constantly can accomplish the work as well as more engines
:h are not hard worked for a considerable proportion of the
, the economy is very apparent and unquestionable. Wel-
:on expresses it concisely: "It is a truth of the first importance
the objection to high gradients is not the work which the
nes have to do on them, but it is the work which they do
lo when they thunder over the track with a light train be-
l them, from end to end of a division, in order that the
led power may be at hand at a few scattered points where
e it is needed."

14. Balance of grades for pusher service. Assume that both
ıer and through engines are the Mikado engine with dimen-
s already given (§ 453), and that they will be operated at
r most effective velocity, $M=6.167$ m.p.h., and that the
:tive draw-bar pull of each is $37190-1771=35419$ lbs.,
the locomotive grade resistance, which on a 1.9% grade
$0\times1.9\times157.5=5985$ lbs. The net draw-bar pull on this
le for each engine is, therefore, 29434 lbs. Assume that
train considered is made up of coal cars weighing 40000 lbs.
and carrying 100,000 lbs. each; also a caboose weighing 12
ı. Utilizing Eq. 121, the tractive resistance of a loaded
car will be $2.2\times70+122=276$, and the grade resistance
$1.9\times70=2660$, making a total of 2936. The total for the
ɔose is $148+456=604$. The two engines have a net draw-
pull of $2\times29434=58868$ lbs. Subtracting 604 for the
ɔose, there is left 58264 for coal cars. $58264\div2936=19.84$,
number of cars. Although the number of cars must, of
'se, be a whole number, the computation of the relative
ıugh and pusher grades requires that we use the fractional
ıber. The tractive resistance of the 19.84 cars and caboose
$2\ [(19.84\times70)+12]+(122\times20.84)=5624$. The force avail-
for grade is $35419-5624=29795$. The tonnage on the
le engine grade is 157.5 (engine) plus $19.84\times70=1388.8$
l cars), plus 12 (caboose), or 1558.3 tons. $29795\div1558.3$
).12 lbs. per ton, which is the grade resistance for a 0.956%
le. This means that the through grade can be made 0.956%
the corresponding pusher grade may be 1.9%. If the same
ɔlem is worked out on the basis of some other type of engine,
ch, perhaps, weighs considerably less, very nearly the same
ıugh grade to correspond with the pusher grade will be

obtained. The above combination of unit car weights n worked as 19 coal cars and a caboose and have a consi margin of unused power. A different combination weights would use up the power with less or no margin, any case the computation of the corresponding lower gr the computation of an allowable pusher grade on the l a given through grade, should be made by using a fra number of cars.

Since the pusher engine service is intermittent, and sin working at full power for much less than half the tim practicable for the fireman to feed coal faster than the st of 4000 lbs. of coal per hour while going up the pusher The above computation was made on the basis of pow duction at the 4000-lb. rate. In § 457, it is shown that i ing the rate of coal consumption increases the value of I conversely when the locomotive is run at a velocity less t the tractive power is increased, although the increase proportionately small. Increasing the tractive power pusher engine will increase the number of cars, although pr not as much as one car. Then the increase in car numb increase the computed resistance and *decrease* the amount able for grade. This decreased amount is divided by creased number of tons and the amount of available for per ton is less and the computed through grade is less. sidering the very slight and disproportionate difference m increasing the rate of coal consumption beyond the 4 standard, it is, perhaps, wisest to make the ratio of the on the basis of engines of equal power.

525. **Two-pusher grades.** It may happen, although that three systems of ruling grades may be necessary c division, which may be so balanced that one unbroken t handled with equal facility on through grades with one e on one-pusher grades with two engines and on two-p grades with three engines. The relation of these three may be computed on the same principles as are used above

526. **Operation of pusher engines.** The maximum effi in operating pusher engines is obtained when the pusher is kept constantly at work, and this is facilitated when the grade is as long as possible, i.e., when the heavy grades a great bulk of the difference of elevation to be surmoun at one place. For example, a pusher grade of three mil

lowed by a comparatively level stretch of three miles and then by another pusher grade of two miles cannot all be operated as cheaply as a continuous pusher grade of five miles. Either the two grades must be operated as a continuous grade of eight miles (sixteen pusher miles per trip) or else as two short pusher grades, in which case there would be a very great loss of time and a difficulty in so arranging the schedules that a train need not wait for a pusher or the pushers need not waste too much time in idleness waiting for trains. If the level stretch were imperative, the two grades would probably be operated as one, but an effort should be made to bring the grades together. It is not necessary to bring the trains to a stop to uncouple the pusher engine, but a stop is generally made for coupling on, and the actual cost in loss of energy and in wear and tear of stopping and starting a heavy train is as great as the cost of running an engine light for several miles.

There are two ways in which it is *possible* to economize in the use of pusher engines. (*a*) When the traffic of a road is so very light that a pusher engine will not be kept reasonably busy on the pusher grade it *may* be worth while to place a siding long enough for the longest trains both at top and bottom of the pusher grade and then take up the train in sections. Perhaps the worst objection to this method is the time lost while the engine runs the extra mileage, but with such very light traffic roads a little time more or less is of small consequence. On light traffic roads this method of surmounting a heavy grade will be occasionally adopted even if pushers are never used. If the traffic is fluctuating, the method has the advantage of only requiring such operation when it is needed and avoiding the purchase and operation of a pusher engine which has but little to do and which might be idle for a considerable proportion of the year. (*b*) The second possible method of economizing is only practicable when a pusher grade begins or ends at or near a station yard where switching-engines are required. In such cases there is a possible economy in utilizing the switching-engines as pushers, especially when the work in each class is small, and thus obtain a greater useful mileage. But such cases are special and generally imply small traffic.

A telegraph-station at top and bottom of a pusher grade is generally indispensable to effective and safe operation.

527. Length of a pusher grade. The virtual length of the

pusher grade, as indicated by the mileage of the pusher engine, is always somewhat in excess of the true length of the grade as shown on the profile, and sometimes the excess length is very great. If a station is located on a lower grade within a mile or so of the top or bottom of a pusher grade, it will ordinarily be advisable to couple or uncouple at or near the station, since the telegraph-station, switching, and signaling may be more economically operated at a regular station. If the extra engine is coupled on ahead of the through engine (as is sometimes required by law for passenger trains) the uncoupling at the top of the grade may be accomplished by running the assistant engine ahead at greater speed after it is uncoupled, and, after running it on a siding, clearing the track for the train. But this requires considerable extra track at the top of the grade. Therefore, when estimating the length of the pusher grade, the most desirable position for the terminal sidings must be studied and the length determined accordingly rather than by measuring the mere length of the grade on the profile. Of course these odd distances are always *excess*; the coupling or uncoupling should not be done while on the grade.

528. **The cost of pusher-engine service.** When we analyze the elements of cost, we will find that many of them are dependent only on time, while others are dependent upon mileage. Still others are dependent on both. Very much will depend on the constancy of the service, and this in turn depends on the train schedule and on a variety of local conditions which must be considered for each particular case. The effect of a pusher-engine on maintenance of way may be considered on the basis that an engine is responsible for one-half of the deterioration of maintenance of way and structures, and, therefore, one-half of the percentage of the first 19 items in Table XLI or 9.06% of the average cost of a train-mile will be considered as chargeable for each mile of pusher engine service. Although the cost of repairs and renewals of engines is evidently a function of the mileage, and would therefore be somewhat less for a pusher-engine which did little work than for an engine which was worked to the limit of its capacity, yet it is only safe to make the same allowance as for other engines. Other items of maintenance of equipment are evidently to be ignored. The item of wages of enginemen will evidently depend upon the system employed on the particular road. Whatever the precise system

TABLE XLIV.—COST FOR EACH MILE OF PUSHER-ENGINE SERVICE.

Item number.	Item (abbreviated).	Normal average.	Per cent affected.	Cost per engine mile, per cent.
1–19	Track material, labor, bridges....	18.12%	50	9.06
25–27	Steam locomotives.....	9.24	100	9.24
80, 81	Road enginemen and engine-house expenses.....	8.12	100	8.12
82–85	Fuel and other engine supplies....	11.27	100	11.27
90, 91, 94	Signaling, flagmen, and telegraph..	1.21	100	1.21
		.....	.....	38.90

the general result is to pay the enginemen as much in wages as the average payment for regular service, and therefore the full allowance for Item 80 will be made. Similarly we must allow the full cost of the items for engine supplies. While the engine is doing its heavy work in climbing up the grade, the consumption of fuel and water is certainly greater than the average; but, on the other hand, on the return trip, when the engine is running light, it probably runs for a considerable portion of the distance actually without steam, and therefore the consumption of fuel and water will nearly, if not quite, average the consumption for an engine running up and down grade along the whole line. That portion of fuel consumption which is due to radiation, blowing-off steam, and the many other causes previously enumerated, will be the same regardless of the work done. We therefore allow 100% for all of these items of engine supplies. In general we must add 100% for Items 90, 91, and 94, the cost of switchmen and telegraphic service. While there might be cases where there would be no actual addition to the pay-rolls or the operating expenses on account of these items, we are not justified in general in neglecting to add the full quota for such service. Collecting these items we will have 38.90% of the average cost of a train-mile for the cost of each mile run by the pusher engine. On the basis that the average cost of a train mile is $1.60, the cost of one mile of pusher engine service would be $.3890 \times \$1.60 = 62.24$ cents. Assume that the pusher engine grade is five miles long but that the engine actually runs 11 miles on a round trip and that it makes 5 round trips or 55 miles per day. Then the daily cost would be $.6224 \times 55 =$ $34.23 per day. Probably $25 to $30 per day should be charged

up even if the mileage did not amount to as much, since ma
the items in the cost of service are largely independent of mil
On the other hand the pusher engine service renders unnece
the extra trains which would have been required to handl
traffic with one engine over the steeper grades. The co
these must be computed for each particular case.

BALANCE OF GRADES FOR UNEQUAL TRAFFIC.

529. **Nature of the subject.** It sometimes happens, as
a road runs into a mountainous country for the purpo
hauling therefrom the natural products of lumber or min
that the heavy grades are all in one direction—that the
line consists of a more or less unbroken climb having pe
a few comparatively level stretches, but no down grade (e
possibly a slight sag) in the direction of the general up g
With such lines this present topic has no concern. Bu
majority of railroads have termini at nearly the same
(500 feet in 500 miles has no practical effect on grade)
consist of up and down grades in nearly equal amounts
rates. The general rate of ruling grade is determined b
character of the country and the character and financial ba
of the road to be built. It is always possible to reduce the
at some point by "development" or in general by the e
diture of more money. It has been tacitly assumed in
previous discussions that when the ruling grade has bee
termined all grades in either direction are cut down to
limit. If the traffic in both directions were the same this
be the proper policy and sometimes is so. But it has develo
especially on the great east and west trunk lines, that the
of the eastbound freight traffic is enormously greater than
of the westbound—that westbound trains consist very large
"empties" and that an engine which could haul twenty lo
cars up a given grade in eastbound traffic could haul the
cars empty up a much higher grade when running west.
an illustration of the large disproportion which may exist,
eastbound ton-mileage on the P. R. R. between the years
and 1885 was 3.7 times the westbound ton-mileage. Bet
the years 1876 and 1880 the ratio rose to more than 4.5
On such a basis it is as important and necessary to obtain,
a 0.6% ruling grade against the eastbound traffic as to h

say, a 1.0% grade against the westbound traffic. This is the basis of the following discussion. It now remains to estimate the probable ratio of the traffic in the two directions and from that to determine the proper "balance" of the opposite ruling grades.

530. Computation of the theoretical balance. Assume first, for simplicity, that the exact business in either direction is accurately known. A little thought will show the truth of the following statements.

1. The locomotive and passenger-car traffic in both directions is equal.

2. Except as a road may carry emigrants, the passenger traffic in both directions is equal. Of course there are innumerable individual instances in which the return trip is made by another route, but it is seldom if ever that there is any marked tendency to uniformity in this. Considering that a car load of, say, 50 passengers at 150 pounds apiece weigh but 7500 pounds, which is $\frac{1}{10}$ of the 75000 pounds which the car may weigh, even a considerable variation in the number of passengers will not appreciably affect the hauling of cars on grades. On parlor-cars and sleepers the ratio of live load to dead load (say 20 passengers, 3000 pounds, and the car, 125000 pounds) is even more insignificant. The effect of passenger traffic on balance of grades may therefore be disregarded.

3. Empty cars have a greater resistance *per ton* than loaded cars. Therefore in computing the hauling capacity of a locomotive hauling so many tons of "empties," a larger figure must be used for the ordinary tractive resistances—say four pounds per ton greater.

4. Owing to greater or less imperfections of management a small percentage of cars will run empty or but partly full in the direction of greatest traffic.

5. Freight having great bulk and weight (such as grain, lumber, coal, etc.) is run from the rural districts toward the cities and manufacturing districts.

6. The return traffic—manufactured products—although worth as much or more, do not weigh as much.

As a simple numerical illustration assume that the weight of the cars is $\frac{1}{3}$ and the live load $\frac{2}{3}$ of the total load when the cars are "full"—although not loaded to their absolute limit of capacity. Assume that the relative weight of live load

to be hauled in the other direction is but ⅓; assume that the grade against the heaviest traffic is 0.9%. Since the tractive resistance per ton is considerably greater in the case of unloaded cars than it is in the case of loaded cars, allowance must be made for this in calculating the train resistance. Assuming the use of the Mikado locomotive described in § 453, its rating on a 0.9% grade, see § 467, equals

$$A = \frac{35758}{.009+.0011} - 315{,}000 = 3{,}230{,}000 = 1615 \text{ tons, the "rating."}$$

Call W_E the total weight, live and dead, of the cars in an *eastbound* train, and W_W the corresponding weight for a west-bound train. F_E and F_W are the weights of live freight; w the dead weight of a car, which for simplicity is considered in this case to be uniformly a 100,000-lb. capacity car, weighing 20 tons or 40,000 lbs. The problem assumes that $F_W = \frac{1}{3}F_E$. Then $W_W = \frac{1}{3}F_E + nw$.

$$W_E = 1615 - 6.0\,n \quad \text{(for a 0.9\% grade—see Table XL, § 467).}$$

By trial, it is found that for $n=24$, $W_E = 1615 - 144 = 1471$, which means a total weight of 61.3 tons per car, or a net load of 41.3 tons or 82,600 lbs. live load per car. This fulfils the condition that the live load is ⅔ of the total load as nearly as possible for an even number of car loads. ⅓ of 41.3 tons, or 13.8 tons, plus 20 tons, gives an average load of 33.8 tons per car for west-bound trains, and for a train of 24 cars = 811.2 tons per train, or 1,622,400 lbs. Substituting in Eq. 122, § 467,

$$\frac{35758}{r+.0011} - 315{,}000 = 1{,}622{,}400 + 24\frac{122}{r+.0011}.$$

Solving, $r = .0169$, or a 1.69% grade, which, *under the above assumptions and conditions*, is the grade on which the given type of locomotive could handle one-third of the live load which could be hauled up a 0.9% grade, in the same number of cars, by that same locomotive. It is interesting to note that the solution of this problem, given in a previous edition, using a more approximate method, and based on the use of a much lighter consolidation locomotive, weighing only 107 tons, gave 1.60% as the grade corresponding to 0.9% against east bound traffic. This substantial agreement, in spite of the difference in operating conditions, shows the substantial accuracy of the method for

the solution of a problem for which the varying conditions of traffic in the two directions render useless any very precise solution.

Of course the actual traffic in the two directions, and their ratio, will vary from time to time, and the actual operation of trains will vary accordingly, and therefore the relation of ruling grades in the two directions, for maximum efficiency of operation, will fluctuate accordingly, while the ruling grades, once established, are practically finalities. Therefore any close precision in the computation of these relative grades is useless. Nevertheless the above calculation shows unmistakably that under the given conditions, a very considerable variation in the rate of grade in opposite directions is not only justifiable, but a neglect to allow for it would be a great economic error.

531. Computation of relative traffic. Some of the principal elements have already been referred to, but in addition the following facts should be considered.

(*a*) The greatest disparity in traffic occurs through the handling of large amounts of coal, lumber, iron ore, grain, etc. On roads which handle but little of these articles or on which for local reasons coal is hauled one way and large shipments of grain the other way the disparity will be less and will perhaps be insignificant.

(*b*) A marked change in the development of the country may, and often does, cause a marked difference in the disparity of traffic. The heaviest traffic (in mere weight) is always toward manufacturing regions and away from agricultural regions. But when a region, from being purely agricultural or mineral, becomes largely manufacturing, or when a manufacturing region develops an industry which will cause a growth of heavy freight traffic from it, a marked change in the relative freight movement will be the result.

(*c*) Very great fluctuations in the relative traffic may be expected for prolonged intervals.

(*d*) An estimate of the relative traffic may be formed by the same general method used in computing the total traffic of the road (see § 473, Chapter XIX) or by noting the relative traffic on existing roads which may be assumed to have practically the same traffic as the proposed road will obtain.

CHAPTER XXIV.

THE IMPROVEMENT OF OLD LINES.

532. **Classification of improvements.** The improvements here considered are only those of alignment—horizontal and vertical. Strictly there is no definite limit, either in kind or magnitude, to the improvements which may be made. But since a railroad cannot ordinarily obtain money, even for improvements, to an amount greater than some small proportion of the previously invested capital, it becomes doubly necessary to expend such money to the greatest possible advantage. It has been previously shown that securing additional business and increasing the train load are the two most important factors in increasing dividends. After these, and of far less importance, come reductions of curvature, reductions of distance (frequently of doubtful policy, see Chap. XXI, § 503), and elimination of sags and humps. These various improvements will be briefly discussed.

(a) **Securing additional business.** It is not often possible by any small modification of alignment to materially increase the business of a road. The cases which do occur are usually those in which a gross error of judgment was committed during the original construction. For instance, in the early history of railroad construction many roads were largely aided by the towns through which the road passed, part of the money necessary for construction being raised by the sale of bonds, which were assumed or guaranteed and subsequently paid by the towns. Such aid was often demanded and exacted by the promoters. Instances are not unknown where a failure to come to an agreement has caused the promoters to deliberately pass by the town at a distance of some miles, to the mutual disadvantage of the road and the town. If the town subsequently grew in spite of this disadvantage, the *annual* [illegible] of business might readily amount to more than the original sum in dispute.

Such an instance would be a legitimate opportunity for study of the advisability of re-location.

As another instance (the original location being justifiable) a railroad might have been located along the bank of a considerable river too wide to be crossed except at considerable expense. When originally constructed the enterprise would not justify the two extra bridges needed to reach the town. A growth in prosperity and in the business obtainable might subsequently make such extra expense a profitable investment.

(b) **Increasing the train load.** On account of its importance this will be separately considered in § 535 *et seq.*

(c) **Reduction in curvature and distance and the elimination of sags and humps.** Such improvements are constantly being made by all progressive roads. The need for such changes occurs in some cases because the original location was very faulty, the revised location being no more expensive than the original, and in other cases because the original location was the best that was then financially possible and because the present expanded business will justify a change.

(d) **Changing the location of stations or of passing sidings.** The station may sometimes be re-located so as to bring it nearer to the business center and thus increase the business done. But the principal reasons for re-locating stations or passing sidings is that starting trains may have an easier grade on which to overcome the additional resistances of starting. Such changes will be discussed in detail in § 537.

533. **Advantages of re-locations.** There are certain undoubted advantages possessed by the engineer who is endeavoring to improve an old line.

(*a*) The gross traffic to be handled is definitely known.

(*b*) The actual cost per train-mile for that road (which *may* differ very greatly from the average) is also known, and therefore the value of the proposed improvement can be more accurately determined.

(*c*) The actual performance of such locomotives as are used on the road may be studied at leisure and more reliable data may be obtained for the computations.

534. **Disadvantages of re-locations.** The disadvantages are generally more apparent and frequently appear practically insuperable—more so than they prove to be on closer inspection

(*a*) It frequently means the abandonment of a greater or less length of old line and the construction of new line. At first thought it might seem as if a change of line such as would permit an increase of train-load of 50 or perhaps 100% could never be obtained, or at least that it could not be done except at an impracticable expense. On the contrary a change of 10% of the old line is frequently all that is necessary to reduce the grades so that the train-loads hauled by one engine may be nearly if not quite doubled. And when it is considered that the cost of a road to sub-grade is generally not more than one-third of the total cost of construction and equipment per mile, it becomes plain that an expenditure of but a small percentage of the original outlay, expended where it will do the most good, will often suffice to increase enormously the earning capacity.

(*b*) One of the most difficult matters is to convince the financial backers of the road that the proposed improvement will be justifiable. The cause is simple. The disadvantages of the original construction lie in the large increase of certain items of expense which are necessary to handle a given traffic. And yet the fact that the expenditures are larger than they need be are only apparent to the expert, and the fact that a saving may be made is considered to be largely a matter of opinion until it is demonstrated by actual trial. On the other hand the cost of the proposed changes is definite, and the very fact that the road has been uneconomically worked and is in a poor financial condition makes it difficult to obtain money for improvements.

(*c*) The legal right to abandon a section of operated line and thus reduce the value of some adjoining property has sometimes been successfully attacked. A common instance would be that of a factory which was located adjoining the right of way for convenience of transportation facilities. The abandonment of that section of the right of way would probably be fatal to the successful operation of the factory. The objection may be largely eliminated by the maintenance of the old right of way as a long siding (although the business of the factory might not be worth it), but it is not always so easy of solution, and this phase of the question must always be considered.

REDUCTION OF VIRTUAL GRADE.

535. Obtaining data for computations. As developed in the last chapter (§§ 515–517) the real object to be attained is the reduction of the *virtual grade*. The method of comparing grades under various assumed conditions was there discussed. When the road is still "on paper" some such method is all that is possible; but when the road is in actual operation the virtual grade of the road at various critical points, with the rolling stock actually in use, may be determined by a simple test and the effect of a proposed change may be reliably computed. Bearing in mind the general principle that the virtual grade line is the locus of points determined by adding to the actual grade profile ordinates equal to the velocity head of the train, it only becomes necessary to measure the velocity at various points. Since the velocity is *not* usually uniform, its precise determination at any instant is almost impossible, but it will generally be found to be sufficiently precise to assume the velocity to be uniform for a short distance, and then observe the time required to pass that short space. Suppose that an ordinary watch is used and the time taken to the nearest second. At 30 miles per hour, the velocity is 44 feet per second. To obtain the time to within 1%, the time would need to be 100 seconds and the space 4400 feet. But with variable velocity there would be too great error in assuming the velocity as uniform for 4400 feet or for the time of 100 seconds. Using a stop-watch registering fifths of a second, a 1% accuracy would require but 20 seconds and a space of 880 feet, at 30 miles per hour. Wellington suggests that the space be made 293 feet 4 inches, or $\frac{1}{18}$ of a mile; then the speed in miles per hour equals $200 \div s$, in which s is the time in seconds required to traverse the 293′ 4″. For instance, suppose the time required to pass the interval is 12.5 seconds. $\frac{1}{18}$ mile in 12.5 seconds = one mile in 225 seconds, or 16 miles per hour. But likewise $200 \div 12.5 = 16$, the required velocity. The following features should be noted when obtaining data for the computations:

(*a*) All critical grades on the road should be located and their profiles obtained—by a survey if necessary.

(*b*) At the bottom and top of all long grades (and perhaps at intermediate points if the grades are very long) spaces of known

length (preferably 293⅓ feet) should be measured off and marked by flags, painted boards, or any other serviceable targets.

(*c*) Provided with a stop-watch marking fifths of seconds, the observer should ride on the trains affected by these grades and note the exact interval of time required to pass these spaces. If the space is 293⅓ feet, the velocity in miles per hour = 200 ÷ interval in seconds. In general, the velocity in miles per hour,

$$V = \frac{\text{distance in feet} \times 3600}{\text{time in seconds} \times 5280}.$$

(*d*) Since these critical grades are those which require the greatest tax on the power of the locomotive, the conditions under which the locomotive is working must be known—i.e., the steam pressure, point of cut-off, and position of the throttle. Economy of coal consumption as well as efficient working at high speeds requires that steam be used expansively (using an early cut-off), and even that the throttle be partly closed; but when an engine is slowly climbing up a maximum grade with a full load it is not exerting its maximum tractive power unless it has its maximum steam pressure, wide-open throttle, and is cutting off nearly at full stroke. These data must therefore be obtained so as to know whether the engine is developing at a critical place all the tractive force of which it is capable. The condition of the track (wet and slippery or dry) and the approximate direction and force of the wind should be noted with sufficient accuracy to judge whether the test has been made under ordinary conditions rather than under conditions which are exceptionally favorable or unfavorable.

(*e*) The train-loading should be obtained as closely as possible. Of course the dead weight of the cars is easily found, and the records of the freight department will usually give the live load with all sufficient accuracy.

536. Use of the data obtained. A very brief inspection of the results, freed from refined calculations or uncertainties, will demonstrate the following truths:

(*a*) If, on a uniform grade, the velocity increases, it shows that, under those conditions of engine working, the load is less than the engine can handle on that grade.

(*b*) If the velocity decreases, it shows that the load is greater than the engine can handle on an indefinite length of such

grade. It shows that such a grade is being operated by momentum. From the rate of decrease of velocity the maximum practicable length of such a grade (starting with a given velocity) may be easily computed.

(*c*) By combining results under different conditions of grade but with practically the same engine working, the tractive power of the engine may be determined (according to the principles previously demonstrated) for any grade and velocity. For example: On an examination of the profile of a division of a road the maximum grade was found to be 1.62% (85.54 feet per mile). At the bottom and near the top of this grade two lengths of 293′ 4″ are laid off. The distance between the centers of these lengths is 6000 feet. A freight train moving up the grade is timed at $9\frac{3}{5}$ seconds on the lower stretch and $7\frac{3}{5}$ seconds on the upper. These times correspond to $\frac{200}{9.4}$ and $\frac{200}{7.6}$ or 21.3 and 26.3 miles per hour respectively. It is at once observed that the velocity has increased and that the engine could draw even a heavier load up such a grade for an indefinite distance. How much heavier might the load be?

For simplicity we will assume that the conditions were normal, neither exceptionally favorable nor unfavorable, and that the engine was worked to its maximum capacity. The engine is a "consolidation" weighing 128700 pounds, with 112600 pounds on the drivers. The train-load behind the engine consists of ten loaded cars weighing 465 tons and eleven empties weighing 183 tons, thus making a total train-weight of 712 tons. Applying Eq. 106, we find that the *additional* force which the engine has actually exerted per ton in increasing the velocity from 21.3 to 26.3 miles per hour in a distance of 6000 feet is

$$P = \frac{70}{6000}(26.3^2 - 21.3^2) = 2.78 \text{ pounds } \textit{per ton}.$$

The grade resistance on a 1.62% grade is 32.4 pounds per ton. The average train resistance may be computed from §§ 429 and 439.

Engine resistance, at say 8 m.p.h. (§ 429)	= 1615 lbs.
Cars resistance, $(648 \times 2.2) + (21 \times 122)$	= 3988 lbs.
Total tractive resistance on level	= 5603 lbs.

The average tractive resistance is therefore 5603 ÷ 712 = 7.87 pounds per ton. Adding the grade resistance (32.4) we have a total train resistance of 40.27 pounds per ton. But, computing from the increase in velocity, the locomotive is evidently exerting a pull of 2.78 pounds per ton in excess of the computed required pull on that grade, or a total pull of 43.05 pounds per ton. Therefore the train load might have been increased proportionately and might have been made

$$712 \times \frac{2.78+40.27}{40.27} = 761 \text{ tons.}$$

This shows that 49 tons additional might have been loaded on to the train, or say, three more empties or one additional loaded car.

A pull of 43.05 pounds per ton means a total adhesion at the drivers of 30,652 pounds, which is about 27% of the weight on the drivers—112,600 pounds. This indicates average conditions as to traction, and as good as can be depended on for regular service.

The above calculation should of course be considered simply as a "single observation." The performance of the same engine on the same grade (as well as on many other grades) on succeeding days should also be noted. It may readily happen that variations in the condition of the track or of the handling of the engine may make considerable variation in the results of the several calculations, but when the work is properly done it is always possible to draw definite and very positive deductions.

537. **Reducing the starting grade at stations.** The resistance to starting a train is augmented from two causes: (*a*) the tractive resistances are usually about 20 pounds per ton instead of, say, 6 pounds, and (*b*) the inertia resistance must be overcome. The inertia resistance of a freight train (see § 435) which is expected to attain a velocity of 15 miles per hour in a distance of 1000 feet is (see Eq. 140)

$$P = \frac{70}{1000}(15^2 - 0) = 15.8 \text{ pounds per ton,}$$

which is the equivalent of a 0.79% grade. Adding this to a grade which nearly or quite equals the ruling grade, it virtually creates a new and higher ruling grade. Of course that additional force can be greatly reduced at the expense of slower acceleration, but even

this cannot be done indefinitely, and an acceleration to only 15 miles per hour in 1000 feet is as slow as should be allowed for. With perhaps 14 pounds per ton additional tractive resistance, we have about 30 pounds per ton additional—equiva-

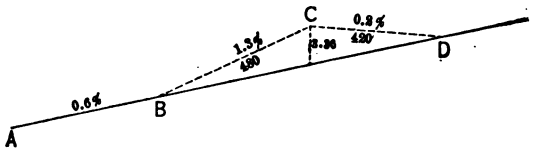


FIG. 216.

lent to a 1.5% grade. Instances are known where it has proven wise to create a *hump* (in what was otherwise a uniform grade) at a station. The effect of this on high-speed passenger trains moving *up* the grade would be merely to reduce their speed very slightly. No harm is done to trains moving *down* the grade. Freight trains moving *up* the grade and intending to stop at the station will merely have their velocity reduced as they approach the station and will actually save part of the wear and tear otherwise resulting from applying brakes. When the trains start they are assisted by the short down grade, just where they need assistance most. Even if the grade *CD* is still an up grade, the pull required at starting is *less* than that required on the uniform grade by an amount equal to 20 times the difference of the grade in per cent.

CHAPTER XXV.

STRESSES IN TRACK.

538. Nature of the subject. The character and amount of the stresses in the rails, rail fastenings and ties, which make up the track, and the intensity and distribution of the pressure which is transmitted by the ties through the ballast and embankment to the subsoil, have long been a subject of investigation by railroad engineers. The complexity of the subject is too great for a dependence on mere theoretical analysis. Even experimental work must be so elaborate that no one person or single individual railroad have hitherto obtained conclusive results, except upon isolated details.

In 1913, a committee was appointed by the Amer. Rwy. Eng. Assoc. who cooperated with a similar committee appointed by the Amer. Soc. of Civil Engineers. Both societies appropriated money for the large expenses involved. Several railroads cooperated by furnishing facilities for experimental work. Several steel-rail corporations contributed funds. Special instruments were designed for experimental use. After five years of work, a progress report, covering 184 pages, was made to the 1919 convention of the Amer. Rwy. Eng. Assoc. The second progress report (170 pp.) was made to the 1920 convention. The investigation is not yet (1921) complete. But from these two voluminous reports, which indicate the magnitude of the problem, the following very condensed summary has been compiled. The thoroughness of the investigation is indicated by the fact that the number of observations for rail strain only, made, read, recorded and reduced, and on which the first progress report was partly based, is more than 250,000. The conclusions, which can be drawn from the tests made, have already had their effect in modifying track construction, and will probably have still greater effect when the principles underlying the stresses in track, due to rapidly moving and very heavy rolling-stock, are more thoroughly comprehended and when these principles have crystallized into definite rules of practice.

539. Action of track as an elastic structure. Wheel loads bear vertically, but usually with some horizontal component, on a rail. The rails are flexible beams, supported by flexible ties, which are supported by a more or less yielding but elastic ballast, which rests on a more or less yielding subsoil. For convenience, the term **modulus of elasticity of rail support** is used as a measure of the vertical stiffness of the rail support, and is defined as "the pressure per unit of length of each rail required to depress the track one unit." For example, a series of wheel loads, equivalent to 10,000 pounds per tie for each rail, depress the track an average of 0.3 inch. Then, on the basis of proportionality of depression to pressure, 33,333 lbs. would produce one inch of depression, which for a tie spacing of 22 inches would require a pressure of $33{,}333 \div 22 = 1515$ lbs. per inch of length of rail per inch of depression. The elasticity and flexibility of these various materials affects the stresses to which they are subject. The spacing of wheels along a rail also affects very greatly the intensity and character of the stresses produced in the rails and ties. Although a purely theoretical solution is unsatisfactory and inadequate, a theoretical study makes it possible to limit the scope of the necessary experimental work. Theoretical analysis shows that the bending moment of a rail will be comparatively large for a single concentrated load with no appreciable loads sufficiently near to hold the rail down and produce a negative bending moment at an adjacent point, thus reducing the positive moment immediately under the concentrated load. But railroad loadings are always in groups. A heavy driver-load is almost invariably preceded and followed by a comparatively light truck-wheel load, if not by another driver. The variation in operating conditions as to spacing and intensity of wheel loads limits the use of precise calculations for purposes of generalization, but analysis (which is substantially confirmed by experimental tests) shows that "the assumption of a continuous elastic support under the rail is by far the most convenient, most easily applied and most comprehensive in its application to the questions involved in the work of the committee."

540. **Typical track depression profile for static load on one or two axles.** See Fig. 217. Note that the depression for one axle extends for about ten tie spaces and that the rail is somewhat raised *above* the normal height beyond a distance of about

5 tie spaces from the load even if there is a comparatively light wheel load on the rail at that point. The deflection is of course maximum directly under the load and it makes almost

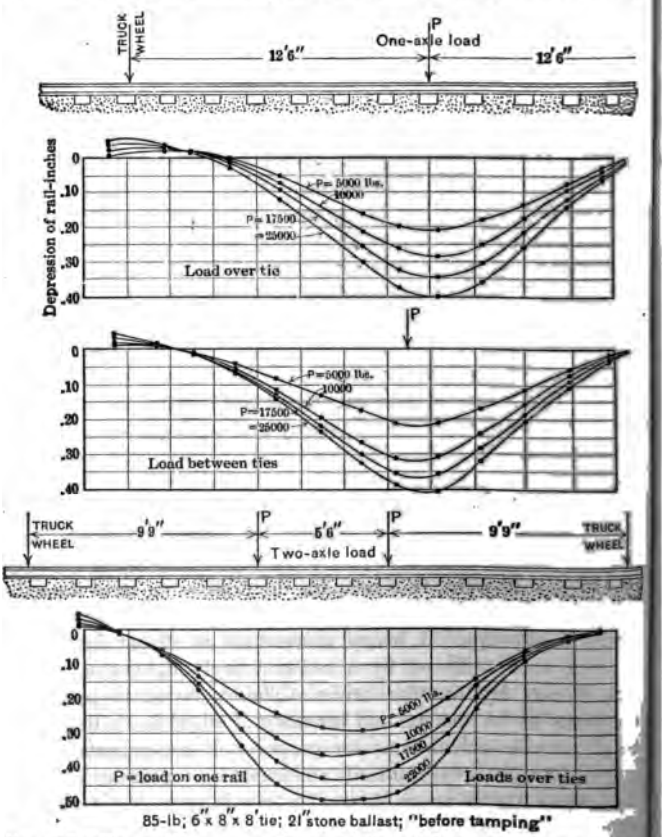


FIG. 217.—TRACK DEPRESSION PROFILES, STATIC LOAD ON ONE AND TWO AXLES.

no difference whether the load is directly above a tie or between two ties. The curves are substantially identical, merely moved along as the load moves. The amount of the depression for a given load varies with the character of the ballast and the tamp-

ing, whether recent or old, as was shown by the numerous other similar profiles given in the report. The effect of recent tamping was investigated and it was shown that the depression under a load on recently tamped track is nearly proportional to the loading, which implies a nearly constant "modulus of elasticity of rail support." On the other hand, if track has not been tamped for several months, there is a comparatively deep depression for the first 5000 lbs., proportionately less for the next additional

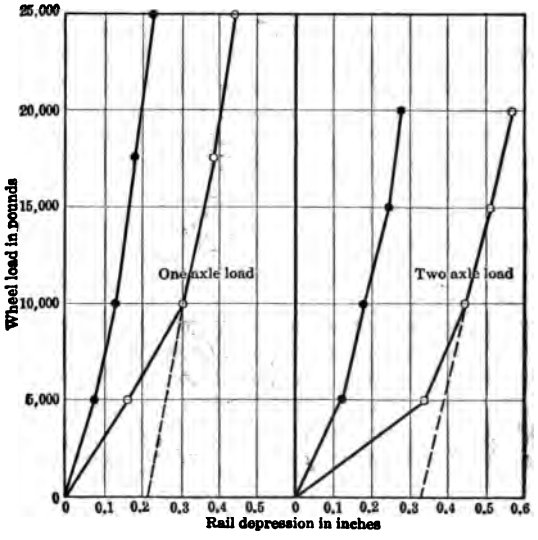


FIG. 218.—TIE DEPRESSION DIAGRAM, STATIC LOADS.

5000 lbs., and perhaps still less for additional increments. This is also shown by Fig. 218, which shows the "after tamping" curve to be nearly a straight line; the "before tamping" curve is much more curved. It should be noted that the "before tamping" depression line is nearly a straight line *after* it is loaded to about 10,000 lbs. In later investigations this fact was utilized by producing this nearly straight line back to the line of zero pressure, as shown by the dotted line. The intercept on the line of zero wheel-load is a measure of the depression of the tie before it has its full bearing on the ballast. As a part of the

investigation on the stresses and the elastic curve of a tie under load, the depression of a tie was very accurately measured at several points along the tie and for a regular series of light to heavy loads. For all cases where the tamping had not been recent (or "before tamping") a curve, similar to those of Fig. 218, was drawn for each point along the tie. Producing the depression line backward to the point of zero loading gives an intercept which is called "the initial position of the ballast bed with respect to the bottom of the tie for the compact condition of ballast existing in the track." Of course this does not mean that there is such an actual gap between the under side of the tie and the ballast, but such gap as may exist at some points along the tie will make up a large part of this initial depression. A comparison of similar curves for light and heavy rails proves what might have been predicted, that the depression under a heavy rail for a given load is less than that under a light load. The heavy rail, by its extra stiffness, distributes the loading over a greater number of ties and the one or two ties nearly under the load do not need to carry such a large proportion of the total.

A broad general idea of the depressions due to track loading and of the proportions of the total depression due to rail, tie, ballast and sub-soil, may be obtained from the following figures, which, however, must be considered as very approximate and subject to great variation.

Division of depressions of track under drivers of Mikado locomotive:

1. Compression of tie under rail, plus effect of bending of tie to bring it to full bearing on the ballast along its length	0.05 in.
2. Compression of 24" of stone ballast immediately under the rail	0.15 in.
3. Compression of roadway immediately under the rail,	0.15 in.
	0.35 in.

Bending of 85-lb. rail between ties spaced 22" c.c. by a Mikado locomotive not more than 0.01."

541. Bending moment and depression in a r to a group of loads. Fig. 219 shows graphically the relat e bending moments under each wheel of a Mikado loc tive. The

light lines show the curves of moments due to each wheel; the heavier lines give the algebraic summation of the effects of all the wheels. Note (*a*) that the effect of each wheel is maximum directly under that wheel but the effect continues even beyond adjacent wheels; (*b*) a wheel usually develops a *negative* moment under an adjacent wheel, which reduces the positive moment developed by the adjacent wheel; (*c*) as an example, calling the moment developed by the second driver (counting

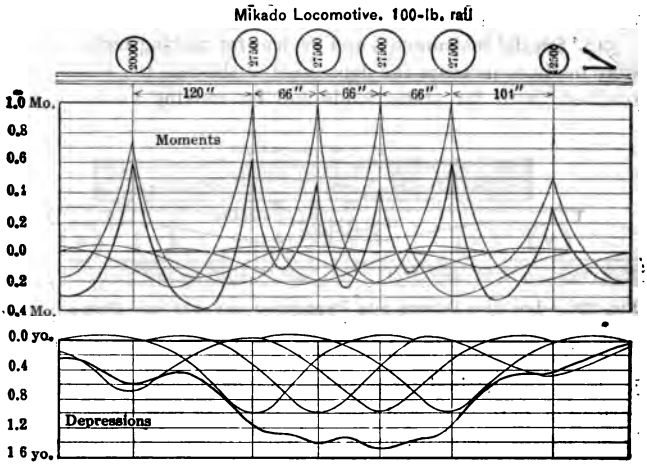


FIG. 219.—BENDING MOMENTS AND DEPRESSIONS. COMBINATION OF LOADS.

from right to left) under the second driver = +1.00, the effect of the first driver is −0.20; the effect of the third driver is −0.20; that of the fourth driver is −0.05; the effect of the pilot is zero and likewise the effect of the trailer. The net effect is that the combination of wheels develops a moment under the second driver of only 55% of that due to the second driver if it acted alone. Similarly the *depression* of rail produced by a wheel is maximum under that wheel, but it develops an upward force which may reduce the depression under some other wheel, although probably not the adjacent wheel. For example, calling the depression produced by the first driver = +1.00, the

effect of the second driver is to cause a further depression under the first driver equal to +0.23; the third driver has an added effect of − 0.04; the effect of the pilot truck is negligible. The net effect is a depression under the first driver which is 1.19 times the depression which the first driver alone would cause. Note that, although there is depression under all the wheels, the depression between the fourth driver and the trailer is less than that under the trailer. Between the two trucks of a car, the depression is usually negative, i.e., the rails are curved upward *above* their normal position.

542. Special instruments and devices for making tests. Tests were made to measure the depression of the rail, tie, ballast and roadbed, both for static loads and for moving loads. Static

FIG. 220.—LOADING DEVICE FOR PRODUCING ANY DESIRED ONE-AXLE OR TWO-AXLE LOADS.

loads of any desired magnitude were produced by spotting a carload of 25 to 50 tons of rails over the track to be tested. Two H-beams (for two-axle loads) or one H-beam (for single-axle loads) were placed under the load of rails, each H-beam being supported by two struts having load-indicating screw jacks, which were carried on curved bearing blocks, placed on the track rails, the blocks having the same radius as car wheels but without coning. Since the bearing blocks were under the center of the car and were 12 to 15 feet in either direction from the car wheels, the effect of the car wheels was nearly negligible and was so considered.

Unit rail stress. The stretching of the base of the rail under a static load was measured with a Berry strain gage just as any such stress in metal is measured in a testing laboratory. The Berry strain gage is not applicable for observing the rapidly changing stresses due to moving loads, which therefore require the use of a stremmatograph. The form used will record at any instant, on a revolving disk any minute variation in the

distance apart of a pair of gage points drilled in the base of the rail exactly 4″ apart.

Unit pressures. The unit pressure exerted at any depth of the ballast was measured by a **pressure capsule.** As shown in Fig. 221, the ballast bears on a circular bearing plate, having an area of 5 sq. in., which transmits the pressure to a thin

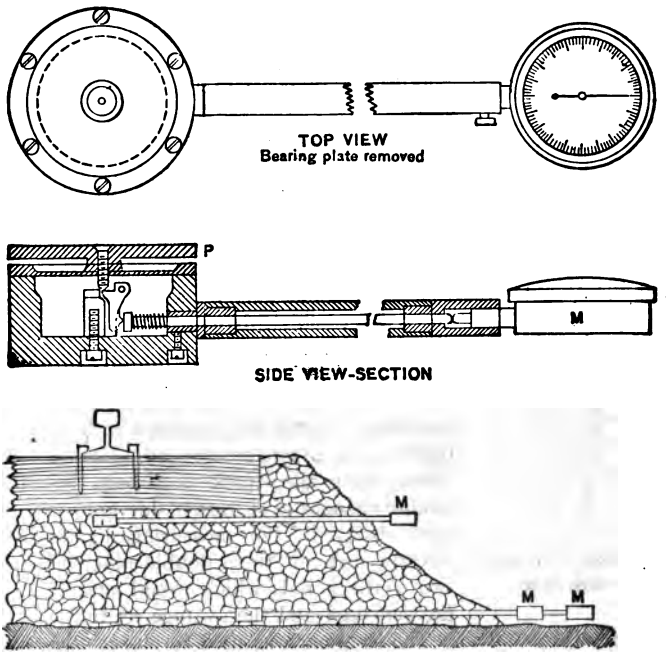


FIG. 221.—PRESSURE CAPSULES.

steel diaphragm. The movement of the diaphragm actuates a simple mechanism which pushes a rod enclosed in a pipe leading to a dial located outside the ballast. The mechanism is calibrated by observing the readings for known pressures. Several of these capsules are inserted in the ballast almost immediately under the tie, and also at the bottom of the ballast just above sub-grade, as shown in Fig. 221. The simple dial form is used to measure the pressure produced by static loads;

For moving loads, the mechanism operates a stylus which makes a record on a revolving disk.

Depression plugs. The actual depression of the ballast at any depth, or of the sub-soil at subgrade, was measured by locating a horizontal plate at the desired point. A vertical $\frac{1}{2}$″ tube, enclosing a $\frac{5}{16}$″ rod, with a set screw for adjustment, (see Fig. 222) is attached to this plate. To avoid any binding action of the ballast through which it passes, the vertical rod and tube is surrounded by another $\frac{3}{4}$″ tube. The top of the rod is adjusted to be above the ballast and at a convenient height for comparison of elevation with a fixed reference plug. Of course the plate will follow the strata in which it is placed in any change of elevation which may occur. The fixed plugs were located in the ground far enough away from the track so that they would not be appreciably influenced by track depression, and at nearly the same elevation as the tops of the vertical rods. The relative elevations were observed very accurately by means of a level-bar, a metal bar provided with a level bubble and a micrometer adjusting screw. Then, after the track had been loaded, minute changes of elevation, due to pressure, were observed. For measuring depressions directly under a tie, a double plug, having two vertical rods which would straddle a tie, were used, and the average reading of the two rods was taken. The level bar was used to observe depressions of rail, tie, ballast or subsoil, but only as to the effect of static loading. The depression of the rail under moving load was measured by a double exposure photograph. Pieces of black paper, with white crosses on them, having one line vertical, were pasted on the web of the rail. A camera was focused on the rail about 10 feet away. An initial exposure was taken of the unloaded rail. Then, without disturbance of the camera, the desired train load was run over the track at the desired speed. When the train (or locomotive) was at the desired point, it closed an electric circuit which operated

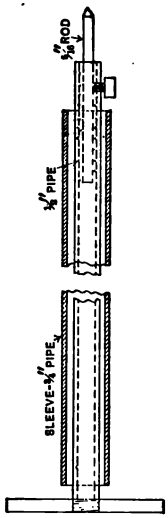


FIG. 222.—DEPRESSION PLUG.

the shutter for a .001 second exposure. The resultant photograph showed for each cross a double cross with one vertical and two horizontal lines whose distance apart represented, after suitable reduction, the depression of the rail. Using a magnifying micrometer microscope, and a computed constant multiplier, it was possible to measure from the photographic plate the actual deflection of the rail with a precision of about 0.01 inch.

543. Pressure transmitted from tie to ballast. This subject was investigated both theoretically and experimentally. The

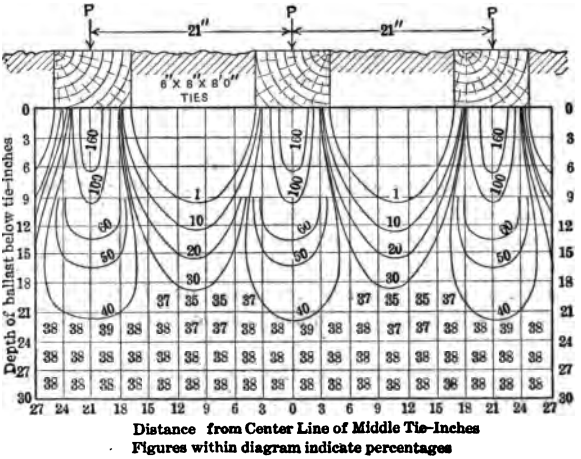


FIG. 223.—LINES OF EQUAL VERTICAL PRESSURE IN BALLAST FOR EQUAL LOADS ON TIES, SPACED 21" C.C.

experimental work included not only track tests but also an extensive series of laboratory tests using sand ballast, pebbles, and broken stone. If ballast consists of absolutely clean spheres of perfectly elastic material, whose mutual actions and reactions are only pressure without friction, a definite theoretical solution as to distribution of pressure is possible, although complicated. The equation of pressure is a logarithmic equation which is chiefly useful in interpreting experimental results. Both theory and experimental tests demonstrate that:

(*a*) "The bearing pressure of the tie varies in intensity from its edge to its middle line." This is shown in Fig. 223, where

the numbers within the diagram give *percentages* of the *average* tie pressure. The figure also shows that if the ballast is only 6 inches thick the entire pressure on the subsoil is concentrated on a comparatively narrow area under the tie and that a considerable part of the subsoil between the ties carries but little pressure. The ballast must be nearly 24 inches deep (if the ties are spaced 21″) before the load is distributed with substantial uniformity. If the ties are spaced further apart, the depth for uniform pressure must be still greater. In a very approximate way, it may be said that the pressure becomes substantially uniform at a depth equal to the tie spacing.

(*b*) "The pressures which react from the lower face of the tie act in other than vertical lines, the greatest variation from the vertical direction being at the edge of the tie." Fig. 223 also shows this.

(*c*) "The variation in intensity of pressure in the ballast lengthwise of the tie (which is dependent upon size and stiffness of tie, quality of tamping, and condition of the bed on which the tie rests) becomes less and less with increase in depth and it may be expected that the variations will be smoothed out at a depth equal to the ordinary tie spacing, or a few inches below, where there will be fairly uniform pressure over the horizontal plane."

(*d*) "For quiescent loading there is little difference in the manner and rate of transmission and distribution of pressure for broken stone, pebbles, and sand ballasts; that is, at a given depth the intensities of pressure will be approximately the same, provided, of course, the ultimate carrying capacity of the ballast is not exceeded; and this conclusion may properly be extended to other non-cohesive materials. It will require less load to force the tie into sand ballast than into broken stone; the ultimate carrying capacity of the broken stone ballast under tie pressure is much greater than that of the sand ballast—the particles of sand ballast are more easily moved and rearrange themselves under lighter loads. For the different kinds of ballast there are great differences in the ultimate load which can be carried on a tie before ballast movement begins. The ultimate carrying capacity depends upon size of particle, smoothness of surface and degree of angularity. A material whose mobility under pressure is increased by the action of water or by mixture with other materials may thereby have its carrying

capacity decreased. For heavy loading the ultimate carrying capacity of a ballast material is especially important."

(*e*) For quiescent loads the presence of ballast above the level of the bottom of the tie has little or no effect in increasing the maximum load which can be carried without forcing the ballast from under the tie and allowing the tie to settle. For moving loads which produce vibration, the presence of ballast up to the top of the tie, and particularly at the tie ends, increases considerably the resistance to lateral displacement. The greater the velocity of trains, the greater the necessity for such lateral reinforcement.

544. Transverse stresses in the tie. The character and distribution of transverse stresses in the tie depend very largely on the tamping. If the tamping were absolutely uniform throughout the length of the tie, the upward pressure would be uniform and there would be a maximum positive moment under each rail, a maximum negative moment in the center of the tie, and points of inflection between the center and each rail. If the tie is very strongly tamped under the center and tamped very little if at all under the ends, making it "center-bound," there will be a severe negative moment in the center, and little or none under the rails. Concentrating the tamping for a short space on each side of each rail, and leaving the center almost clear of ballast, relieves the center of any transverse stress and even minimizes that under the rails. From the standpoint of stress in the tie, it is desirable, but it makes an undesirable concentration of pressure on the ballast and roadbed. Probably the ballast would soon crush down under such a concentrated pressure. The best method of tamping is that which makes the tamping firmest on either side of each rail, with enough tamping in the center to give good support and yet not so much that a negative moment would be developed which would be in excess of the positive moment under the rail. Since the amounts of these moments depend on the tamping and since the effect of the tamping may be more or less altered with the passage of each train, due to a slight settlement of the ballast, any attempt at precise quantitative computation of moment is fruitless. Nevertheless tests were made to determine the moments under a variety of conditions (center-bound ties, end-bound ties, etc.) so as to determine maximum and minimum values for the moments under the rails and in the center. Fig.

224 is a composite of the deflections of three ties on Class A track on the Ch. M. & St. P. Rwy. The vertical scale is 500 times the horizontal scale. The curve represents the depression of the tie and may also be considered to represent the deformed neutral axis and that the curvature indicates the character of the bending. The curve shows the usual case of a negative moment in the center and positive moments under each rail. Static tests under a truck load of 100,000 lbs., on poorly ballasted track, showed a negative bending moment in the center of as much as $-4.5\ W$ inch-pounds, in which W = the load in pounds carried by one tie. This was observed to be about 15,000 lbs. $4.5 \times 15{,}000 = 67{,}500$ in. lbs. For a 6″×8″ tie, a moment of 67,500 in. lbs. means a maximum unit stress

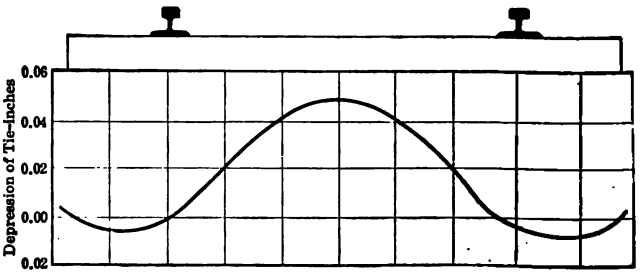


FIG. 224.—COMPOSITE DIAGRAM OF TIE FLEXURE.

of 1406 lbs. per sq. in. But this stress was produced by a static load. The effect of speed and dynamic augment (see § 413) would largely increase this figure and perhaps make it exceed the safe working stress for even an oak tie. On the other hand, for track in good condition, a negative bending moment of $-2.0\ W$ in the center is as much as should be expected.

545. Effect of counterbalancing. In § 413 there is given an elementary explanation of the necessity for counterbalancing and some of the rules for accomplishing it. It was also explained that perfect counterbalancing is necessarily impossible and that there is always an unbalanced dynamic augment which produces an increased pressure on the rails at some part of the revolution of the driver, or a racking of the locomotive frame at each half-stroke of each piston. The dynamic augment increases as the square of the velocity, and its effect is therefore

very great and serious at high speeds. It is sometimes found impracticable to make the counterweight on the main driver sufficiently large and heavy to balance the effect of the very great weight of the side rods, main rod, etc., of a very heavy locomotive. In such a case, the driver is said to be **underbalanced** and then the greatest stress in the rail may occur when the counterweight is up rather than when it is down. The underbalance of the main driver is made up by overbalancing

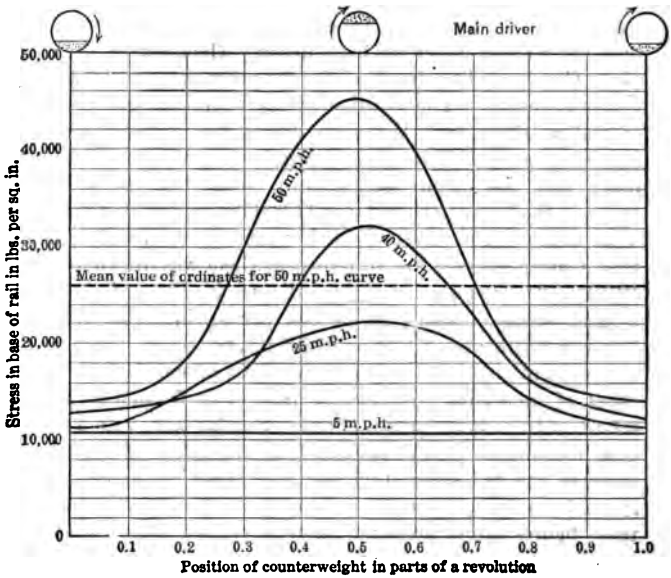


FIG. 225.—COUNTERBALANCING STRESS IN RAIL, UNDER MAIN DRIVER, DURING ONE REVOLUTION.

the other drivers, and this increases the pressure under them when the counterweight is down. Although the dynamic augment may be computed numerically, as illustrated in § 413, its real effect on the rail is modified by the action of the equalizing levers and also by the effect that a change of pressure by the other drivers has on the rail and on the reaction of the rail on the driver considered.

Another item which increases the pressure exerted by the

main driver on the rail is the vertical component of the pull (or push) of the main rod. This component acts downward both when the crank pin is up and when it is down. For one case this was computed to be about 12% of the cylinder pressure at mid-stroke. This is a very significant addition to the rail pressure. It was not included in the figures observed in the tests since steam was shut off when the locomotive passed over the test track.

In Fig. 225 are shown plotted results of tests with a locomotive of the Santa Fé type (2-10-2). The diagram shows only the stresses under the main driver—which carries the crank pin. Corresponding diagrams for the other drivers showed very different results. The position of the counterweight is shown, indicating that, since the main driver was *underbalanced*, the maximum stress in the rail occurred when the crank pin was down and the counterweight up. Note that the minimum stress in the base of the rail was 14,000 lbs. per sq. in. for a speed of 50 m.p.h. The pressures for 5 m.p.h. were so nearly uniform that a mean average line to represent them was drawn at about 10,700. The mean value of the ordinates for 50 m.p.h. indicated a mean stress of about 26,000 lbs. per sq. in. The difference between 10,700 and 26,000, or 15,300 lbs. per sq. in., is considered to represent the mean value of the effect of speed alone, or the effect of increasing the velocity from 5 to 50 m.p.h. without reference to counter-weight effects. Similar tests with a Pacific type locomotive (4-6-2) showed that the effect of speed is to increase the rail stress 1.95 and 2.25 times by increasing the speed from 5 m.p.h. to 45 and to 60 m.p.h. respectively. But these figures are of academic interest chiefly, since the critical figure is the maximum actual stress in the rail at high speed. The average maximum shown by the tests was 44,700 lbs. per sq. in. during each revolution of the drivers. One observed stress was as high as 52,000 lbs. per sq. in.

It should also be noted that, for the type of rail used in this test, the maximum stress in the head of the rail is about 10% greater than that in the base for vertical loads. In these tests the strain measurements were taken in the base of the rail since the lateral stresses are greater there than in the head and are "great enough to be significant."

Trailer. These tests developed some very unexpected results with respect to the stresses under the trailers—the compar-

ively small loose wheels under the firebox and next behind the drivers. These wheels are presumably perfectly balanced and normally carry a definite proportion of the load of the engine. It might have been expected that the rail pressure would be substantially uniform and that any variation in pressure would be due to some accidental unevenness in the track. On the contrary the variations were quite marked, especially at high speeds, and the positions of maximum stress seem to bear a definite relation to the position of the counterweight on the drivers. If this relation were constant for all locomotives, its analysis would be simplified. For a locomotive of the Santa Fé type, the maximum stress occurred when the counterweight was at a position from 0.6 to 0.8 of a revolution after the low position. For a locomotive of the Pacific type, the maximum effect occurred at about 0.4 of a revolution after the low position. In each case the various observations of the tests were so consistent that the conclusions are indisputable. The differences in results for different locomotives shows that it depends on the relative weights on the wheels and on the equalizer system. The systematic variation from uniformity shows the effect of variable pressure of adjacent drivers, acting through the equalizing levers, and also through the rails, to modify what would otherwise be a uniform pressure. It also helps to explain certain apparent inconsistencies in the results for the driver pressures. Evidently there is a large field for future investigation, and it is to be expected that succeeding reports from this committee will throw more light on this phase of the subject.

APPENDIX.

THE ADJUSTMENTS OF INSTRUMENTS.

THE accuracy of instrumental work may be vitiated by any one of a large number of inaccuracies in the geometrical relations of the parts of the instruments. Some of these relations are apt to be altered by ordinary usage of the instrument that the makers have provided adjusting-screws so that the inaccuracies may be readily corrected. There are other possible defects which, however, will seldom be found to exist, provided the instrument was properly made and has never been subjected to treatment sufficiently rough to distort it. Such defects, when found, can only be corrected by a competent instrument-maker or repairer.

A WARNING is necessary to those who would test the accuracy of instruments, and especially to those whose experience in such work is small. Lack of skill in handling an instrument will often indicate an apparent error of adjustment when the actual error is very different or perhaps non-existent. It is always a safe plan when testing an adjustment to note the amount of the apparent error; then, beginning anew, make another independent determination of the amount of the error. When two or more *perfectly independent* determinations of such an error are made it will generally be found that they differ by an appreciable amount. The *differences* may be due in variable measure to careless inaccurate manipulation and to instrumental defects which are wholly independent of the particular test being made. Such careful determinations of the amounts of the errors are generally advisable in view of the next paragraph.

DO NOT DISTURB THE ADJUSTING-SCREWS ANY MORE THAN NECESSARY. Although metals are apparently rigid, they are really elastic and yielding. If some parts of a complicated mechanism, which is held together largely by friction, are subjected to greater internal stresses than other parts of the mech-

anism, the jarring resulting from handling will frequently cause a slight readjustment in the parts which will tend to more nearly equalize the internal stresses. Such action frequently occurs with the adjusting mechanism of instruments. One screw may be strained more than others. The friction of parts may prevent the opposing screw from *immediately* taking up an equal stress. Perhaps the adjustment appears perfect under these conditions. Jarring diminishes the friction between the parts, and the unequal stresses tend to equalize. A motion takes place which, although microscopically minute, is sufficient to indicate an error of adjustment. A readjustment made by unskillful hands may not make the final adjustment any more perfect. The frequent shifting of adjusting-screws wears them badly, and when the screws are worn it is still more difficult to keep them from moving enough to vitiate the adjustments. It is therefore preferable in many cases to refrain from disturbing the adjusting-screws, especially as the accuracy of the work done is not *necessarily* affected by errors of adjustment, as may be illustrated:

(*a*) Certain operations are *absolutely* unaffected by certain errors of adjustment.

(*b*) Certain operations are so slightly affected by certain *small* errors of adjustment that their effect may properly be neglected.

(*c*) Certain errors of adjustment may be readily allowed for and neutralized so that no error results from the use of the maladjusted instrument. Illustrations of all these cases will be given under their proper heads.

ADJUSTMENTS OF THE TRANSIT.

1. *To have the plate-bubbles in the center of the tubes when the axis is vertical.* Clamp the upper plate and, with the lower clamp loose, swing the instrument so that the plate-bubbles are parallel to the lines of opposite leveling-screws. Level up until both bubbles are central. Swing the instrument 180°. If the bubbles again settle at the center, the adjustment is perfect. If either bubble does not settle in the center, move the leveling-screws until the bubble is *half-way* back to the center. Then, before touching the adjusting-screws, note carefully the position of the bubbles and observe whether the bubbles always settle at the *same* place in the tube, no matter to what position the in-

strument may be rotated. When the instrument is so leveled, the axis is truly vertical and the discrepancies between this constant position of the bubbles and the centers of the tubes measure the errors of adjustment. By means of the adjusting-screws bring each bubble to the center of the tube. If this is done so skillfully that the true level of the instrument is not disturbed, the bubbles should settle in the center for all positions of the instrument. Under unskillful hands, two or more such trials may be necessary.

When the plates are not horizontal, the measured angle is greater than the true horizontal angle by the difference between the measured angle and its projection on a horizontal plane. When this angle of inclination is small, the difference is insignificant. Therefore when the plate-bubbles are *very nearly* in adjustment, the error of measurement of horizontal angles may be far within the lowest unit of measurement used. A small error of adjustment of the plate-bubble *perpendicular* to the telescope will affect the horizontal angles by only a small proportion of the error, which will be perhaps imperceptible. Vertical angles will be affected by the same insignificant amount. A *small* error of adjustment of the plate-bubble *parallel* to the telescope will affect horizontal angles very slightly, but will affect vertical angles by the full amount of the error.

All error due to unadjusted plate-bubbles may be avoided by noting in what positions in the tubes the bubbles will remain fixed for all positions of azimuth and then keeping the bubbles adjusted to these positions, for the axis is then truly vertical. It will often save time to work in this way temporarily rather than to stop to make the adjustments. This should especially be done when accurate vertical angles are required.

When the bubbles are truly adjusted, they should remain stationary regardless of whether the telescope is revolved with the upper plate loose and the lower plate clamped or whether the whole instrument is revolved, the plates being clamped together. If there is any appreciable difference, it shows that the two vertical axes or "centers" of the plates are not concentric. This may be due to cheap and faulty construction or to the excessive wear that may be sometimes observed in an old instrument originally well made. In either case it can only be corrected by a maker.

2. *To make the revolving axis of the telescope perpendicular to the vertical axis of the instrument.* This is best tested by using a long plumb-line, so placed that the telescope must be pointed upward at an angle of about 45° to sight at the top of the plumb-line and downward about the same amount, if possible, to sight at the lower end. The vertical axis of the transit must be made truly vertical. Sight at the upper part of the line clamping the horizontal plates. Swing the telescope down and see if the cross-wire again bisects the cord. If so, the adjustment is *probably* perfect (a conceivable exception will be

oted later); if not, raise or lower one end of the axis by means
the adjusting-screws, placed at the top of one of the standards,
ntil the cross-wire will bisect the cord both at top and bottom.
he plumb-bob may be steadied, if necessary, by hanging it
a pail of water. As many telescopes cannot be focused
an object nearer than 6 or 8 feet from the telescope, this
ethod requires a long plumb-line swung from a high point,
hich may be inconvenient.

Another method is to set up the instrument about 10 feet
om a high wall. After leveling, sight at some convenient
ark high up on the wall. Swing the telescope down and make
mark (when working alone some convenient natural mark may
nerally be found) low down on the wall. Plunge the telescope
id revolve the instrument about its vertical axis and again sight
the upper mark. Swing down to the lower mark. If the
ire again bisects it, the adjustment is perfect. If not, fix a
oint *half-way* between the two positions of the lower mark.
he plane of this point, the upper point, and the center of the
strument is truly vertical. Adjust the axis to these upper and
wer points as when using the plumb-line.

3. *To make the line of collimation perpendicular to the revolving xis of the telescope.* With the instrument level and the telescope
early horizontal point at some well-defined point at a distance
200 feet or more. Plunge the telescope and establish a point
the opposite direction. Turn the whole instrument about the
ertical axis until it again points at the first mark. Again
lunge to "direct position" (*i.e.*, with the level-tube *under* ie telescope). If the vertical cross-wire again points at the
cond mark, the adjustment is perfect. If not, the error is *ne-fourth* of the distance between the two positions of the
cond mark. Loosen the capstan screw on one side of the
lescope and tighten it on the other side until the vertical
ire is set at the one-fourth mark. Turn the whole instrument
y means of the tangent screw until the vertical wire is *midway* etween the two positions of the second mark. Plunge the
lescope. If the adjusting has been skillfully done, the cross-
ire should come exactly to the first mark. As an "erecting yepiece" reinverts an image already inverted, the ring carrying ie cross-wires must be moved in the *same* direction as the *pparent* error in order to correct that error.

The necessity for the third adjustment lies principally in the p of producing a line by plunging the telescope, but when this is requ be done with great accuracy it is always better to obtain the forward by reversion (as described above for making the test) and take th of the two forward points. Horizontal and vertical angles are pra unaffected by *small* errors of this adjustment, unless, in the case o zontal angles, the vertical angles to the points observed are very di

Unnecessary motion of the adjusting-screws may sometimes be a by carefully establishing the forward point on line by repeated rev of the instrument, and thus determining by repeated trials the exact a of the error. *Differences* in the amount of error determined wo evidence of inaccuracy in manipulating the instrument, and would that an adjustment based on the first trial would *probably* prove s factory.

The 2d and 3d adjustments are mutually dependent. If either ment is badly out, the other adjustment cannot be made except as f

(*a*) The second adjustment can be made regardless of the third the lines to the high point and the low point make *equal* angles wi horizontal.

(*b*) The third adjustment can be made regardless of the second the front and rear points are *on a level* with the instrument.

When both of these requirements are *nearly* fulfilled, and esp when the error of either adjustment is small, no trouble will be fo perfecting either adjustment on account of a small error in the oth justment.

If the test for the second adjustment is made by means of the p line and the vertical cross-wire intersects the line at all points as th scope is raised or lowered, it not only demonstrates at once the ac of that adjustment, but also shows that the third adjustment is perfect or has so small an error that it does not affect the second.

4. *To have the bubble of the telescope-level in the center tube when the line of collimation is horizontal.* The line of mation should coincide with the optical axis of the tele If the object-glass and eyepiece have been properly cent the previous adjustment will have brought the vertical c wire to the center of the field of view. The horizontal c wire should also be brought to the center of the field of and the bubble should be adjusted to it.

a. Peg method. Set up the transit at one end of a s level stretch of about 300 feet. Clamp the telescope wi bubble in the center. Drive a stake vertically under the piece of the transit, and another about 300 feet away. Ob the height of the center of the eyepiece (the telescope l level) above the stake (calling it *a*); observe the reading o rod when held on the other stake (calling it *b*); take the in ment to the other stake and set it up so that the eyepi

ertically over the stake, observing the height, c; take a reading n the first stake, calling it d. If this adjustment is perfect, hen

$$a-d=b-c,$$

or $$(a-d)-(b-c)=0.$$

Call $$(a-d)-(b-c)=2m.$$

When m is positive, the line points downward;
" m " negative, " " " upward.

To adjust: if the line points *up*, sight the horizontal cross-ire (by moving the vertical tangent screw) at a point which is lower, then adjust the bubble so that it is in the center.

By taking several independent values for a, b, c, and d, a mean value r m is obtained, which is more reliable and which may save much un-ecessary working of the adjusting-screws.

b. Using an auxiliary level. When a carefully adjusted level s at hand, this adjustment may sometimes be more easily ıade by setting up the transit and level, so that their lines of ollimation are as nearly as possible at the same height If a oint may be found which is half a mile or more away and hich is on the horizontal cross-wire of the level, the horizontal ross-wire of the transit may be pointed directly at it, and the ubble adjusted accordingly. Any slight difference in the eights of the lines of collimation of the transit and level (say ") may almost be disregarded at a distance of ½ mile or more, r, if the difference of level would have an appreciable effect, ven this may be practically eliminated by making an estimated llowance when sighting at the distant point. Or, if a distant oint is not available, a level-rod with target may be used at a istance of (say) 300 feet, making allowance for the carefully etermined difference of elevation of the two lines of collimation.

5. *Zero of vertical circle.* When the line of collimation is truly orizontal and the vertical axis is truly vertical, the reading f the vertical circle should be 0°. If the arc is adjustable, t should be brought to 0°. If it is not adjustable, the *index rror* should be observed, so that it may be applied to all readings f vertical angles.

ADJUSTMENTS OF THE WYE LEVEL.

1. *To make the line of collimation coincide with the center of he rings.* Point the intersection of the cross-wires at some

well-defined point which is at a considerable distance. strument need not be level, which allows much great in choosing a convenient point. The vertical axis clamped, and the clips over the wyes should be loos raised. Rotate the telescope in the wyes. The inter the cross-wires should be continually on the point. If it requires adjustment. Rotate the telescope 180° a *one-half* of the error by means of the capstan-headed sc move the cross-wire ring. It should be remembered t an erecting telescope, on account of the inversion of th the ring should be moved in the direction of the *appar* Adjust the other half of the error with the levelin Then rotate the telescope 90° from its usual positi accurately at the point, and then rotate 180° from that and adjust any error as before. It may require seve but it is necessary to adjust the ring until the inter the cross-wires will remain on the point for any po rotation.

If such a test is made on a very distant point and again on a 10 or 15 feet from the instrument, the adjustment may be fou for one point and incorrect for the other. This indicates that slide is improperly centered. Usually this defect can only be co an instrument-maker. If the difference is very small it may b but the adjustment should then be made on a point which is at mean distance for usual practice—say 150 feet.

If the whole image appears to shift as the telescope is rotat cates that the eyepiece is improperly adjusted. This defect usually corrected only by the maker. It does not interfere w mental accuracy, but it usually causes the intersection of the to be eccentric with the field of view.

2. *To make the axis of the level-tube parallel to the line mation.* Raise the clips as far as possible. Swing so that it is parallel to a pair of opposite leveling-sc clamp it. Bring the bubble to the middle of the tube of the leveling-screws. Take the telescope out of the replace it end for end, using *extreme care* that the wye jarred by the action. If the bubble does not come to th correct *one-half* of the error by the vertical adjusting- one end of the bubble. Correct the other half by the screws. Test the work by again changing the telescop end in the wyes.

Care should be taken while making this adjustme

that the level-tube is vertically under the telescope. With the bubble in the center of the tube, rotate the telescope in the wyes for a considerable angle each side of the vertical. If the first half of the adjustment has been made and the bubble moves, it shows that the axis of the wyes and the axis of the level-tube are not in the same vertical plane although both have been made horizontal. By moving one end of the level-tube *sidewise* by means of the horizontal screws at one end of the tube, the two axes may be brought into the same plane. As this adjustment is liable to disturb the other, both should be alternately tested until both requirements are complied with.

By these methods the axis of the bubble is made parallel to the axis of the wyes; and as this has been made parallel to the lines of collimation by means of the previous adjustment, the axis of the bubble is therefore parallel to the line of collimation.

3. *To make the line of collimation perpendicular to the vertical axis.* Level up so that the instrument is approximately level over both sets of leveling-screws. Then, after leveling carefully over one pair of screws, revolve the telescope 180°. If it is not level, adjust half of the error by means of the capstan-headed screw under one of the wyes, and the other half by the leveling-screws. Reverse again as a test.

When the first two adjustments have been accurately made, good leveling may always be done by bringing the bubble to the center by means of the leveling-screws, at every sight if necessary, even if the third adjustment is not made. Of course this third adjustment should be made as a matter of convenience, so that the line of collimation may be always level no matter in what direction it may be pointed, but it is not *necessary* to stop work to make this adjustment every time it is found to be defective.

ADJUSTMENTS OF THE DUMPY LEVEL.

1. *To make the axis of the level-tube perpendicular to the vertical axis.* Level up so that the instrument is approximately level over both sets of leveling-screws. Then, after leveling carefully over one pair of screws, revolve the telescope 180°. If it is not level, adjust *one-half* of the error by means of the adjusting-screws at one end of the bubble, and the other half by means of the leveling-screws. Reverse again as a test.

2. *To make the line of collimation perpendicular to the vertical axis.* The method of adjustment is identical with that for the transit (No. 4, pl. 505) except that the cross-wire must be

adjusted to agree with the level-bubble rather than *vice versa*, as is the case with the corresponding adjustment of the transit, i.e., with the level-bubble in the center, raise or lower the horizontal cross-wire until it points at the mark known to be on a level with the center of the instrument.

If the instrument has been well made and has not been distorted by rough usage, the cross-wires will intersect at the center of the field of view when adjusted as described. If they do not, it indicates an error which ordinarily can only be corrected by an instrument-maker. The error may be due to any one of several causes, which are

(*a*) faulty centering of object-slide;

(*b*) faulty centering of eyepiece;

(*c*) distortion of instrument so that the geometric axis of the telescope is not perpendicular to the vertical axis. If the error is only just perceptible, it will not probably cause any error in the work.

AZIMUTH.

The azimuth of a line on the surface of the earth is its angle with a true meridian through a point on the line. It is the true bearing as distinguished from "magnetic bearing." Federal law requires that all surveys of government lands shall be made by "Solar Observations" (rather than with the magnetic needle) so as to obtain true bearings.

Solar Azimuth may be obtained in two general ways, (*a*) by direct observation on the sun with an ordinary "complete" transit, provided with a colored glass shade, and (*b*) by the use of a "solar attachment" or a solar compass. The first method only requires as special equipment a colored glass shade costing but a few dollars, but it requires the separate solution of a formula for each observation made. Even the colored glass shade is not always necessary—as when the disc of the sun is just

through thin clouds and is not too bright to be observed with the naked eye. The " colored glass shade " may be merely a piece of colored glass fitted over the eye-piece, or the glass may be set into a frame very similar to the object glass cover and readily taken off and put on. In the latter case the glass must be " optically perfect," i.e., with the sides perfectly plane and parallel, so that there shall be no refraction of the image, or such glass as is used for the sun shade of a sextant.

The second method (*b*) does not require any calculation of a formula; the true meridian is given directly but it requires the use of a special instrument, whose adjustments must be made with great care or the resulting azimuth will often be in error by a *much larger* amount than the error in the adjustment. A proper appreciation of either method requires an understanding of certain astronomical relations.

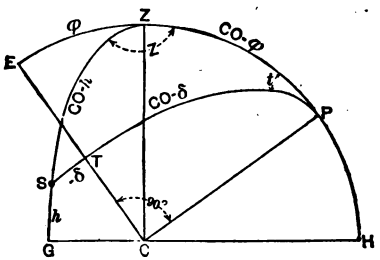


FIG. 1.

Fig. 1 represents the orthographic projection of the celestial sphere, projected on the plane of the meridian of the observer.

H P Z E represents the meridian of the observer.

Z = the zenith.

CP = the polar axis of the earth.

CE = the plane of the equator.

S = the position of the sun.

EZ = the latitude of the observer = ϕ.

ZP = 90° − ϕ = co ϕ.

SG = the true altitude of the sun = *h*.

SZ = 90° − *h* = co *h*.

ST = the declination of the sun, north or south of the equator = δ.

SP = 90° − δ = co δ, also called *p* = polar distance.

The essential sign of δ must be considered. If the sun is south of the equator (as it is from about September 21 to March 21), δ is negative and if the declination is (say) S 20°, $\delta = -20°$. Then co $\delta = 90° - \delta = 90° - (-20°) = 110°$.

Z = the angle from the position of the sun to the true north = the spherical angle SZP. A is its supplement = $180° - Z$.

Of several possible formulae, the U. S. Coast and Geodetic Survey prefer the following:

$$\text{Cot } \tfrac{1}{2} A = \sqrt{\frac{\sin (S-\phi) \sin (S-h)}{\cos S \cos (S-p)}},$$

in which $S = \frac{1}{2}(\phi + h + p)$.

The sun describes each day a path which is approximately parallel with the equator, the change in declination being very small during June and December and fastest when the sun is crossing the equator in March and September, the greatest rate of change being about 59 seconds of arc per hour. The declination of the sun must be known for the time of observation. This is obtainable from the Nautical Almanac or Ephemeris.

Example.—Declination for Philadelphia, Feb. 20, 1914, at 8:10 A. M., standard time, 75th meridian. Since "standard time" is a definite time interval from Greenwich mean local time, we may use it here regardless of precise longitude or mean local time, 8:10 A. M. on the 75° meridian is 1:10 P. M. mean time, at Greenwich. $1.17h \times 53''.64 = 62''.58 = 1'\ 2''.6$ and $-11°\ 7'\ 1''.1 + 0°\ 1'\ 2''.6 = -11°\ 5'\ 58''.5$ which is south declination.

Refraction. Refraction causes the sun to appear higher than it actually is. Therefore when the altitude of the sun is observed the computed refraction should be subtracted from the apparent altitude to obtain the true altitude. The amount of the refraction is a very complicated function of the temperature and the barometric pressure. For refined astronomical work, large refraction tables should be used, making due allowances for temperature and pressure, but for such work as may be done with an ordinary transit the values given in the following table will suffice.

Angular diameter of sun. The sun's angular diameter is about 0° 32′. With the comparatively high power telescopes now generally used on transits, this fills a large part of the field of view and it is impossible to accurately

EAN REFRACTIONS—[BESSEL] TRUE FOR BAROMETER AT 29″.6, TEMP. 48° F.

Alt.	Refr.	Alt.	Refr.	Alt.	Refr.
0° 0′	34′ 54″	1° 30′	20′ 51″	5° 0′	9′ 46″
10	32 49	40	19 52	30	9 02
20	30 52	50	18 58	6 0	8 23
30	29 03	2 0	18 09	30	7 49
40	27 23	30	16 01	7 0	7 20
50	25 50	3 0	14 15	30	6 53
1° 0	24 25	30	12 48	8 0	6 30
10	23 07	4 0	11 39	30	6 08
20	21 56	30	10 40	9 0	5 49

Alt.	Refr.	Alt.	Refr.	Alt.	Refr.
9° 30′	5′ 32″	18°	2′ 56″	30°	1′ 40″
10 0	5 16	19	2 46	35	1 22
11 0	4 48	20	2 37	40	1 09
12 0	4 25	21	2 29	45	0 58
13 0	4 05	22	2 22	50	0 48
14 0	3 47	23	2 15	60	0 33
15 0	3 32	24	2 09	70	0 21
16 0	3 19	26	1 58	80	0 10
17 0	3 07	28	1 48	90	0 0

gular width especially as the apparent motion of the sun across e field of view is very rapid. It therefore becomes advisable hen sighting directly at the sun with the transit telescope) to ght the cross wires on the edges of the n, as shown in Fig. 2, and make due owance for the semi-diameter of the n. The effect of this is to obtain an itude which differs from the true altide by the angular value of the semiameter. The observed azimuth differs m the true azimuth by the semiameter ÷ cos h. When the sun is at e horizon, cos $h = 1$, and the allowance equals the semi-diameter th for altitude and azimuth. For higher altitudes the allowce for azimuth is much larger than the semi-diameter, since e divisor (cos h) is small. If several observations are taken thin a short interval, the *change* in this allowance for azimuth ring this short interval may be too small for notice and one lue may be sufficiently accurate for all the observations.

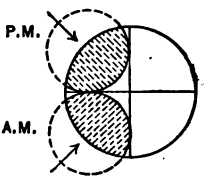


FIG. 2.

There is a slight variation in the semi-diameter as is shown in e accompanying tabular form, giving average values, which

may be used by interpolation, if a closer value than t minute is desired.

Time.	Semi-diam. of the Sun in minutes of arc.
Jan. 1....	16′.30 (max)
April 1...	16 .03
July 1....	15 .76 (min)
Oct. 1....	16 .01

Latitude. If the latitude of the place of observat known to the nearest minute, it may readily be ob observing the altitude of the sun at culmination at n horizontal cross wire should be sighted at the upper (or edge of the disc of the sun.

If d = angular diameter of sun — r = refr
ϕ = latitude — δ = decl
h' = observed angle of elevation

$$\text{then } \phi = 90° - [h' - r - \delta \pm \tfrac{1}{2}d]$$

in which $\frac{1}{2}d$ is + for an observation on the lower edge, and $\frac{1}{2}d$ is − for an observation on the upper edge.

Set up the transit several minutes before noon, taking time to level up with the utmost care. Set the horizo wire on the upper (or lower) edge of the sun and with th screw follow the motion of the sun. As the required found at culmination, the motion of the telescope sho when the highest altitude is obtained and the sun descend.

Azimuth by an Observation with the transit telesc up the transit at a convenient station from which an unol view of the sun may be obtained at all times and from convenient permanent azimuth mark (e.g., a distant s chimney) may be observed. Point at the azimuth m the horizontal plates reading zero. With the upper pl point at the sun observing the time, altitude and the h angle from the azimuth mark. Three or more such obs are generally advisable, especially as they are so ea quickly taken and are such a valuable check on each o observation may be vitiated by some inaccuracy o

in manipulation or reading which would not be discovered unless more than one observation is taken, in which case the error would hardly be precisely repeated both in nature and amount. Finally, point at the azimuth mark to test whether the lower plate has slipped. The reading on the azimuth mark should be 0°.

Reducing the Observations. Compute the declinations for the given times of observation. If several observations are taken, it is generally best to compute the declinations for the times of the first and last observations and interpolate for the others. The observations may most readily be reduced by using a regular form as given below. The six observations quoted were taken in 15 minutes by one of the author's students.

Time	Apparent Altitude	α	h	δ	Z	Semi-diam. / cos.ap. alt.	True Azi. of Mark.
4:50	22° 48′.5	237° 41′	22° 30′.3	14° 45′.6	89° 16′.6	17′.2	213° 19′.6
4:53	22 12 .5	238 11	21 54 .3	45 .6	88 46 .6	17 .2	19 .6
4:55	21 44 .5	238 34	21 26 .2	45 .6	88 23 .3	17 .1	19 .8
4:58	21 19 .0	238 55	21 0 .7	45 .7	88 02 .4	17 .1	19 .7
5:00	20 49 .5	239 19 .5	20 31 .1	45 .7	87 38 .0	17 .0	19 .5
5:03	20 28 .0	239 38 .0	20 9 .5	14 45 .7	87 19 .9	17 .0	213 19 .1

Mean = 213°19′.55.

Observations taken Apr. 29, 1897: Semi-diam. of Sun 15′.9. Sun observed in lower left-hand corner.

α = horizontal angle to azimuth mark, the angle being measured to the right.

h = app. alt. − refraction − semi-diam. of sun; semi-diam. is +when sun is **above** hor. cross wire, −when below.

δ = declination, and Z = computed angle (as illustrated below).

True azimuth of mark $= 540° \pm \frac{\text{Semi-diam.}}{\text{cos. app. alt.}} \pm Z - \alpha$, in which Z is +for A. M. and −for P. M. and the $\frac{\text{Semi-diam.}}{\text{cos. app. alt.}}$ is +when the sun is on the **left** of the middle wire (as above); $\frac{\text{Semi-diam.}}{\text{cos. app. alt.}}$ is − when the sun is on the **right** of the middle wire.

TABLE I.—RADII OF CURVES.

Deg	0°		1°		2°		3°	
Min	Radius.	Log R	Radius.	Log R	Radius.	Log R	Radius.	Log
0	∞	∞	5729·6	3·75813	2864·9	3·4571$\bar{1}$	1910·1	3·28
1	343775	5·53627	5635·7	·75095	2841·3	·45351	1899·5	·27
2	171887	5·23524	5544·8	·74389	2818·0	·44993	1889·1	·27
3	114592	5·05915	5456·8	·73694	2795·1	·44639	1878·8	·27
4	85944	4·93421	5371·6	·73010	2772·5	·44287	1868·6	·27
5	68755	4·83730	5288·9	·72336	2750·4	·43939	1858·5	·26
6	57296	4·75812	5208·8	3·71673	2728·5	3·43593	1848·5	3·26
7	49111	·69117	5131·0	·71020	2707·0	·43249	1838·6	·26
8	42972	·63318	5055·6	·70377	2685·9	·42909	1828·8	·26
9	38197	·58203	4982·3	·69743	2665·1	·42571	1819·1	·26
10	34377	·53627	4911·2	·69118	2644·6	·42235	1809·6	·25
11	31252	4·49488	4842·0	3·68502	2624·4	3·41903	1800·1	3·25
12	28648	·45709	4774·7	·67895	2604·5	·41572	1790·7	·25
13	26444	·42233	4709·3	·67296	2584·9	·41245	1781·5	·25
14	24555	·39014	4645·7	·66705	2565·6	·40919	1772·3	·24
15	22918	·36018	4583·8	·66122	2546·6	·40597	1763·2	·24
16	21486	4·33215	4523·4	3·65547	2527·9	3·40276	1754·2	3·24
17	20222	·30582	4464·7	·64979	2509·5	·39958	1745·3	·24
18	19099	·28100	4407·5	·64419	2491·3	·39642	1736·5	·23
19	18093	·25752	4351·7	·63865	2473·4	·39329	1727·8	·23
20	17189	·23524	4297·3	·63319	2455·7	·39017	1719·1	·23
21	16370	4·21405	4244·2	3·62780	2438·3	3·38708	1710·6	3·23
22	15626	·19385	4192·5	·62247	2421·1	·38401	1702·1	·23
23	14947	·17454	4142·0	·61720	2404·2	·38097	1693·7	·23
24	14324	·15606	4092·7	·61200	2387·5	·37794	1685·4	·22
25	13751	·13833	4044·5	·60686	2371·0	·37494	1677·2	·22
26	13222	4·12130	3997·5	3·60178	2354·8	3·37195	1669·1	3·22
27	12732	·10491	3951·5	·59676	2338·8	·36899	1661·0	·22
28	12278	·08911	3906·6	·59180	2323·0	·36604	1653·0	·21
29	11854	·07387	3862·7	·58689	2307·4	·36312	1645·1	·21
30	11459	·05915	3819·8	·58204	2292·0	·36021	1637·3	·21
31	11090	4·04491	3777·9	3·57724	2276·8	3·35733	1629·5	3·21
32	10743	·03112	3736·8	·57250	2261·9	·35446	1621·8	·21
33	10417	·01776	3696·6	·56780	2247·1	·35162	1614·2	·20
34	10111	4·00479	3657·3	·56316	2232·5	·34879	1606·7	·20
35	9822·2	3·99221	3618·8	·55856	2218·1	·34598	1599·2	·20
36	9549·3	3·97997	3581·1	3·55401	2203·9	3·34318	1591·8	3·20
37	9291·3	·96807	3544·2	·54951	2189·8	·34041	1584·5	·19
38	9046·7	·95649	3508·0	·54506	2176·0	·33765	1577·2	·19
39	8814·8	·94521	3472·6	·54065	2162·3	·33491	1570·0	·19
40	8594·4	·93421	3437·9	·53629	2148·8	·33219	1562·9	·19
41	8384·8	3·92349	3403·8	3·53197	2135·4	3·32949	1555·8	3·19
42	8185·2	·91302	3370·5	·52769	2122·3	·32680	1548·8	·18
43	7994·8	·90281	3337·7	·52345	2109·2	·32412	1541·9	·18
44	7813·1	·89282	3305·7	·51925	2096·4	·32147	1535·0	·18
45	7639·5	·88306	3274·2	·51510	2083·7	·31883	1528·2	·18
46	7473·4	3·87352	3243·3	3·51098	2071·1	3·31621	1521·4	3·18
47	7314·4	·86418	3213·0	·50691	2058·7	·31360	1514·7	·18
48	7162·0	·85503	3183·2	·50287	2046·5	·31101	1508·1	·17
49	7015·9	·84608	3154·0	·49885	2034·4	·30843	1501·5	·17
50	6875·6	·83731	3125·4	·49490	2022·4	·30587	1495·0	·17
51	6740·7	3·82871	3097·2	3·49097	2010·6	3·30332	1488·5	3·17
52	6611·1	·82027	3069·6	·48707	1998·9	·30079	1482·1	·17
53	6486·4	·81200	3042·4	·48321	1987·3	·29827	1475·7	·16
54	6366·3	·80388	3015·7	·47939	1975·9	·29577	1469·4	·16
55	6250·5	·79591	2989·5	·47559	1964·6	·29328	1463·2	·16
56	6138·9	3·78809	2963·7	3·47183	1953·5	3·29081	1457·0	3·16
57	6031·2	·78040	2938·4	·46811	1942·4	·28835	1450·8	·16
58	5927·2	·77285	2913·5	·46441	1931·5	·28590	1444·7	·15
59	5826·8	·76542	2889·0	·46075	1920·7	·28347	1438·7	·15
	5729·6	·75813	2864·9	·4571$\bar{1}$	1910·1	·28105	1432·7	·15

4°		5°		6°		7°		Deg
Radius.	Log R	Radius.	Log R	Radius.	Log R	Radius.	Log R	Min
1432·7	3·15615	1146·3	3·05929	955·37	2·98017	819·02	2·91325	0
1426·7	·15434	1142·5	·05784	952.72	·97896	817.08	·91226	1
1420·8	·15255	1138·7	·05640	950·09	·97776	815.14	·91128	2
1415·0	·15076	1134·9	·05497	947·48	·97657	813·22	·91021	3
1409·2	·14897	1131·2	·05354	944·88	·97537	811·30	·90916	4
1403·5	·14720	1127·5	·05211	942·29	·97418	809·40	·90816	5
1397·8	3·14543	1123·8	3·05069	939·72	2·97300	807·50	2·90714	6
1392·1	·14367	1120·2	·04928	937·16	·97181	805·61	·90612	7
1386·5	·14191	1116·5	·04787	934·62	·97063	803·73	·90511	8
1380·9	·14017	1112·9	·04646	932·09	·96945	801·86	·90410	9
1375·4	·13843	1109·3	·04506	929·57	·96828	800·00	·90309	10
1369·9	3·13669	1105·8	3·04366	927·07	2·96711	798·14	2·90208	11
1364·5	·13497	1102·2	·04227	924·58	·96594	796·30	·90107	12
1359·1	·13325	1098·7	·04088	922·10	·96478	794·46	·90007	13
1353·8	·13154	1095·2	·03950	919·64	·96361	792·63	·89907	14
1348·4	·12983	1091·7	·03811	917·19	·96246	790·81	·89807	15
1343·2	3·12813	1088·3	3·03674	914·75	2·96130	789·00	2·89708	16
1338·0	·12644	1084·8	·03537	912·33	·96015	787·20	·89608	17
1332·8	·12475	1081·4	·03400	909·92	·95900	785·41	·89509	18
1327·6	·12307	1078·1	·03264	907·52	·95785	783·62	·89410	19
1322·5	·12140	1074·7	·03128	905·13	·95671	781·84	·89312	20
1317·5	3·11974	1071·3	3·02992	902·76	2·95557	780·07	2·89213	21
1312·4	·11808	1068·0	·02857	900·40	·95443	778·31	·89115	22
1307·4	·11642	1064·7	·02723	898·05	·95330	776·55	·89017	23
1302·5	·11477	1061·4	·02589	895·71	·95217	774·81	·88919	24
1297·6	·11313	1058·2	·02455	893·39	·95104	773·07	·88821	25
1292·7	3·11150	1054·9	3·02322	891·08	2·94991	771·34	2·88724	26
1287·9	·10987	1051·7	·02189	888·78	·94879	769·61	·88627	27
1283·1	·10825	1048·5	·02056	886·49	·94767	767·90	·88530	28
1278·3	·10663	1045·3	·01924	884·21	·94655	766·19	·88433	29
1273·6	·10502	1042·1	·01792	881·95	·94544	764·49	·88337	30
1268·9	3·10341	1039·0	3·01661	879·69	2·94433	762·80	2·88241	31
1264·2	·10182	1035·9	·01530	877·45	·94322	761·11	·88145	32
1259·6	·10022	1032·8	·01400	875·22	·94212	759·43	·88049	33
1255·0	·09864	1029·7	·01270	873·00	·94101	757·76	·87953	34
1250·4	·09705	1026·6	·01140	870·80	·93991	756·10	·87858	35
1245·9	3·09548	1023·5	3·01010	868·60	2·93882	754·44	2·87762	36
1241·4	·09391	1020·5	·00882	866·41	·93772	752·80	·87668	37
1236·9	·09234	1017·5	·00753	864·24	·93663	751·16	·87573	38
1232·5	·09079	1014·5	·00625	862·07	·93554	749·52	·87478	39
1228·1	·08923	1011·5	·00497	859·92	·93446	747·89	·87384	40
1223·7	3·08769	1008·6	3·00370	857·78	2·93337	746.27	2·87290	41
1219·4	·08614	1005·6	·00242	855·65	·93229	744.66	·87196	42
1215·1	·08461	1002·7	3·00116	853·53	·93122	743.06	·87102	43
1210·8	·08308	999·76	2·99989	851·42	·93014	741·46	·87008	44
1206·6	·08155	996·87	·99863	849·32	·92907	739·83	·86915	45
1202·4	3·08003	993·99	2·99738	847·23	2·92800	738.28	2·86822	46
1198·2	·07852	991·13	·99613	845.15	·92693	736.70	·86729	47
1194·0	·07701	988·28	·99488	843.08	·92587	735.13	·86636	48
1189·9	·07550	985·45	·99363	841.02	·92480	733.57	·86544	49
1185·8	·07400	982·64	·99239	838·97	·92374	732.01	·86451	50
1181·7	3·07251	979·84	2·99115	836·93	2·92269	730.45	2·86359	51
1177·7	·07102	977·06	·98992	834.90	·92163	728.91	·86267	52
1173·6	·06954	974.29	·98869	832.89	·92058	727.37	·86175	53
1169·7	·06806	971·54	·98746	830·88	·91953	725.84	·86084	54
1165·7	·06658	968·81	·98624	828·88	·91849	724.31	·85992	55
1161·8	3·06511	966·09	2·98501	826·89	2·91744	722·79	2·85901	56
1157·9	·06365	963·39	·98380	824·91	·91640	721.28	·85810	57
1154·0	·06219	960·70	·98258	822·93	·91536	719.77	·85719	58
1150·1	·06074	958·03	·98137	820·97	·91433	718.27	·85629	59
1146·3	·05929	955·37	·98017	819·02	·91329	716.78	·85538	60

TABLE I.—RADII OF CURVES.

Deg.	8°		9°		10°		11°	
Min.	Radius.	Log R	Radius.	Log R	Radius.	Log R	Radius.	Lo
0	716·78	2·85538	637·27	2·80432	573·69	2·75867	521·67	2·7
1	715·29	·85448	636·10	·80352	572·73	·75795	520·88	·7
2	713·81	·85358	634·93	·80272	571·78	·75723	520·10	·7
3	712·34	·85268	633·76	·80192	570·84	·75651	519·32	·7
4	710·87	·85178	632·60	·80113	569·90	·75579	518·54	·7
5	709·40	·85089	631·44	·80033	568·96	·75508	517·76	·7
6	707·95	2·85000	630·29	2·79954	568·02	2·75436	516·99	2·7
7	706·49	·84911	629·14	·79874	567·09	·75365	516·21	·7
8	705·05	·84822	627·99	·79795	566·16	·75293	515·44	·7
9	703·61	·84733	626·85	·79716	565·23	·75222	514·68	·7
10	702·17	·84644	625·71	·79637	564·31	·75151	513·91	·7
11	700·75	2·84556	624·58	2·79558	563·38	2·75080	513·15	2·7
12	699·33	·84468	623·45	·79480	562·47	·75009	512·38	·7
13	697·91	·84380	622·32	·79401	561·55	·74939	511·63	·7
14	696·50	·84292	621·20	·79323	560·64	·74868	510·87	·7
15	695·09	·84204	620·09	·79245	559·73	·74798	510·11	·7
16	693·70	2·84117	618·97	2·79167	558·82	2·74727	509·36	2·7
17	692·30	·84029	617·87	·79089	557·92	·74657	508·61	·7
18	690·91	·83942	616·76	·79011	557·02	·74587	507·86	·7
19	689·53	·83855	615·66	·78934	556·12	·74517	507·12	·7
20	688·16	·83768	614·56	·78856	555·23	·74447	506·38	·7
21	686·78	2·83682	613·47	2·78779	554·34	2·74377	505·64	2·7
22	685·42	·83595	612·38	·78702	553·45	·74307	504·90	·7
23	684·06	·83509	611·30	·78625	552·56	·74238	504·16	·7
24	682·70	·83423	610·21	·78548	551·68	·74168	503·42	·7
25	681·35	·83337	609·14	·78471	550·80	·74099	502·69	·7
26	680·01	2·83251	608·06	2·78395	549·92	2·74030	501·96	2·7
27	678·67	·83166	606·99	·78318	549·05	·73961	501·23	·7
28	677·34	·83080	605·93	·78242	548·17	·73892	500·51	·8
29	676·01	·82995	604·86	·78165	547·30	·73823	499·78	·8
30	674·69	·82910	603·80	·78089	546·44	·73754	499·06	·8
31	673·37	2·82825	602·75	2·78013	545·57	2·73685	498·34	2·8
32	672·06	·82740	601·70	·77938	544·71	·73617	497·62	·8
33	670·75	·82656	600·65	·77862	543·86	·73548	496·91	·8
34	669·45	·82571	599·61	·77786	543·00	·73480	496·19	·8
35	668·15	·82487	598·57	·77711	542·15	·73412	495·48	·8
36	666·86	2·82403	597·53	2·77636	541·30	2·73343	494·77	2·8
37	665·57	·82319	596·50	·77561	540·45	·73275	494·07	·8
38	664·29	·82235	595·47	·77486	539·61	·73207	493·36	·8
39	663·01	·82152	594·44	·77411	538·76	·73140	492·66	·8
40	661·74	·82068	593·42	·77336	537·92	·73072	491·96	·8
41	660·47	2·81985	592·40	2·77261	537·09	2·73004	491·26	2·8
42	659·21	·81902	591·38	·77187	536·25	·72937	490·56	·8
43	657·95	·81819	590·37	·77112	535·42	·72869	489·86	·8
44	656·69	·81736	589·36	·77038	534·59	·72802	489·17	·8
45	655·45	·81653	588·36	·76964	533·77	·72735	488·48	·8
46	654·20	2·81571	587·36	2·76890	532·94	2·72668	487·79	2·8
47	652·96	·81489	586·36	·76816	532·12	·72601	487·10	·8
48	651·73	·81406	585·36	·76742	531·30	·72534	486·42	·8
49	650·50	·81324	584·37	·76669	530·49	·72467	485·73	·8
50	649·27	·81243	583·38	·76595	529·67	·72401	485·05	·8
51	648·05	2·81161	582·40	2·76522	528·86	2·72334	484·37	2·8
52	646·84	·81079	581·42	·76449	528·05	·72267	483·69	·8
53	645·63	·80998	580·44	·76376	527·25	·72201	483·02	·8
54	644·42	·80917	579·47	·76303	526·44	·72135	482·34	·8
55	643·22	·80836	578·49	·76230	525·64	·72069	481·67	·8
56	642·02	2·80755	577·53	2·76157	524·84	2·72003	481·00	2·8
57	640·83	·80674	576·56	·76084	524·05	·71937	480·33	·8
58	639·64	·80593	575·60	·76012	523·25	·71871	479·67	·8
59	·45	·80513	574·64	·75939	522·46	·71805	479·00	·8
60	·27	·80432	573·69	·75867	521·67	·71739	478·34	·8

TABLE I.—RADII OF CURVES.

Radius.	Log R	Deg.	Radius.	Log R	Deg.	Radius.	Log R	Deg.	Radius	Log R
8·34	2·67973	14°	410·28	2·61807	16°	359·26	2·55541	21°	274·37	2·43833
7·02	·67853	2	409·31	·61205	5	357·42	·55317	10	272·23	·43494
5·71	·67734	4	408·34	·61102	10	355·59	·55094	20	270·13	·43157
4·40	·67614	6	407·38	·61000	15	353·77	·54872	30	268·06	·42823
3·10	·67495	8	406·42	·60898	20	351·98	·54652	40	266·02	·42492
1·81	2·67376	10	405·47	2·60796	25	350·21	·54432	50	264·02	·42163
0·53	·67258	12	404·53	·60694	30	348·45	2·54214	22°	262·04	2·41837
9·25	·67140	14	403·58	·60593	35	346·71	·53997	10	260·10	·41513
7·98	·67022	16	402·65	·60492	40	344·99	·53780	20	258·18	·41192
6·72	·66905	18	401·71	·60391	45	343·29	·53565	30	256·29	·40873
5·46	2·66788	20	400·78	2·60291	50	341·60	·53351	40	254·43	·40557
4·21	·66671	22	399·86	·60190	55	339·93	·53138	50	252·60	·40243
2·97	·66555	24	398·94	·60090	17°	338·27	2·52927	23°	250·79	2·39931
1·73	·66439	26	398·02	·59990	5	336·64	·52716	10	249·01	·39622
0·50	·66323	28	397·11	·59891	10	335·01	·52506	20	247·26	·39315
9·28	2·66207	30	396·20	2·59791	15	333·41	·52297	30	245·53	·39010
8·06	·66092	32	395·30	·59692	20	331·82	·52090	40	243·82	·38707
6·85	·65977	34	394·40	·59593	25	330·24	·51883	50	242·14	·38407
5·65	·65863	36	393·50	·59494	30	328·68	2·51677	24°	240·49	2·38109
4·45	·65748	38	392·61	·59396	35	327·13	·51472	10	238·85	·37813
3·26	2·65634	40	391·72	2·59298	40	325·60	·51269	20	237·24	·37519
2·07	·65521	42	390·84	·59199	45	324·09	·51066	30	235·65	·37227
0·89	·65407	44	389·96	·59102	50	322·59	·50864	40	234·08	·36937
9·72	·65294	46	389·08	·59004	55	321·10	·50663	50	232·54	·36649
8·56	·65181	48	388·21	·58907	18°	319·62	2·50464	25°	231·01	2·36363
7·40	2·65069	50	387·34	2·58809	5	318·16	·50265	30	226·55	·35517
6·24	·64957	52	386·48	·58713	10	316·71	·50067	26°	222·27	·34688
5·09	·64845	54	385·62	·58616	15	315·28	·49869	30	218·15	·33875
3·95	·64733	56	384·77	·58519	20	313·86	·49673	27°	214·18	2·33078
2·81	·64622	58	383·91	·58423	25	312·45	·49478	30	210·36	·32296
1·68	2·64511	15°	383·06	2·58327	30	311·06	2·49284	28°	206·68	·31529
0·56	·64400	2	382·22	·58231	35	309·67	·49090	30	203·13	·30776
9·44	·64290	4	381·38	·58135	40	308·30	·48898	29°	199·70	2·30037
8·33	·64180	6	380·54	·58040	45	306·95	·48706	30	196·38	·29310
7·22	·64070	8	379·71	·57945	50	305·60	·48515	30°	193·19	·28597
6·12	2·63960	10	378·88	2·57850	55	304·27	·48325	30	190·09	·27896
5·02	·63851	12	378·05	·57755	19°	302·94	2·48136	31°	187·10	2·27207
3·93	·63742	14	377·23	·57661	5	301·63	·47948	32	181·40	·25863
2·84	·63633	16	376·41	·57566	10	300·33	·47760	33	176·05	·24563
1·76	·63524	18	375·60	·57472	15	299·04	·47573	34	171·02	·23303
0·69	2·63416	20	374·79	2·57378	20	297·77	·47388	35	166·28	·22083
9·62	·63308	22	373·98	·57284	25	296·50	·47203	36	161·80	2·20899
8·56	·63201	24	373·17	·57191	30	295·25	2·47018	37	157·58	·19745
7·50	·63093	26	372·37	·57097	35	294·00	·46835	38	153·58	·18623
6·44	·62986	28	371·57	·57004	40	292·77	·46652	39	149·79	·17547
5·40	2·62879	30	370·78	2·56911	45	291·55	·46471	40	146·19	·16492
4·35	·62773	32	369·99	·56819	50	290·33	·46289	41	142·77	2·15464
3·32	·62666	34	369·20	·56726	55	289·13	·46109	42	139·52	·14464
2·28	·62560	36	368·42	·56634	20°	287·94	2·45930	43	136·43	·13489
1·26	·62454	38	367·64	·56542	5	286·76	·45751	44	133·47	·12536
0·23	2·62349	40	366·86	2·56450	10	285·58	·45573	45	130·66	·11613
9·22	·62243	42	366·09	·56358	15	284·42	·45396	46	127·97	2·10709
8·20	·62138	44	365·31	·56266	20	283·27	·45219	47	125·39	·09827
7·19	·62034	46	364·55	·56175	25	282·12	·45044	48	122·93	·08965
6·19	·61929	48	363·78	·56084	30	280·99	2·44869	49	120·57	·08124
5·19	2·61825	50	363·02	2·55993	35	279·86	·44694	50	118·31	·07302
4·20	·61721	52	362·26	·55902	40	278·75	·44521	52	114·06	2·05713
3·21	·61617	54	361·51	·55812	45	277·64	·44348	54	110·13	·04192
2·23	·61514	56	360·76	·55721	50	276·54	·44176	56	106·50	·02736
1·25	·61410	58	360·01	·55631	55	275·45	·44004	58	103·13	·01340
0·28	2·61307	16°	359·26	2·55541	21°	274·37	2·43833	60	100·00	2·00000

TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND LONG CH
FOR A 1° CURVE.

Δ	Tang. T.	Ext. Dist. E.	Long Chord LC.	Δ	Tang. T.	Ext. Dist. E.	Long Chord LC.	Δ	Tang. T.	Ext. Dist. E.
1°	50.00	0.218	100.00	11°	551.70	26.500	1098.3	21°	1061.9	97.58
10′	58.34	0.297	116.67	10	560.11	27.313	1114.9	10	1070.6	99.15
20	66.67	0.388	133.33	20	568.53	28.137	1131.5	20	1079.2	100.75
30	75.01	0.491	150.00	30	576.95	28.974	1148.1	30	1087.8	102.35
40	83.34	0.606	166.66	40	585.36	29.824	1164.7	40	1096.4	103.97
50	91.68	0.733	183.33	50	593.79	30.686	1181.2	50	1105.1	105.60
2°	100.01	0.873	199.99	12°	602.21	31.561	1197.8	22°	1113.7	107.24
10	108.35	1.024	216.66	10	610.64	32.447	1214.4	10	1122.4	108.90
20	116.68	1.188	233.32	20	619.07	33.347	1231.0	20	1131.0	110.57
30	125.02	1.364	249.98	30	627.50	34.259	1247.5	30	1139.7	112.25
40	133.36	1.552	266.65	40	635.93	35.183	1264.1	40	1148.4	113.95
50	141.70	1.752	283.31	50	644.37	36.120	1280.7	50	1157.0	115.66
3°	150.04	1.964	299.97	13°	652.81	37.069	1297.2	23°	1165.7	117.38
10	158.38	2.189	316.63	10	661.25	38.031	1313.8	10	1174.4	119.11
20	166.72	2.425	333.29	20	669.70	39.006	1330.3	20	1183.1	120.87
30	175.06	2.674	349.95	30	678.15	39.993	1346.9	30	1191.8	122.63
40	183.40	2.934	366.61	40	686.60	40.992	1363.4	40	1200.5	124.41
50	191.74	3.207	383.27	50	695.06	42.004	1380.0	50	1209.2	126.20
4°	200.08	3.492	399.92	14°	703.51	43.029	1396.5	24°	1217.9	128.00
10	208.43	3.790	416.58	10	711.97	44.066	1413.1	10	1226.6	129.82
20	216.77	4.099	433.24	20	720.44	45.116	1429.6	20	1235.3	131.65
30	225.12	4.421	449.89	30	728.90	46.178	1446.2	30	1244.0	133.50
40	233.47	4.755	466.54	40	737.37	47.253	1462.7	40	1252.8	135.36
50	241.81	5.100	483.20	50	745.85	48.341	1479.2	50	1261.5	137.23
5°	250.16	5.459	499.85	15°	754.32	49.441	1495.7	25°	1270.2	139.11
10	258.51	5.829	516.50	10	762.80	50.554	1512.3	10	1279.0	141.01
20	266.86	6.211	533.15	20	771.29	51.679	1528.8	20	1287.7	142.93
30	275.21	6.606	549.80	30	779.77	52.818	1545.3	30	1296.5	144.85
40	283.57	7.013	566.44	40	788.26	53.969	1561.8	40	1305.3	146.79
50	291.92	7.432	583.09	50	796.75	55.132	1578.3	50	1314.0	148.75
6°	300.28	7.863	599.73	16°	805.25	56.309	1594.8	26°	1322.8	150.71
10	308.64	8.307	616.38	10	813.75	57.498	1611.3	10	1331.6	152.69
20	316.99	8.762	633.02	20	822.25	58.699	1627.8	20	1340.4	154.68
30	325.35	9.230	649.66	30	830.76	59.914	1644.3	30	1349.2	156.70
40	333.71	9.710	666.30	40	839.27	61.141	1660.8	40	1358.0	158.72
50	342.08	10.202	682.94	50	847.78	62.381	1677.3	50	1366.8	160.76
7°	350.44	10.707	699.57	17°	856.30	63.634	1693.8	27°	1375.6	162.81
10	358.81	11.224	716.21	10	864.82	64.900	1710.3	10	1384.4	164.87
20	367.17	11.753	732.84	20	873.35	66.178	1726.8	20	1393.2	166.95
30	375.54	12.294	749.47	30	881.88	67.470	1743.2	30	1402.0	169.04
40	383.91	12.847	766.10	40	890.41	68.774	1759.7	40	1410.9	171.15
50	392.28	13.413	782.73	50	898.95	70.091	1776.2	50	1419.7	173.27
8°	400.66	13.991	799.36	18°	907.49	71.421	1792.6	28°	1428.6	175.41
10	409.03	14.582	815.99	10	916.03	72.764	1809.1	10	1437.4	177.55
20	417.41	15.184	832.61	20	924.58	74.119	1825.5	20	1446.3	179.72
30	425.79	15.799	849.23	30	933.13	75.488	1842.0	30	1455.1	181.89
40	434.17	16.426	865.85	40	941.69	76.869	1858.4	40	1464.0	184.08
50	442.55	17.066	882.47	50	950.25	78.264	1874.9	50	1472.9	186.29
9°	450.93	17.717	899.09	19°	958.81	79.671	1891.3	29°	1481.8	188.51
10	459.32	18.381	915.70	10	967.38	81.092	1907.8	10	1490.7	190.74
20	467.71	19.058	932.31	20	975.96	82.525	1924.2	20	1499.6	192.99
30	476.10	19.746	948.92	30	984.53	83.972	1940.6	30	1508.5	195.25
40	484.49	20.447	965.53	40	993.12	85.431	1957.1	40	1517.4	197.53
50	492.88	21.161	982.14	50	1001.70	86.904	1973.5	50	1526.3	199.82
10°	501.28	21.886	998.74	20°	1010.29	88.389	1989.9	30°	1535.3	202.12
10	509.68	22.624	1015.35	10	1018.89	89.888	2006.3	10	1544.2	204.44
20	518.08	23.375	1031.95	20	1027.49	91.399	2022.7	20	1553.1	206.77
30	526.48	24.138	1048.54	30	1036.09	92.924	2039.1	30	1562.1	209.12
	34.88	24.913	1065.14	40	1044.70	94.462	2055.5	40	1571.0	211.48
	29	25.700	1081.73	50	1053.31	96.013	2071.9	50	1580.0	213.86
	70	26.500	1098.33	21°	1061.93	97.577	2088.3	31°	1589.0	216.25

TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND LONG CHORDS FOR A 1° CURVE.

Δ	Tang. T.	Ext. Dist. E.	Long Chord LC.	Δ	Tang. T.	Ext. Dist. E.	Long Chord LC.	Δ	Tang. T.	Ext. Dist. E.	Long Chord LC.
31°	1589.0	216.25	3062.4	41°	2142.2	387.38	4013.1	51°	2732.9	618.39	4933.4
10′	1598.0	218.66	3078.4	10	2151.7	390.71	4028.7	10	2743.1	622.81	4948.4
20	1606.9	221.08	3094.5	20	2161.2	394.06	4044.3	20	2753.4	627.24	4963.4
30	1615.9	223.51	3110.5	30	2170.8	397.43	4059.9	30	2763.7	631.69	4978.4
40	1624.9	225.96	3126.6	40	2180.3	400.82	4075.5	40	2773.9	636.16	4993.4
50	1633.9	228.42	3142.6	50	2189.9	404.22	4091.1	50	2784.2	640.66	5008.4
32°	1643.0	230.90	3158.6	42°	2199.4	407.64	4106.6	52°	2794.5	645.17	5023.4
10	1652.0	233.39	3174.6	10	2209.0	411.07	4122.2	10	2804.9	649.70	5038.4
20	1661.0	235.90	3190.6	20	2218.6	414.52	4137.7	20	2815.2	654.25	5053.4
30	1670.0	238.43	3206.6	30	2228.1	417.99	4153.3	30	2825.6	658.83	5068.3
40	1679.1	240.96	3222.6	40	2237.7	421.48	4168.8	40	2835.9	663.42	5083.3
50	1688.1	243.52	3238.6	50	2247.3	424.98	4184.3	50	2846.3	668.03	5098.2
33°	1697.2	246.08	3254.6	43°	2257.0	428.50	4199.8	53°	2856.7	672.66	5113.1
10	1706.3	248.66	3270.6	10	2266.6	432.04	4215.3	10	2867.1	677.32	5128.0
20	1715.3	251.26	3286.6	20	2276.2	435.59	4230.8	20	2877.5	681.99	5142.9
30	1724.4	253.87	3302.5	30	2285.9	439.16	4246.3	30	2888.0	686.68	5157.8
40	1733.5	256.50	3318.5	40	2295.6	442.75	4261.8	40	2898.4	691.40	5172.7
50	1742.6	259.14	3334.4	50	2305.2	446.35	4277.3	50	2908.9	696.13	5187.6
34°	1751.7	261.80	3350.4	44°	2314.9	449.98	4292.7	54°	2919.4	700.89	5202.4
10	1760.8	264.47	3366.3	10	2324.6	453.62	4308.2	10	2929.9	705.66	5217.3
20	1770.0	267.16	3382.2	20	2334.3	457.27	4323.6	20	2940.4	710.46	5232.1
30	1779.1	269.86	3398.2	30	2344.1	460.95	4339.0	30	2951.0	715.28	5246.9
40	1788.2	272.58	3414.1	40	2353.8	464.64	4354.5	40	2961.5	720.11	5261.7
50	1797.4	275.31	3430.0	50	2363.5	468.35	4369.9	50	2972.1	724.97	5276.5
35°	1806.6	278.05	3445.9	45°	2373.3	472.08	4385.3	55°	2982.7	729.85	5291.3
10	1815.7	280.82	3461.8	10	2383.1	475.82	4400.7	10	2993.3	734.76	5306.1
20	1824.9	283.60	3477.7	20	2392.8	479.59	4416.1	20	3003.9	739.68	5320.9
30	1834.1	286.39	3493.5	30	2402.6	483.37	4431.4	30	3014.5	744.62	5335.6
40	1843.3	289.20	3509.4	40	2412.4	487.16	4446.8	40	3025.2	749.59	5350.4
50	1852.5	292.02	3525.3	50	2422.3	490.98	4462.2	50	3035.8	754.57	5365.1
36°	1861.7	294.86	3541.1	46°	2432.1	494.82	4477.5	56°	3046.5	759.58	5379.8
10	1870.9	297.72	3557.0	10	2441.9	498.67	4492.8	10	3057.2	764.61	5394.5
20	1880.1	300.59	3572.8	20	2451.8	502.54	4508.2	20	3067.9	769.66	5409.2
30	1889.4	303.47	3588.6	30	2461.7	506.42	4523.5	30	3078.7	774.73	5423.9
40	1898.6	306.37	3604.5	40	2471.5	510.33	4538.8	40	3089.4	779.83	5438.6
50	1907.9	309.29	3620.3	50	2481.4	514.25	4554.1	50	3100.2	784.94	5453.3
37°	1917.1	312.22	3636.1	47°	2491.3	518.20	4569.4	57°	3110.9	790.08	5467.9
10	1926.4	315.17	3651.9	10	2501.2	522.16	4584.7	10	3121.7	795.24	5482.5
20	1935.7	318.13	3667.7	20	2511.2	526.13	4599.9	20	3132.6	800.42	5497.2
30	1945.0	321.11	3683.5	30	2521.1	530.13	4615.2	30	3143.4	805.62	5511.8
40	1954.3	324.11	3699.3	40	2531.1	534.15	4630.4	40	3154.2	810.85	5526.4
50	1963.6	327.12	3715.0	50	2541.0	538.18	4645.7	50	3165.1	816.10	5541.0
38°	1972.9	330.15	3730.8	48°	2551.0	542.23	4660.9	58°	3176.0	821.37	5555.6
10	1982.2	333.19	3746.5	10	2561.0	546.30	4676.1	10	3186.9	826.66	5570.2
20	1991.5	336.25	3762.3	20	2571.0	550.39	4691.3	20	3197.8	831.98	5584.7
30	2000.9	339.32	3778.0	30	2581.0	554.50	4706.5	30	3208.8	837.31	5599.3
40	2010.2	342.41	3793.8	40	2591.1	558.63	4721.7	40	3219.7	842.67	5613.8
50	2019.6	345.52	3809.5	50	2601.1	562.77	4736.9	50	3230.7	848.06	5628.3
39°	2029.0	348.64	3825.2	49°	2611.2	566.94	4752.1	59°	3241.7	853.46	5642.8
10	2038.4	351.78	3840.9	10	2621.2	571.12	4767.3	10	3252.7	858.89	5657.3
20	2047.8	354.94	3856.6	20	2631.3	575.32	4782.4	20	3263.7	864.34	5671.8
30	2057.2	358.11	3872.3	30	2641.4	579.54	4797.5	30	3274.8	869.82	5686.2
40	2066.6	361.29	3888.0	40	2651.5	583.78	4812.7	40	3285.8	875.32	5700.8
50	2076.0	364.50	3903.6	50	2661.6	588.04	4827.8	50	3296.9	880.84	5715.2
40°	2085.4	367.72	3919.3	50°	2671.8	592.32	4842.9	60°	3308.0	886.38	5729.7
10	2094.9	370.95	3935.0	10	2681.9	596.62	4858.0	10	3319.1	891.95	5744.1
20	2104.3	374.20	3950.6	20	2692.1	600.93	4873.1	20	3330.3	897.54	5758.5
30	2113.8	377.47	3966.3	30	2702.3	605.27	4888.2	30	3341.4	903.15	5772.9
40	2123.3	380.76	3981.9	40	2712.5	609.62	4903.2	40	3352.6	908.79	5787.3
50	2132.7	384.06	3997.5	50	2722.7	614.00	4918.3	50	3363.8	914.45	5801.7
41°	2142.2	387.38	4013.1	51°	2732.9	618.39	4933.4	61°	3375.0	920.14	5816.0

TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND CHORDS FOR A 1° CURVE.

Δ	Tang. *T.*	Ext. Dist. *E.*	Long Chord *LC.*	Δ	Tang. *T.*	Ext. Dist. *E.*	Long Chord *LC.*	Δ	Tang. *T.*	Ex Dis *E*
61°	3375.0	920.14	5816.0	68°	3864.7	1181.6	6408.0	75°	4396.5	1492
10′	3386.3	925.85	5830.4	10′	3876.8	1188.4	6421.8	10′	4409.8	1500
20	3397.5	931.58	5844.7	20	3889.0	1195.2	6435.6	20	4423.1	1508
30	3408.8	937.34	5859.1	30	3901.2	1202.0	6449.4	30	4436.4	1516
40	3420.1	943.12	5873.4	40	3913.4	1208.9	6463.1	40	4449.7	1524
50	3431.4	948.92	5887.7	50	3925.6	1215.8	6476.9	50	4463.1	1533
62°	3442.7	954.75	5902.0	69°	3937.9	1222.7	6490.6	76°	4476.5	1541
10	3454.1	960.60	5916.3	10	3950.2	1229.7	6504.4	10	4489.9	1549
20	3465.4	966.48	5930.5	20	3962.5	1236.7	6518.1	20	4503.4	1558
30	3476.8	972.39	5944.8	30	3974.8	1243.7	6531.8	30	4516.9	1566
40	3488.2	978.31	5959.0	40	3987.2	1250.8	6545.5	40	4530.4	1574
50	3499.7	984.27	5973.3	50	3999.5	1257.9	6559.1	50	4544.0	1583
63°	3511.1	990.24	5987.5	70°	4011.9	1265.0	6572.8	77°	4557.6	1591
10	3522.6	996.24	6001.7	10	4024.4	1272.1	6586.4	10	4571.2	1600
20	3534.1	1002.3	6015.9	20	4036.8	1279.3	6600.1	20	4584.8	1608
30	3545.6	1008.3	6030.0	30	4049.3	1286.5	6613.7	30	4598.5	1617
40	3557.2	1014.4	6044.2	40	4061.8	1293.7	6627.3	40	4612.2	1625
50	3568.7	1020.5	6058.4	50	4074.4	1300.9	6640.9	50	4626.0	1634
64°	3580.3	1026.6	6072.5	71°	4086.9	1308.2	6654.4	78°	4639.8	1643
10	3591.9	1032.8	6086.6	10	4099.5	1315.5	6668.0	10	4653.6	1651
20	3603.5	1039.0	6100.7	20	4112.1	1322.9	6681.6	20	4667.4	1660
30	3615.1	1045.2	6114.8	30	4124.8	1330.3	6695.1	30	4681.3	1669
40	3626.8	1051.4	6128.9	40	4137.4	1337.7	6708.6	40	4695.2	1678
50	3638.5	1057.7	6143.0	50	4150.1	1345.1	6722.1	50	4709.2	1688
65°	3650.2	1063.9	6157.1	72°	4162.8	1352.6	6735.6	79°	4723.2	1695
10	3661.9	1070.2	6171.1	10	4175.6	1360.1	6749.1	10	4737.2	1704
20	3673.7	1076.6	6185.2	20	4188.4	1367.6	6762.5	20	4751.2	1713
30	3685.4	1082.9	6199.2	30	4201.2	1375.2	6776.0	30	4765.3	1722
40	3697.2	1089.3	6213.2	40	4214.0	1382.8	6789.4	40	4779.4	1731
50	3709.0	1095.7	6227.2	50	4226.8	1390.4	6802.8	50	4793.6	1740
66°	3720.9	1102.2	6241.2	73°	4239.7	1398.0	6816.3	80°	4807.7	1749
10	3732.7	1108.6	6255.2	10	4252.6	1405.7	6829.6	10	4822.0	1759
20	3744.6	1115.1	6269.1	20	4265.6	1413.5	6843.0	20	4836.2	1768
30	3756.5	1121.7	6283.1	30	4278.5	1421.2	6856.4	30	4850.5	1777
40	3768.5	1128.2	6297.0	40	4291.5	1429.0	6869.7	40	4864.8	1786
50	3780.4	1134.8	5310.9	50	4304.6	1436.8	6883.1	50	4879.2	1796
67°	3792.4	1141.4	6324.8	74°	4317.6	1444.6	6896.4	81°	4893.6	1805
10	3804.4	1148.0	6338.7	10	4330.7	1452.5	6909.7	10	4908.0	1814
20	3816.4	1154.7	6352.6	20	4343.8	1460.4	6923.0	20	4922.5	1824
30	3828.4	1161.3	6366.4	30	4356.9	1468.4	6936.2	30	4937.0	1833
40	3840.5	1168.1	6380.3	40	4370.1	1476.4	6949.5	40	4951.5	1843
50	3852.6	1174.8	6394.1	50	4383.3	1484.4	6962.8	50	4966.1	1852
68°	3864.7	1181.6	6408.0	75°	4396.5	1492.4	6976.0	82°	4980.7	1862

Correction Table (always additive)

Δ	Degree of curve.											
	5°			10°			15°			2		
	T	E	LC	T	E	LC	T	E	LC	T		
10°	.03	.001	.06	.06	.003	.13	.10	.004	.17	.13	.0	
20	.06	.005	.12	.13	.011	.25	.19	.017	.38	.26	.0	
30	.09	.012	.18	.19	.025	.37	.29	.038	.56	.39	.0	
40	.13	.022	.24	.26	.046	.49	.40	.070	.74	.53	.0	
50	.16	.036	.30	.34	.075	.61	.51	.112	.92	.68	.1	
60	.20	.054	.35	.42	.111	.72	.63	.168	1.09	.84	.2	
	.24	.077	.40	.50	.159	.83	.76	.240	1.25	1.02	.3	
	29	.107	.45	.60	.220	.93	.91	.332	1.40	1.22	.4	
	35	.145	.49	.72	.298	1.02	1.09	.451	1.54	1.46	.6	

TABLE IIA. EXCESS LENGTH OF SUB CHORDS. SEE § 48.

Degree of Curve	Nominal length of sub chord.														
	10	20	30	40	45	50	55	60	65	70	75	80	85	90	95
5	.003	.006	.009	.011	.011	.012	.012	.012	.012	.011	.010	.009	.007	.005	.003
6	.005	.009	.012	.015	.016	.017	.018	.018	.017	.016	.015	.013	.011	.008	.004
7	.006	.012	.017	.021	.022	.023	.024	.024	.023	.022	.020	.018	.015	.011	.006
8	.008	.016	.022	.027	.029	.030	.031	.031	.030	.029	.027	.023	.019	.014	.008
9	.010	.020	.028	.035	.037	.038	.039	.039	.039	.037	.034	.030	.024	.018	.010
10	.013	.024	.035	.043	.046	.048	.049	.049	.048	.045	.042	.037	.030	.022	.012
11	.015	.029	.042	.052	.055	.058	.059	.059	.058	.055	.051	.044	.036	.026	.014
12	.018	.035	.050	.062	.066	.069	.070	.070	.069	.066	.060	.053	.043	.031	.017
13	.021	.041	.059	.072	.077	.080	.082	.083	.081	.077	.071	.062	.051	.037	.020
14	.025	.048	.068	.084	.090	.094	.096	.096	.094	.089	.082	.072	.059	.043	.023
15	.028	.055	.079	.097	.103	.108	.110	.110	.108	.103	.094	.083	.068	.049	.027
16	.032	.063	.089	.109	.117	.122	.125	.125	.122	.116	.107	.094	.077	.056	.030
17	.036	.071	.100	.123	.132	.138	.141	.141	.138	.131	.120	.106	.087	.063	.034
18	.041	.079	.113	.139	.148	.155	.158	.158	.155	.147	.135	.119	.097	.070	.038
19	.045	.088	.125	.154	.165	.172	.176	.177	.172	.164	.151	.132	.108	.079	.043
20	.050	.098	.139	.171	.183	.191	.195	.196	.191	.182	.167	.147	.120	.087	.047
21	.056	.108	.153	.189	.202	.211	.215	.216	.211	.200	.184	.162	.132	.096	.052
22	.061	.118	.168	.207	.221	.231	.237	.237	.231	.220	.202	.177	.145	.105	.057
23	.067	.129	.184	.226	.242	.253	.259	.259	.253	.241	.221	.194	.159	.115	.062
24	.073	.141	.201	.247	.264	.275	.282	.282	.276	.262	.241	.211	.173	.125	.068
25	.079	.153	.218	.268	.286	.299	.306	.306	.299	.284	.261	.229	.188	.136	.074
26	.085	.166	.236	.290	.310	.324	.331	.331	.324	.308	.283	.248	.203	.147	.080
27	.092	.179	.254	.313	.334	.349	.357	.357	.349	.332	.305	.268	.219	.159	.086
28	.099	.192	.273	.337	.359	.375	.384	.384	.376	.357	.328	.288	.236	.171	.093
29	.107	.207	.293	.361	.386	.403	.412	.412	.403	.383	.352	.309	.253	.183	.099
30	.114	.221	.314	.387	.413	.431	.441	.442	.432	.410	.377	.331	.271	.196	.109

TABLE III.—SWITCH LEADS AND DISTANCES.

A. TRIGONOMETRICAL FUNCTIONS OF THE FROG ANGLES.

Frog No. (n)	Frog Angle (F).	Nat. sin F.	Nat. cos F.	Log sin F.	Log cos F.	Log cot F.	Log vers F.	Frog No. (n)
5	11° 25′ 16″	.19802	.98020	$9.29670\overline{0}$	$9.99131\overline{1}$	10.69461	$8.29670\overline{0}$	5
6	9 31 38	.16552	.98621	$.21884\overline{4}$	$.99397$	$.77513$	$.13966\overline{6}$	6
7	8 10 16	.14213	.98985	$.15268\overline{3}$	$.99557$	$.84288\overline{5}$	$8.00655\overline{5}$	7
8	7 09 10	.12452	.99222	$.09522\overline{2}$	$.99660\overline{0}$	$.90138\overline{8}$	$7.89110\overline{0}$	8
9	6 21 35	.11077	.99385	9.04442	$.99732$	$.95289$	$.78915$	9
10	5 43 29	.09975	.99501	8.99891	$.99783$	10.99892	$.69787$	10
11	5 12 18	.09072	.99588	$.95770$	$.99820$	11.04050	$.61527$	11
12	4 46 19	.08319	.99653	$.92007$	$.99849$	$.07842$	$.53986$	12
14	4 05 27	.07134	.99745	$.8533\overline{1}$	$.99889$	$.14557$	$.40616$	14
15	3 49 06	.06659	.99778	$.82343$	$.99903$	$.17560$	$.34631$	15
16	3 34 47	.06244	.99805	$.79543$	$.99915$	$.20370$	$.29028$	16
18	3 10 56	.05551	.99846	$.7443\overline{8}$	$.99933$	$.25494$	$.18807$	18
20	2° 51′ 51″	.04997	.99875	8.69869	9.99945	11.30076	7.09663	20

TABLE III.—SWITCH LEADS AND DISTANCES—*Continued*

B. THEORETICAL LEADS, USING STRAIGHT POINT-RAILS AND STRAIGHT FROG RAILS; GAUGE 4′ 8½″. See §§ 305 and 312.

Frog No. (n)	Frog bluntness.	Frog				Switch Rail.				Switch Dimensions.					
		Toe length to theoret. pt. of frog. (W)		Heel to theoret. pt. of frog. (K)		Length. (S)		Angle. (α)		Radius. (r)		Degree of lead curve. (D)			Ac. pt. of sw. rail to ac. pt. frog. (L')
		ft.	in.	ft.	in.	ft.	in.	°	′	″	ft.	°	′	″	ft.
5	0·21	3	4	5	8	11	0	2	36	19	185·59	31	15	28	43·15
6	0·25	3	6	6	6	11	0	2	36	19	280·48	20	32	14	48·66
7	0·29	4	5	7	7	16	6	1	44	11	364·88	15	47	19	62·28
8	0·33	4	9	8	3	16	6	1	44	11	488·71	11	44	40	67·80
9	0·37	6	0	10	0	16	6	1	44	11	616·27	9	18	27	72·61
10	0·42	6	0	10	6	16	6	1	44	11	790·25	7	15	18	77·93
11	0·46	6	0	11	0	22	0	1	18	08	940·21	6	05	48	92·52
12	0·50	6	5	12	1	22	0	1	18	08	1136·34	5	02	38	97·75
14	0·58	7	3	14	3	22	0	1	18	08	1600·73	3	34	48	107·74
15	0·62	7	8	14	10	30	0	0	57	18	1764·69	3	14	50	126·49
16	0·67	8	0	16	0	30	0	0	57	18	2032·74	2	49	08	131·82
18	0·75	8	10	17	8	30	0	0	57	18	2632·76	2	10	35	141·98
20	0·83	9	8	19	4	30	0	0	57	18	3334·16	1	43	06	151·60

C. PRACTICAL LEADS, USING STRAIGHT POINT-RAILS AND STRAIGHT FROG RAILS; GAUGE 4′ 8½″. See §§ 305–307.

Frog No. (n)	Radius of center line. (r)	Degree of lead curve. (D)			Tangent adjacent to switch rail. (T_s)	Tangent adjacent to toe of frog. (T_f)	Actual point of switch rail to act. pt. of frog. (L')	Closure for straight rail.			Closure for curved rail.	
	ft.	°	′	″	ft.	ft.	ft.					
5	175·40	33	07	28	0·00	0·97	42·54		1–28·0			1–28·31
6	254·00	22	42	20	0·00	2·00	47·50		1–32·75	1–33		
7	361·69	15	53	30	0·00	0·22	62·08	1–26	1–14·87	1–26		1–15·12
8	487·37	11	46	36	0·32	0·00	68·00	1–30	1–16·42	1–30		1–16·58
9	605·18	9	28	42	0·00	0·57	72·28	1–33	1–16·41	1–33		1–16·59
10	779·82	7	21	08	1·56	0·00	78·75	1–28	1–27·83	2–28		
11	922·65	6	12	47	2·99	0·00	94·31	1–33	1–32·85	2–33		
12	1098·73	5	12	59	5·33	0·00	100·80	2–24	1–28·88	3–24		
14	1512·14	3	47	23	0·00	2·84	106·27	2–30	1–16·44	2–30		1–16·56
15	1748·29	3	16	40	0·00	0·51	126·19	2–30	1–27·90	2–30		1–28
16	2019·18	2	50	16	0·00	0·40	131·56	2–30	1–32·90	2–30		1–33
18	2380·47	2	24	26	0·00	6·38	138·50	2–33	1–32·92	3–33		
20	3322·13	1	43	29	0·00	0·27	151·46	2–33	1–30	2–33		1–30
									1–14·96			1–15·02

The lengths of switch rail used with each frog are the same as those ed for theoretical leads.

TABLE IV.—FUNCTIONS OF THE TEN-CHORD SPIRAL.

PART A.—Coefficients of a_1 for deflection angles to chord points.

Deflection angle to chord-point number.	Transit at chord-point number.										
	0 T. S.	1	2	3	4	5	6	7	8	9	10 S. C.
0 T. S.	0	2	8	18	32	50	72	98	128	162	200
1	1	0	5	14	27	44	65	90	119	152	189
2	4	4	0	8	20	36	56	80	108	140	176
3	9	10	7	0	11	26	45	68	95	126	161
4	16	18	16	10	0	14	32	54	80	110	144
5	25	28	27	22	13	0	17	38	63	92	125
6	36	40	40	36	28	16	0	20	44	72	104
7	49	54	55	52	45	34	19	0	23	50	81
8	64	70	72	70	64	54	40	22	0	26	56
9	81	88	91	90	85	76	63	46	25	0	29
10 S. C.	100	108	112	112	108	100	88	72	52	28	0

PART B.—Values of $\frac{U}{L}$ and $\frac{V}{L}$.

ϕ	$\frac{U}{L}$	$\frac{V}{L}$	ϕ	$\frac{U}{L}$	$\frac{V}{L}$
0°	.666 667	.333 333	23°	.672 428	.338 586
1	.666 678	.333 343	24	.672 948	.339 061
2	.666 710	.333 372	25	.673 486	.339 559
3	.666 763	.333 421	26	.674 054	.340 078
4	.666 838	.333 490	27	.674 645	.340 619
5	.666 935	.333 578	28	.675 261	.341 183
6	.667 053	.333 685	29	.675 901	.341 769
7	.667 193	.333 812	30	.676 566	.342 378
8	.667 354	.333 959	31	.677 256	.343 011
9	.667 537	.334 126	32	.677 971	.343 667
10	.667 742	.334 313	33	.678 712	.344 346
11	.667 968	.334 519	34	.679 478	.345 050
12	.668 216	.334 746	35	.680 270	.345 777
13	.668 487	.334 992	36	.681 089	.346 529
14	.668 779	.335 259	37	.681 936	.347 307
15	.669 094	.335 546	38	.682 808	.348 109
16	.669 431	.335 853	39	.683 708	.348 937
17	.669 790	.336 181	40	.684 636	.349 791
18	.670 172	.336 529	41	.685 592	.350 671
19	.670 576	.336 899	42	.686 577	.351 578
20	.671 003	.337 289	43	.687 590	.352 513
21	.671 453	.337 700	44	.688 633	.353 474
22	.671 926	.338 132	45	.689 706	.354 464

Table IV, of which Part C is condensed, was computed by the Track Committee of the American Railway Engineering Association and is taken from the Proceedings of the Association.

TABLE IV.—FUNCTIONS OF THE TEN-CHORD SPIRAL.

Part C.

Total spiral angle, ϕ	A	$\frac{C}{L}$	$\frac{X}{L}$	$\frac{Y}{L}$
0° 0′	0° 00′ 00″	1.000 000	1.000 000	.000 000
30	0 10 00	.999 997	.999 993	.002 909
1 0	0 20 00	.999 987	.999 970	.005 818
30	0 30 00	.999 970	.999 932	.008 726
2 0	0 40 00	.999 947	.999 879	.011 635
30	0 50 00	.999 916	.999 811	.014 543
3 0	1 00 00	.999 880	.999 727	.017 450
30	1 10 00	.999 836	.999 629	.020 357
4 00	1 20 00	.999 786	.999 515	.023 263
30	1 30 00	.999 729	.999 387	.026 169
5 00	1 40 00	.999 666	.999 243	.029 073
30	1 50 00	.999 596	.999 084	.031 977
6 00	1 59 59	.999 519	.998 910	.034 880
30	2 09 59	.999 435	.998 721	.037 781
7 00	2 19 59	.999 345	.998 517	.040 681
30	2 29 59	.999 248	.998 298	.043 581
8 00	2 39 58	.999 145	.998 063	.046 478
30	2 49 58	.999 035	.997 814	.049 374
9 00	2 59 58	.998 918	.997 549	.052 269
30	3 09 57	.998 794	.997 270	.055 162
10 00	3 19 57	.998 664	.996 975	.058 053
30	3 29 57	.998 527	.996 666	.060 942
11 00	3 39 56	.998 384	.996 341	.063 829
30	3 49 55	.998 233	.996 002	.066 714
12 00	3 59 55	.998 077	.995 647	.069 598
30	4 09 54	.997 913	.995 278	.072 478
13 00	4 19 53	.997 743	.994 893	.075 357
30	4 29 53	.997 566	.994 494	.078 233
14 00	4 39 52	.997 383	.994 079	.081 108
30	4 49 51	.997 192	.993 650	.083 977
15 00	4 59 50	.996 996	.993 206	.086 843
30	5 09 49	.996 792	.992 747	.089 711
16 00	5 19 48	.996 582	.992 273	.092 574
30	5 29 47	.996 366	.991 785	.095 433
17 00	5 39 45	.996 142	.991 281	.098 290
30	5 49 44	.995 912	.990 763	.101 142
18 00	5 59 43	.995 676	.990 230	.103 993
30	6 09 41	.995 432	.989 682	.106 840
19 00	6 19 40	.995 183	.989 120	.109 683
30	6 29 38	.994 926	.988 543	.112 523
20 00	6 39 36	.994 663	.987 951	.115 360
30	6 49 34	.994 393	.987 344	.118 192
21 00	6 59 32	.994 117	.986 723	.121 021
30	7 09 30	.993 834	.986 088	.123 846
22 00	7 19 28	.993 545	.985 437	.126 667
22° 30′	7° 29′ 26″	.993 248	.984 772	.129 483

TABLE IV.—FUNCTIONS OF THE TEN-CHORD SPIRAL.

PART C.—*Con.*

Total spiral angle, ϕ	A	$\frac{C}{L}$	$\frac{X}{L}$	$\frac{Y}{L}$
22° 30′	7° 29′ 26″	.993 248	.984 772	.129 483
23 00	7 39 24	.992 946	.984 093	.132 296
30	7 49 21	.992 636	.983 399	.135 105
24 00	7 59 19	.992 321	.982 691	.137 909
30	8 09 16	.991 998	.981 968	.140 708
25 00	8 19 14	.991 669	.981 231	.143 504
30	8 29 11	.991 333	.980 479	.146 294
26 00	8 39 08	.990 991	.979 714	.149 080
30	8 49 05	.990 642	.978 933	.151 861
27 00	8 59 02	.990 287	.978 139	.154 638
30	9 08 58	.989 925	.977 330	.157 409
28 00	9 18 55	.989 557	.976 508	.160 176
30	9 28 51	.989 182	.975 670	.162 937
29 00	9 38 48	.988 800	.974 819	.165 693
30	9 48 44	.988 412	.973 954	.168 444
30 00	9 58 40	.988 018	.973 074	.171 189
30	10 08 36	.987 617	.972 181	.173 929
31 00	10 18 32	.987 209	.971 273	.176 664
30	10 28 27	.986 795	.970 352	.179 392
32 00	10 38 23	.986 375	.969 417	.182 116
30	10 48 18	.985 948	.968 468	.184 833
33 00	10 58 13	.985 514	.967 504	.187 544
30	11 08 08	.985 074	.966 528	.190 250
34 00	11 18 03	.984 627	.965 537	.192 949
30	11 27 58	.984 174	.964 532	.195 643
35 00	11 37 53	.983 715	.963 515	.198 330
30	11 47 47	.983 249	.962 483	.201 010
36 00	11 57 41	.982 777	.961 438	.203 685
30	12 07 36	.982 298	.960 379	.206 353
37 00	12 17 30	.981 813	.959 306	.209 014
30	12 27 23	.981 321	.958 221	.211 669
38 00	12 37 17	.980 823	.957 121	.214 317
30	12 47 11	.980 318	.956 009	.216 959
39 00	12 57 04	.979 807	.954 883	.219 593
30	13 06 57	.979 290	.953 744	.222 221
40 00	13 16 50	.978 766	.952 591	.224 841
30	13 26 43	.978 236	.951 426	.227 455
41 00	13 36 35	.977 700	.950 247	.230 061
30	13 46 28	.977 157	.949 055	.232 660
42 00	13 56 20	.976 608	.947 850	.235 252
30	14 06 12	.976 053	.946 632	.237 836
43 00	14 16 04	.975 491	.945 402	.240 413
30	14 25 56	.974 923	.944 158	.242 982
44 00	14 35 47	.974 348	.942 901	.245 544
30	14 45 38	.973 768	.941 632	.248 098
45° 00′	14° 55′ 29″	.973 181	.940 350	.250 644

N.	0	1	2	3	4	5	6	7	8	9	P. P.
100	00 000	043	087	130	173	216	260	303	346	389	
											43 43 42 41
101	432	475	518	561	604	646	689	732	775	817	.1 4.3 4.3 4.2 4.1
102	860	902	945	987	*030	*072	*114	*157	*199	*241	.2 8.7 8.6 8.4 8.2
103	01 283	326	368	410	452	494	536	578	619	661	.3 13.0 12.9 12.6 12.3
104	703	745	787	828	870	911	953	994	*036	*077	.4 17.4 17.2 16.8 16.4
105	02 119	160	201	243	284	325	366	407	448	489	.5 21.7 21.5 21.0 20.5
106	530	571	612	653	694	735	775	816	857	898	.6 26.1 25.8 25.2 24.6
107	938	979	*019	*060	*100	*141	*181	*221	*262	*302	.7 30.4 30.1 29.4 28.7
108	03 342	382	422	463	503	543	583	623	663	703	.8 34.8 34.4 33.6 32.8
109	742	782	822	862	901	941	981	*020	*060	*100	.9 39.1 38.7 37.8 36.9
											40 40 39 38
110	04 139	178	218	257	297	336	375	415	454	493	
111	532	571	610	649	688	727	766	805	844	883	.1 4.0 4.0 3.9 3.8
112	922	960	999	*038	*076	*115	*154	*192	*231	*269	.2 8.1 8.0 7.8 7.6
113	05 308	346	384	423	461	499	538	576	614	652	.3 12.1 12.0 11.7 11.4
114	690	728	766	804	842	880	918	956	994	*032	.4 16.2 16.0 15.6 15.2
115	06 070	107	145	183	220	258	296	333	371	408	.5 20.2 20.0 19.5 19.0
116	446	483	520	558	595	632	670	707	744	781	.6 24.3 24.0 23.4 22.8
117	818	855	893	930	967	*004	*040	*077	*114	*151	.7 28.3 28.0 27.3 26.6
118	07 188	225	261	298	335	372	408	445	481	518	.8 32.4 32.0 31.2 30.4
119	554	591	627	664	700	737	773	809	845	882	.9 36.4 36.0 35.1 34.2
											37 37 36 35
120	918	954	990	*026	*062	*098	*134	*170	*206	*242	
121	08 278	314	350	386	422	457	493	529	564	600	.1 3.7 3.7 3.6 3.5
122	636	671	707	742	778	813	849	884	920	955	.2 7.5 7.4 7.2 7.0
123	990	*026	*061	*096	*131	*166	*202	*237	*272	*307	.3 11.2 11.1 10.8 10.5
124	09 342	377	412	447	482	517	552	586	621	656	.4 15.0 14.8 14.4 14.0
125	691	725	760	795	830	864	899	933	968	*002	.5 18.7 18.5 18.0 17.5
126	10 037	071	106	140	174	209	243	277	312	346	.6 22.5 22.2 21.6 21.0
127	380	414	448	483	517	551	585	619	653	687	.7 26.2 25.9 25.2 24.5
128	721	755	789	822	856	890	924	958	991	*025	.8 30.0 29.6 28.8 28.0
129	11 059	092	126	160	193	227	260	294	327	361	.9 33.7 33.3 32.4 31.5
											34 34 33 32
130	394	427	461	494	528	561	594	627	661	694	
131	727	760	793	826	859	892	925	958	991	*024	.1 3.4 3.4 3.3 3.2
132	12 057	090	123	156	189	221	254	287	320	352	.2 6.8 6.8 6.6 6.4
133	385	418	450	483	515	548	580	613	645	678	.3 10.2 10.2 9.9 9.6
134	710	743	775	807	840	872	904	937	969	*001	.4 13.6 13.6 13.2 12.8
135	13 033	065	097	130	162	194	226	258	290	322	.5 17.0 17.0 16.5 16.0
136	354	386	417	449	481	513	545	577	608	640	.6 20.4 20.4 19.8 19.2
137	672	703	735	767	798	830	862	893	925	956	.7 24.1 23.8 23.1 22.4
138	988	*019	*051	*082	*113	*145	*176	*207	*239	*270	.8 27.6 27.2 26.4 25.6
139	14 301	332	364	395	426	457	488	519	550	582	.9 31.0 30.6 29.7 28.8
											31 31 30 29
140	613	644	675	706	737	767	798	829	860	891	
141	922	952	983	*014	*045	*075	*106	*137	*167	*198	.1 3.1 3.1 3.0 2.9
142	15 229	259	290	320	351	381	412	442	473	503	.2 6.2 6.2 6.0 5.8
143	533	564	594	624	655	685	715	745	776	806	.3 9.4 9.3 9.0 8.7
144	836	866	896	926	956	987	*017	*047	*077	*107	.4 12.6 12.4 12.0 11.6
145	16 137	166	196	226	256	286	316	346	376	405	.5 15.7 15.5 15.0 14.5
146	435	465	494	524	554	584	613	643	672	702	.6 18.9 18.6 18.0 17.4
147	731	761	791	820	848	879	908	938	967	997	.7 22.0 21.7 21.0 20.3
148	17 026	055	085	114	143	172	202	231	260	289	.8 25.2 24.8 24.0 23.2
149	318	348	377	406	435	464	493	522	551	580	.9 28.3 27.9 27.0 26.1
150	609	638	667	696	725	753	782	811	840	869	

N.	0	1	2	3	4	5	6	7	8	9	P. P.

	0	1	2	3	4	5	6	7	8	9	P. P.
0	17 609	638	667	696	725	753	782	811	840	869	
											29 28 27
	897	926	955	984	*013	*041	*070	*098	*127	*156	.1 2.9 2.8 2.7
	18 184	213	241	270	298	327	355	384	412	440	.2 5.8 5.6 5.4
	469	497	526	554	582	611	639	667	695	724	.3 8.7 8.4 8.1
	752	780	808	836	864	893	921	949	977	*005	.4 11.6 11.2 10.8
	19 033	061	089	117	145	173	201	229	256	284	.5 14.5 14.0 13.5
	312	340	368	396	423	451	479	507	534	562	.6 17.4 16.8 16.2
	590	617	645	673	700	728	755	783	810	838	.7 20.3 19.6 18.9
	865	893	920	948	975	*003	*030	*057	*085	*112	.8 23.2 22.4 21.6
	20 139	167	194	221	249	276	303	330	357	385	.9 26.1 25.2 24.3
0	412	439	466	493	520	547	574	601	628	655	
											26 26
	682	709	736	763	790	817	844	871	898	924	.1 2.6 2.6
	951	978	*005	*032	*058	*085	*112	*139	*165	*192	.2 5.3 5.2
	21 219	245	272	298	325	352	378	405	431	458	.3 7.9 7.8
	484	511	537	564	590	616	643	669	695	722	.4 10.6 10.4
	748	774	801	827	853	880	906	932	958	984	.5 13.2 13.0
	22 011	037	063	089	115	141	167	193	219	245	.6 15.9 15.6
	271	297	323	349	375	401	427	453	479	505	.7 18.5 18.2
	531	557	582	608	634	660	686	711	737	763	.8 21.2 20.8
	788	814	840	865	891	917	942	968	994	*019	.9 23.8 23.4
0	23 045	070	096	121	147	172	198	223	249	274	
											25 25 24
	299	325	350	375	401	426	451	477	502	527	.1 2.5 2.5 2.4
	553	578	603	628	653	679	704	729	754	779	.2 5.1 5.0 4.8
	804	829	855	880	905	930	955	980	*005	*030	.3 7.6 7.5 7.2
	24 055	080	105	129	154	179	204	229	254	279	.4 10.2 10.0 9.6
	304	325	353	378	403	427	452	477	502	526	.5 12.7 12.5 12.0
	551	576	600	625	650	674	699	723	748	773	.6 15.3 15.0 14.4
	797	822	846	871	895	920	944	969	993	*017	.7 17.8 17.5 16.8
	25 042	066	091	115	139	164	188	212	237	261	.8 20.4 20.0 19.2
	285	309	334	358	382	406	430	455	479	503	.9 22.9 22.5 21.6
0	527	551	575	599	623	647	672	696	720	744	
											23 23
	768	792	816	840	863	887	911	935	959	983	.1 2.3 2.3
	26 007	031	055	078	102	126	150	174	197	221	.2 4.7 4.6
	245	269	292	316	340	363	387	411	434	458	.3 7.0 6.9
	482	505	529	552	576	599	623	646	670	693	.4 9.4 9.2
	717	740	764	787	811	834	858	881	904	928	.5 11.7 11.5
	951	974	998	*021	*044	*068	*091	*114	*137	*161	.6 14.1 13.8
	27 184	207	230	254	277	300	323	346	369	392	.7 16.4 16.1
	416	439	462	485	508	531	554	577	600	623	.8 18.8 18.4
	646	669	692	715	738	761	784	806	829	852	.9 21.1 20.7
0	875	898	921	944	966	989	*012	*035	*058	*080	
											22 22 21
	28 103	126	149	171	194	217	239	262	285	307	.1 2.2 2.2 2.1
	330	352	375	398	420	443	465	488	510	533	.2 4.5 4.4 4.3
	555	578	600	623	645	668	690	713	735	758	.3 6.7 6.6 6.4
	780	802	825	847	869	892	914	936	959	981	.4 9.0 8.8 8.6
	29 003	025	048	070	092	114	137	159	181	203	.5 11.2 11.0 10.7
	225	248	270	292	314	336	358	380	402	424	.6 13.5 13.2 12.9
	446	468	490	512	534	556	578	600	622	644	.7 15.7 15.4 15.0
	666	688	710	732	754	776	798	820	841	863	.8 18.0 17.6 17.2
	885	907	929	950	972	994	*016	*038	*059	*081	.9 20.2 19.8 19.3
0	30 103	124	146	168	190	211	233	254	276	298	
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200	30 103	124	146	168	190	211	233	254	276	298	
											22 21
201	319	341	363	384	406	427	449	470	492	513	.1 2.2 2.1
202	535	556	578	599	621	642	664	685	707	728	.2 4.4 4.2
203	749	771	792	813	835	856	878	899	920	941	.3 6.6 6.3
204	963	984	*005	*027	*048	*069	*090	*112	*133	*154	.4 8.8 8.4
205	31 175	196	217	239	260	281	302	323	344	365	.5 11.0 10.5
206	386	408	429	450	471	492	513	534	555	576	.6 13.2 12.6
207	597	618	639	660	681	702	722	743	764	785	.7 15.4 14.7
208	806	827	848	869	890	910	931	952	973	994	.8 17.6 16.8
209	32 014	035	056	077	097	118	139	160	180	201	.9 19.8 18.9
210	222	242	263	284	304	325	346	366	387	407	
											20 19
211	428	449	469	490	510	531	551	572	592	613	.1 2.0 1.9
212	633	654	674	695	715	736	756	776	797	817	.2 4.1 3.8
213	838	858	878	899	919	940	960	980	*001	*021	.3 6.1 5.7
214	33 041	061	082	102	122	142	163	183	203	223	.4 8.2 7.6
215	244	264	284	304	324	344	365	385	405	425	.5 10.2 9.5
216	445	465	485	505	525	546	566	586	606	626	.6 12.3 11.4
217	646	666	686	706	726	746	766	786	806	825	.7 14.3 13.3
218	845	865	885	905	925	945	965	985	*004	*024	.8 16.4 15.2
219	34 044	064	084	104	123	143	163	183	203	223	.9 18.4 17.1
220	242	262	281	301	321	341	360	380	400	419	
											19 18
221	439	459	478	498	518	537	557	576	596	615	.1 1.9 1.8
222	635	655	674	694	713	733	752	772	791	811	.2 3.9 3.6
223	830	850	869	889	908	928	947	966	986	*005	.3 5.8 5.4
224	35 025	044	063	083	102	121	141	160	179	199	.4 7.8 7.2
225	218	237	257	276	295	314	334	353	372	391	.5 9.7 9.0
226	411	430	449	468	487	507	526	545	564	583	.6 11.7 10.8
227	602	621	641	660	679	698	717	736	755	774	.7 13.6 12.6
228	793	812	831	850	869	888	907	926	945	964	.8 15.6 14.4
229	983	*002	*021	*040	*059	*078	*097	*116	*135	*154	.9 17.5 16.2
230	36 173	191	210	229	248	267	286	305	323	342	
											18 17
231	361	380	399	417	436	455	474	492	511	530	.1 1.8 1.7
232	549	567	586	605	623	642	661	679	698	717	.2 3.7 3.4
233	735	754	773	791	810	828	847	866	884	903	.3 5.5 5.1
234	921	940	958	977	996	*014	*033	*051	*070	*088	.4 7.4 6.8
235	37 107	125	143	162	180	199	217	236	254	273	.5 9.2 8.5
236	291	309	328	346	364	383	401	420	438	456	.6 11.1 10.2
237	475	493	511	530	548	566	584	603	621	639	.7 12.9 11.9
238	657	676	694	712	730	749	767	785	803	821	.8 14.8 13.6
239	840	858	876	894	912	930	948	967	985	*003	.9 16.6 15.3
240	38 021	039	057	075	093	111	129	147	165	183	
											17 16
241	201	219	237	255	273	291	309	327	345	363	.1 1.7 1.6
242	381	399	417	435	453	471	489	507	525	543	.2 3.5 3.2
243	560	578	596	614	632	650	667	685	703	721	.3 5.2 4.8
244	739	757	774	792	810	828	845	863	881	899	.4 7.0 6.4
245	916	934	952	970	987	*005	*023	*040	*058	*076	.5 8.7 8.0
246	39 093	111	129	146	164	181	199	217	234	252	.6 10.5 9.6
247	269	287	305	322	340	357	375	392	410	427	.7 12.2 11.2
248	445	462	480	497	515	532	550	567	585	602	.8 14.0 12.8
249	620	637	655	672	689	707	724	742	759	776	.9 15.7 14.4
250	794	811	828	846	863	881	898	915	933	950	
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39	794	81$\bar{1}$	82$\bar{8}$	846	86$\bar{3}$	881	898	91$\bar{5}$	933	950	
	967	98$\bar{4}$	*00$\bar{2}$	*01$\bar{9}$	*03$\bar{6}$	*054	*07$\bar{1}$	*08$\bar{8}$	*10$\bar{5}$	*123	
40	140	157	17$\bar{4}$	19$\bar{1}$	209	226	243	260	277	295	17 17
	312	329	346	36$\bar{3}$	380	398	415	432	449	466	.1 1.7 1.7
	48$\bar{3}$	500	517	534	55$\bar{1}$	569	586	603	620	637	.2 3.5 3.4
	654	671	688	705	722	739	756	773	790	807	.3 5.2 5.1
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	993	*010	*027	*044	*061	*077	*094	*11$\bar{1}$	*128	*145	.5 8.7 8.5
41	162	179	195	212	229	246	263	279	296	312	.6 10.5 10.2
	330	346	363	380	397	413	430	447	464	480	.7 12.2 11.9
											.8 14.0 13.6
	497	514	530	547	564	581	597	614	631	647	.9 15.7 15.3
	664	680	697	714	730	747	764	780	797	813	
	830	846	863	880	896	913	929	946	962	979	
	995	*012	*028	*045	*061	*078	*094	*111	*127	*144	
42	160	177	193	209	226	242	259	275	292	308	
	324	341	357	373	390	406	423	439	455	472	16 16
	488	504	521	537	553	569	586	602	618	635	.1 1.6 1.6
	651	667	683	700	716	732	748	765	781	797	.2 3.3 3.2
	813	829	846	862	878	894	910	927	943	959	.3 4.9 4.8
	975	991	*007	*023	*040	*056	*072	*088	*104	*120	.4 6.6 6.4
											.5 8.2 8.0
43	136	152	168	184	200	216	233	249	265	281	.6 9.9 9.6
											.7 11.5 11.2
	297	313	329	345	361	377	393	409	425	441	.8 13.2 12.8
	457	473	489	505	520	536	552	568	584	600	.9 14.8 14.4
	616	632	648	664	680	695	711	727	743	759	
	775	791	806	822	838	854	870	886	901	917	
	933	949	965	980	996	*012	*028	*043	*059	*075	
44	091	106	122	138	154	169	185	201	216	232	
	248	263	279	295	310	326	342	357	373	389	
	404	420	435	451	467	482	498	513	529	545	15 15
	560	576	591	607	622	638	653	669	685	700	.1 1.5 1.5
											.2 3.1 3.0
	716	731	747	762	778	793	809	824	839	855	.3 4.6 4.5
											.4 6.2 6.0
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45	025	040	055	071	086	102	117	132	148	163	.6 9.3 9.0
	178	194	209	224	240	255	270	286	301	316	.7 10.8 10.5
	332	347	362	377	393	408	423	438	454	469	.8 12.4 12.0
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	636	652	667	682	697	712	727	743	758	773	
	788	803	818	833	848	864	879	894	909	924	
	939	954	969	984	999	*014	*029	*044	*059	*075	
46	090	105	120	135	150	165	180	195	210	225	
											14 14
	240	255	269	284	299	314	329	344	359	374	.1 1.4 1.4
	38$\bar{9}$	404	419	434	449	464	479	493	508	523	.2 2.9 2.8
	538	553	568	583	597	612	627	642	657	672	.3 4.3 4.2
	687	701	716	731	746	761	775	790	805	820	.4 5.8 5.6
	834	849	864	879	894	908	923	938	952	967	.5 7.2 7.0
	982	997	*011	*026	*041	*055	*070	*085	*100	*114	.6 8.7 8.4
47	129	144	158	173	188	202	217	232	246	261	.7 10.1 9.8
	27$\bar{5}$	290	305	31$\bar{9}$	334	348	363	378	392	407	.8 11.6 11.2
	42$\bar{1}$	436	451	46$\bar{5}$	480	492	509	523	538	552	.9 13.0 12.6
	567	58$\bar{1}$	596	610	625	639	654	668	683	697	
	712	726	741	755	770	784	799	813	828	842	
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300	47 712	726	741	755	770	784	799	813	828	842	
301	856	871	885	900	914	928	943	957	972	986	
302	48 000	015	029	044	058	072	087	101	116	130	
303	144	159	173	187	201	216	230	244	259	273	
304	287	301	316	330	344	358	373	387	401	416	
305	430	444	458	472	487	501	515	529	543	558	
306	572	586	600	614	629	643	657	671	685	699	
307	714	728	742	756	770	784	798	812	827	841	
308	855	869	883	897	911	925	939	953	967	982	
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122	
310	49 136	150	164	178	192	206	220	234	248	262	
311	276	290	304	318	332	346	360	374	387	401	
312	415	429	443	457	471	485	499	513	526	540	
313	554	568	582	596	610	624	637	651	665	679	
314	693	707	720	734	748	762	776	789	803	817	
315	831	845	858	872	886	900	913	927	941	955	
316	968	982	996	*010	*023	*037	*051	*065	*079	*092	
317	50 106	119	133	147	160	174	188	201	215	229	
318	242	256	270	283	297	311	324	338	352	365	
319	379	393	406	420	433	447	460	474	488	501	
320	515	528	542	555	569	583	595	610	623	637	
321	650	664	677	691	704	718	731	745	758	772	
322	785	799	812	826	839	853	866	880	893	907	
323	920	933	947	960	974	987	*001	*014	*027	*041	
324	51 054	068	081	094	108	121	135	148	161	175	
325	188	201	215	228	242	255	268	282	295	308	
326	322	335	348	361	375	388	401	415	428	441	
327	455	468	481	494	508	521	534	547	561	574	
328	587	600	614	627	640	653	667	680	693	706	
329	719	733	746	759	772	785	798	812	825	838	
330	851	864	877	891	904	917	930	943	956	969	
331	983	996	*009	*022	*035	*048	*061	*074	*087	*100	
332	52 114	127	140	153	166	179	192	205	218	231	
333	244	257	270	283	296	309	322	335	348	361	
334	374	387	400	413	426	439	452	465	478	491	
335	504	517	530	543	556	569	582	595	608	621	
336	634	647	660	673	686	699	711	724	737	750	
337	763	776	789	801	814	827	840	853	866	879	
338	891	904	917	930	943	956	968	981	994	*007	
339	53 020	033	045	058	071	084	097	109	122	135	
340	148	160	173	186	199	211	224	237	250	262	
341	275	288	301	313	326	339	352	364	377	390	
342	402	415	428	440	453	466	478	491	504	516	
343	529	542	554	567	580	592	605	618	630	643	
344	656	668	681	694	706	719	732	744	757	769	
345	782	794	807	819	832	845	857	870	882	895	
346	907	920	932	945	958	970	983	995	*008	*020	
347	54 033	045	058	070	083	095	108	120	133	145	
348	158	170	183	195	208	220	232	245	257	270	
349	282	295	307	320	332	344	357	369	382	394	
350	407	419	431	444	456	469	481	493	506	518	
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54	407	419	431	444	456	469	481	493	506	518	
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	530	543	555	568	580	592	605	617	629	642	.1 1.2
	654	666	679	691	705	716	728	740	753	765	.2 2.5
	777	790	802	814	826	839	851	863	876	888	.3 3.7
	900	912	925	937	949	961	974	986	998	*010	.4 5.0
55	023	035	047	059	071	084	096	108	120	133	.5 6.2
	145	157	169	181	194	206	218	230	242	254	.6 7.5
	267	279	291	305	315	327	340	352	364	376	.7 8.7
	388	400	412	424	437	449	461	473	485	497	.8 10.0
	509	521	533	545	558	570	582	594	606	618	.9 11.2
	630	642	654	666	678	690	702	714	726	738	
											12
	750	762	775	787	799	811	823	835	847	859	.1 1.2
	871	883	895	907	919	931	943	955	966	978	.2 2.4
	990	*002	*014	*026	*038	*050	*062	*074	*086	*098	.3 3.6
56	110	122	134	146	158	170	181	193	205	217	.4 4.8
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	585	596	608	620	632	643	655	667	679	691	.8 9.6
	702	714	726	738	749	761	773	785	796	808	.9 10.8
	820	832	843	855	867	879	890	902	914	925	
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	937	949	961	972	984	996	*007	*019	*031	*042	.1 1.1
57	054	066	077	089	101	112	124	136	147	159	.2 2.3
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	287	299	310	322	333	345	357	368	380	391	.4 4.6
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	519	530	542	553	565	576	588	599	611	622	.6 6.9
	634	645	657	668	680	691	703	714	726	737	.7 8.0
	749	760	772	783	795	806	818	829	841	852	.8 9.2
	864	875	887	898	908	921	932	944	955	967	.9 10.3
	978	990	*001	*012	*024	*035	*047	*058	*069	*081	
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58	092	104	115	126	138	149	161	172	183	195	.1 1.1
	206	217	229	240	252	263	274	286	297	308	.2 2.2
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	883	894	905	916	928	939	950	961	972	984	.8 8.8
	995	*006	*017	*028	*039	*050	*062	*073	*084	*095	.9 9.9
59	106	117	128	140	151	162	173	184	195	206	
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	217	229	240	251	262	273	284	295	306	317	.1 1.0
	328	339	351	362	373	384	395	406	417	428	.2 2.1
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	549	560	571	582	593	604	615	626	637	648	.4 4.2
	659	670	681	692	703	714	725	736	747	758	.5 5.2
	769	780	791	802	813	824	835	846	857	868	.6 6.3
	879	890	901	912	923	933	944	955	966	977	.7 7.3
	988	999	*010	*021	*032	*043	*053	*064	*075	*086	.8 8.4
60	097	108	119	130	141	151	162	173	184	195	.9 9.4
	206	217	227	238	249	260	271	282	293	303	
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400	60 206	217	227	238	249	260	271	282	293	303	
401	314	325	336	347	357	368	379	390	401	412	
402	423	433	444	455	466	476	487	498	509	519	
403	530	541	552	563	573	584	595	606	616	627	
404	638	649	659	670	681	692	702	713	724	735	11
405	745	756	767	777	788	799	810	820	831	842	.1 | 1.1
406	852	863	874	884	895	906	916	927	938	949	.2 | 2.2
407	959	970	981	991	*002	*013	*023	*034	*044	*055	.3 | 3.3
408	61 066	076	087	098	108	119	130	140	151	161	.4 | 4.4
409	172	183	193	204	215	225	236	246	257	268	.5 | 5.5
											.6 | 6.6
410	278	289	299	310	320	331	342	352	363	373	.7 | 7.7
											.8 | 8.8
411	384	394	405	416	426	437	447	458	468	479	.9 | 9.9
412	489	500	511	521	532	542	553	563	574	584	
413	595	605	616	626	637	647	658	668	679	689	
414	700	710	721	731	742	752	763	773	784	794	
415	805	815	825	836	846	857	867	878	888	899	10
416	909	920	930	940	951	961	972	982	993	*003	.1 | 1.0
417	62 013	024	034	045	055	065	076	086	097	107	.2 | 2.1
418	117	128	138	149	159	169	180	190	200	211	.3 | 3.1
419	221	232	242	252	263	273	283	294	304	314	.4 | 4.2
											.5 | 5.2
420	325	335	345	356	366	376	387	397	407	418	.6 | 6.3
											.7 | 7.3
421	428	438	449	459	469	480	490	500	510	521	.8 | 8.4
422	531	541	552	562	572	582	593	603	613	624	.9 | 9.4
423	634	644	654	665	675	685	695	706	716	726	
424	737	747	757	767	777	788	798	808	818	828	
425	839	849	859	869	879	890	900	910	920	931	
426	941	951	961	971	981	992	*002	*012	*022	*032	
427	63 043	053	063	073	083	093	104	114	124	134	10
428	144	154	164	175	185	195	205	215	225	235	.1 | 1.0
429	245	256	266	276	286	296	306	316	326	336	.2 | 2.0
											.3 | 3.0
430	347	357	367	377	387	397	407	417	427	437	.4 | 4.0
											.5 | 5.0
431	447	458	468	478	488	498	508	518	528	538	.6 | 6.0
432	548	558	568	578	588	598	608	618	628	639	.7 | 7.0
433	649	659	669	679	689	699	709	719	729	739	.8 | 8.0
434	749	759	769	779	789	799	809	819	829	839	.9 | 9.0
435	849	859	869	879	889	899	909	919	928	938	
436	948	958	968	978	988	998	*008	*018	*028	*038	
437	64 048	058	068	078	088	098	107	117	127	137	
438	147	157	167	177	187	197	207	217	226	236	
439	246	256	266	276	286	296	306	315	325	335	
											9
440	345	355	365	375	384	394	404	414	424	434	.1 | 0.9
											.2 | 1.8
441	444	453	463	473	483	493	503	512	522	532	.3 | 2.7
442	542	552	562	571	581	591	601	611	621	630	.4 | 3.6
443	640	650	660	670	679	689	699	709	718	728	.5 | 4.5
444	738	748	758	767	777	787	797	806	816	826	.6 | 5.4
445	836	846	855	865	875	885	894	904	914	923	.7 | 6.3
446	933	943	953	962	972	982	992	*001	*011	*021	.8 | 7.2
447	65 031	040	050	060	069	079	089	098	108	118	.9 | 8.1
448	128	137	147	157	166	176	186	195	205	215	
449	224	234	244	253	263	273	282	292	302	311	
450	321	331	340	350	360	365	379	389	398	408	
	0	1	2	3	4	5	6	7	8	9	P.P.

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65	32$\bar{1}$	331	340	350	360	369	379	389	398	408	
	417	427	437	446	456	466	475	485	494	504	
	514	523	533	542	552	562	571	581	590	600	**10**
	610	619	629	638	648	657	667	677	686	696	·1 | 1·0
	705	715	724	734	744	753	763	772	782	791	·2 | 2·0
	801	810	820	830	839	849	858	868	877	887	·3 | 3·0
	896	906	915	925	934	944	953	963	972	982	·4 | 4·0
	991	*001	*010	*020	*029	*039	*048	*058	*067	*077	·5 | 5·0
66	086	096	105	115	124	134	143	153	162	172	·6 | 6·0
	181	190	200	209	219	228	238	247	257	266	·7 | 7·0
											·8 | 8·0
	276	285	294	304	313	323	332	342	351	360	·9 | 9·0
	370	379	389	398	408	417	426	436	445	455	
	464	473	483	492	502	511	520	530	539	548	
	558	567	577	586	595	605	614	623	633	642	
	652	661	670	680	689	698	708	717	726	736	
	745	754	764	773	782	792	801	810	820	829	**9**
	838	848	857	867	876	885	894	904	913	922	·1 | 0·9
	931	941	950	959	969	978	987	996	*006	*015	·2 | 1·8
67	024	034	043	052	061	071	080	089	099	108	·3 | 2·7
	117	126	136	145	154	163	173	182	191	200	·4 | 3·6
											·5 | 4·5
	210	219	228	237	246	256	265	274	283	293	·6 | 5·4
											·7 | 6·3
	302	311	320	329	339	348	357	366	376	385	·8 | 7·2
	394	403	412	422	431	440	449	458	467	477	·9 | 8·1
	486	495	504	513	523	532	541	550	559	568	
	578	587	596	605	614	623	633	642	651	660	
	669	678	687	697	706	715	724	733	742	751	
	760	770	779	788	797	806	815	824	833	842	
	852	861	870	879	888	897	906	915	924	933	
	943	952	961	970	979	988	997	*006	*015	*024	
68	033	042	051	060	070	079	088	097	106	115	**9**
											·1 | 0·9
	124	133	142	151	160	169	178	187	196	205	·2 | 1·8
											·3 | 2·7
	214	223	232	241	250	259	268	277	286	295	·4 | 3·6
	304	313	322	331	340	349	358	367	376	385	·5 | 4·5
	394	403	412	421	430	439	448	457	466	475	·6 | 5·4
	484	493	502	511	520	529	538	547	556	565	·7 | 6·3
	574	583	592	601	610	619	628	637	646	654	·8 | 7·2
	663	672	681	690	699	708	717	726	735	744	·9 | 8·1
	753	762	771	779	788	797	806	815	824	833	
	842	851	860	868	877	886	895	904	913	922	
	931	940	948	957	966	975	984	993	*002	*010	
											8
69	019	028	037	046	055	064	073	081	090	099	·1 | 0·8
	108	117	126	134	143	152	161	170	179	187	·2 | 1·7
	196	205	214	223	232	240	249	258	267	276	·3 | 2·5
	284	293	302	311	320	328	337	346	355	364	·4 | 3·4
	372	381	390	399	408	416	425	434	443	451	·5 | 4·2
	460	469	478	487	495	504	513	522	530	539	·6 | 5·1
	548	557	565	574	583	592	600	609	618	627	·7 | 5·9
	635	644	653	662	670	679	688	697	705	714	·8 | 6·8
	723	731	740	749	758	766	775	784	792	801	·9 | 7·6
	810	819	827	836	845	853	862	871	879	888	
	897	905	914	923	931	940	949	958	966	975	
	0	1	2	3	4	5	6	7	8	9	P. P.

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500	69 897	905	914	923	931	940	949	958	966	975	
501	984	992	*001	*010	*018	*027	*036	*044	*053	*061	
502	70 070	079	087	096	105	113	122	131	139	148	9
503	157	165	174	183	191	200	208	217	226	234	.1 0.9
504	243	251	260	269	277	286	294	303	312	320	.2 1.8
505	329	337	346	355	363	372	380	389	398	406	.3 2.7
506	415	423	432	441	449	458	466	475	483	492	.4 3.6
507	501	509	518	526	535	543	552	560	569	578	.5 4.5
508	586	595	603	612	620	629	637	646	654	663	.6 5.4
509	672	680	689	697	706	714	723	731	740	748	.7 6.3
510	757	765	774	782	791	799	808	816	825	833	.8 7.2 .9 8.1
511	842	850	859	867	876	884	893	901	910	918	
512	927	935	944	952	961	969	978	986	995	*003	
513	71 011	020	028	037	045	054	062	071	079	088	
514	096	105	113	121	130	138	147	155	164	172	
515	180	189	197	206	214	223	231	239	248	256	8
516	265	273	282	290	298	307	315	324	332	340	.1 0.8
517	349	357	366	374	382	391	399	408	416	424	.2 1.6
518	433	441	449	458	466	475	483	491	500	508	.3 2.4
519	516	525	533	542	550	558	567	575	583	592	.4 3.2 .5 4.0
520	600	608	617	625	633	642	650	659	667	675	.6 4.8 .7 5.6
521	684	692	700	709	717	725	734	742	750	758	.8 6.4 .9 7.2
522	767	775	783	792	800	808	817	825	833	842	
523	850	858	867	875	883	891	900	908	916	925	
524	933	941	949	958	966	974	983	991	999	*007	
525	72 016	024	032	040	049	057	065	074	082	090	
526	098	107	115	123	131	140	148	156	164	173	
527	181	189	197	206	214	222	230	238	247	255	
528	263	271	280	288	296	304	312	321	329	337	8
529	345	354	362	370	378	386	395	403	411	419	.1 0.8 .2 1.6
530	427	436	444	452	460	468	476	485	493	501	.3 2.4 .4 3.2
531	505	517	526	534	542	550	558	566	575	583	.5 4.0 .6 4.8
532	591	599	607	615	624	632	640	648	656	664	.7 5.6 .8 6.4
533	672	681	689	697	705	713	721	729	738	746	.9 7.2
534	754	762	770	778	786	795	803	811	819	827	
535	835	843	851	859	868	876	884	892	900	908	
536	916	924	932	941	949	957	965	973	981	989	
537	997	*005	*013	*021	*030	*038	*046	*054	*062	*070	
538	73 078	086	094	102	110	118	126	134	143	151	
539	159	167	175	183	191	199	207	215	223	231	
540	239	247	255	263	271	279	287	295	303	311	7
541	319	328	336	344	352	360	368	376	384	392	.1 0.7 .2 1.4
542	400	408	416	424	432	440	448	456	464	472	.3 2.1 .4 2.8
543	480	488	496	504	512	520	528	536	544	552	.5 3.0 .6 3.7
544	560	568	576	584	592	600	608	615	623	631	.7 4.5 .8 5.2
545	639	647	655	663	671	679	687	695	703	711	.9 6.0
546	719	727	735	743	751	759	767	775	783	791	
547	798	806	814	822	830	838	846	854	862	870	
548	878	886	894	902	909	917	925	933	941	949	
549	957	965	973	981	989	997	*004	*012	*020	*028	
550	74 036	044	052	060	068	075	083	091	099	107	

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	0	1	2	3	4	5	6	7	8	9	P. P.
74	0386	044	052	060	068	0755	0853	0911	0990	1077	
	115	123	131	139	1465	1544	1622	1700	178	186	
	194	202	209	217	225	233	241	249	257	264	
	272	280	288	296	304	312	319	327	335	343	
	351	359	366	374	382	390	398	406	413	421	
	429	437	445	453	460	468	476	484	492	499	
	507	515	523	531	538	546	554	562	570	577	
	585	593	601	609	616	624	632	640	648	655	8
	663	671	679	687	694	702	710	718	725	733	.1 0.8
	741	749	756	764	772	780	788	795	803	811	.2 1.6
											.3 2.4
	819	826	834	842	850	857	865	873	881	888	.4 3.2
											.5 4.0
	896	904	912	919	927	935	942	950	958	966	.6 4.8
	973	981	989	997	*004	*012	*020	*027	*035	*043	.7 5.6
75	051	058	066	074	081	089	097	105	112	120	.8 6.4
	128	135	143	151	158	166	174	182	189	197	.9 7.2
	205	212	220	228	235	243	251	258	266	274	
	281	289	297	304	312	320	327	335	343	350	
	358	366	373	381	389	396	404	412	419	427	
	435	442	450	458	465	473	480	488	496	503	
	511	519	526	534	541	549	557	564	572	580	
	587	595	602	610	618	625	633	641	648	656	
											7
	663	671	679	686	694	701	709	717	724	732	.1 0.7
	739	747	755	762	770	777	785	792	800	808	.2 1.5
	815	823	830	838	846	853	861	868	876	883	.3 2.2
	891	899	906	914	921	929	936	944	951	959	.4 3.0
	967	974	982	989	997	*004	*012	*019	*027	*034	.5 3.7
76	042	050	057	065	072	080	087	095	102	110	.6 4.5
	117	125	132	140	147	155	162	170	178	185	.7 5.2
	193	200	208	215	223	230	238	245	253	260	.8 6.0
	268	275	283	290	298	305	313	320	328	335	.9 6.7
	343	350	358	365	372	380	387	395	402	410	
	417	425	432	440	447	455	462	470	477	485	
	492	500	507	514	522	529	537	544	552	559	
	567	574	582	589	596	604	611	619	626	634	
	641	648	656	663	671	678	686	693	700	708	
	715	723	730	738	745	752	760	767	775	782	
	790	797	804	812	819	827	834	841	849	856	
	864	871	878	886	893	901	908	915	923	930	7
	937	945	952	960	967	974	982	989	997	*004	.1 0.7
77	011	019	026	033	041	048	055	063	070	078	.2 1.4
											.3 2.1
	085	092	100	107	114	122	129	136	144	151	.4 2.8
											.5 3.5
	158	166	173	181	188	195	203	210	217	225	.6 4.2
	232	239	247	254	261	269	276	283	291	298	.7 4.9
	305	313	320	327	335	342	349	356	364	371	.8 5.6
	378	386	393	400	408	415	422	430	437	444	.9 6.3
	451	459	466	473	481	488	495	503	510	517	
	524	532	539	546	554	561	568	575	583	590	
	597	604	612	619	626	634	641	648	655	663	
	670	677	684	692	699	706	713	721	728	735	
	742	750	757	764	771	779	786	793	800	808	
	815	822	829	837	844	851	858	866	873	880	

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N.	0	1	2	3	4	5	6	7	8	9	P. P.
600	77 815	822	829	837	844	851	858	866	873	880	
601	887	894	902	909	916	923	931	938	945	952	
602	959	967	974	981	988	995	*003	*010	*017	*024	
603	78 031	039	046	053	060	067	075	082	089	096	
604	103	111	118	125	132	139	147	154	161	168	
605	175	182	190	197	204	211	218	226	233	240	
606	247	254	261	269	276	283	290	297	304	311	
607	319	326	333	340	347	354	362	369	376	383	7
608	390	397	404	412	419	426	433	440	447	454	·1 0·7
609	461	469	476	483	490	497	504	511	518	526	·2 1·5
											·3 2·2
610	533	540	547	554	561	568	575	583	590	597	·4 3·0
											·5 3·7
611	604	611	618	625	632	639	646	654	661	668	·6 4·5
612	675	682	689	696	703	710	717	725	732	739	·7 5·2
613	746	753	760	767	774	781	788	795	802	810	·8 6·0
614	817	824	831	838	845	852	859	866	873	880	·9 6·7
615	887	894	901	908	915	923	930	937	944	951	
616	958	965	972	979	986	993	*000	*007	*014	*021	
617	79 028	035	042	049	056	063	070	078	085	092	
618	099	106	113	120	127	134	141	148	155	162	
619	169	176	183	190	197	204	211	218	225	232	
620	239	246	253	260	267	274	281	288	295	302	
621	309	316	323	330	337	344	351	358	365	372	7
622	379	386	393	400	407	414	421	428	435	442	·1 0·7
623	449	456	462	469	476	483	490	497	504	511	·2 1·4
624	518	525	532	539	546	553	560	567	574	581	·3 2·1
625	588	595	602	609	616	622	629	636	643	650	·4 2·8
626	657	664	671	678	685	692	699	706	713	720	·5 3·5
627	727	733	740	747	754	761	768	775	782	789	·6 4·2
628	796	803	810	816	823	830	837	844	851	858	·7 4·9
629	865	872	879	886	892	899	906	913	920	927	·8 5·6
											·9 6·3
630	934	941	948	954	961	968	975	982	989	996	
631	80 003	010	017	023	030	037	044	051	058	065	
632	071	078	085	092	099	106	113	120	126	133	
633	140	147	154	161	168	174	181	188	195	202	
634	209	216	222	229	236	243	250	257	263	270	
635	277	284	291	298	304	311	318	325	332	339	
636	345	352	359	366	373	380	386	393	400	407	
637	414	421	427	434	441	448	455	461	468	475	6
638	482	489	495	502	509	516	523	529	536	543	·1 0·6
639	550	557	563	570	577	584	591	597	604	611	·2 1·2
											·3 1·8
640	618	625	631	638	645	652	658	665	672	679	·4 2·4
											·5 3·0
641	686	692	699	706	713	719	726	733	740	746	·6 3·6
642	753	760	767	774	780	787	794	801	807	814	·7 4·2
643	821	828	834	841	848	855	861	868	875	882	·8 4·8
644	888	895	902	909	915	922	929	936	942	949	·9 5·4
645	956	962	969	976	983	989	996	*003	*010	*016	
646	81 023	030	036	043	050	057	063	070	077	083	
647	090	097	104	110	117	124	130	137	144	151	
648	157	164	171	177	184	191	197	204	211	218	
649	224	231	238	244	251	258	264	271	278	284	
650	291	298	304	311	318	324	331	338	345	351	
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81	291	298	304	311	318	324	331	338	345	351	
	358	365	371	378	385	391	398	405	411	418	
	425	431	438	444	451	458	464	471	478	484	
	491	498	504	511	518	524	531	538	544	551	
	558	564	571	577	584	591	597	604	611	617	
	624	631	637	644	650	657	664	670	677	684	
	690	697	703	710	717	723	730	736	743	750	
	756	763	770	776	783	789	796	803	809	816	7
	822	829	836	842	849	855	862	869	875	882	.1 | 0·7
	888	895	901	908	915	921	928	934	941	948	.2 | 1·4
											.3 | 2·1
	954	961	967	974	980	987	994	*000	*007	*013	.4 | 2·8
82	020	026	033	040	046	053	059	066	072	079	.5 | 3·5
	086	092	099	105	112	118	125	131	138	145	.6 | 4·2
	151	158	164	171	177	184	190	197	203	210	.7 | 4·9
	217	223	230	236	243	249	256	262	269	275	.8 | 5·6
	282	288	295	302	308	315	321	328	334	341	.9 | 6·3
	347	354	360	367	373	380	386	393	399	406	
	412	419	425	432	438	445	451	458	464	471	
	477	484	490	497	503	510	516	523	529	536	
	542	549	555	562	568	575	581	588	594	601	
	607	614	620	627	633	640	646	653	659	666	
	672	678	685	691	698	704	711	717	724	730	6
	737	743	750	756	763	769	775	782	788	795	.1 | 0·6
	801	808	814	821	827	834	840	846	853	859	.2 | 1·3
	866	872	879	885	892	898	904	911	917	924	.3 | 1·9
	930	937	943	949	956	962	969	975	982	988	.4 | 2·6
	994	*001	*007	*014	*020	*027	*033	*039	*046	*052	.5 | 3·2
83	059	065	071	078	084	091	097	103	110	116	.6 | 3·9
	123	129	136	142	148	155	161	168	174	180	.7 | 4·5
	187	193	200	206	212	219	225	231	238	244	.8 | 5·2
											.9 | 5·8
	251	257	263	270	276	283	289	295	302	308	
	314	321	327	334	340	346	353	359	365	372	
	378	385	391	397	404	410	416	423	429	435	
	442	448	455	461	467	474	480	486	493	499	
	505	512	518	524	531	537	543	550	556	562	
	569	575	581	588	594	600	607	613	619	626	
	632	638	645	651	657	664	670	676	683	689	
	695	702	708	714	721	727	733	740	746	752	
	759	765	771	778	784	790	796	803	809	815	6
	822	828	834	841	847	853	859	866	872	878	.1 | 0·6
											.2 | 1·2
	885	891	897	904	910	916	922	929	935	941	.3 | 1·8
											.4 | 2·4
	948	954	960	966	973	979	985	992	998	*004	.5 | 3·0
84	010	017	023	029	035	042	048	054	061	067	.6 | 3·6
	073	079	086	092	098	104	111	117	123	129	.7 | 4·2
	136	142	148	154	161	167	173	179	186	192	.8 | 4·8
	198	204	211	217	223	229	236	242	248	254	.9 | 5·4
	261	267	273	279	286	292	298	304	311	317	
	323	329	335	342	348	354	360	367	373	379	
	385	392	398	404	410	416	423	429	435	441	
	447	454	460	466	472	479	485	491	497	503	
	510	516	522	528	534	541	547	553	559	565	
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700	34 510	516	522	528	534	541	547	553	559	565	
701	572	578	584	590	595	603	609	615	621	627	
702	633	640	646	652	658	664	671	677	683	689	
703	695	701	708	714	720	726	732	739	745	751	
704	757	763	769	776	782	788	794	800	806	813	
705	819	825	831	837	843	849	856	862	868	874	
706	880	886	893	899	905	911	917	923	929	936	
707	942	948	954	930	966	972	979	985	991	997	
708	85 003	009	015	021	028	034	040	046	052	058	
709	064	070	077	083	089	095	101	107	113	119	
710	126	132	138	144	150	156	162	168	174	181	
711	187	193	199	205	211	217	223	229	236	242	
712	248	254	260	266	272	278	284	290	297	303	
713	309	315	321	327	333	339	345	351	357	363	
714	370	376	382	388	394	400	406	412	418	424	
715	430	436	443	449	455	461	467	473	479	485	
716	491	497	503	509	515	521	527	533	540	546	
717	552	558	564	570	576	582	588	594	600	606	
718	612	618	624	630	636	642	648	655	661	667	
719	673	679	685	691	697	703	709	715	721	727	
720	733	739	745	751	757	763	769	775	781	787	
721	793	799	805	811	817	823	829	835	841	847	
722	853	859	865	872	878	884	890	896	902	908	
723	914	920	926	932	938	944	950	956	962	968	
724	974	980	986	992	998	*004	*010	*016	*022	*028	
725	86 034	040	046	052	058	063	069	075	081	087	
726	093	099	105	111	117	123	129	135	141	147	
727	153	159	165	171	177	183	189	195	201	207	
728	213	219	225	231	237	243	249	255	261	267	
729	273	278	284	290	296	302	308	314	320	326	
730	332	338	344	350	356	362	368	374	380	386	
731	391	397	403	409	415	421	427	433	439	445	
732	451	457	463	469	475	481	486	492	498	504	
733	510	516	522	528	534	540	546	552	558	563	
734	569	575	581	587	593	599	605	611	617	623	
735	628	634	640	646	652	658	664	670	676	682	
736	688	693	699	705	711	717	723	729	735	741	
737	746	752	758	764	770	776	782	788	794	800	
738	805	811	817	823	829	835	841	847	852	858	
739	864	870	876	882	888	894	895	905	911	917	
740	923	929	935	941	946	952	958	964	970	976	
741	982	987	993	999	*005	*011	*017	*023	*028	*034	
742	87 040	046	052	058	064	069	075	081	087	093	
743	099	104	110	116	122	128	134	140	145	151	
744	157	163	169	175	180	186	192	198	204	210	
745	215	221	227	233	239	245	250	256	262	268	
746	274	279	285	291	297	303	309	314	320	326	
747	332	338	343	349	355	361	367	372	378	384	
748	390	396	402	407	413	419	425	431	436	442	
749	448	454	460	465	471	477	483	489	494	500	
750	506	512	517	523	529	535	541	548	552	558	
N.	0	1	2	3	4	5	6	7	8	9	P. P.

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0	87 506	512	517	523	529	535	541	546	552	558	
	564	570	575	581	587	593	598	604	610	616	
	622	627	633	639	645	650	656	662	668	673	
	679	685	691	697	702	708	714	720	725	731	
	737	743	748	754	760	766	771	777	783	789	
	794	800	806	812	817	823	829	835	840	846	
	852	858	863	869	875	881	886	892	898	904	
	909	915	921	927	932	938	944	949	955	961	6
	967	972	978	984	990	995	*001	*007	*012	*018	.1 | 0·6
	88 024	030	035	041	047	053	058	064	070	075	.2 | 1·2
0	081	087	093	098	104	110	115	121	127	133	.3 | 1·8
	138	144	150	155	161	167	172	178	184	190	.4 | 2·4
	195	201	207	212	218	224	229	235	241	247	.5 | 3·0
	252	258	264	269	275	281	286	292	298	303	.6 | 3·6
	309	315	320	326	332	337	343	349	355	360	.7 | 4·2
	366	372	377	383	389	394	400	406	411	417	.8 | 4·8
	423	428	434	440	445	451	457	462	468	474	.9 | 5·4
	479	485	491	496	502	508	513	519	525	530	
	536	542	547	553	558	564	570	575	581	587	
	592	598	604	609	615	621	626	632	638	643	
)	649	654	660	666	671	677	683	688	694	700	
	705	711	716	722	728	733	739	745	750	756	$\overline{5}$
	761	767	773	778	784	790	795	801	806	812	.1 | 0·5
	818	823	829	835	840	846	851	857	863	868	.2 | 1·1
	874	879	885	891	896	902	907	913	919	924	.3 | 1·6
	930	936	941	947	952	958	964	969	975	980	.4 | 2·2
	986	992	997	*003	*008	*014	*019	*025	*081	*036	.5 | 2·7
	89 042	047	053	059	064	070	075	081	087	092	.6 | 3·3
	098	103	109	114	120	126	131	137	142	148	.7 | 3·8
	153	159	165	170	176	181	187	193	198	204	.8 | 4·4
)	209	215	220	226	231	237	243	248	254	259	.9 | 4·9
	265	270	276	282	287	293	298	304	309	315	
	320	326	332	337	343	348	354	359	365	370	
	376	381	387	393	398	404	409	415	420	426	
	431	437	442	448	454	459	465	470	476	481	
	487	492	498	503	509	514	520	525	531	536	
	542	548	553	559	564	570	575	581	586	592	
	597	603	608	614	619	625	630	636	641	647	5
	652	658	663	669	674	680	685	691	696	702	.1 | 0·5
	707	713	718	724	729	735	740	746	751	757	.2 | 1·0
)	762	768	773	779	784	790	795	801	806	812	.3 | 1·5
	817	823	828	834	839	845	850	856	861	867	.4 | 2·0
	872	878	883	889	894	900	905	911	916	922	.5 | 2·5
	927	933	938	943	949	954	960	965	971	976	.6 | 3·0
	982	987	993	998	*004	*009	*015	*020	*026	*031	.7 | 3·5
	90 036	042	047	053	058	064	069	075	080	086	.8 | 4·0
	091	097	102	107	113	118	124	129	135	140	.9 | 4·5
	146	151	156	162	167	173	178	184	189	195	
	200	205	211	216	222	227	233	238	244	249	
	254	260	265	271	276	282	287	292	298	303	
	309	314	320	325	330	336	341	347	352	358	
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N.	0	1	2	3	4	5	6	7	8	9	P. P.
800	90 309	314	320	325	330	336	341	347	352	358	
801	363	368	374	379	385	390	396	401	406	412	
802	417	423	428	433	439	444	450	455	460	466	
803	471	477	482	488	493	498	504	509	515	520	
804	525	531	536	542	547	552	558	563	569	574	
805	579	585	590	596	601	606	612	617	622	628	
806	633	639	644	649	655	660	666	671	675	682	
807	687	692	698	703	709	714	719	725	730	736	
808	741	746	752	757	762	768	773	778	784	789	
809	795	800	805	811	816	821	827	832	838	843	
810	848	854	859	864	870	875	880	886	891	896	
811	902	907	913	918	923	929	934	939	945	950	
812	955	961	966	971	977	982	987	993	998	*003	
813	91 009	014	019	025	030	036	041	046	052	057	
814	062	068	073	078	084	089	094	100	105	110	
815	116	121	126	131	137	142	147	153	158	163	
816	169	174	179	185	190	195	201	206	211	217	
817	222	227	233	238	243	249	254	259	264	270	
818	275	280	286	291	296	302	307	312	318	323	
819	328	333	339	344	349	355	360	365	371	376	
820	381	386	392	397	402	408	413	418	423	429	
821	434	439	445	450	455	461	466	471	476	482	
822	487	492	497	503	508	513	519	524	529	534	
823	540	545	550	556	561	566	571	577	582	587	
824	592	598	603	608	614	619	624	629	635	640	
825	645	650	656	661	666	671	677	682	687	692	
826	698	703	708	714	719	724	729	735	740	745	
827	750	756	761	766	771	777	782	787	792	798	
828	803	808	813	819	824	829	834	839	845	850	
829	855	860	866	871	876	881	887	892	897	902	
830	908	913	918	923	928	934	939	944	949	955	
831	960	965	970	976	981	986	991	996	*002	*007	
832	92 012	017	023	028	033	038	043	049	054	059	
833	064	069	075	080	085	090	096	101	106	111	
834	116	122	127	132	137	142	148	153	158	163	
835	168	174	179	184	189	194	200	205	210	215	
836	220	226	231	236	241	246	252	257	262	267	
837	272	277	283	288	293	298	303	309	314	319	
838	324	329	335	340	345	350	355	360	366	371	
839	376	381	386	391	397	402	407	412	417	423	
840	428	433	438	443	448	454	459	464	469	474	
841	479	485	490	495	500	505	510	515	521	526	
842	531	536	541	546	552	557	562	567	572	577	
843	583	588	593	598	603	608	613	619	624	629	
844	634	639	644	649	655	660	665	670	675	680	
845	685	691	696	701	706	711	716	721	727	732	
846	737	742	747	752	757	762	768	773	778	783	
847	788	793	798	803	809	814	819	824	829	834	
848	835	844	850	855	860	865	870	875	880	885	
849	891	896	901	906	911	916	921	926	931	937	
850	942	947	952	957	962	967	972	977	982	988	

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92	942	947	952	957	962	967	972	977	982	988	
	993	998	*003	*008	*013	*018	*023	*028	*034	*039	
93	044	049	054	059	064	069	074	079	084	090	
	095	100	105	110	115	120	125	130	135	140	
	146	151	156	161	166	171	176	181	186	191	
	196	201	207	212	217	222	227	232	237	242	
	247	252	257	262	267	272	278	283	288	293	
	298	303	308	313	318	323	328	333	338	343	5
	348	354	359	364	369	374	379	384	389	394	.1 | 0.5
	399	404	409	414	419	424	429	434	439	445	.2 | 1.1
											.3 | 1.6
	450	455	460	465	470	475	480	485	490	495	.4 | 2.2
											.5 | 2.7
	500	505	510	515	520	525	530	535	540	545	.6 | 3.3
	550	556	561	566	571	576	581	586	591	596	.7 | 3.8
	601	606	611	616	621	626	631	636	641	646	.8 | 4.4
	651	656	661	666	671	676	681	686	691	696	.9 | 4.9
	701	706	711	716	721	726	731	736	742	747	
	752	757	762	767	772	777	782	787	792	797	
	802	807	812	817	822	827	832	837	842	847	
	852	857	862	867	872	877	882	887	892	897	
	902	907	912	917	922	927	932	937	942	947	
	952	957	962	967	972	977	982	987	992	997	
											5
94	002	007	012	017	022	026	031	036	041	046	.1 | 0.5
	051	056	061	066	071	076	081	086	091	096	.2 | 1.0
	101	106	111	116	121	126	131	136	141	146	.3 | 1.5
	151	156	161	166	171	176	181	186	191	196	.4 | 2.0
	201	206	210	215	220	225	230	235	240	245	.5 | 2.5
	250	255	260	265	270	275	280	285	290	295	.6 | 3.0
	300	305	310	315	320	324	329	334	339	344	.7 | 3.5
	349	354	359	364	369	374	379	384	389	394	.8 | 4.0
	399	404	409	413	418	423	428	433	438	443	.9 | 4.5
	448	453	458	463	468	473	478	483	487	492	
	497	502	507	512	517	522	527	532	537	542	
	547	552	556	561	566	571	576	581	586	591	
	596	601	606	611	615	620	625	630	635	640	
	645	650	655	660	665	670	674	679	684	689	
	694	699	704	709	714	719	724	728	733	738	
	743	748	753	758	763	768	773	777	782	787	
	792	797	802	807	812	817	821	826	831	836	4
	841	846	851	856	861	865	870	875	880	885	.1 | 0.4
	890	895	900	905	909	914	919	924	929	934	.2 | 0.9
											.3 | 1.3
	939	944	949	953	958	963	968	973	978	983	.4 | 1.8
											.5 | 2.2
	988	992	997	*002	*007	*012	*017	*022	*026	031	.6 | 2.7
95	036	041	046	051	056	061	065	070	075	080	.7 | 3.1
	085	090	095	099	104	109	114	119	124	129	.8 | 3.6
	134	138	143	148	153	158	163	167	172	177	.9 | 4.0
	182	187	192	197	201	206	211	216	221	226	
	231	235	240	245	250	255	260	264	269	274	
	279	284	289	294	298	303	308	313	318	323	
	327	332	337	342	347	352	356	361	366	371	
	376	381	385	390	395	400	405	410	414	419	
	424	429	434	438	443	448	453	458	463	467	
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N.	0	1	2	3	4	5	6	7	8	9	P. P.
900	95 424$\bar{1}$	429	434	438	443	448	453	458	463	467	
901	472	477	482	487	492	496	501	506	511	516	
902	520	525	530	535	540	544	549	554	559	564	
903	569	573	578	583	588	593	597	602	607	612	
904	617	621	626	631	636	641	645	650	655	660	
905	665	669	674	679	684	689	693	698	703	708	
906	713	717	722	727	732	737	741	746	751	756	
907	760	765	770	775	780	784	789	794	799	804	
908	808	813	818	823	827	832	837	842	847	851	
909	856	861	866	870	875	880	885	890	894	899	
910	904	909	913	918	923	928	933	937	942	947	
911	952	956	961	966	971	975	980	985	990	994	5
912	999	*004	*009	*014	*018	*023	*028	*033	*037	*042	.1 | 0.5
913	96 047	052	056	061	066	071	075	080	085	090	.2 | 1.0
914	094	099	104	109	113	118	123	128	132	137	.3 | 1.5
915	142	147	151	156	161	166	170	175	180	185	.4 | 2.0
916	189	194	199	204	208	213	218	222	227	232	.5 | 2.5
917	237	241	246	251	256	260	265	270	275	279	.6 | 3.0
918	284	289	293	298	303	308	312	317	322	327	.7 | 3.5
919	331	336	341	345	350	355	360	364	369	374	.8 | 4.0
											.9 | 4.5
920	379	383	388	393	397	402	407	412	416	421	
921	426	430	435	440	445	449	454	459	463	468	
922	473	478	482	487	492	496	501	506	511	515	
923	520	525	529	534	539	543	548	553	558	562	
924	567	572	576	581	586	590	595	600	605	609	
925	614	619	623	628	633	637	642	647	651	656	
926	661	666	670	675	680	684	689	694	698	703	
927	708	712	717	722	726	731	736	741	745	750	
928	755	759	764	769	773	778	783	787	792	797	
929	801	806	811	815	820	825	829	834	839	843	
930	848	853	857	862	867	871	876	881	885	890	
931	895	899	904	909	913	918	923	927	932	937	4
932	941	946	951	955	960	965	969	974	979	983	.1 | 0.4
933	988	993	997	*002	*007	*011	*016	*020	*025	*030	.2 | 0.8
934	97 034	039	044	048	053	058	062	067	072	076	.3 | 1.2
935	081	086	090	095	099	104	109	113	118	123	.4 | 1.6
936	127	132	137	141	146	151	155	160	164	169	.5 | 2.0
937	174	178	183	188	192	197	202	206	211	215	.6 | 2.4
938	220	225	229	234	239	243	248	252	257	262	.7 | 2.8
939	266	271	276	280	285	289	294	299	303	308	.8 | 3.2
											.9 | 3.6
940	313	317	322	326	331	336	340	345	349	354	
941	359	363	368	373	377	382	386	391	396	400	
942	405	409	414	419	423	428	432	437	442	446	
943	451	456	460	465	469	474	479	483	488	492	
944	497	502	506	511	515	520	525	529	534	538	
945	543	548	552	557	561	566	570	575	580	584	
946	589	593	598	603	607	612	616	621	626	630	
947	635	639	644	649	653	658	662	667	671	676	
948	681	685	690	694	699	703	708	713	717	722	
949	726	731	736	740	745	749	754	758	763	768	
950	772	777	781	786	790	795	800	804	809	813	
	0	1	2	3	4	5	6	7	8	9	P. P.

	0	1	2	3	4	5	6	7	8	9	P. P.
	97 772	777	781	786	790	795	800	804	809	813	
1	818	822	827	831	836	841	845	850	854	859	
2	863	868	873	877	882	886	891	895	900	904	
3	909	914	918	923	927	932	936	941	945	950	
4	955	959	964	968	973	977	982	986	991	996	
5	98 000	005	009	014	018	023	027	032	036	041	
6	046	050	055	059	064	068	073	077	082	086	
7	091	095	100	105	109	114	118	123	127	132	5
8	136	141	145	150	154	159	163	168	173	177	.1 | 0.5
9	182	186	191	195	200	204	209	213	218	222	.2 | 1.0
											.3 | 1.5
0	227	231	236	240	245	249	254	259	263	268	.4 | 2.0
											.5 | 2.5
	272	277	281	286	290	295	299	304	308	313	.6 | 3.0
	317	322	326	331	335	340	344	349	353	358	.7 | 3.5
	362	367	371	376	380	385	389	394	398	403	.8 | 4.0
	407	412	416	421	425	430	434	439	443	448	.9 | 4.5
	452	457	461	466	470	475	479	484	488	493	
	497	502	506	511	515	520	524	529	533	538	
	542	547	551	556	560	565	569	574	578	583	
	587	592	596	601	605	610	614	619	623	628	
	632	637	641	646	650	655	659	663	668	672	
0	677	681	686	690	695	699	704	708	713	717	
											4
	722	726	731	735	740	744	749	753	757	762	.1 | 0.4
	766	771	775	780	784	789	793	798	802	807	.2 | 0.9
	811	815	820	824	829	833	838	842	847	851	.3 | 1.3
	856	860	865	869	873	878	882	887	891	896	.4 | 1.8
	900	905	909	914	918	922	927	931	936	940	.5 | 2.2
	945	949	954	958	963	967	971	976	980	985	.6 | 2.7
	989	994	998	*003	*007	*011	*016	*020	*025	*029	.7 | 3.1
	99 034	038	043	047	051	056	060	065	069	074	.8 | 3.6
	078	082	087	091	096	100	105	109	113	118	.9 | 4.0
0	122	127	131	136	140	145	149	153	158	162	
	167	171	176	180	184	189	193	198	202	206	
	211	215	220	224	229	233	237	242	246	251	
	255	260	264	268	273	277	282	286	290	295	
	299	304	308	312	317	321	326	330	335	339	
	343	348	352	357	361	365	370	374	379	383	
	387	392	396	401	405	409	414	418	423	427	
	431	438	440	445	449	453	458	462	467	471	4
	475	480	484	489	493	497	502	506	511	515	.1 | 0.4
	519	524	528	533	537	541	546	550	554	559	.2 | 0.8
											.3 | 1.2
0	563	568	572	576	581	585	590	594	598	603	.4 | 1.6
											.5 | 2.0
	607	611	616	620	625	629	633	638	642	647	.6 | 2.4
	651	655	660	664	668	673	677	682	686	690	.7 | 2.8
	695	699	703	708	712	717	721	725	730	734	.8 | 3.2
	738	743	747	751	756	760	765	769	773	778	.9 | 3.6
	782	786	791	795	800	804	808	813	817	821	
	826	830	834	839	843	847	852	856	861	865	
	869	874	878	882	887	891	895	900	904	908	
	913	917	922	926	930	935	939	943	948	952	
	956	961	965	969	974	978	982	987	991	995	
0	00 000	004	008	013	017	021	026	030	034	039	
	0	1	2	3	4	5	6	7	8	9	P. P.

N.	0	1	2	3	4	5	6	7	8	9	P. P.
1000	000 000	043	087	130	173	217	260	304	347	390	
01	434	477	521	564	607	651	694	737	781	824	
02	867	911	954	997	*041	*084	*127	*171	*214	*257	
03	001 301	344	387	431	474	517	560	604	647	690	
04	733	777	820	863	906	950	993	*036	*079	*123	
05	002 166	209	252	295	339	382	425	468	511	555	
06	598	641	684	727	770	814	857	900	943	986	
07	003 029	072	115	159	202	245	288	331	374	417	43 43
08	460	503	546	590	633	676	719	762	805	848	·1 4·3 4·3
09	891	934	977	*020	*063	*106	*149	*192	*235	*278	·2 8·7 8·6
											·3 13·0 12·9
1010	004 321	364	407	450	493	536	579	622	665	708	·4 17·4 17·2
											·5 21·7 21·5
11	751	794	837	880	923	966	*009	*052	*094	*137	·6 26·1 25·8
12	005 180	223	266	309	352	395	438	481	523	566	·7 30·4 30·1
13	609	652	695	738	781	824	866	909	952	995	·8 34·8 34·4
14	006 038	081	123	166	209	252	295	337	380	423	·9 39·1 38·7
15	466	509	551	594	637	680	722	765	808	851	
16	893	936	979	*022	*064	*107	*150	*193	*235	*278	
17	007 321	363	406	449	491	534	577	620	662	705	
18	748	790	833	875	918	961	*003	*046	*089	131	
19	008 174	217	259	302	344	387	430	472	515	557	
1020	600	642	685	728	770	813	855	898	940	983	
											42 42
21	009 025	068	111	153	196	238	281	323	366	408	·1 4·2 4·2
22	451	493	536	578	621	663	706	748	790	833	·2 8·5 8·4
23	875	918	960	*003	*045	*088	*130	*173	*215	*257	·3 12·7 12·6
24	010 300	342	385	427	469	512	554	596	639	681	·4 17·0 16·8
25	724	766	808	851	893	935	978	*020	*062	*105	·5 21·2 21·0
26	011 147	189	232	274	316	359	401	443	486	528	·6 25·5 25·2
27	570	612	655	697	739	782	824	866	908	951	·7 29·7 29·4
28	993	*035	*077	*120	*162	*204	*246	*288	*331	*373	·8 34·0 33·6
29	012 415	457	500	542	584	626	668	710	753	795	·9 38·2 37·8
1030	837	879	921	963	*006	*048	*090	*132	174	216	
31	013 258	301	343	385	427	469	511	553	595	637	
32	679	722	764	806	848	890	932	974	*016	*058	
33	014 100	142	184	226	268	310	352	394	436	478	
34	520	562	604	646	688	730	772	814	856	898	
35	940	982	*024	*066	*108	*150	*192	*234	*276	*318	
36	015 360	401	443	485	527	569	611	653	695	737	
37	779	820	862	904	946	988	*030	*072	*113	155	41 41
38	016 197	239	281	323	364	406	448	490	532	573	·1 4·1 4·1
39	615	657	699	741	782	824	866	908	950	991	·2 8·3 8·2
											·3 12·4 12·3
1040	017 033	075	117	158	200	242	284	325	367	408	·4 16·6 16·4
											·5 20·7 20·5
41	450	492	534	576	617	659	701	742	784	826	·6 24·9 24·6
42	867	909	951	992	*034	*076	*117	*159	*201	*242	·7 29·0 28·7
43	018 284	326	367	409	451	492	534	575	617	659	·8 33·2 32·8
44	700	742	783	825	867	908	950	991	*033	*074	·9 37·3 36·9
45	019 116	158	199	241	282	324	365	407	448	490	
46	531	573	614	656	697	739	780	822	863	905	
47	946	988	*029	*071	*112	*154	*195	*237	*278	*320	
48	020 361	402	444	485	527	568	610	651	692	734	
49	775	817	858	899	941	982	*024	*065	*106	*148	
1050	021 189	230	272	313	354	396	437	478	520	561	
	0	1	2	3	4	5	6	7	8	9	P. P.

N.	0	1	2	3	4	5	6	7	8	9	P. P.
1050	021 189	230	272	313	354	396	437	478	520	561	41
51	602	644	685	726	768	809	850	892	933	974	.1 4.1
52	022 015	057	098	139	181	222	263	304	346	387	.2 8.2
53	428	469	511	552	593	634	676	717	758	799	.3 12.3
54	840	882	923	964	*005	*046	*088	*129	*170	*211	.4 16.4
55	023 252	293	335	376	417	458	499	540	581	623	.5 20.5
56	664	705	746	787	828	869	910	951	993	*034	.6 24.6
57	024 075	116	157	198	239	280	321	362	403	444	.7 28.7
58	485	526	568	609	650	691	732	773	814	855	.8 32.8
59	896	937	978	*019	*060	*101	*142	*183	*224	*265	.9 36.9
1060	025 306	347	388	429	469	510	551	592	633	674	41
61	715	755	797	838	879	920	961	*002	*042	*083	.1 4.1
62	026 124	165	206	247	288	329	370	410	451	492	.2 8.2
63	533	574	615	656	696	737	778	819	860	901	.3 12.3
64	941	982	*023	*064	*105	*145	*186	*227	*268	*309	.4 16.4
65	027 349	390	431	472	512	553	594	635	675	716	.5 20.5
66	757	798	838	879	920	961	*001	*042	*083	*123	.6 24.6
67	028 164	205	246	286	327	368	408	449	490	530	.7 28.7
68	571	612	652	693	734	774	815	856	896	937	.8 32.8
69	977	*018	*059	*099	*140	*181	*221	*262	*302	*343	.9 36.9
1070	029 384	424	465	505	546	586	627	668	708	749	40
71	789	830	870	911	951	992	*032	*073	*114	*154	.1 4.0
72	030 195	235	276	316	357	397	438	478	519	559	.2 8.0
73	599	640	680	721	761	802	842	883	923	964	.3 12.0
74	031 004	044	085	125	166	206	247	287	327	368	.4 16.0
75	408	449	489	529	570	610	651	691	731	772	.5 20.0
76	812	852	893	933	973	*014	*054	*094	*135	*175	.6 24.0
77	032 215	256	296	336	377	417	457	498	538	578	.7 28.0
78	619	659	699	739	780	820	860	900	941	981	.8 32.0
79	033 021	061	102	142	182	222	263	303	343	383	.9 36.0
1080	424	464	504	544	584	625	665	705	745	785	40
81	825	866	906	946	986	*026	*066	*107	147	187	.1 4.0
82	034 227	267	307	347	388	428	468	508	548	588	.2 8.0
83	628	668	708	748	789	829	869	909	949	989	.3 12.0
84	035 029	069	109	149	189	229	269	309	349	389	.4 16.0
85	429	470	510	550	590	630	670	710	750	790	.5 20.0
86	830	870	910	950	990	*029	*069	*109	*149	*189	.6 24.0
87	036 229	269	309	349	389	429	469	509	549	589	.7 28.0
88	629	669	708	748	789	828	868	908	948	988	.8 32.0
89	037 028	068	107	147	187	227	267	307	347	386	.9 36.0
1090	426	466	506	546	586	625	665	705	745	785	39
91	825	864	904	944	984	*023	*063	*103	143	183	.1 3.9
92	038 222	262	302	342	381	421	461	501	540	580	.2 7.8
93	620	660	699	739	779	819	858	898	938	977	.3 11.8
94	039 017	057	096	136	176	216	255	295	335	374	.4 15.8
95	414	454	493	533	572	612	652	691	731	771	.5 19.7
96	810	850	890	929	969	*008	*048	*088	*127	*167	.6 23.7
97	040 206	246	286	325	365	404	444	483	523	563	.7 27.6
98	602	642	681	721	760	800	839	879	918	958	.8 31.6
99	997	*037	*076	*116	*155	*195	*234	*274	*313	*353	.9 35.5
1100	041 392	432	471	511	550	590	629	669	708	748	

N.	0	1	2	3	4	5	6	7	8	9	P. P.

Log sin ϕ = log ϕ'' + S. Log tan ϕ = log ϕ'' + T. **0°** log ϕ'' = log sin ϕ + S'. log ϕ'' = log tan ϕ + T'.

$''$	$'$	S	T	Log. Sin.	S′	T′	Log. Tan.
0	0	4·685 57	57	— ∞	5·314 43	43	— ∞
60	1	57	57	6·46 37	43	43	6·46 37
120	2	57	57	·76 475	43	43	·76 475
180	3	57	57	·94 084	43	43	·94 084
240	4	57	57	7·06 578	43	43	7·06 578
300	5	4·685 57	57	7·16 269	5·314 43	43	7·16 269
360	6	57	57	·24 187	43	43	·24 188
420	7	57	57	·30 882	43	43	·30 882
480	8	57	57	·36 681	43	43	·36 681
540	9	57	57	·41 797	43	43	·41 797
600	10	4·685 57	57	7·46 372	5·314 43	43	7·46 372
660	11	57	57	·50 512	43	43	·50 512
720	12	57	57	·54 290	43	43	·54 291
780	13	57	57	·57 767	43	43	·57 767
840	14	57	57	·60 985	43	43	·60 985
900	15	4·685 57	58	7·63 981	5·314 43	43	7·63 982
960	16	57	58	·66 784	43	43	·66 785
1020	17	57	58	·69 417	43	43	·69 418
1080	18	57	58	·71 899	43	43	·71 900
1140	19	57	58	·74 248	43	43	·74 249
1200	20	4·685 57	58	7·76 475	5·314 43	43	7·76 476
1260	21	57	58	·78 594	43	43	·78 595
1320	22	57	58	·80 614	43	43	·80 615
1380	23	57	58	·82 545	43	43	·82 546
1440	24	57	58	·84 393	43	43	·84 394
1500	25	4·685 57	58	7·86 166	5·314 43	43	7·86 167
1560	26	57	58	·87 869	43	43	·87 871
1620	27	57	58	·89 508	43	43	·89 510
1680	28	57	58	·91 088	43	43	·91 089
1740	29	57	58	·92 612	43	43	·92 613
1800	30	4·685 57	58	7·94 084	5·314 43	43	7·94 086
1860	31	57	58	·95 508	43	43	·95 510
1920	32	57	58	·96 887	43	43	·96 889
1980	33	57	59	·98 223	43	43	·98 226
2040	34	57	59	·99 520	43	43	·99 522
2100	35	4·685 56	59	8·00 778	5·314 43	43	8·00 781
2160	36	56	59	·02 002	43	43	·02 004
2220	37	56	59	·03 192	43	43	·03 194
2280	38	56	59	·04 350	43	43	·04 352
2340	39	56	59	·05 478	43	43	·05 481
2400	40	4·685 56	59	8·06 577	5·314 43	43	8·06 580
2460	41	56	59	·07 650	43	43	·07 653
2520	42	56	59	·08 696	43	43	·08 699
2580	43	56	60	·09 718	43	43	·09 721
2640	44	56	60	·10 716	43	43	·10 720
2700	45	4·685 56	60	8·11 692	5·314 44	43	8·11 695
2760	46	56	60	·12 647	44	43	·12 651
2820	47	56	60	·13 581	44	43	·13 585
2880	48	56	60	·14 495	44	43	·14 499
2940	49	56	60	·15 390	44	43	·15 395
3000	50	4·685 56	60	8·16 268	5·314 44	43	8·16 272
3060	51	56	60	·17 128	44	43	·17 132
3120	52	56	61	·17 971	44	43	·17 975
3180	53	56	61	·18 798	44	43	·18 803
3240	54	55	61	·19 610	44	43	·19 615
3300	55	4·685 55	61	8·20 407	5·314 44	43	8·20 412
3360	56	55	61	·21 189	44	43	·21 194
3420	57	55	61	·21 958	44	43	·21 962
3480	58	55	61	·22 713	44	43	·22 719
3540	59	55	62	·23 455	44	43	·23 462

$\sin \phi = \log \phi'' + S.$ $\quad$ 1° $\quad$ $\log \phi'' = \log \sin \phi + S'.$
$\tan \phi = \log \phi'' + T.$ $\quad$ $\log \phi'' = \log \tan \phi + T'.$

′	″	S	T	Log. Sin.	S′	T′	Log. Tan.
)	0	4.685 55	62	8.24 185	5.314 44	38	8.24 192
)	1	55	62	.24 903	45	38	.24 910
)	2	55	62	.25 609	45	38	.25 616
)	3	55	62	.26 304	45	37	.26 311
)	4	55	62	.26 988	45	37	.26 995
)	5	4.685 55	62	8.27 661	5.314 45	37	8.27 669
)	6	55	63	.28 324	45	37	.28 332
)	7	54	63	.28 977	45	37	.28 985
)	8	54	63	.29 620	45	37	.29 629
)	9	54	63	.30 254	45	36	.30 263
)	10	4.685 54	63	8.30 879	5.314 45	36	8.30 888
)	11	54	63	.31 495	45	36	.31 504
)	12	54	64	.32 102	45	36	.32 112
)	13	54	64	.32 701	46	36	.32 711
)	14	54	64	.33 292	46	36	.33 302
)	15	4.685 54	64	8.33 875	5.314 46	35	8.33 885
)	16	54	64	.34 450	46	35	.34 461
)	17	54	65	.35 018	46	35	.35 029
)	18	54	65	.35 578	46	35	.35 589
)	19	53	65	.36 131	46	35	.36 143
)	20	4.685 53	65	8.36 677	5.314 46	34	8.36 689
)	21	53	65	.37 217	46	34	.37 229
)	22	53	65	.37 750	46	34	.37 762
)	23	53	66	.38 276	46	34	.38 289
)	24	53	66	.38 796	47	34	.38 809
)	25	4.685 53	66	8.39 310	5.314 47	33	8.39 323
)	26	53	66	.39 818	47	33	.39 831
)	27	53	67	.40 320	47	33	.40 334
)	28	52	67	.40 816	47	33	.40 830
)	29	52	67	.41 307	47	33	.41 321
)	30	4.685 52	67	8.41 792	5.314 47	32	8.41 807
)	31	52	67	.42 271	47	32	.42 287
)	32	52	68	.42 746	47	32	.42 762
)	33	52	68	.43 215	48	32	.43 231
)	34	52	68	.43 680	48	31	.43 696
)	35	4.685 52	68	8.44 139	5.314 48	31	8.44 156
)	36	52	69	.44 594	48	31	.44 611
)	37	51	69	.45 044	48	31	.45 061
)	38	51	69	.45 489	48	30	.45 507
)	39	51	69	.45 930	48	30	.45 948
)	40	4.685 51	69	8.46 366	5.314 48	30	8.46 385
)	41	51	70	.46 798	49	30	.46 817
)	42	51	70	.47 226	49	30	.47 245
)	43	51	70	.47 650	49	29	.47 669
)	44	51	70	.48 069	49	29	.48 089
)	45	4.685 50	71	8.48 485	5.314 49	29	8.48 505
)	46	50	71	.48 896	49	28	.48 917
)	47	50	71	.49 304	49	28	.49 325
)	48	50	72	.49 708	49	28	.49 729
)	49	50	72	.50 108	50	28	.50 130
)	50	4.685 50	72	8.50 504	5.314 50	27	8.50 526
)	51	50	72	.50 897	50	27	.50 920
)	52	50	73	.51 286	50	27	.51 310
)	53	49	73	.51 672	50	27	.51 696
)	54	49	73	.52 055	50	26	.52 079
)	55	4.685 49	73	8.52 434	5.314 50	26	8.52 458
)	56	49	74	.52 810	51	26	.52 835
)	57	49	74	.53 183	51	25	.53 208
)	58	49	74	.53 552	51	25	.53 578
)	59	49	75	.53 918	51	25	.53 944

Log sin ϕ = log ϕ'' + S. Log tan ϕ = log ϕ'' + T. **2°** log ϕ'' = log sin ϕ + S'. log ϕ'' = log tan ϕ + T'.

$''$	$'$	S	T	Log. Sin.	S'	T'	Log. Tan.
7200	0	4.685 48	75	8.54 282	5.314 52	25	8.54 308
7260	1	48	75	.54 642	52	25	.54 669
7320	2	48	75	.54 999	52	24	.55 027
7380	3	48	76	.55 354	52	24	.55 381
7440	4	48	76	.55 705	52	23	.55 733
7500	5	4.685 48	76	8.56 054	5.314 52	23	8.56 083
7560	6	48	77	.56 400	52	23	.56 429
7620	7	47	77	.56 743	52	22	.56 772
7680	8	47	77	.57 083	52	22	.57 113
7740	9	47	78	.57 421	52	22	.57 452
7800	10	4.685 47	78	8.57 756	5.314 53	22	8.57 787
7860	11	47	78	.58 089	53	21	.58 121
7920	12	47	79	.58 419	53	21	.58 451
7980	13	46	79	.58 747	53	21	.58 779
8040	14	46	79	.59 072	53	20	.59 105
8100	15	4.685 46	80	8.59 395	5.314 53	20	8.59 428
8160	16	46	80	.59 715	54	20	.59 749
8220	17	46	80	.60 033	54	19	.60 067
8280	18	46	81	.60 349	54	19	.60 384
8340	19	45	81	.60 662	54	19	.60 698
8400	20	4.685 45	81	8.60 973	5.314 54	18	8.61 009
8460	21	45	82	.61 282	54	18	.61 319
8520	22	45	82	.61 589	55	18	.61 626
8580	23	45	82	.61 893	55	17	.61 931
8640	24	45	83	.62 196	55	17	.62 234
8700	25	4.685 44	83	8.62 496	5.314 55	16	8.62 535
8760	26	44	83	.62 795	55	16	.62 834
8820	27	44	84	.63 091	55	16	.63 131
8880	28	44	84	.63 385	56	15	.63 425
8940	29	44	84	.63 677	56	15	.63 718
9000	30	4 685 43	85	8.63 968	5.314 56	15	8.64 009
9060	31	43	85	.64 256	56	14	.64 298
9120	32	43	86	.64 543	56	14	.64 585
9180	33	43	86	.64 827	57	14	.64 870
9240	34	43	86	.65 110	57	13	.65 153
9300	35	4.685 43	87	8.65 391	5.314 57	13	8.65 435
9360	36	42	87	.65 670	57	13	.65 715
9420	37	42	87	.65 947	57	12	.65 993
9480	38	42	88	.66 223	58	12	.66 269
9540	39	42	88	.66 497	58	11	.66 543
9600	40	4.685 42	89	8.66 769	5.314 58	11	8.66 816
9660	41	42	89	.67 039	58	10	.67 087
9720	42	41	89	.67 308	58	10	.67 355
9780	43	41	90	.67 575	59	10	.67 624
9840	44	41	90	.67 840	59	09	.67 889
9900	45	4.685 41	91	8.68 104	5.314 59	09	8.68 154
9960	46	40	91	.68 366	59	08	.68 417
10020	47	40	91	.68 627	59	08	.68 678
10080	48	40	92	.68 886	60	08	.68 938
10140	49	40	92	.69 144	60	07	.69 196
10200	50	4.685 40	93	8.69 400	5.314 60	07	8.69 453
10260	51	39	93	.69 654	60	06	.69 708
10320	52	39	93	.69 907	60	05	.69 961
10380	53	39	94	.70 159	61	06	.70 214
10440	54	39	94	.70 409	61	05	.70 464
10500	55	4.685 38	95	8.70 657	5.314 61	05	8.70 714
10560	56	38	95	.70 905	61	04	.70 962
10620	57	38	96	.71 150	61	04	.71 208
10680	58	38	96	.71 395	62	03	.71 453
10740	59	38	97	.71 638	62	03	.71 697

Log. Sin.	D	Log. Tan.	Com. D.	Log. Cot.	Log. Cos.	
$-\infty$		$-\infty$		$+\infty$	0.00 000	60
6.46 37$\overline{2}$	30103	6.46 37$\overline{2}$	30103	3.53 627	0.00 000	59
6.76 47$\overline{5}$	17609	6.76 47$\overline{5}$	17609	3.23 524	0.00 000	58
6.94 08$\overline{4}$	12494	6.94 08$\overline{4}$	12494	3.05 91$\overline{5}$	0.00 000	57
7.06 57$\overline{8}$		7.06 57$\overline{8}$		2.93 42$\overline{1}$	0.00 000	56
7.16 26$\overline{9}$	9691	7.16 26$\overline{9}$	9691	2.83 73$\overline{0}$	0.00 000	55
7.24 18$\overline{7}$	7913	7.24 18$\overline{8}$	7918	2.75 81$\overline{2}$	0.00 000	54
7.30 88$\overline{2}$	6695	7.30 88$\overline{2}$	6694	2.69 11$\overline{7}$	0.00 000	53
7.36 68$\overline{1}$	5799	7.36 68$\overline{1}$	5799	2.63 31$\overline{8}$	0.00 000	52
7.41 79$\overline{7}$	5115	7.41 79$\overline{7}$	5115	2.58 203	0.00 000	51
7.46 37$\overline{2}$	4575	7.46 37$\overline{2}$	4575	2.53 627	0.00 000	50
7.50 51$\overline{2}$	4139	7.50 51$\overline{2}$	4139	2.49 488	0.00 000	49
7.54 29$\overline{0}$	3778	7.54 29$\overline{1}$	3779	2.45 709	9.99 999	48
7.57 76$\overline{7}$	3476	7.57 76$\overline{7}$	3476	2.42 233	9.99 999	47
7.60 98$\overline{5}$	3218	7.60 98$\overline{5}$	3218	2.39 014	9.99 999	46
7.63 98$\overline{1}$	2996	7.63 98$\overline{2}$	2996	2.36 018	9.99 999	45
7.66 78$\overline{4}$	2803	7.66 78$\overline{5}$	2803	2.33 215	9.99 999	44
7.69 41$\overline{7}$	2633	7.69 41$\overline{8}$	2633	2.30 582	9.99 999	43
7.71 89$\overline{9}$	2482	7.71 90$\overline{0}$	2482	2.28 09$\overline{9}$	9.99 999	42
7.74 24$\overline{8}$	2348	7.74 24$\overline{8}$	2348	2.25 75$\overline{1}$	9.99 999	41
7.76 47$\overline{5}$	2227	7.76 47$\overline{6}$	2227	2.23 524	9.99 999	40
7.78 59$\overline{4}$	2119	7.78 59$\overline{5}$	2119	2.21 405	9.99 999	39
7.80 61$\overline{4}$	2020	7.80 61$\overline{5}$	2020	2.19 384	9.99 999	38
7.82 54$\overline{5}$	1930	7.82 54$\overline{6}$	1930	2.17 454	9.99 999	37
7.84 39$\overline{3}$	1848	7.84 39$\overline{4}$	1848	2.15 605	9.99 999	36
7.86 16$\overline{6}$	1772	7.86 16$\overline{7}$	1773	2.13 83$\overline{2}$	9.99 999	35
7.87 86$\overline{9}$	1703	7.87 87$\overline{1}$	1703	2.12 129	9.99 999	34
7.89 50$\overline{8}$	1639	7.89 51$\overline{0}$	1639	2.10 490	9.99 998	33
7.91 08$\overline{8}$	1579	7.91 08$\overline{9}$	1579	2.08 91$\overline{0}$	9.99 998	32
7.92 61$\overline{2}$	1524	7.92 61$\overline{3}$	1524	2.07 386	9.99 998	31
7.94 08$\overline{4}$	1472	7.94 08$\overline{6}$	1472	2.05 914	9.99 998	30
7.95 50$\overline{8}$	1424	7.95 51$\overline{0}$	1424	2.04 490	9.99 998	29
7.96 88$\overline{7}$	1379	7.96 88$\overline{9}$	1379	2.03 111	9.99 998	28
7.98 22$\overline{3}$	1336	7.98 22$\overline{5}$	1336	2.01 774	9.99 998	27
7.99 52$\overline{0}$	1296	7.99 52$\overline{2}$	1296	2.00 478	9.99 998	26
8.00 77$\overline{8}$	1258	8.00 78$\overline{1}$	1259	1.99 219	9.99 997	25
8.02 00$\overline{2}$	1223	8.02 00$\overline{4}$	1223	1.97 995	9.99 997	24
8.03 19$\overline{2}$	1190	8.03 19$\overline{4}$	1190	1.96 80$\overline{5}$	9.99 997	23
8.04 35$\overline{0}$	1158	8.04 35$\overline{2}$	1158	1.95 647	9.99 997	22
8.05 47$\overline{8}$	1128	8.05 48$\overline{1}$	1128	1.94 519	9.99 997	21
8.06 57$\overline{7}$	1099	8.06 58$\overline{0}$	1099	1.93 41$\overline{9}$	9.99 997	20
8.07 65$\overline{0}$	1072	8.07 65$\overline{3}$	1072	1.92 347	9.99 997	19
8.08 69$\overline{6}$	1046	8.08 69$\overline{9}$	1046	1.91 30$\overline{0}$	9.99 997	18
8.09 71$\overline{8}$	1022	8.09 72$\overline{1}$	1022	1.90 27$\overline{8}$	9.99 996	17
8.10 71$\overline{6}$	998	8.10 72$\overline{0}$	999	1.89 27$\overline{6}$	9.99 996	16
8.11 69$\overline{2}$	976	8.11 69$\overline{6}$	976	1.88 30$\overline{3}$	9.99 996	15
8.12 64$\overline{7}$	954	8.12 65$\overline{1}$	954	1.87 349	9.99 996	14
8.13 58$\overline{1}$	934	8.13 58$\overline{5}$	934	1.86 415	9.99 996	13
8.14 49$\overline{5}$	914	8.14 49$\overline{9}$	914	1.85 50$\overline{0}$	9.99 996	12
8.15 39$\overline{0}$	895	8.15 39$\overline{5}$	895	1.84 605	9.99 99$\overline{5}$	11
8.16 28$\overline{8}$	877	8.16 27$\overline{3}$	877	1.83 72$\overline{7}$	9.99 99$\overline{5}$	10
8.17 12$\overline{8}$	860	8.17 13$\overline{3}$	860	1.82 867	9.99 995	9
8.17 97$\overline{1}$	843	8.17 97$\overline{6}$	843	1.82 02$\overline{3}$	9.99 995	8
8.18 79$\overline{8}$	827	8.18 80$\overline{3}$	827	1.81 19$\overline{6}$	9.99 995	7
8.19 61$\overline{0}$	811	8.19 61$\overline{5}$	812	1.80 384	9.99 994	6
8.20 40$\overline{7}$	797	8.20 41$\overline{2}$	797	1.79 58$\overline{7}$	9.99 994	5
8.21 18$\overline{9}$	782	8.21 19$\overline{5}$	783	1.78 80$\overline{4}$	9.99 994	4
8.21 95$\overline{8}$	768	8.21 96$\overline{4}$	768	1.78 036	9.99 994	3
8.22 71$\overline{3}$	755	8.22 71$\overline{9}$	755	1.77 28$\overline{0}$	9.99 994	2
8.23 45$\overline{5}$	742	8.23 46$\overline{3}$	742	1.76 538	9.99 993	1
8.24 18$\overline{5}$	730	8.24 19$\overline{2}$	730	1.75 808	9.99 993	0
Log. Cos.	D	Log. Cot.	Com. D.	Log. Tan.	Log. Sin.	′

′	Log. Sin.	D	Log. Tan.	Com. D.	Log. Cot.	Log. Cos.	
0	8.24 185	718	8.24 192	718	1.75 808	9.99 995	60
1	8.24 903	706	8.24 910	706	1.75 090	9.99 993	59
2	8.25 609	694	8.25 616	695	1.74 383	9.99 993	58
3	8.26 304	684	8.26 311	684	1.73 688	9.99 992	57
4	8.26 988	673	8.26 995	673	1.73 004	9.99 992	56
5	8.27 661	663	8.27 669	663	1.72 331	9.99 992	55
6	8.28 324	653	8.28 332	653	1.71 667	9.99 992	54
7	8.28 977	643	8.28 985	643	1.71 014	9.99 992	53
8	8.29 620	634	8.29 629	634	1.70 371	9.99 991	52
9	8.30 254	625	8.30 263	625	1.69 736	9.99 991	51
10	8.30 879	616	8.30 888	616	1.69 111	9.99 991	50
11	8.31 495	607	8.31 504	607	1.68 495	9.99 990	49
12	8.32 102	599	8.32 112	599	1.67 888	9.99 990	48
13	8.32 701	591	8.32 711	591	1.67 288	9.99 990	47
14	8.33 292	583	8.33 302	582	1.66 697	9.99 990	46
15	8.33 875	575	8.33 885	575	1.66 114	9.99 989	45
16	8.34 450	567	8.34 461	568	1.65 539	9.99 989	44
17	8.35 018	560	8.35 029	560	1.64 971	9.99 989	43
18	8.35 578	553	8.35 589	553	1.64 410	9.99 989	42
19	8.36 131	546	8.36 143	546	1.63 857	9.99 988	41
20	8.36 677	539	8.36 689	539	1.63 310	9.99 988	40
21	8.37 217	533	8.37 229	533	1.62 771	9.99 988	39
22	8.37 750	526	8.37 762	527	1.62 238	9.99 987	38
23	8.38 276	520	8.38 289	520	1.61 711	9.99 987	37
24	8.38 796	514	8.38 809	514	1.61 191	9.99 987	36
25	8.39 310	508	8.39 323	508	1.60 676	9.99 986	35
26	8.39 818	502	8.39 831	502	1.60 168	9.99 986	34
27	8.40 320	496	8.40 334	496	1.59 666	9.99 986	33
28	8.40 816	491	8.40 830	491	1.59 169	9.99 986	32
29	8.41 307	485	8.41 321	485	1.58 678	9.99 985	31
30	8.41 792	479	8.41 807	480	1.58 193	9.99 985	30
31	8.42 271	474	8.42 287	475	1.57 713	9.99 985	29
32	8.42 746	469	8.42 762	469	1.57 238	9.99 984	28
33	8.43 215	464	8.43 231	464	1.56 768	9.99 984	27
34	8.43 680	459	8.43 696	460	1.56 304	9.99 984	26
35	8.44 139	454	8.44 156	455	1.55 844	9.99 983	25
36	8.44 594	450	8.44 611	450	1.55 389	9.99 983	24
37	8.45 044	445	8.45 061	445	1.54 938	9.99 982	23
38	8.45 489	440	8.45 507	441	1.54 493	9.99 982	22
39	8.45 930	436	8.45 948	437	1.54 052	9.99 982	21
40	8.46 366	432	8.46 385	432	1.53 615	9.99 981	20
41	8.46 798	428	8.46 817	428	1.53 183	9.99 981	19
42	8.47 226	423	8.47 245	424	1.52 754	9.99 981	18
43	8.47 650	419	8.47 669	419	1.52 330	9.99 980	17
44	8.48 069	415	8.48 089	416	1.51 911	9.99 980	16
45	8.48 485	411	8.48 505	412	1.51 495	9.99 979	15
46	8.48 896	407	8.48 917	408	1.51 083	9.99 979	14
47	8.49 304	404	8.49 325	404	1.50 675	9.99 979	13
48	8.49 708	400	8.49 729	400	1.50 270	9.99 978	12
49	8.50 108	396	8.50 130	396	1.49 870	9.99 978	11
50	8.50 504	393	8.50 926	393	1.49 473	9.99 978	10
51	8.50 897	389	8.51 320	390	1.49 080	9.99 977	9
52	8.51 286	386	8.51 310	386	1.48 690	9.99 977	8
53	8.51 672	382	8.51 696	383	1.48 304	9.99 976	7
54	8.52 055	379	8.52 079	379	1.47 921	9.99 976	6
55	8.52 434	375	8.52 458	376	1.47 541	9.99 975	5
56	8.52 810	373	8.52 835	373	1.47 165	9.99 975	4
57	8.53 183	369	8.53 208	370	1.46 792	9.99 975	3
58	8.53 552	366	8.53 578	366	1.46 422	9.99 974	2
59	8.53 918	363	8.53 944	364	1.46 055	9.99 974	1
60	8.54 282		8.54 308		1.45 691	9.99 973	0
	Log. Cos.	D	Log. Cot.	Com. D.	Log. Tan.	Log. Sin.	′

Log. Sin.	D	Log. Tan.	Com. D.	Log. Cot.	Log. Cos.	
8.54 282	360	8.54 308	360	1.45 691	9.99 973	60
8.54 642	357	8.54 669	358	1.45 331	9.99 973	59
8.54 999	354	8.55 027	354	1.44 973	9.99 972	58
8.55 354	351	8.55 381	352	1.44 618	9.99 972	57
8.55 705	348	8.55 733	349	1.44 266	9.99 971	56
8.56 054	346	8.56 083	346	1.43 917	9.99 971	55
8.56 400	343	8.56 429	343	1.43 571	9.99 971	54
8.56 743	340	8.56 772	341	1.43 227	9.99 970	53
8.57 083	338	8.57 113	338	1.42 886	9.99 970	52
8.57 421	335	8.57 452	335	1.42 548	9.99 969	51
8.57 756	332	8.57 787	333	1.42 212	9.99 969	50
8.58 089	330	8.58 121	330	1.41 879	9.99 968	49
8.58 419	327	8.58 451	328	1.41 548	9.99 968	48
8.58 747	325	8.58 779	325	1.41 220	9.99 967	47
8.59 072	323	8.59 105	323	1.40 895	9.99 967	46
8.59 395	320	8.59 428	320	1.40 571	9.99 966	45
8.59 715	318	8.59 749	318	1.40 251	9.99 966	44
8.60 033	316	8.60 067	316	1.39 932	9.99 965	43
8.60 349	313	8.60 384	314	1.39 616	9.99 965	42
8.60 662	311	8.60 698	311	1.39 302	9.99 964	41
8.60 973	309	8.61 009	309	1.38 990	9.99 964	40
8.61 282	306	8.61 319	307	1.38 681	9.99 963	39
8.61 589	304	8.61 626	305	1.38 374	9.99 963	38
8.61 893	302	8.61 931	303	1.38 068	9.99 962	37
8.62 196	300	8.62 234	300	1.37 765	9.99 962	36
8.62 496	298	8.62 535	299	1.37 465	9.99 961	35
8.62 795	296	8.62 834	297	1.37 166	9.99 961	34
8.63 091	294	8.63 131	294	1.36 869	9.99 960	33
8.63 385	292	8.63 425	293	1.36 574	9.99 959	32
8.63 677	290	8.63 718	291	1.36 281	9.99 959	31
8.63 968	288	8.64 009	288	1.35 990	9.99 958	30
8.64 256	286	8.64 298	287	1.35 702	9.99 958	29
8.64 543	284	8.64 585	285	1.35 414	9.99 957	28
8.64 827	282	8.64 870	283	1.35 129	9.99 957	27
8.65 110	281	8.65 153	281	1.34 846	9.99 956	26
8.65 391	279	8.65 435	280	1.34 565	9.99 956	25
8.65 670	277	8.65 715	278	1.34 285	9.99 955	24
8.65 947	275	8.65 993	276	1.34 007	9.99 954	23
8.66 223	274	8.66 269	274	1.33 731	9.99 954	22
8.66 497	272	8.66 543	272	1.33 456	9.99 953	21
8.66 769	270	8.66 816	271	1.33 184	9.99 953	20
8.67 039	268	8.67 087	269	1.32 913	9.99 952	19
8.67 308	267	8.67 356	267	1.32 643	9.99 952	18
8.67 575	265	8.67 624	268	1.32 376	9.99 951	17
8.67 840	264	8.67 890	266	1.32 110	9.99 950	16
8.68 104	262	8.68 154	264	1.31 845	9.99 950	15
8.68 366	260	8.68 417	262	1.31 583	9.99 949	14
8.68 627	259	8.68 678	261	1.31 321	9.99 948	13
8.68 886	257	8.68 938	259	1.31 062	9.99 948	12
8.69 144	256	8.69 196	258	1.30 803	9.99 947	11
8.69 400	254	8.69 453	256	1.30 547	9.99 947	10
8.69 654	253	8.69 708	255	1.30 292	9.99 946	9
8.69 907	251	8.69 961	253	1.30 038	9.99 945	8
8.70 159	250	8.70 214	252	1.29 786	9.99 945	7
8.70 409	248	8.70 464	250	1.29 535	9.99 944	6
8.70 657	247	8.70 714	249	1.29 286	9.99 943	5
8.70 905	245	8.70 962	248	1.29 038	9.99 943	4
8.71 150	244	8.71 208	246	1.28 791	9.99 942	3
8.71 395	243	8.71 453	245	1.28 546	9.99 942	2
8.71 638	241	8.71 697	243	1.28 303	9.99 941	1
8.71 880		8.71 939	242	1.28 060	9.99 940	0
Log. Cos.	D	Log. Cot.	Com. D.	Log. Tan.	Log. Sin.	′

′	Log. Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	
0	8·71 880	240	8·71 939	241	1·28 060	9·99 943	60
1	8·72 120	239	8·72 180	240	1·27 819	9·99 940	59
2	8·72 359	237	8·72 420	238	1·27 579	9·99 939	58
3	8·72 597	236	8·72 659	237	1·27 341	9·99 938	57
4	8·72 833	235	8·72 896	235	1·27 104	9·99 938	56
5	8·73 069	233	8·73 131	235	1·26 868	9·99 937	55
6	8·73 302	233	8·73 366	233	1·26 633	9·99 936	54
7	8·73 535	231	8·73 599	232	1·26 400	9·99 935	53
8	8·73 766	230	8·73 831	231	1·26 168	9·99 935	52
9	8·73 997	229	8·74 062	229	1·25 937	9·99 934	51
10	8·74 226	227	8·74 292	228	1·25 708	9·99 933	50
11	8·74 453	226	8·74 520	227	1·25 479	9·99 933	49
12	8·74 680	225	8·74 748	226	1·25 252	9·99 932	48
13	8·74 905	224	8·74 974	225	1·25 026	9·99 931	47
14	8·75 129	223	8·75 199	223	1·24 801	9·99 931	46
15	8·75 353	221	8·75 422	223	1·24 577	9·99 930	45
16	8·75 574	221	8·75 645	221	1·24 354	9·99 929	44
17	8·75 795	219	8·75 867	220	1·24 133	9·99 928	43
18	8·76 015	218	8·76 087	219	1·23 913	9·99 928	42
19	8·76 233	217	8·76 306	218	1·23 693	9·99 927	41
20	8·76 451	216	8·76 524	217	1·23 475	9·99 926	40
21	8·76 667	215	8·76 741	216	1·23 258	9·99 925	39
22	8·76 883	215	8·76 959	214	1·23 042	9·99 925	38
23	8·77 097	214	8·77 172	214	1·22 827	9·99 924	37
24	8·77 310	213	8·77 386	213	1·22 613	9·99 923	36
25	8·77 522	211	8·77 599	212	1·22 400	9·99 922	35
26	8·77 733	210	8·77 811	210	1·22 188	9·99 922	34
27	8·77 943	209	8·78 022	210	1·21 978	9·99 921	33
28	8·78 152	208	8·78 232	209	1·21 768	9·99 920	32
29	8·78 360	207	8·78 441	209	1·21 559	9·99 919	31
30	8·78 567	206	8·78 648	207	1·21 351	9·99 919	30
31	8·78 773	205	8·78 855	207	1·21 144	9·99 918	29
32	8·78 978	204	8·79 061	206	1·20 938	9·99 917	28
33	8·79 183	203	8·79 266	204	1·20 734	9·99 916	27
34	8·79 386	202	8·79 470	204	1·20 530	9·99 916	26
35	8·79 588	201	8·79 673	203	1·20 327	9·99 915	25
36	8·79 789	200	8·79 875	202	1·20 126	9·99 914	24
37	8·79 989	199	8·80 076	201	1·19 923	9·99 913	23
38	8·80 189	198	8·80 276	200	1·19 723	9·99 912	22
39	8·80 387	197	8·80 476	199	1·19 524	9·99 912	21
40	8·80 585	197	8·80 674	198	1·19 326	9·99 911	20
41	8·80 782	195	8·80 871	197	1·19 128	9·99 910	19
42	8·80 977	195	8·81 068	195	1·18 931	9·99 909	18
43	8·81 172	194	8·81 264	196	1·18 736	9·99 908	17
44	8·81 366	193	8·81 459	195	1·18 541	9·99 907	16
45	8·81 560	192	8·81 653	194	1·18 347	9·99 907	15
46	8·81 752	191	8·81 846	193	1·18 154	9·99 906	14
47	8·81 943	191	8·82 038	192	1·17 961	9·99 905	13
48	8·82 134	191	8·82 230	191	1·17 770	9·99 904	12
49	8·82 324	189	8·82 420	190	1·17 579	9·99 903	11
50	8·82 513	189	8·82 610	190	1·17 389	9·99 902	10
51	8·82 701	188	8·82 799	189	1·17 201	9·99 902	9
52	8·82 888	187	8·82 987	188	1·17 012	9·99 901	8
53	8·83 075	187	8·83 175	187	1·16 825	9·99 900	7
54	8·83 260	185	8·83 361	186	1·16 639	9·99 899	6
55	8·83 445	185	8·83 547	186	1·16 453	9·99 898	5
56	8·83 629	184	8·83 732	185	1·16 268	9·99 897	4
57	8·83 813	183	8·83 916	184	1·16 083	9·99 896	3
58	8·83 995	182	8·84 100	183	1·15 900	9·99 896	2
59	8·84 177	182	8·84 282	182	1·15 717	9·99 895	1
60	8·84 358	181	8·84 464	182	1·15 535	9·99 894	0
	Log. Cos.	d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	′

P. P.

	330	320	310	300
6	33.0	32.0	31.0	30.0
7	38.5	37.3	36.1	35.0
8	44.0	42.6	41.3	40.0
9	49.5	48.0	46.5	45.0
10	55.0	53.3	51.6	50.0
20	110.0	106.6	103.3	100.0
30	165.0	160.0	155.0	150.0
40	220.0	213.3	206.6	200.0
50	275.0	266.6	258.3	250.0

	290	280	270	260
6	29.0	28.0	27.0	26.0
7	33.8	32.6	31.5	30.3
8	38.6	37.3	36.0	34.6
9	43.5	42.0	40.5	39.0
10	48.3	46.6	45.0	43.3
20	96.6	93.3	90.0	86.6
30	145.0	140.0	135.0	130.0
40	193.3	186.6	180.0	173.3
50	241.6	233.3	225.0	216.6

	250	240	230	220
6	25.0	24.0	23.0	22.0
7	29.1	28.0	26.8	25.6
8	33.3	32.0	30.6	29.3
9	37.5	36.0	34.5	33.0
10	41.6	40.0	38.3	36.6
20	83.3	80.0	76.6	73.3
30	125.0	120.0	115.0	110.0
40	166.6	160.0	153.3	146.6
50	208.3	200.0	191.6	183.3

	210	200	190	180
6	21.0	20.0	19.0	18.0
7	24.5	23.3	22.1	21.0
8	28.0	26.6	25.3	24.0
9	31.5	30.0	28.5	27.0
10	35.0	33.3	31.6	30.0
20	70.0	66.6	63.3	60.0
30	105.0	100.0	95.0	90.0
40	140.0	133.3	126.6	120.0
50	175.0	166.6	158.3	150.0

	9	8	7	6	5
6	0.9	0.8	0.7	0.6	0.5
7	1.0	0.9	0.8	0.7	0.6
8	1.2	1.1	1.0	0.8	0.7
9	1.4	1.3	1.2	1.0	0.8
10	1.6	1.5	1.3	1.1	1.0
20	3.1	3.0	2.6	2.3	2.0
30	4.7	4.5	4.0	3.5	3.0
40	6.3	6.0	5.3	4.6	4.0
50	7.9	7.5	6.6	5.8	5.0

	4	3	2	1	0
6	0.4	0.3	0.2	0.1	0.0
7	0.5	0.4	0.3	0.2	0.1
8	0.6	0.5	0.4	0.2	0.1
9	0.7	0.6	0.4	0.3	0.1
10	0.7	0.6	0.5	0.3	0.1
20	1.5	1.3	1.0	0.6	0.3
30	2.2	2.0	1.5	1.0	0.5
40	3.0	2.6	2.0	1.3	0.6
50	3.7	3.3	2.5	1.6	0.8

P. P.

′	Log. Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.		P. P.			
								181	180	178	176
0	8.84 358	180	8.84 464	181	1.15 535	9.99 894	60				
1	8.84 538	180	8.84 645	180	1.15 354	9.99 893	59	6	18.1	18.0	17.8	17.6
2	8.84 718	178	8.84 826	179	1.15 174	9.99 892	58	7	21.1	21.0	20.7	20.5
3	8.84 897	178	8.85 005	179	1.14 994	9.99 891	57	8	24.1	24.0	23.7	23.4
4	8.85 075	177	8.85 184	178	1.14 815	9.99 890	56	9	27.1	27.0	26.7	26.4
5	8 85 252	176	8.85 363	177	1.14 637	9.99 889	55	10	30.1	30.0	29.6	29.3
6	8.85 429	176	8.85 540	177	1.14 459	9.99 888	54	20	60.3	60.0	59.3	58.6
7	8.85 605	175	8.85 717	176	1.14 283	9.99 888	53	30	90.5	90.0	89.0	88.0
8	8.85 780	175	8.85 893	176	1.14 107	9.99 887	52	40	120.6	120.0	118.6	117.3
9	8.85 954	174	8.86 068	175	1.13 931	9.99 886	51	50	150.8	150.0	148.3	146.6
		174		175							
10	8 86 128	173	8.86 243	174	1.13 756	9.99 885	50		174	172	170	168
11	8.86 301	173	8.86 417	174	1.13 582	9.99 884	49	6	17.4	17.2	17.0	16.8
12	8.86 474	172	8.86 590	173	1.13 409	9.99 883	48	7	20.3	20.0	19.8	19.6
13	8.86 645	171	8.86 763	172	1.13 237	9.99 882	47	8	23.2	22.9	22.6	22.4
14	8.86 816	171	8.86 935	172	1.13 065	9.99 881	46	9	26.1	25.8	25.5	25.2
		170		171							
15	8.86 987	169	8.87 106	170	1.12 893	9.99 880	45	10	29.0	28.6	28.3	28.0
16	8.87 156	169	8.87 277	170	1.12 723	9.99 879	44	20	58.0	57.3	56.6	56.0
17	8.87 325	169	8.87 447	170	1.12 553	9.99 878	43	30	87.0	86.0	85.0	84.0
18	8.87 494	168	8.87 616	169	1.12 384	9.99 877	42	40	116.0	114.6	113.3	112.0
19	8.87 661	167	8.87 785	169	1.12 215	9.99 876	41	50	145.0	143.3	141.6	140.0
20	8.87 828	167	8.87 953	168	1.12 047	9.99 875	40					
21	8.87 995	166	8.88 120	167	1.11 880	9.99 874	39		166	164	162	160
22	8.88 160	165	8.88 287	167	1.11 713	9.99 874	38	6	16.6	16.4	16.2	16.0
23	8.88 326	166	8.88 453	166	1.11 547	9.99 873	37	7	19.3	19.1	18.9	18.6
24	8.88 490	164	8.88 618	165	1.11 381	9.99 872	36	8	22.1	21.8	21.6	21.3
		163		165				9	24.9	24.6	24.3	24.0
25	8.88 654	163	8.88 783	164	1.11 216	9.99 871	35	10	27.6	27.3	27.0	26.6
26	8.88 817	162	8.88 947	163	1.11 052	9.99 870	34	20	55.3	54.6	54.0	53.3
27	8.88 980	162	8.89 111	163	1.10 889	9.99 869	33	30	83.0	82.0	81.0	80.0
28	8.89 142	161	8.89 274	162	1.10 726	9.99 868	32	40	110.6	109.3	108.0	106.6
29	8.89 303	161	8.89 436	162	1.10 563	9.99 867	31	50	138.3	136.6	135.0	133.3
30	8.89 464	161	8.89 598	162	1.10 401	9.99 866	30					
31	8.89 624	160	8.89 759	161	1.10 240	9.99 865	29		158	156	154	152
32	8.89 784	159	8.89 920	161	1.10 079	9.99 864	28	6	15.8	15.6	15.4	15.2
33	8.89 943	159	8.90 080	160	1.09 919	9.99 863	27	7	18.4	18.2	17.9	17.7
34	8.90 101	158	8.90 240	159	1.09 760	9.99 862	26	8	21.0	20.8	20.5	20.2
		158		158				9	23.7	23.4	23.1	22.8
35	8.90 259	157	8.90 398	158	1.09 601	9.99 861	25	10	26.3	26.0	25.6	25.3
36	8.90 417	156	8.90 557	157	1.09 443	9.99 860	24	20	52.6	52.0	51.3	50.6
37	8.90 573	156	8.90 714	157	1.09 285	9.99 859	23	30	79.0	78.0	77.0	76.0
38	8.90 729	156	8.90 872	158	1.09 128	9.99 858	22	40	105.3	104.0	102.6	101.3
39	8.90 885	155	8.91 028	156	1.08 971	9.99 857	21	50	131.6	130.0	128.3	126.6
40	8.91 040	154	8.91 184	156	1.08 815	9.99 856	20					
41	8.91 195	154	8.91 340	155	1.08 660	9.99 855	19		150	149	148	147
42	8.91 349	154	8.91 495	155	1.08 505	9.99 853	18	6	15.0	14.9	14.8	14.7
43	8.91 502	153	8.91 649	154	1.08 350	9.99 852	17	7	17.5	17.4	17.2	17.1
44	8.91 655	153	8.91 803	154	1.08 196	9.99 851	16	8	20.0	19.8	19.7	19.6
		152		153				9	22.5	22.3	22.2	22.0
45	8.91 807	151	8.91 957	152	1.08 043	9.99 850	15	10	25.0	24.8	24.6	24.5
46	8.91 959	151	8.92 109	152	1.07 890	9.99 849	14	20	50.0	49.6	49.3	49.0
47	8.92 110	150	8 92 262	151	1.07 738	9.99 848	13	30	75.0	74.5	74.0	73.5
48	8.92 261	150	8.92 413	151	1.07 586	9.99 847	12	40	100.0	99.3	98.6	98.0
49	8.92 411	150	8.92 565	150	1.07 435	9.99 846	11	50	125.0	124.1	123.3	122.5
50	8.92 561	149	8.92 715	150	1.07 284	9.99 845	10					
51	8.92 710	148	8.92 866	149	1.07 134	9.99 844	9		146	145	1	1	0
52	8.92 858	148	8.93 015	149	1.06 984	9.99 843	8	6	14.6	14.5	0.1	0.1	0.0
53	8.93 007	147	8.93 164	149	1.06 835	9.99 842	7	7	17.0	16.9	0.2	0.1	0.0
54	8.93 154	147	8.93 313	149	1.06 686	9.99 841	6	8	19.4	19.3	0.2	0.1	0.0
		147		148				9	21.9	21.7	0.2	0.1	0.1
55	8.93 301	146	8.93 461	148	1.06 538	9.99 840	5	10	24.3	24.1	0.2	0.1	0.1
56	8.93 448	146	8.93 609	147	1.06 390	9.99 839	4	20	48.6	48.3	0.5	0.3	0.1
57	8.93 594	146	8.93 756	148	1.06 243	9.99 837	3	30	73.0	72.5	0.7	0.5	0.2
58	8.93 740	145	8.93 903	146	1.06 097	9.99 836	2	40	97.3	96.6	1.0	0.6	0.3
59	8.93 885	144	8.94 049	145	1.05 950	9.99 835	1	50	121.6	120.8	1.2	0.8	0.4
60	8.94 029		8.94 195		1.05 805	9.99 834	0						
	Log. Cos.	d.	Log. Cot	c.d.	Log. Tan.	Log. Sin.	′	P. P.					

Log sin ϕ = log ϕ'' + S. Log tan ϕ = log ϕ'' + T.

0°

log ϕ'' = log sin ϕ + S'. log ϕ'' = log tan ϕ + T'.

$''$	$'$	S	T	Log. Sin.	S′	T′	Log. Tan.
0	0	4·685 57	57	— ∞	5·314 43	43	— ∞
60	1	57	57	6·46 373	43	43	6·46 373
120	2	57	57	·76 475	43	43	·76 475
180	3	57	57	·84 084	43	43	·84 084
240	4	57	57	7·06 578	42	42	7·06 578
300	5	4·685 57	57	7·16 269	5·314 43	43	7·16 269
360	6	57	57	·24 188	43	43	·24 188
420	7	57	57	·30 882	43	43	·30 882
480	8	57	57	·36 681	42	43	·36 681
540	9	57	57	·41 797	42	42	·41 797
600	10	4·685 57	57	7·46 372	5·314 43	43	7·46 372
660	11	57	57	·50 512	43	43	·50 512
720	12	57	57	·54 290	43	43	·54 291
780	13	57	57	·57 767	43	43	·57 767
840	14	57	57	·60 985	43	43	·60 985
900	15	4·685 57	58	7·63 981	5·314 42	42	7·63 982
960	16	57	58	·66 784	42	42	·66 785
1020	17	57	58	·69 417	42	42	·69 418
1080	18	57	58	·71 899	42	42	·71 900
1140	19	57	58	·74 248	42	42	·74 248
1200	20	4·685 57	58	7·76 475	5·314 43	42	7·76 478
1260	21	57	58	·78 594	43	42	·78 598
1320	22	57	58	·80 614	43	42	·80 615
1380	23	57	58	·82 545	43	42	·82 546
1440	24	57	58	·84 393	43	42	·84 394
1500	25	4·685 57	58	7·86 166	5·314 43	41	7·86 167
1560	26	57	58	·87 869	43	41	·87 871
1620	27	57	58	·89 508	43	41	·89 510
1680	28	57	58	·91 088	43	41	·91 089
1740	29	57	58	·92 612	43	41	·92 813
1800	30	4·685 57	58	7·94 084	5·314 43	41	7·94 086
1860	31	57	58	·95 508	43	41	·95 510
1920	32	57	58	·96 887	43	41	·96 889
1980	33	57	59	·98 223	43	41	·98 225
2040	34	57	59	·99 520	43	41	·99 522
2100	35	4·685 56	59	8·00 778	5·314 43	41	8·00 781
2160	36	56	59	·02 002	43	41	·02 004
2220	37	56	59	·03 192	43	41	·03 194
2280	38	56	59	·04 350	43	40	·04 352
2340	39	56	59	·05 478	43	40	·05 481
2400	40	4·685 56	59	8·06 577	5·314 43	40	8·06 580
2460	41	56	59	·07 650	43	40	·07 653
2520	42	56	59	·08 696	43	40	·08 699
2580	43	56	60	·09 718	43	40	·09 721
2640	44	56	60	·10 716	43	40	·10 720
2700	45	4·685 56	60	8·11 692	5·314 44	40	8·11 694
2760	46	56	60	·12 647	44	40	·12 651
2820	47	56	60	·13 581	44	40	·13 585
2880	48	56	60	·14 495	44	39	·14 499
2940	49	56	60	·15 390	44	39	·15 394
3000	50	4·685 56	60	8·16 268	5·314 44	39	8·16 272
3060	51	56	60	·17 128	44	39	·17 132
3120	52	56	61	·17 971	44	39	·17 97[illegible]
3180	53	56	61	·18 798	44	39	·18 80[illegible]
3240	54	55	61	·19 610	44	39	·19 61[illegible]
3300	55	4·685 55	61	8·20 407	5·314 44	39	8·20 41[illegible]
3360	56	55	61	·21 189	44	38	·21 19[illegible]
3420	57	55	61	·21 958	44	38	·21 96[illegible]
[illegible]	58	55	61	·22 713	44	38	·22 71[illegible]
	59	55	62	·23 455	44	38	·23 46[illegible]

$\log \sin \phi = \log \phi'' + S.$ $\log \tan \phi = \log \phi'' + T.$ **1°** $\log \phi'' = \log \sin \phi + S'.$ $\log \phi'' = \log \tan \phi + T'.$

′	″	S	T	Log. Sin.	S′	T′	Log. Tan.
0	0	4.685 55	62	8.24 185	5.314 45	38	8.24 192
0	1	55	62	.24 903	45	38	.24 910
0	2	55	62	.25 609	45	38	.25 616
0	3	55	62	.26 304	45	37	.26 311
0	4	55	62	.26 988	45	37	.26 995
0	5	4.685 55	62	8.27 661	5.314 45	37	8.27 669
0	6	55	63	.28 324	45	37	.28 332
0	7	54	63	.28 977	45	37	.28 985
0	8	54	63	.29 620	45	37	.29 629
0	9	54	63	.30 254	45	36	.30 263
0	10	4.685 54	63	8.30 879	5.314 45	36	8.30 888
0	11	54	63	.31 495	45	36	.31 504
0	12	54	64	.32 102	45	36	.32 112
0	13	54	64	.32 701	46	36	.32 711
0	14	54	64	.33 292	46	36	.33 302
0	15	4.685 54	64	8.33 875	5.314 46	35	8.33 885
0	16	54	64	.34 450	46	35	.34 461
0	17	54	65	.35 018	46	35	.35 029
0	18	54	65	.35 578	46	35	.35 589
0	19	53	65	.36 131	46	35	.36 143
0	20	4.685 53	65	8.36 677	5.314 46	34	8.36 689
0	21	53	65	.37 217	46	34	.37 229
0	22	53	65	.37 750	46	34	.37 762
0	23	53	66	.38 276	46	34	.38 289
0	24	53	66	.38 796	47	34	.38 809
0	25	4.685 53	66	8.39 310	5.314 47	33	8.39 323
0	26	53	66	.39 818	47	33	.39 831
0	27	53	67	.40 320	47	33	.40 334
0	28	52	67	.40 816	47	33	.40 830
0	29	52	67	.41 307	47	33	.41 321
0	30	4.685 52	67	8.41 792	5.314 47	32	8.41 807
0	31	52	67	.42 271	47	32	.42 287
0	32	52	68	.42 746	47	32	.42 762
0	33	52	68	.43 215	48	32	.43 231
0	34	52	68	.43 680	48	31	.43 696
0	35	4.685 52	68	8.44 139	5.314 48	31	8.44 156
0	36	52	69	.44 594	48	31	.44 611
0	37	51	69	.45 044	48	31	.45 061
0	38	51	69	.45 489	48	30	.45 507
0	39	51	69	.45 930	48	30	.45 948
0	40	4.685 51	69	8.46 366	5.314 48	30	8.46 385
0	41	51	70	.46 798	49	30	.46 817
0	42	51	70	.47 226	49	30	.47 245
0	43	51	70	.47 650	49	29	.47 669
0	44	51	70	.48 069	49	29	.48 089
0	45	4.685 50	71	8.48 485	5.314 49	29	8.48 505
0	46	50	71	.48 896	49	28	.48 917
0	47	50	71	.49 304	49	28	.49 325
0	48	50	72	.49 708	49	28	.49 729
0	49	50	72	.50 108	50	28	.50 130
0	50	4.685 50	72	8.50 504	5.314 50	27	8.50 526
0	51	50	72	.50 897	50	27	.50 920
0	52	50	73	.51 286	50	27	.51 310
0	53	49	73	.51 672	50	27	.51 696
0	54	49	73	.52 055	50	26	.52 079
0	55	4.685 49	73	8.52 434	5.314 50	26	8.52 458
0	56	49	74	.52 810	51	26	.52 835
0	57	49	74	.53 183	51	25	.53 208
0	58	49	74	.53 552	51	25	.53 579
0	59	49	75	.53 918	51	25	.53 944

TABLE VI.—LOGARITHMIC SINES AND TANGENTS OF SMALL ANGLES

Log sin ϕ = log ϕ'' + S. Log tan ϕ = log ϕ'' + T. **2°** log ϕ'' = log sin ϕ + S', log ϕ'' = log tan ϕ + T'.

$''$	$'$	S	T	Log. Sin.	S'	T'	Log. Tan.
7200	0	4.685 48	75	8.54 282	5.314 52	25	8.54 308
7260	1	48	75	.54 642	52	24	.54 668
7320	2	48	75	.54 999	51	24	.55 027
7380	3	48	76	.55 354	52	24	.55 381
7440	4	48	76	.55 705	52	23	.55 733
7500	5	4.685 48	76	8.56 054	5.314 52	23	8.56 083
7560	6	48	77	.56 400	52	23	.56 429
7620	7	47	77	.56 743	52	23	.56 772
7680	8	47	77	.57 083	52	22	.57 113
7740	9	47	78	.57 421	52	22	.57 452
7800	10	4.685 47	78	8.57 756	5.314 53	22	8.57 787
7860	11	47	78	.58 089	53	21	.58 121
7920	12	47	79	.58 419	53	21	.58 451
7980	13	46	79	.58 747	53	21	.58 779
8040	14	46	79	.59 072	53	20	.59 106
8100	15	4.685 46	80	8.59 395	5.314 53	20	8.59 428
8160	16	46	80	.59 715	54	20	.59 749
8220	17	46	80	.60 033	54	19	.60 067
8280	18	46	81	.60 349	54	19	.60 384
8340	19	45	81	.60 662	54	19	.60 698
8400	20	4.685 45	81	8.60 973	5.314 54	18	8.61 009
8460	21	45	82	.61 282	54	18	.61 318
8520	22	45	82	.61 589	55	18	.61 626
8580	23	45	82	.61 893	55	17	.61 931
8640	24	45	83	.62 196	55	17	.62 234
8700	25	4.685 44	83	8.62 496	5.314 55	16	8.62 535
8760	26	44	83	.62 795	55	16	.62 834
8820	27	44	84	.63 091	55	16	.63 131
8880	28	44	84	.63 385	56	15	.63 425
8940	29	44	84	.63 677	56	15	.63 718
9000	30	4 685 43	85	8.63 968	5.314 56	15	8.64 009
9060	31	43	85	.64 256	56	14	.64 298
9120	32	43	86	.64 543	56	14	.64 585
9180	33	43	86	.64 827	57	14	.64 870
9240	34	43	86	.65 110	57	13	.65 153
9300	35	4.685 43	87	8.65 391	5.314 57	13	8.65 435
9360	36	42	87	.65 670	57	12	.65 715
9420	37	42	87	.65 947	57	12	.65 993
9480	38	42	88	.66 223	58	12	.66 269
9540	39	42	88	.66 497	58	11	.66 543
9600	40	4.685 42	89	8.66 769	5.314 58	11	8.66 816
9660	41	41	89	.67 039	58	10	.67 087
9720	42	41	89	.67 308	58	10	.67 356
9780	43	41	90	.67 575	59	10	.67 624
9840	44	41	90	.67 840	59	09	.67 890
9900	45	4.685 41	91	8.68 104	5.314 59	09	8.68 154
9960	46	40	91	.68 366	59	08	.68 417
10020	47	40	91	.68 627	59	08	.68 678
10080	48	40	92	.68 886	60	08	.68 938
10140	49	40	92	.69 144	60	07	.69 196
10200	50	4.685 40	93	8.69 400	5.314 60	07	8.69 453
10260	51	39	93	.69 654	60	06	.69 709
10320	52	39	93	.69 907	60	05	.69 961
10380	53	39	94	.70 159	61	06	.70 214
10440	54	39	94	.70 409	61	05	.70 464
10500	55	4.685 38	95	8.70 657	5.314 61	05	8.70 714
10560	56	38	95	.70 905	61	04	.70 962
10620	57	38	96	.71 150	61	04	.71 208
	58	38	96	.71 395	62	03	.71 453
	59	38	97	.71 638	62	03	.71 697

′	Log. Sin.	D	Log. Tan.	Com. D.	Log. Cot.	Log. Cos.	
0	$-\infty$		$-\infty$		$+\infty$	0.00 000	60
1	6.46 372	30103	6.46 372	30103	3.53 628	0.00 000	59
2	6.76 475	17609	6.76 475	17609	3.23 525	0.00 000	58
3	6.94 084	12494	6.94 084	12494	3.05 916	0.00 000	57
4	7.06 578	9691	7.06 578	9691	2.93 422	0.00 000	56
5	7.16 269	7918	7.16 269	7918	2.83 731	0.00 000	55
6	7.24 187	6695	7.24 188	6694	2.75 812	0.00 000	54
7	7.30 882	5799	7.30 882	5799	2.69 118	0.00 000	53
8	7.36 681	5115	7.36 681	5115	2.63 319	0.00 000	52
9	7.41 797	4575	7.41 797	4575	2.58 203	0.00 000	51
10	7.46 372	4139	7.46 372	4139	2.53 628	0.00 000	50
11	7.50 512	3778	7.50 512	3779	2.49 488	0.00 000	49
12	7.54 290	3476	7.54 291	3476	2.45 709	9.99 999	48
13	7.57 767	3218	7.57 767	3218	2.42 233	9.99 999	47
14	7.60 985	2996	7.60 985	2996	2.39 014	9.99 999	46
15	7.63 981	2803	7.63 982	2803	2.36 018	9.99 999	45
16	7.66 784	2633	7.66 785	2633	2.33 215	9.99 999	44
17	7.69 417	2482	7.69 418	2482	2.30 582	9.99 999	43
18	7.71 899	2348	7.71 900	2348	2.28 099	9.99 999	42
19	7.74 248	2227	7.74 248	2227	2.25 751	9.99 999	41
20	7.76 475	2119	7.76 476	2119	2.23 524	9.99 999	40
21	7.78 594	2020	7.78 595	2020	2.21 405	9.99 999	39
22	7.80 614	1930	7.80 615	1930	2.19 384	9.99 999	38
23	7.82 545	1848	7.82 546	1848	2.17 454	9.99 999	37
24	7.84 393	1772	7.84 394	1773	2.15 605	9.99 999	36
25	7.86 166	1703	7.86 167	1703	2.13 833	9.99 999	35
26	7.87 869	1639	7.87 871	1639	2.12 129	9.99 999	34
27	7.89 508	1579	7.89 510	1579	2.10 490	9.99 998	33
28	7.91 088	1524	7.91 089	1524	2.08 910	9.99 998	32
29	7.92 612	1472	7.92 613	1472	2.07 386	9.99 998	31
30	7.94 084	1424	7.94 086	1424	2.05 914	9.99 998	30
31	7.95 508	1379	7.95 510	1379	2.04 490	9.99 998	29
32	7.96 887	1336	7.96 889	1336	2.03 111	9.99 998	28
33	7.98 223	1296	7.98 225	1296	2.01 774	9.99 998	27
34	7.99 520	1258	7.99 522	1259	2.00 478	9.99 998	26
35	8.00 778	1223	8.00 781	1223	1.99 219	9.99 997	25
36	8.02 002	1190	8.02 004	1190	1.97 995	9.99 997	24
37	8.03 192	1158	8.03 194	1158	1.96 805	9.99 997	23
38	8.04 350	1128	8.04 352	1128	1.95 647	9.99 997	22
39	8.05 478	1099	8.05 481	1099	1.94 519	9.99 997	21
40	8.06 577	1072	8.06 580	1072	1.93 418	9.99 997	20
41	8.07 650	1046	8.07 653	1046	1.92 347	9.99 997	19
42	8.08 696	1022	8.08 699	1022	1.91 300	9.99 997	18
43	8.09 718	998	8.09 721	999	1.90 278	9.99 996	17
44	8.10 716	976	8.10 720	976	1.89 279	9.99 996	16
45	8.11 692	954	8.11 696	954	1.88 303	9.99 996	15
46	8.12 647	934	8.12 651	934	1.87 349	9.99 996	14
47	8.13 581	914	8.13 585	914	1.86 415	9.99 996	13
48	8.14 495	895	8.14 499	895	1.85 500	9.99 996	12
49	8.15 390	877	8.15 395	877	1.84 605	9.99 995	11
50	8.16 268	860	8.16 272	860	1.83 727	9.99 995	10
51	8.17 128	843	8.17 133	843	1.82 867	9.99 995	9
52	8.17 971	827	8.17 976	827	1.82 023	9.99 995	8
53	8.18 798	811	8.18 803	812	1.81 196	9.99 995	7
54	8.19 610	797	8.19 615	797	1.80 384	9.99 994	6
55	8.20 407	782	8.20 412	783	1.79 587	9.99 994	5
56	8.21 189	768	8.21 195	768	1.78 804	9.99 994	4
57	8.21 958	755	8.21 964	755	1.78 036	9.99 994	3
58	8.22 713	742	8.22 719	742	1.77 280	9.99 994	2
59	8.23 455	730	8.23 462	730	1.76 538	9.99 993	1
60	8.24 185		8.24 192		1.75 808	9.99 993	0
	Log. Cos.	D	Log. Cot.	Com. D.	Log. Tan.	Log. Sin.	′

′	Log. Sin.	D	Log. Tan.	Com. D.	Log. Cot.	Log. Cos.	
0	8.24 185	718	8.24 192	718	1.75 808	9.99 993	60
1	8.24 903	706	8.24 910	706	1.75 090	9.99 993	59
2	8.25 609	694	8.25 616	695	1.74 383	9.99 993	58
3	8.26 304	684	8.26 311	684	1.73 688	9.99 992	57
4	8.26 988	673	8.26 995	673	1.73 004	9.99 992	56
5	8.27 661	663	8.27 669	663	1.72 331	9.99 992	55
6	8.28 324	653	8.28 332	653	1.71 667	9.99 992	54
7	8.28 977	643	8.28 985	643	1.71 014	9.99 992	53
8	8.29 620	634	8.29 629	634	1.70 371	9.99 991	52
9	8.30 254	625	8.30 263	625	1.69 736	9.99 991	51
10	8.30 879	616	8.30 888	616	1.69 111	9.99 991	50
11	8.31 495	607	8.31 504	607	1.68 495	9.99 990	49
12	8.32 102	599	8.32 112	599	1.67 888	9.99 990	48
13	8.32 701	591	8.32 711	591	1.67 288	9.99 990	47
14	8.33 292	583	8.33 302	583	1.66 697	9.99 990	46
15	8.33 875	575	8.33 885	575	1.66 114	9.99 989	45
16	8.34 450	567	8.34 461	568	1.65 539	9.99 989	44
17	8.35 018	560	8.35 029	560	1.64 971	9.99 989	43
18	8.35 578	553	8.35 589	553	1.64 410	9.99 989	42
19	8.36 131	546	8.36 143	546	1.63 857	9.99 988	41
20	8.36 677	539	8.36 689	539	1.63 310	9.99 988	40
21	8.37 217	533	8.37 229	533	1.62 771	9.99 988	39
22	8.37 750	526	8.37 762	527	1.62 238	9.99 987	38
23	8.38 276	520	8.38 289	520	1.61 711	9.99 987	37
24	8.38 796	514	8.38 809	514	1.61 191	9.99 987	36
25	8.39 310	508	8.39 323	508	1.60 676	9.99 986	35
26	8.39 818	502	8.39 831	502	1.60 168	9.99 986	34
27	8.40 320	496	8.40 334	496	1.59 666	9.99 986	33
28	8.40 816	491	8.40 830	491	1.59 169	9.99 986	32
29	8.41 307	485	8.41 321	485	1.58 678	9.99 985	31
30	8.41 792	479	8.41 807	480	1.58 193	9.99 985	30
31	8.42 271	474	8.42 287	475	1.57 713	9.99 985	29
32	8.42 746	469	8.42 762	469	1.57 238	9.99 984	28
33	8.43 215	464	8.43 231	464	1.56 768	9.99 984	27
34	8.43 680	459	8.43 696	460	1.56 304	9.99 984	26
35	8.44 139	454	8.44 156	455	1.55 844	9.99 983	25
36	8.44 594	450	8.44 611	450	1.55 389	9.99 983	24
37	8.45 044	445	8.45 061	445	1.54 938	9.99 982	23
38	8.45 489	440	8.45 507	441	1.54 493	9.99 982	22
39	8.45 930	436	8.45 948	437	1.54 052	9.99 982	21
40	8.46 366	432	8.46 385	432	1.53 615	9.99 981	20
41	8.46 798	428	8.46 817	428	1.53 183	9.99 981	19
42	8.47 226	423	8.47 245	424	1.52 754	9.99 981	18
43	8.47 650	419	8.47 669	419	1.52 330	9.99 980	17
44	8.48 069	415	8.48 089	416	1.51 911	9.99 980	16
45	8.48 485	411	8.48 505	412	1.51 495	9.99 979	15
46	8.48 896	407	8.48 917	408	1.51 083	9.99 979	14
47	8.49 304	404	8.49 325	404	1.50 675	9.99 979	13
48	8.49 708	400	8.49 729	400	1.50 270	9.99 978	12
49	8.50 108	396	8.50 130	396	1.49 870	9.99 978	11
50	8.50 504	393	8.50 526	393	1.49 473	9.99 978	10
51	8.50 897	389	8.51 320	390	1.49 080	9.99 977	9
52	8.51 286	386	8.51 310	386	1.48 690	9.99 977	8
53	8.51 672	382	8.51 696	383	1.48 304	9.99 976	7
54	8.52 055	379	8.52 079	379	1.47 921	9.99 976	6
55	8.52 434	375	8.52 458	376	1.47 541	9.99 975	5
56	8.52 810	373	8.52 835	373	1.47 165	9.99 975	4
57	8.53 183	369	8.53 208	370	1.46 792	9.99 975	3
58	8.53 552	366	8.53 578	366	1.46 422	9.99 974	2
	8.53 918	363	8.53 944	364	1.46 055	9.99 974	1
	8.54 282		8.54 308		1.45 691	9.99 973	0
	Log. Cos.	D	Log. Cot.	Com. D.	Log. Tan.	Log. Sin.	′

′	Log. Sin.	D	Log. Tan.	Com. D.	Log. Cot.	Log. Cos.	
0	8.54 282	360	8.54 308	360	1.45 691	9.99 973	60
1	8.54 642	357	8.54 669	358	1.45 331	9.99 973	59
2	8.54 999	354	8.55 027	354	1.44 973	9.99 972	58
3	8.55 354	351	8.55 381	352	1.44 618	9.99 972	57
4	8.55 705	348	8.55 733	349	1.44 266	9.99 971	56
5	8.56 054	346	8.56 083	346	1.43 917	9.99 971	55
6	8.56 400	343	8.56 429	343	1.43 571	9.99 971	54
7	8.56 743	340	8.56 772	341	1.43 227	9.99 970	53
8	8.57 083	338	8.57 113	338	1.42 886	9.99 970	52
9	8.57 421	335	8.57 452	335	1.42 548	9.99 969	51
10	8.57 756	332	8.57 787	333	1.42 212	9.99 969	50
11	8.58 088	330	8.58 121	330	1.41 879	9.99 968	49
12	8.58 419	327	8.58 451	328	1.41 548	9.99 968	48
13	8.58 747	325	8.58 779	325	1.41 220	9.99 967	47
14	8.59 072	323	8.59 105	323	1.40 895	9.99 967	46
15	8.59 395	320	8.59 428	320	1.40 571	9.99 966	45
16	8.59 715	318	8.59 749	318	1.40 251	9.99 966	44
17	8.60 033	316	8.60 067	316	1.39 932	9.99 965	43
18	8.60 349	313	8.60 384	314	1.39 616	9.99 965	42
19	8.60 662	311	8.60 698	311	1.39 302	9.99 964	41
20	8.60 973	309	8.61 009	309	1.38 990	9.99 964	40
21	8.61 282	306	8.61 319	307	1.38 681	9.99 963	39
22	8.61 589	304	8.61 626	305	1.38 374	9.99 963	38
23	8.61 893	302	8.61 931	303	1.38 068	9.99 962	37
24	8.62 196	300	8.62 234	300	1.37 765	9.99 962	36
25	8.62 496	298	8.62 535	299	1.37 465	9.99 961	35
26	8.62 795	296	8.62 834	297	1.37 166	9.99 961	34
27	8.63 091	294	8.63 131	294	1.36 869	9.99 960	33
28	8.63 385	292	8.63 425	293	1.36 574	9.99 959	32
29	8.63 677	290	8.63 718	291	1.36 281	9.99 959	31
30	8.63 968	288	8.64 009	288	1.35 990	9.99 958	30
31	8.64 256	286	8.64 298	287	1.35 702	9.99 958	29
32	8.64 543	284	8.64 585	285	1.35 414	9.99 957	28
33	8.64 827	282	8.64 870	283	1.35 129	9.99 957	27
34	8.65 110	281	8.65 153	284	1.34 846	9.99 956	26
35	8.65 391	279	8.65 435	280	1.34 565	9.99 956	25
36	8.65 670	277	8.65 715	278	1.34 285	9.99 955	24
37	8.65 947	275	8.65 993	276	1.34 007	9.99 954	23
38	8.66 223	274	8.66 269	274	1.33 731	9.99 954	22
39	8.66 497	272	8.66 543	272	1.33 456	9.99 953	21
40	8.66 769	270	8.66 816	271	1.33 184	9.99 953	20
41	8.67 039	268	8.67 087	269	1.32 913	9.99 952	19
42	8.67 308	267	8.67 356	267	1.32 643	9.99 952	18
43	8.67 575	265	8.67 624	266	1.32 376	9.99 951	17
44	8.67 840	264	8.67 890	264	1.32 110	9.99 950	16
45	8.68 104	262	8.68 154	262	1.31 845	9.99 950	15
46	8.68 366	260	8.68 417	261	1.31 583	9.99 949	14
47	8.68 627	259	8.68 678	259	1.31 321	9.99 948	13
48	8.68 886	257	8.68 938	258	1.31 062	9.99 948	12
49	8.69 144	256	8.69 196	256	1.30 803	9.99 947	11
50	8.69 400	254	8.69 453	255	1.30 547	9.99 947	10
51	8.69 654	253	8.69 708	253	1.30 292	9.99 946	9
52	8.69 907	251	8.69 961	252	1.30 038	9.99 945	8
53	8.70 159	250	8.70 214	250	1.29 786	9.99 945	7
54	8.70 409	248	8.70 464	249	1.29 535	9.99 944	6
55	8.70 657	247	8.70 714	248	1.29 286	9.99 943	5
56	8.70 905	245	8.70 962	246	1.29 038	9.99 943	4
57	8.71 150	244	8.71 208	245	1.28 791	9.99 942	3
58	8.71 395	243	8.71 453	243	1.28 546	9.99 942	2
59	8.71 638	241	8.71 697	242	1.28 303	9.99 941	1
60	8.71 880		8.71 939		1.28 060	9.99 940	0
	Log. Cos.	D	Log. Cot.	Com. D.	Log. Tan.	Log. Sin.	′

′	Log. Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	
0	8·71 880		8·71 939		1·28 060	9·99 943	60
1	8·72 120	240	8·72 180	241	1·27 819	9·99 940	59
2	8·72 355	239	8·72 420	240	1·27 579	9·99 939	58
3	8·72 597	237	8·72 659	238	1·27 341	9·99 938	57
4	8·72 833	236	8·72 896	237	1·27 104	9·99 938	56
5	8·73 069	235	8·73 131	235	1·26 868	9·99 937	55
6	8·73 302	233	8·73 366	235	1·26 633	9·99 936	54
7	8·73 535	233	8·73 599	233	1·26 400	9·99 935	53
8	8·73 766	231	8·73 831	232	1·26 168	9·99 935	52
9	8·73 997	230	8·74 062	231	1·25 937	9·99 934	51
10	8·74 226	229	8·74 292	229	1·25 708	9·99 933	50
11	8·74 453	227	8·74 520	228	1·25 479	9·99 933	49
12	8·74 680	227	8·74 748	228	1·25 252	9·99 932	48
13	8·74 905	225	8·74 974	226	1·25 026	9·99 931	47
14	8·75 129	224	8·75 199	225	1·24 801	9·99 931	46
15	8·75 353	224	8·75 422	223	1·24 577	9·99 930	45
16	8·75 574	221	8·75 645	223	1·24 354	9·99 929	44
17	8·75 795	221	8·75 867	222	1·24 133	9·99 928	43
18	8·76 015	220	8·76 087	220	1·23 913	9·99 928	42
19	8·76 233	218	8·76 306	219	1·23 693	9·99 927	41
20	8·76 451	218	8·76 524	218	1·23 475	9·99 926	40
21	8·76 667	216	8·76 741	217	1·23 258	9·99 925	39
22	8·76 883	216	8·76 959	216	1·23 042	9·99 925	38
23	8·77 097	214	8·77 172	214	1·22 827	9·99 924	37
24	8·77 310	213	8·77 386	214	1·22 613	9·99 923	36
25	8·77 522	212	8·77 599	213	1·22 400	9·99 922	35
26	8·77 733	211	8·77 811	212	1·22 188	9·99 922	34
27	8·77 943	210	8·78 022	211	1·21 978	9·99 921	33
28	8·78 152	209	8·78 232	210	1·21 768	9·99 920	32
29	8·78 360	208	8·78 441	209	1·21 559	9·99 919	31
30	8·78 567	207	8·78 648	207	1·21 351	9·99 919	30
31	8·78 773	206	8·78 855	207	1·21 144	9·99 918	29
32	8·78 978	205	8·79 061	206	1·20 938	9·99 917	28
33	8·79 183	204	8·79 266	204	1·20 734	9·99 916	27
34	8·79 386	203	8·79 470	204	1·20 530	9·99 916	26
35	8·79 588	202	8·79 673	203	1·20 327	9·99 915	25
36	8·79 789	201	8·79 875	202	1·20 125	9·99 914	24
37	8·79 989	200	8·80 076	201	1·19 923	9·99 913	23
38	8·80 189	199	8·80 276	200	1·19 723	9·99 912	22
39	8·80 387	198	8·80 476	199	1·19 524	9·99 912	21
40	8·80 585	197	8·80 674	198	1·19 326	9·99 911	20
41	8·80 782	197	8·80 871	197	1·19 128	9·99 910	19
42	8·80 977	195	8·81 068	197	1·18 931	9·99 909	18
43	8·81 172	195	8·81 264	196	1·18 736	9·99 908	17
44	8·81 366	194	8·81 459	195	1·18 541	9·99 907	16
45	8·81 560	194	8·81 653	194	1·18 347	9·99 907	15
46	8·81 752	192	8·81 846	193	1·18 154	9·99 906	14
47	8·81 943	191	8·82 038	192	1·17 961	9·99 905	13
48	8·82 134	191	8·82 230	192	1·17 770	9·99 904	12
49	8·82 324	190	8·82 420	190	1·17 575	9·99 903	11
50	8·82 513	189	8·82 610	190	1·17 389	9·99 902	10
51	8·82 701	188	8·82 799	189	1·17 201	9·99 902	9
52	8·82 888	187	8·82 987	188	1·17 012	9·99 901	8
53	8·83 075	187	8·83 175	187	1·16 825	9·99 900	7
54	8·83 260	185	8·83 361	186	1·16 638	9·99 899	6
55	8·83 445	185	8·83 547	186	1·16 453	9·99 898	5
56	8·83 629	184	8·83 732	185	1·16 268	9·99 897	4
57	8·83 813	184	8·83 916	184	1·16 083	9·99 896	3
58	8·83 995	182	8·84 100	184	1·15 900	9·99 896	2
59	8·84 177	182	8·84 282	182	1·15 717	9·99 895	1
60	8·84 358	181	8·84 464	182	1·15 535	9·99 894	0
	Log. Cos.	d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	′

P. P.				
	330	320	310	300
6	33.0	32.0	31.0	30.0
7	38.5	37.3	36.1	35.0
8	44.0	42.7	41.3	40.0
9	49.5	48.0	46.5	45.0
10	55.0	53.3	51.7	50.0
20	110.0	106.7	103.3	100.0
30	165.0	160.0	155.0	150.0
40	220.0	213.3	206.7	200.0
50	275.0	266.7	258.3	250.0

P. P.				
	290	280	270	260
6	29.0	28.0	27.0	26.0
7	33.8	32.7	31.5	30.3
8	38.7	37.3	36.0	34.7
9	43.5	42.0	40.5	39.0
10	48.3	46.7	45.0	43.3
20	96.7	93.3	90.0	86.7
30	145.0	140.0	135.0	130.0
40	193.3	186.7	180.0	173.3
50	241.7	233.3	225.0	216.7

P. P.				
	250	240	230	220
6	25.0	24.0	23.0	22.0
7	29.2	28.0	26.8	25.7
8	33.3	32.0	30.7	29.3
9	37.5	36.0	34.5	33.0
10	41.7	40.0	38.3	36.7
20	83.3	80.0	76.7	73.3
30	125.0	120.0	115.0	110.0
40	166.7	160.0	153.3	146.7
50	208.3	200.0	191.7	183.3

P. P.				
	210	200	190	180
6	21.0	20.0	19.0	18.0
7	24.5	23.3	22.2	21.0
8	28.0	26.7	25.3	24.0
9	31.5	30.0	28.5	27.0
10	35.0	33.3	31.7	30.0
20	70.0	66.7	63.3	60.0
30	105.0	100.0	95.0	90.0
40	140.0	133.3	126.7	120.0
50	175.0	166.7	158.3	150.0

P. P.							
	9	8	7	6	5		
6	0.9	0.8	0.7	0.6	0.5		
7	1.1	1.0	0.9	0.7	0.6		
8	1.2	1.1	1.0	0.8	0.7		
9	1.4	1.3	1.2	1.0	0.8		
10	1.5	1.3	1.2	1.0	0.8		
20	3.0	2.7	2.3	2.0	1.7		
30	4.5	4.0	3.5	3.0	2.5		
40	6.0	5.3	4.7	4.0	3.3		
50	7.5	6.7	5.8	5.0	4.2		
	4	3	2	1	0		
6	0.4	0.3	0.2	0.1	0.0		
7	0.5	0.4	0.3	0.2	0.1		
8	0.6	0.5	0.4	0.2	0.1		
9	0.7	0.6	0.4	0.2	0.1		
10	0.7	0.5	0.3	0.2	0.1		
20	1.3	1.0	0.7	0.3	0.2		
30	2.0	1.5	1.0	0.5	0.2		
40	2.7	2.0	1.3	0.7	0.3		
50	3.3	2.5	1.7	0.8	0.4		

′	Log. Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	
0	8.84 358	180	8.84 464	181	1.15 535	9.99 894	60
1	8.84 538	180	8.84 645	180	1.15 354	9.99 893	59
2	8.84 718	178	8.84 826	179	1.15 174	9.99 892	58
3	8.84 897	178	8.85 005	179	1.14 994	9.99 891	57
4	8.85 075	177	8.85 184	178	1.14 815	9.99 890	56
5	8.85 252	176	8.85 363	177	1.14 637	9.99 889	55
6	8.85 429	176	8.85 540	177	1.14 459	9.99 888	54
7	8.85 605	175	8.85 717	176	1.14 283	9.99 888	53
8	8.85 780	174	8.85 893	176	1.14 107	9.99 887	52
9	8.85 954	174	8.86 068	175	1.13 931	9.99 886	51
10	8.86 128	173	8.86 243	175	1.13 756	9.99 885	50
11	8.86 301	173	8.86 417	174	1.13 582	9.99 884	49
12	8.86 474	171	8.86 590	173	1.13 409	9.99 883	48
13	8.86 645	171	8.86 763	172	1.13 237	9.99 882	47
14	8.86 816	170	8.86 935	172	1.13 065	9.99 881	46
15	8.86 987	169	8.87 106	171	1.12 893	9.99 880	45
16	8.87 156	169	8.87 277	170	1.12 723	9.99 879	44
17	8.87 325	168	8.87 447	170	1.12 553	9.99 878	43
18	8.87 494	167	8.87 616	169	1.12 384	9.99 877	42
19	8.87 661	167	8.87 785	169	1.12 215	9.99 876	41
20	8.87 828	166	8.87 953	168	1.12 047	9.99 875	40
21	8.87 995	165	8.88 120	167	1.11 880	9.99 874	39
22	8.88 160	165	8.88 287	167	1.11 713	9.99 874	38
23	8.88 326	164	8.88 453	166	1.11 547	9.99 873	37
24	8.88 490	163	8.88 618	165	1.11 381	9.99 872	36
25	8.88 654	163	8.88 783	165	1.11 217	9.99 871	35
26	8.88 817	162	8.88 947	164	1.11 052	9.99 870	34
27	8.88 980	162	8.89 111	163	1.10 889	9.99 869	33
28	8.89 142	161	8.89 274	163	1.10 726	9.99 868	32
29	8.89 303	161	8.89 436	162	1.10 563	9.99 867	31
30	8.89 464	160	8.89 598	162	1.10 401	9.99 866	30
31	8.89 624	159	8.89 759	161	1.10 240	9.99 865	29
32	8.89 784	159	8.89 920	161	1.10 079	9.99 864	28
33	8.89 943	158	8.90 080	160	1.09 919	9.99 863	27
34	8.90 101	158	8.90 240	159	1.09 760	9.99 862	26
35	8.90 259	157	8.90 398	158	1.09 601	9.99 861	25
36	8.90 417	156	8.90 557	158	1.09 443	9.99 860	24
37	8.90 573	156	8.90 714	157	1.09 285	9.99 859	23
38	8.90 729	156	8.90 872	157	1.09 128	9.99 858	22
39	8.90 885	155	8.91 028	156	1.08 971	9.99 857	21
40	8.91 040	154	8.91 184	156	1.08 815	9.99 856	20
41	8.91 195	154	8.91 340	155	1.08 660	9.99 855	19
42	8.91 349	153	8.91 495	155	1.08 505	9.99 853	18
43	8.91 502	153	8.91 649	154	1.08 350	9.99 852	17
44	8.91 655	153	8.91 803	154	1.08 196	9.99 851	16
45	8.91 807	152	8.91 957	153	1.08 043	9.99 850	15
46	8.91 959	151	8.92 109	152	1.07 890	9.99 849	14
47	8.92 110	151	8.92 262	153	1.07 738	9.99 848	13
48	8.92 261	150	8.92 413	151	1.07 586	9.99 847	12
49	8.92 411	150	8.92 565	152	1.07 435	9.99 846	11
50	8.92 561	150	8.92 715	150	1.07 284	9.99 845	10
51	8.92 710	149	8.92 866	151	1.07 134	9.99 844	9
52	8.92 858	148	8.93 015	149	1.06 984	9.99 843	8
53	8.93 007	148	8.93 164	149	1.06 835	9.99 842	7
54	8.93 154	147	8.93 313	149	1.06 686	9.99 841	6
55	8.93 301	147	8.93 461	148	1.06 538	9.99 840	5
56	8.93 448	146	8.93 609	148	1.06 390	9.99 839	4
57	8.93 594	146	8.93 756	147	1.06 243	9.99 837	3
58	8.93 740	146	8.93 903	147	1.06 097	9.99 836	2
59	8.93 885	145	8.94 049	146	1.05 950	9.99 835	1
60	8.94 029	144	8.94 195	146	1.05 805	9.99 834	0
	Log. Cos.	d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	′

P. P.				
	181	180	178	176
6	18.1	18.0	17.8	17.6
7	21.1	21.0	20.7	20.5
8	24.1	24.0	23.7	23.4
9	27.1	27.0	26.7	26.4
10	30.1	30.0	29.6	29.3
20	60.3	60.0	59.3	58.6
30	90.5	90.0	89.0	88.0
40	120.6	120.0	118.6	117.3
50	150.8	150.0	148.3	146.6
	174	172	170	168
6	17.4	17.2	17.0	16.8
7	20.3	20.0	19.8	19.6
8	23.2	22.9	22.6	22.4
9	26.1	25.8	25.5	25.2
10	29.0	28.6	28.3	28.0
20	58.0	57.3	56.6	56.0
30	87.0	86.0	85.0	84.0
40	116.0	114.6	113.3	112.0
50	145.0	143.3	141.6	140.0
	166	164	162	160
6	16.6	16.4	16.2	16.0
7	19.3	19.1	18.9	18.8
8	22.1	21.8	21.6	21.3
9	24.9	24.6	24.3	24.0
10	27.6	27.3	27.0	26.6
20	55.3	54.6	54.0	53.3
30	83.0	82.0	81.0	80.0
40	110.6	109.3	108.0	106.6
50	138.3	136.6	135.0	133.3
	158	156	154	152
6	15.8	15.6	15.4	15.2
7	18.4	18.2	17.9	17.7
8	21.0	20.8	20.5	20.2
9	23.7	23.4	23.1	22.8
10	26.3	26.0	25.6	25.3
20	52.6	52.0	51.3	50.6
30	79.0	78.0	77.0	76.0
40	105.3	104.0	102.6	101.3
50	131.6	130.0	128.3	126.6
	150	149	148	147
6	15.0	14.9	14.8	14.7
7	17.5	17.4	17.2	17.1
8	20.0	19.8	19.7	19.6
9	22.5	22.3	22.2	22.0
10	25.0	24.8	24.6	24.5
20	50.0	49.6	49.3	49.0
30	75.0	74.5	74.0	73.5
40	100.0	99.3	98.6	98.0
50	125.0	124.1	123.3	122.5
	146	145	1	1	0
6	14.6	14.5	0.1	0.1	0.0
7	17.0	16.9	0.2	0.1	0.0
8	19.4	19.3	0.2	0.1	0.0
9	21.9	21.7	0.2	0.1	0.1
10	24.3	24.1	0.2	0.1	0.1
20	48.6	48.3	0.5	0.3	0.1
30	73.0	72.5	0.7	0.5	0.2
40	97.3	96.6	1.0	0.6	0.3
50	121.6	120.8	1.2	0.8	0.4

P. P.

′	Log. Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	
0	8.94 029	144	8.94 195	145	1.05 805	9.99 834	60
1	8.94 173	143	8.94 340	144	1.05 659	9.99 833	59
2	8.94 317	143	8.94 485	144	1.05 515	9.99 832	58
3	8.94 460	143	8.94 629	144	1.05 370	9.99 831	57
4	8.94 603	142	8.94 773	143	1.05 226	9.99 830	56
5	8.94 745	142	8.94 917	142	1.05 083	9.99 829	55
6	8.94 887	141	8.95 059	142	1.04 940	9.99 827	54
7	8.95 028	141	8.95 202	142	1.04 798	9.99 826	53
8	8.95 169	140	8.95 344	141	1.04 656	9.99 825	52
9	8.95 310	140	8.95 485	141	1.04 514	9.99 824	51
10	8.95 450	139	8.95 626	141	1.04 373	9.99 823	50
11	8.95 589	139	8.95 767	140	1.04 232	9.99 822	49
12	8.95 728	139	8.95 907	140	1.04 092	9.99 821	48
13	8.95 867	138	8.96 047	139	1.03 952	9.99 819	47
14	8.96 005	138	8.96 186	139	1.03 813	9.99 818	46
15	8.96 143	137	8.96 325	139	1.03 674	9.99 817	45
16	8.96 280	137	8.96 464	138	1.03 536	9.99 816	44
17	8.96 417	137	8.96 602	138	1.03 398	9.99 815	43
18	8.96 553	136	8.96 739	137	1.03 260	9.99 814	42
19	8.96 689	136	8.96 876	137	1.03 123	9.99 813	41
20	8.96 825	135	8.97 013	137	1.02 986	9.99 811	40
21	8.96 960	135	8.97 149	136	1.02 850	9.99 810	39
22	8.97 094	134	8.97 285	136	1.02 714	9.99 809	38
23	8.97 229	134	8.97 421	135	1.02 579	9.99 808	37
24	8.97 363	134	8.97 556	135	1.02 444	9.99 807	36
25	8.97 496	133	8.97 690	134	1.02 309	9.99 805	35
26	8.97 629	133	8.97 825	135	1.02 175	9.99 804	34
27	8.97 762	133	8.97 958	133	1.02 041	9.99 803	33
28	8.97 894	132	8.98 092	134	1.01 908	9.99 802	32
29	8.98 026	132	8.98 225	133	1.01 775	9.99 801	31
30	8.98 157	131	8.98 357	132	1.01 642	9.99 799	30
31	8.98 288	131	8.98 490	133	1.01 510	9.99 798	29
32	8.98 419	131	8.98 621	131	1.01 378	9.99 797	28
33	8.98 549	130	8.98 753	132	1.01 247	9.99 796	27
34	8.98 679	130	8.98 884	131	1.01 116	9.99 794	26
35	8.98 808	129	8.99 015	131	1.00 985	9.99 793	25
36	8.98 937	129	8.99 145	130	1.00 855	9.99 792	24
37	8.99 066	129	8.99 275	130	1.00 725	9.99 791	23
38	8.99 194	128	8.99 404	129	1.00 595	9.99 789	22
39	8.99 322	128	8.99 533	129	1.00 466	9.99 788	21
40	8.99 449	127	8.99 662	129	1.00 337	9.99 787	20
41	8.99 577	127	8.99 791	129	1.00 209	9.99 786	19
42	8.99 703	126	8.99 919	128	1.00 081	9.99 784	18
43	8.99 830	126	9.00 046	127	0.99 953	9.99 783	17
44	8.99 956	126	9.00 174	128	0.99 826	9.99 782	16
45	9.00 081	125	9.00 300	126	0.99 699	9.99 781	15
46	9.00 207	126	9.00 427	127	0.99 573	9.99 779	14
47	9.00 332	125	9.00 553	126	0.99 446	9.99 778	13
48	9.00 456	124	9.00 679	126	0.99 321	9.99 777	12
49	9.00 580	124	9.00 804	125	0.99 195	9.99 776	11
50	9.00 704	124	9.00 930	126	0.99 070	9.99 774	10
51	9.00 828	123	9.01 054	124	0.98 945	9.99 773	9
52	9.00 951	123	9.01 179	125	0.98 821	9.99 772	8
53	9.01 075	122	9.01 303	124	0.98 697	9.99 770	7
54	9.01 196	121	9.01 427	124	0.98 573	9.99 769	6
55	9.01 318	122	9.01 550	123	0.98 450	9.99 768	5
56	9.01 440	122	9.01 673	123	0.98 327	9.99 766	4
57	9.01 561	121	9.01 796	123	0.98 204	9.99 765	3
58	9.01 682	121	9.01 918	122	0.98 081	9.99 764	2
59	9.01 803	120	9.02 040	122	0.97 959	9.99 763	1
60	9.01 923	120	9.02 162	121	0.97 838	9.99 761	0
	Log. Cos.	d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	′

	P. P.				
	145	144	143	142	141
6	14.5	14.4	14.3	14.2	14.1
7	16.9	16.8	16.7	16.5	16.4
8	19.3	19.2	19.0	18.9	18.8
9	21.7	21.6	21.4	21.3	21.2
10	24.1	24.0	23.8	23.6	23.5
20	48.3	48.0	47.6	47.3	47.0
30	72.5	72.0	71.5	71.0	70.5
40	96.6	96.0	95.3	94.6	94.0
50	120.8	120.0	119.1	118.3	117.5
	140	139	138	137	136
6	14.0	13.9	13.8	13.7	13.6
7	16.3	16.2	16.1	16.0	15.9
8	18.6	18.5	18.4	18.2	18.1
9	21.0	20.8	20.7	20.5	20.4
10	23.3	23.1	23.0	22.8	22.7
20	46.6	46.3	46.0	45.6	45.3
30	70.0	69.5	69.0	68.5	68.0
40	93.3	92.6	92.0	91.3	90.7
50	116.6	115.8	115.0	114.1	113.3
	135	134	133	132	
6	13.5	13.4	13.3	13.2	
7	15.7	15.6	15.5	15.4	
8	18.0	17.8	17.7	17.6	
9	20.2	20.1	19.9	19.8	
10	22.5	22.3	22.1	22.0	
20	45.0	44.6	44.3	44.0	
30	67.5	67.0	66.5	66.0	
40	90.0	89.3	88.6	88.0	
50	112.5	111.6	110.8	110.0	
	131	130	129	128	
6	13.1	13.0	12.9	12.8	
7	15.3	15.1	15.0	14.9	
8	17.4	17.3	17.2	17.0	
9	19.6	19.5	19.3	19.2	
10	21.8	21.6	21.5	21.3	
20	43.6	43.3	43.0	42.6	
30	65.5	65.0	64.5	64.0	
40	87.3	86.6	86.0	85.3	
50	109.1	108.3	107.5	106.6	
	127	126	125	124	123
6	12.7	12.6	12.5	12.4	12.3
7	14.8	14.7	14.6	14.4	14.3
8	16.9	16.8	16.6	16.5	16.4
9	19.0	18.9	18.7	18.6	18.4
10	21.1	21.0	20.8	20.6	20.5
20	42.3	42.0	41.6	41.3	41.0
30	63.5	63.0	62.5	62.0	61.5
40	84.6	84.0	83.3	82.6	82.0
50	105.8	105.0	104.1	103.3	102.5
	122	121	120	1	0
6	12.2	12.1	12.0	0.1	0.0
7	14.2	14.1	14.0	0.2	0.1
8	16.2	16.1	16.0	0.2	0.1
9	18.3	18.1	18.0	0.2	0.1
10	20.3	20.1	20.0	0.2	0.1
20	40.6	40.3	40.0	0.5	0.3
30	61.0	60.5	60.0	0.8	0.5
40	81.3	80.6	80.0	1.0	0.7
50	101.6	100.8	100.0	1.2	0.8

P

′	Log. Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	
0	9.01 923		9.02 162		0.97 838	9.99 761	60
1	9.02 043	120	9.02 283	121	0.97 717	9.99 760	59
2	9.02 163	119	9.02 404	121	0.97 595	9.99 759	58
3	9.02 282	119	9.02 525	120	0.97 475	9.99 757	57
4	9.02 401	119	9.02 645	120	0.97 354	9.99 756	56
5	9.02 520	119	9.02 765	120	0.97 234	9.99 754	55
6	9.02 638	118	9.02 885	119	0.97 115	9.99 753	54
7	9.02 756	118	9.03 004	119	0.96 995	9.99 752	53
8	9.02 874	118	9.03 123	119	0.96 876	9.99 750	52
9	9.02 992	117	9.03 242	118	0.96 757	9.99 749	51
10	9.03 109	117	9.03 361	118	0.96 639	9.99 748	50
11	9.03 225	116	9.03 479	118	0.96 521	9.99 746	49
12	9.03 342	116	9.03 597	117	0.96 403	9.99 745	48
13	9.03 458	116	9.03 714	117	0.96 285	9.99 744	47
14	9.03 574	115	9.03 831	117	0.96 168	9.99 742	46
15	9.03 689	115	9.03 948	117	0.96 057	9.99 741	45
16	9.03 805	114	9.04 065	116	0.95 935	9.99 739	44
17	9.03 919	114	9.04 181	116	0.95 818	9.99 738	43
18	9.04 034	114	9.04 297	116	0.95 702	9.99 737	42
19	9.04 148	114	9.04 413	115	0.95 587	9.99 735	41
20	9.04 262	113	9.04 528	115	0.95 471	9.99 734	40
21	9.04 376	113	9.04 643	115	0.95 356	9.99 732	39
22	9.04 489	113	9.04 758	114	0.95 242	9.99 731	38
23	9.04 602	113	9.04 872	114	0.95 127	9.99 730	37
24	9.04 715	113	9.04 987	114	0.95 013	9.99 728	36
25	9.04 828	112	9.05 101	114	0.94 899	9.99 727	35
26	9.04 940	112	9.05 214	113	0.94 785	9.99 725	34
27	9.05 052	112	9.05 328	113	0.94 672	9.99 724	33
28	9.05 163	111	9.05 440	113	0.94 559	9.99 723	32
29	9.05 275	111	9.05 553	113	0.94 446	9.99 721	31
30	9.05 386	111	9.05 666	112	0.94 334	9.99 720	30
31	9.05 496	110	9.05 778	112	0.94 222	9.99 718	29
32	9.05 607	110	9.05 890	112	0.94 110	9.99 717	28
33	9.05 717	110	9.06 001	111	0.93 998	9.99 715	27
34	9.05 827	110	9.06 113	111	0.93 887	9.99 714	26
35	9.05 937	109	9.06 224	111	0.93 776	9.99 712	25
36	9.06 046	109	9.06 335	111	0.93 665	9.99 711	24
37	9.06 155	109	9.06 445	110	0.93 554	9.99 710	23
38	9.06 264	109	9.06 555	110	0.93 444	9.99 708	22
39	9.06 373	108	9.06 665	110	0.93 334	9.99 707	21
40	9.06 480	108	9.06 775	109	0.93 225	9.99 705	20
41	9.06 588	108	9.06 884	109	0.93 115	9.99 704	19
42	9.06 696	107	9.06 994	109	0.93 006	9.99 702	18
43	9.06 803	107	9.07 102	108	0.92 897	9.99 701	17
44	9.06 910	107	9.07 211	109	0.92 788	9.99 699	16
45	9.07 017	106	9.07 319	108	0.92 680	9.99 698	15
46	9.07 124	106	9.07 428	108	0.92 572	9.99 696	14
47	9.07 230	106	9.07 535	107	0.92 464	9.99 695	13
48	9.07 336	106	9.07 643	107	0.92 357	9.99 693	12
49	9.07 442	106	9.07 750	107	0.92 249	9.99 692	11
50	9.07 548	105	9.07 857	107	0.92 142	9.99 690	10
51	9.07 653	105	9.07 964	107	0.92 035	9.99 689	9
52	9.07 758	105	9.08 071	106	0.91 929	9.99 687	8
53	9.07 863	104	9.08 177	106	0.91 822	9.99 686	7
54	9.07 967	104	9.08 283	106	0.91 716	9.99 684	6
55	9.08 072	104	9.08 389	105	0.91 611	9.99 683	5
56	9.08 176	104	9.08 494	105	0.91 505	9.99 681	4
57	9.08 279	103	9.08 600	105	0.91 400	9.99 679	3
58	9.08 383	103	9.08 705	105	0.91 295	9.99 678	2
59	9.08 486	103	9.08 810	105	0.91 190	9.99 676	1
30	9.08 589	103	9.08 914	104	0.91 085	9.99 675	0
	Log. Cos.	d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	′

P. P.					
	121	121	120	119	118
6	12.1	12.1	12.0	11.9	11.8
7	14.2	14.1	14.0	13.9	13.7
8	16.2	16.1	16.0	15.9	15.7
9	18.2	18.1	18.0	17.8	17.7
10	20.2	20.1	20.0	19.8	19.6
20	40.5	40.3	40.0	39.6	39.3
30	60.7	60.5	60.0	59.5	59.0
40	81.0	80.6	80.0	79.3	78.6
50	101.2	100.8	100.0	99.1	98.3

P. P.					
	117	117	116	115	
6	11.7	11.7	11.6	11.5	
7	13.7	13.6	13.5	13.4	
8	15.6	15.6	15.4	15.3	
9	17.6	17.5	17.4	17.2	
10	19.6	19.5	19.3	19.1	
20	39.1	39.0	38.6	38.3	
30	58.7	58.5	58.0	57.5	
40	78.3	78.0	77.3	76.6	
50	97.9	97.5	96.6	95.8	

P. P.					
	114	114	113	112	111
6	11.4	11.4	11.3	11.2	11.1
7	13.3	13.3	13.2	13.0	12.9
8	15.2	15.2	15.0	14.9	14.8
9	17.2	17.1	16.9	16.8	16.6
10	19.1	19.0	18.8	18.6	18.5
20	38.1	38.0	37.6	37.3	37.0
30	57.2	57.0	56.5	56.0	55.5
40	76.3	76.0	75.3	74.6	74.0
50	95.4	95.0	94.1	93.3	92.5

P. P.					
	110	110	109	108	
6	11.0	11.0	10.9	10.8	
7	12.9	12.8	12.7	12.6	
8	14.7	14.6	14.5	14.4	
9	16.6	16.5	16.3	16.2	
10	18.4	18.3	18.1	18.0	
20	36.8	36.6	36.3	36.0	
30	55.2	55.0	54.5	54.0	
40	73.6	73.3	72.6	72.0	
50	92.1	91.6	90.8	90.0	

P. P.					
	107	107	106	105	104
6	10.7	10.7	10.6	10.5	10.4
7	12.5	12.5	12.3	12.2	12.1
8	14.3	14.2	14.1	14.0	13.8
9	16.1	16.0	15.9	15.7	15.6
10	17.9	17.8	17.6	17.5	17.3
20	35.8	35.6	35.3	35.0	34.6
30	53.7	53.5	53.0	52.5	52.0
40	71.6	71.3	70.6	70.0	69.3
50	89.6	89.1	88.3	87.5	86.6

P. P.					
	103	103	2	1	1
6	10.3	10.3	0.2	0.1	0.1
7	12.1	12.0	0.2	0.2	0.1
8	13.8	13.7	0.2	0.2	0.1
9	15.5	15.4	0.3	0.2	0.1
10	17.2	17.1	0.3	0.2	0.1
20	34.5	34.3	0.6	0.5	0.3
30	51.7	51.5	1.0	0.7	0.5
40	69 0	68.6	1.3	1.0	0.6
50	86.2	85.8	1.6	1.2	0.8

P. P.

′	Log. Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	
0	9.08 585	103	9.08 314	104	0.91 085	9.99 675	60
1	9.08 692	102	9.09 018	104	0.90 981	9.99 673	59
2	9.08 794	102	9.09 123	103	0.90 877	9.99 672	58
3	9.08 897	102	9.09 226	103	0.90 773	9.99 670	57
4	9.08 999	102	9.09 330	103	0.90 670	9.99 669	56
5	9.09 101	101	9.09 433	103	0.90 566	9.99 667	55
6	9.09 202	101	9.09 536	103	0.90 463	9.99 665	54
7	9.09 303	101	9.09 639	103	0.90 360	9.99 664	53
8	9.09 404	101	9.09 742	103	0.90 258	9.99 662	52
9	9.09 505	100	9.09 844	102	0.90 155	9.99 661	51
10	9.09 603	100	9.09 947	101	0.90 053	9.99 659	50
11	9.09 706	100	9.10 048	102	0.89 951	9.99 658	49
12	9.09 805	100	9.10 150	101	0.89 849	9.99 656	48
13	9.09 906	99	9.10 252	101	0.89 748	9.99 654	47
14	9.10 006	99	9.10 353	101	0.89 647	9.99 653	46
15	9.10 105	99	9.10 454	101	0.89 546	9.99 651	45
16	9.10 205	98	9.10 555	101	0.89 445	9.99 650	44
17	9.10 303	98	9.10 655	100	0.89 344	9.99 648	43
18	9.10 402	99	9.10 756	100	0.89 244	9.99 646	42
19	9.10 501	98	9.10 856	100	0.89 144	9.99 645	41
20	9.10 599	98	9.10 956	99	0.89 044	9.99 643	40
21	9.10 697	98	9.11 055	99	0.88 944	9.99 641	39
22	9.10 795	97	9.11 155	99	0.88 845	9.99 640	38
23	9.10 892	97	9.11 254	99	0.88 745	9.99 638	37
24	9.10 990	97	9.11 353	99	0.88 64	9.99 637	36
25	9.11 087	97	9.11 452	98	0.88 548	9.99 635	35
26	9.11 184	97	9.11 550	98	0.88 449	9.99 633	34
27	9.11 281	96	9.11 649	98	0.88 351	9.99 632	33
28	9.11 377	96	9.11 747	98	0.88 253	9.99 630	32
29	9.11 473	96	9.11 845	98	0.88 155	9.99 628	31
30	9.11 570	95	9.11 943	98	0.88 057	9.99 627	30
31	9.11 665	96	9.12 040	97	0.87 959	9.99 625	29
32	9.11 761	95	9.12 137	97	0.87 862	9.99 623	28
33	9.11 856	95	9.12 235	97	0.87 765	9.99 622	27
34	9.11 952	95	9.12 331	96	0.87 668	9.99 620	26
35	9.12 047	94	9.12 428	97	0.87 571	9.99 618	25
36	9.12 141	94	9.12 525	96	0.87 475	9.99 617	24
37	9.12 236	94	9.12 621	96	0.87 379	9.99 615	23
38	9.12 330	94	9.12 717	96	0.87 283	9.99 613	22
39	9.12 425	93	9.12 813	96	0.87 187	9.99 611	21
40	9.12 518	94	9.12 908	95	0.87 091	9.99 610	20
41	9.12 612	93	9.13 004	95	0.86 996	9.99 608	19
42	9.12 706	93	9.13 099	95	0.86 900	9.99 606	18
43	9.12 799	93	9.13 194	95	0.86 805	9.99 605	17
44	9.12 892	93	9.13 289	95	0.86 710	9.99 603	16
45	9.12 985	93	9.13 384	94	0.86 616	9.99 601	15
46	9.13 078	92	9.13 478	94	0.86 521	9.99 600	14
47	9.13 170	93	9.13 572	94	0.86 427	9.99 598	13
48	9 13 263	92	9.13 666	94	0.86 333	9.99 596	12
49	9.13 355	92	9.13 760	94	0.86 239	9.99 594	11
50	9.13 447	92	9.13 854	93	0.86 146	9.99 593	10
51	9.13 538	91	9.13 947	93	0.86 052	9.99 591	9
52	9.13 630	92	9.14 041	93	0.85 959	9.99 589	8
53	9.13 721	91	9.14 134	93	0.85 866	9.99 587	7
54	9.13 813	91	9.14 227	93	0.85 773	9.99 586	6
55	9.13 903	90	9.14 319	92	0.85 680	9.99 584	5
56	9.13 994	91	9.14 412	93	0.85 588	9.99 582	4
57	9.14 085	90	9.14 504	92	0.85 495	9.99 580	3
58	9.14 175	90	9.14 596	92	0.85 403	9.99 579	2
59	9.14 265	90	9.14 688	92	0.85 311	9.99 577	1
60	9.14 355	90	9.14 780	92	0.85 219	9.99 575	0
	Log. Cos.	d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	′

P. P.				
	104	103	102	101
6	10.4	10.3	10.2	10.1
7	12.1	12.0	11.9	11.8
8	13.8	13.7	13.6	13.5
9	15.6	15.4	15.3	15.1
10	17.3	17.1	17.0	16.8
20	34.6	34.3	34.0	33.6
30	52.0	51.5	51.0	50.5
40	69.3	68.6	68.0	67.3
50	86.6	85.8	85.0	84.1

P. P.				
	100	100	99	98
6	10.0	10.0	9.9	9.8
7	11.7	11.6	11.5	11.4
8	13.4	13.3	13.2	13.0
9	15.1	15.0	14.8	14.7
10	16.7	16.6	16.5	16.3
20	33.5	33.3	33.0	32.6
30	50.2	50.0	49.5	49.0
40	67.0	66.6	66.0	65.3
50	83.7	83.3	82.5	81.6

P. P.				
	97	97	96	95
6	9.7	9.7	9.6	9.5
7	11.4	11.3	11.2	11.1
8	13.0	12.9	12.8	12.6
9	14.6	14.5	14.4	14.2
10	16.2	16.1	16.0	15.8
20	32.5	32.3	32.0	31.6
30	48.7	48.5	48.0	47.5
40	65.0	64.6	64.0	63.3
50	81.2	80.8	80.0	79.1

P. P.				
	94	94	93	92
6	9.4	9.4	9.3	9.2
7	11.0	10.9	10.8	10.7
8	12.6	12.5	12.4	12.2
9	14.2	14.1	13.9	13.8
10	15.7	15.6	15.5	15.3
20	31.5	31.3	31.0	30.6
30	47.2	47.0	46.5	46.0
40	63.0	62.6	62.0	61.3
50	78.7	78.3	77.5	76.6

P. P.				
	91	91	90	2
6	9.1	9.1	9.0	0.2
7	10.7	10.6	10.5	0.2
8	12.2	12.1	12.0	0.2
9	13.7	13.6	13.5	0.3
10	15.2	15.1	15.0	0.3
20	30.5	30.3	30.0	0.6
30	45.7	45.5	45.0	1.0
40	61.0	60.6	60.0	1.3
50	76.2	75.8	75.0	1.6

Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.		P. P.
355		9.14 780		0.85 219	9.99 575	60	
445	90	9.14 872	91	0.85 128	9 99 573	59	
535	89	9.14 963	91	0.85 037	9.99 571	58	
624	89	9.15 054	91	0.84 945	9.99 570	57	
713	89	9.15 145	91	0.84 854	9.99 568	56	
802	89	9.15 236	91	0.84 763	9.99 566	55	
891	89	9.15 327	90	0.84 673	9.99 564	54	
980	89	9.15 417	90	0.84 582	9.99 563	53	
068	88	9.15 507	90	0.84 492	9.99 561	52	
157	88	9.15 598	90	0.84 402	9.99 559	51	
245	88	9.15 687	89	0.84 312	9.99 557	50	
333	88	9.15 777	90	0.84 222	9.99 555	49	
421	88	9.15 867	89	0.84 133	9.99 553	48	
508	87	9 15 956	89	0.84 043	9.99 552	47	
595	87	9.16 045	89	0.83 954	9.99 550	46	
683	87	9.16 134	89	0.83 865	9.99 548	45	
770	87	9 16 223	89	0.83 776	9.99 546	44	
857	87	9.16 312	89	0.83 687	9.99 544	43	
943	86	9.16 401	88	0.83 599	9.99 542	42	
030	86	9.16 489	88	0.83 511	9.99 541	41	
116	86	9.16 577	88	0.83 422	9.99 539	40	
202	86	9.16 665	88	0.83 334	9.99 537	39	
288	86	9.16 753	87	0.83 247	9.99 535	38	
374	86	9.16 841	88	0.83 159	9.99 533	37	
460	85	9.16 928	87	0.83 071	9.99 531	36	
545	85	9.17 015	87	0.82 984	9.99 529	35	
630	85	9.17 103	87	0.82 897	9.99 528	34	
716	85	9.17 190	87	0.82 810	9.99 526	33	
801	85	9.17 276	86	0.82 723	9.99 524	32	
885	84	9.17 363	87	0.82 636	9.99 522	31	
970	84	9.17 450	86	0.82 550	9.99 520	30	
054	84	9.17 536	86	0.82 464	9.99 518	29	
139	84	9.17 622	86	0.82 377	9.99 516	28	
223	84	9.17 708	86	0.82 291	9.99 514	27	
307	84	9.17 794	85	0.82 206	9.99 513	26	
391	84	9.17 8 0	86	0.82 120	9.99 511	25	
474	83	9.17 965	85	0.82 034	9.99 509	24	
558	83	9.18 051	85	0.81 949	9.99 507	23	
641	83	9.18 136	85	0.81 864	9.99 505	22	
724	83	9.18 221	85	0.81 779	9.99 503	21	
807	83	9.18 306	84	0.81 694	9.99 501	20	
890	83	9.18 390	84	0.81 609	9.99 499	19	
972	82	9.18 475	84	0.81 525	9.99 497	18	
055	82	9.18 559	84	0.81 440	9.99 495	17	
137	82	9.18 644	84	0.81 356	9.99 493	16	
219	82	9.18 728	84	0.81 272	9.99 491	15	
301	82	9.18 812	84	0.81 188	9.99 489	14	
383	82	9.18 896	84	0.81 104	9.99 487	13	
465	81	9.18 979	83	0.81 020	9.99 485	12	
546	81	9.19 063	83	0.80 937	9.99 484	11	
628	81	9.19 146	83	0.80 854	9.99 482	10	
709	81	9.19 229	83	0.80 770	9.99 480	9	
790	81	9.19 312	83	0.80 687	9.99 478	8	
871	80	9.19 395	83	0.80 604	9.99 476	7	
952	81	9.19 478	82	0.80 522	9.99 474	6	
032	80	9.19 560	82	0.80 439	9.99 472	5	
113	80	9.19 643	82	0.80 357	9 99 470	4	
193	80	9.19 725	82	0.80 274	9.99 468	3	
273	80	9.19 807	82	0.80 192	9.99 466	2	
353	80	9.19 889	82	0.80 110	9.99 464	1	
433	79	9.19 971	82	0.80 028	9.99 462	0	
Cos.	d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	′	P. P.

	9$\bar{1}$	91	90	89
6	9.1	9.1	9.0	8.9
7	10.7	10.6	10.5	10.4
8	12.2	12.1	12.0	11.8
9	13.7	13.6	13.5	13.3
10	15.2	15.1	15.0	14.8
20	30.5	30.3	30.0	29.6
30	45.7	45.5	45.0	44.5
40	61.0	60.6	60.0	59.3
50	76.2	75.8	75.0	74.1

	8$\bar{8}$	88	87	86
6	8.8	8.8	8.7	8.6
7	10.3	10.2	10.1	10.0
8	11.8	11.7	11.6	11.4
9	13.3	13.2	13.0	12.9
10	14.7	14.6	14.5	14.3
20	29.5	29.3	29.0	28.6
30	44.2	44.0	43.5	43.0
40	59.0	58.6	58.0	57.3
50	73.7	73.3	72.5	71.6

	8$\bar{5}$	85	84	83
6	8.5	8.5	8.4	8.3
7	10.0	9.9	9.8	9.7
8	11.4	11.3	11.2	11.0
9	12.8	12.7	12.6	12.4
10	14.2	14.1	14.0	13.8
20	28.5	28.3	28.0	27.6
30	42.7	42.5	42.0	41.5
40	57.0	56.6	56.0	55.3
50	71.2	70.8	70.0	69.1

	8$\bar{2}$	82	81	80
6	8.2	8.2	8.1	8.0
7	9.6	9.5	9.4	9.3
8	11.0	10.9	10.8	10.6
9	12.4	12.3	12.1	12.0
10	13.7	13.6	13.5	13.3
20	27.5	27.3	27.0	26.6
30	41.2	41.0	40.5	40.0
40	55.0	54.6	54.0	53.3
50	68.7	68.3	67.5	66.6

	7$\bar{9}$	2	1$\bar{1}$
6	7.9	0.2	0.1
7	9.3	0.2	0.2
8	10.6	0.2	0.2
9	11.9	0.3	0.2
10	13.2	0.3	0.2
20	26.5	0.6	0.5
30	39.7	1.0	0.7
40	53.0	1.3	1.0
50	66.2	1.6	1.2

′	Log. Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	
0	9·19 433	80	9·19 971	81	0·80 028	9·99 462	60
1	9·19 513	79	9·20 053	81	0·79 947	9·99 460	59
2	9·19 592	79	9·20 134	81	0·79 865	9·99 458	58
3	9·19 672	79	9·20 216	81	0·79 784	9·99 456	57
4	9·19 751	79	9·20 297	81	0·79 703	9·99 454	56
5	9·19 830	79	9·20 378	81	0·79 622	9·99 452	55
6	9·19 909	79	9·20 459	81	0·79 541	9·99 450	54
7	9·19 988	79	9·20 540	80	0·79 460	9·99 448	53
8	9·20 066	78	9·20 620	81	0·79 379	9·99 446	52
9	9·20 145	78	9·20 701	81	0·79 298	9·99 444	51
10	9·20 223	78	9·20 781	80	0·79 218	9·99 442	50
11	9·20 301	78	9·20 862	80	0·79 138	9·99 440	49
12	9·20 379	78	9·20 942	80	0·79 058	9·99 437	48
13	9·20 457	78	9·21 022	80	0·78 978	9·99 435	47
14	9·20 535	78	9·21 102	79	0·78 898	9·99 433	46
15	9·20 613	77	9·21 181	79	0·78 819	9·99 431	45
16	9·20 690	77	9·21 261	79	0·78 739	9·99 429	44
17	9·20 768	77	9·21 340	79	0·78 659	9·99 427	43
18	9·20 845	77	9·21 420	79	0·78 580	9·99 425	42
19	9·20 922	77	9·21 499	78	0·78 501	9·99 423	41
20	9·20 999	77	9·21 578	79	0·78 422	9·99 421	40
21	9·21 076	77	9·21 657	78	0·78 343	9·99 419	39
22	9·21 152	76	9·21 735	78	0·78 264	9·99 417	38
23	9·21 229	76	9·21 814	78	0·78 186	9·99 415	37
24	9·21 305	76	9·21 892	78	0·78 107	9·99 413	36
25	9·21 382	76	9·21 971	78	0·78 029	9·99 411	35
26	9·21 458	76	9·22 049	78	0·77 951	9·99 408	34
27	9·21 534	76	9·22 127	78	0·77 873	9·99 406	33
28	9·21 609	75	9·22 205	78	0·77 795	9·99 404	32
29	9·21 685	76	9·22 283	77	0·77 717	9·99 402	31
30	9·21 761	75	9·22 360	77	0·77 639	9·99 400	30
31	9·21 836	75	9·22 438	77	0·77 562	9·99 398	29
32	9·21 911	75	9·22 515	77	0·77 484	9·99 396	28
33	9·21 987	75	9·22 593	77	0·77 407	9·99 394	27
34	9·22 062	75	9·22 670	77	0·77 330	9·99 392	26
35	9·22 136	74	9·22 747	77	0·77 253	9·99 389	25
36	9·22 211	75	9·22 824	77	0·77 176	9·99 387	24
37	9·22 285	74	9·22 900	76	0·77 099	9·99 385	23
38	9·22 360	75	9·22 977	77	0·77 022	9·99 383	22
39	9·22 435	74	9·23 054	76	0·76 946	9·99 381	21
40	9·22 509	74	9·23 130	76	0·76 870	9·99 379	20
41	9·22 583	74	9·23 206	76	0·76 793	9·99 377	19
42	9·22 657	74	9·23 282	76	0·76 717	9·99 374	18
43	9·22 731	73	9·23 358	76	0·76 641	9·99 372	17
44	9·22 805	74	9·23 434	76	0·76 565	9·99 370	16
45	9·22 878	73	9·23 510	76	0·76 489	9·99 368	15
46	9·22 952	73	9·23 586	75	0·76 414	9·99 366	14
47	9·23 025	73	9·23 661	75	0·76 338	9·99 364	13
48	9·23 098	73	9·23 737	75	0·76 263	9·99 361	12
49	9·23 171	73	9·23 812	75	0·76 188	9·99 359	11
50	9·23 244	73	9·23 887	75	0·76 113	9·99 357	10
51	9·23 317	73	9·23 962	75	0·76 038	9·99 355	9
52	9·23 390	73	9·24 037	75	0·75 963	9·99 353	8
53	9·23 463	72	9·24 112	74	0·75 888	9·99 350	7
54	9·23 535	72	9·24 186	74	0·75 813	9·99 348	6
55	9·23 607	72	9·24 261	74	0·75 739	9·99 346	5
56	9·23 679	72	9·24 335	74	0·75 664	9·99 344	4
57	9·23 751	72	9·24 409	74	0·75 590	9·99 342	3
58	9·23 823	72	9·24 484	74	0·75 516	9·99 339	2
59	9·23 895	72	9·24 558	74	0·75 442	9·99 337	1
60	9·23 967	71	9·24 632	74	0·75 368	9·99 335	0
	Log. Cos.	d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	′

P. P.

	81	81	80	79
6	8.1	8.1	8.0	7.9
7	9.5	9.4	9.3	9.2
8	10.8	10.8	10.7	10.5
9	12.2	12.1	12.0	11.8
10	13.6	13.5	13.3	13.2
20	27.1	27.0	26.7	26.3
30	40.7	40.5	40.0	39.5
40	54.3	54.0	53.3	52.7
50	67.9	67.5	66.7	65.8

	78	78	77
6	7.8	7.8	7.7
7	9.1	9.1	9.0
8	10.4	10.4	10.3
9	11.7	11.7	11.5
10	13.1	13.0	12.8
20	26.1	26.0	25.7
30	39.2	39.0	38.5
40	52.3	52.0	51.3
50	65.4	65.0	64.1

	76	76	75	74
6	7.6	7.6	7.5	7.4
7	8.9	8.8	8.7	8.6
8	10.2	10.1	10.0	9.9
9	11.5	11.4	11.2	11.1
10	12.7	12.6	12.5	12.3
20	25.5	25.3	25.0	24.7
30	38.2	38.0	37.5	37.0
40	51.0	50.6	50.0	49.3
50	63.7	63.3	62.5	61.6

	73	73	72
6	7.3	7.3	7.2
7	8.6	8.5	8.4
8	9.8	9.7	9.6
9	11.0	10.9	10.8
10	12.2	12.1	12.0
20	24.5	24.3	24.0
30	36.7	36.5	36.0
40	49.0	48.6	48.0
50	61.2	60.8	60.0

	71	71	70	69
6	7.1	7.1	7.0	6.9
7	8.3	8.3	8.2	8.1
8	9.5	9.4	9.3	9.2
9	10.7	10.6	10.5	10.4
10	11.9	11.8	11.7	11.5
20	23.8	23.6	23.3	23.0
30	35.7	35.5	35.0	34.5
40	47.6	47.3	46.7	46.1
50	59.6	59.2	58.3	57.6

P. P.

Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.		
967	71	9·24 632	73	0·75 368	9·99 335	60	
038	71	9·24 705	74	0·75 295	9·99 333	59	
110	71	9·24 779	73	0·75 220	9·99 330	58	
181	71	9·24 853	73	0·75 147	9·99 328	57	
252	71	9·24 926	73	0·75 073	9·99 326	56	
323	71	9·25 000	73	0·75 000	9·99 324	55	
394	71	9·25 073	73	0·74 927	9·99 321	54	
465	71	9·25 146	73	0·74 854	9·99 319	53	
536	70	9·25 219	73	0·74 781	9·99 317	52	
607	70	9·25 292	73	0·74 708	9·99 315	51	
677	70	9·25 365	73	0·74 635	9·99 312	50	
748	70	9·25 437	72	0·74 562	9·99 310	49	
818	70	9·25 510	73	0·74 490	9·99 308	48	
888	70	9·25 582	72	0·74 417	9·99 306	47	
958	69	9·25 654	72	0·74 345	9·99 303	46	
028	70	9·25 727	72	0·74 273	9·99 301	45	
098	69	9·25 799	72	0·74 201	9·99 299	44	
167	70	9·25 871	72	0·74 129	9·99 296	43	
237	69	9·25 943	72	0·74 057	9·99 294	42	
306	69	9·26 014	71	0·73 985	9·99 292	41	
376	69	9·26 086	72	0·73 913	9·99 290	40	
445	69	9·26 158	71	0·73 842	9·99 287	39	
514	69	9·26 229	71	0·73 771	9·99 285	38	
583	69	9·26 300	71	0·73 699	9·99 283	37	
652	68	9·26 371	71	0·73 628	9·99 280	36	
721	69	9·26 443	71	0·73 557	9·99 278	35	
790	68	9·26 514	70	0·73 486	9·99 276	34	
858	68	9·26 585	71	0·73 415	9·99 273	33	
927	68	9·26 655	70	0·73 344	9·99 271	32	
995	68	9·26 726	70	0·73 274	9·99 269	31	
063	68	9·26 796	70	0·73 203	9·99 266	30	
131	68	9·26 867	70	0·73 133	9·99 264	29	
199	68	9·26 937	70	0·73 062	9·99 262	28	
267	67	9·27 007	70	0·72 992	9·99 259	27	
335	67	9·27 078	70	0·72 922	9·99 257	26	
403	68	9·27 148	70	0·72 852	9·99 255	25	
470	67	9·27 218	69	0·72 782	9·99 252	24	
537	67	9·27 287	70	0·72 713	9·99 250	23	
605	67	9·27 357	69	0·72 642	9·99 248	22	
672	67	9·27 427	70	0·72 573	9·99 245	21	
739	67	9·27 496	69	0·72 503	9·99 243	20	
806	67	9·27 566	69	0·72 434	9·99 240	19	
873	66	9·27 635	69	0·72 365	9·99 238	18	
940	67	9·27 704	69	0·72 295	9·99 236	17	
007	66	9·27 773	69	0·72 226	9·99 233	16	
073	66	9·27 842	69	0·72 157	9·99 231	15	
140	65	9·27 911	68	0·72 088	9·99 228	14	
206	66	9·27 980	69	0·72 020	9·99 226	13	
272	66	9·28 049	68	0·71 951	9·99 224	12	
339	66	9·28 117	68	0·71 882	9·99 221	11	
405	66	9·28 186	68	0·71 814	9·99 219	10	
471	66	9·28 254	68	0·71 746	9·99 216	9	
538	65	9·28 325	68	0·71 677	9·99 214	8	
602	66	9·28 390	68	0·71 609	9·99 212	7	
668	65	9·28 459	68	0·71 541	9·99 209	6	
733	65	9·28 527	68	0·71 473	9·99 207	5	
799	65	9·28 594	67	0·71 405	9·99 204	4	
864	65	9·28 662	68	0·71 337	9·99 202	3	
929	65	9·28 730	67	0·71 270	9·99 198	2	
995	65	9·28 797	67	0·71 202	9·99 197	1	
060	65	9·28 865	67	0·71 135	9·99 194	0	
Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′	

P. P.				
	74	73	73	
6	7·4	7·3	7·3	
7	8·6	8·6	8·5	
8	9·8	9·8	9·7	
9	11·1	11·0	10·9	
10	12·3	12·2	12·1	
20	24·6	24·5	24·3	
30	37·0	36·7	36·5	
40	49·3	49·0	48·6	
50	61·6	61·2	60·8	

	72	72	71	71
6	7·2	7·2	7·1	7·1
7	8·4	8·4	8·3	8·3
8	9·6	9·6	9·5	9·4
9	10·9	10·8	10·7	10·6
10	12·1	12·0	11·9	11·8
20	24·1	24·0	23·8	23·6
30	36·2	36·0	35·7	35·5
40	48·3	48·0	47·6	47·3
50	60·4	60·0	59·6	59·1

	70	70	69	69
6	7·0	7·0	6·9	6·9
7	8·2	8·1	8·1	8·0
8	9·4	9·3	9·2	9·2
9	10·6	10·5	10·4	10·3
10	11·7	11·6	11·6	11·5
20	23·5	23·3	23·1	23·0
30	35·2	35·0	34·7	34·5
40	47·0	46·6	46·3	46·0
50	58·7	58·3	57·9	57·5

	68	68	67	67
6	6·8	6·8	6·7	6·7
7	8·0	7·9	7·9	7·8
8	9·1	9·0	9·0	8·9
9	10·3	10·2	10·1	10·0
10	11·4	11·3	11·2	11·1
20	22·8	22·6	22·5	22·3
30	34·2	34·0	33·7	33·5
40	45·6	45·3	45·0	44·6
50	57·1	56·6	56·2	55·8

	66	66	65	65
6	6·6	6·6	6·5	6·5
7	7·7	7·7	7·6	7·6
8	8·8	8·8	8·7	8·6
9	10·0	9·9	9·8	9·7
10	11·1	11·0	10·9	10·8
20	22·1	22·0	21·8	21·6
30	33·2	33·0	32·7	32·5
40	44·3	44·0	43·6	43·3
50	55·4	55·0	54·6	54·1

	2	2
6	0·2	0·2
7	0·3	0·2
8	0·3	0·2
9	0·4	0·3
10	0·4	0·3
20	0·8	0·6
30	1·2	1·0
40	1·6	1·3
50	2·1	1·6

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9.51 264	37	9.53 697	41	0.46 303	9.97 567	4	60	
1	9.51 301	36	9.53 738	41	0.46 262	9.97 562	4	59	
2	9.51 337	36	9.53 779	41	0.46 221	9.97 558	4	58	
3	9.51 374	36	9.53 820	41	0.46 180	9.97 554	4	57	
4	9.51 410	36	9.53 861	41	0.46 139	9.97 549	4	56	41 40
5	9.51 447	36	9.53 902	41	0.46 098	9.97 545	4	55	6 4.1 4.0
6	9.51 483	36	9.53 943	40	0.46 057	9.97 541	4	54	7 4.8 4.7
7	9.51 520	36	9.53 983	41	0.46 016	9.97 536	4	53	8 5.4 5.4
8	9.51 555	36	9.54 024	41	0.45 975	9.97 532	4	52	9 6.1 6.1
9	9.51 593	35	9.54 065	41	0.45 934	9.97 527	4	51	10 6.8 6.7
10	9.51 629	36	9.54 106	40	0.45 894	9.97 523	4	50	20 13.6 13.5
11	9.51 665	33	9.54 147	41	0.45 853	9.97 519	4	49	30 20.5 20.2
12	9.51 702	36	9.54 187	40	0.45 812	9.97 514	4	48	40 27.3 27.0
13	9.51 738	36	9.54 228	40	0.45 772	9.97 510	4	47	50 34.1 33.7
14	9.51 774	36	9.54 269	41	0.45 731	9.97 505	4	46	
15	9.51 811	36	9.54 309	40	0.45 690	9.97 501	4	45	39
16	9.51 847	36	9.54 350	40	0.45 650	9.97 497	4	44	6 3.9
17	9.51 883	36	9.54 390	40	0.45 609	9.97 492	4	43	7 4.6
18	9.51 919	36	9.54 431	40	0.45 569	9.97 488	4	42	8 5.2
19	9.51 955	36	9.54 471	40	0.45 528	9.97 483	4	41	9 5.9
20	9.51 991	36	9.54 512	40	0.45 488	9.97 479	4	40	10 6.6
21	9.52 027	36	9.54 552	40	0.45 447	9.97 475	4	39	20 13.1 13
22	9.52 063	36	9.54 593	40	0.45 407	9.97 470	4	38	30 19.7 19
23	9.52 099	36	9.54 633	40	0.45 367	9.97 466	4	37	40 26.3 26
24	9.52 135	36	9.54 673	40	0.45 326	9.97 461	4	36	50 32.9 32
25	9.52 170	35	9.54 714	40	0.45 286	9.97 457	4	35	
26	9.52 206	36	9.54 754	40	0.45 246	9.97 452	4	34	37 36
27	9.52 242	36	9.54 794	40	0.45 205	9.97 448	4	33	6 3.7 3.6
28	9.52 278	35	9.54 834	40	0.45 165	9.97 443	4	32	7 4.3 4.2
29	9.52 314	36	9.54 874	40	0.45 125	9.97 439	4	31	8 4.9 4.8
30	9.52 349	35	9.54 915	40	0.45 085	9.97 434	4	30	9 5.5 5.4
31	9.52 385	36	9.54 955	40	0.45 045	9.97 430	4	29	10 6.1 6.1
32	9.52 421	35	9.54 995	40	0.45 005	9.97 425	4	28	20 12.3 12.1
33	9.52 456	35	9.55 035	40	0.44 965	9.97 421	4	27	30 18.5 18.2
34	9.52 492	35	9.55 075	40	0.44 925	9.97 416	4	26	40 24.6 24.3
35	9.52 527	35	9.55 115	40	0.44 884	9.97 412	4	25	50 30.8 30.4
36	9.52 563	35	9.55 155	40	0.44 845	9.97 407	4	24	
37	9.52 598	35	9.55 195	40	0.44 805	9.97 403	4	23	35 35
38	9.52 634	35	9.55 235	40	0.44 765	9.97 398	4	22	6 3.5 3.5
39	9.52 669	35	9.55 275	40	0.44 725	9.97 394	4	21	7 4.1 4.1
40	9.52 704	35	9.55 315	40	0.44 685	9.97 389	4	20	8 4.7 4.6
41	9.52 740	35	9.55 355	40	0.44 645	9.97 385	4	19	9 5.3 5.2
42	9.52 775	35	9.55 394	39	0.44 605	9.97 380	4	18	10 5.9 5.8
43	9.52 810	35	9.55 434	40	0.44 565	9.97 376	4	17	20 11.8 11.6
44	9.52 846	35	9.55 474	39	0.44 526	9.97 371	4	16	30 17.7 17.4
45	9.52 881	35	9.55 514	40	0.44 486	9.97 367	4	15	40 23.6 23.2
46	9.52 916	35	9.55 553	39	0.44 446	9.97 362	4	14	50 29.6 29.1
47	9.52 951	35	9.55 593	40	0.44 406	9.97 358	4	13	
48	9.52 986	35	9.55 633	39	0.44 367	9.97 353	4	12	5 4
49	9.53 021	35	9.55 672	39	0.44 327	9.97 349	4	11	6 0.5 0.4
50	9.53 056	35	9.55 712	39	0.44 288	9.97 344	4	10	7 0.6 0.5
51	9.53 092	35	9.55 751	39	0.44 248	9.97 340	5	9	8 0.7 0.6
52	9.53 126	35	9.55 791	40	0.44 208	9.97 335	4	8	9 0.7 0.7
53	9.53 161	35	9.55 831	39	0.44 169	9.97 330	4	7	10 0.8 0.8
54	9.53 196	35	9.55 870	39	0.44 129	9.97 326	4	6	20 1.6 1.5
55	9.53 231	34	9.55 909	39	0.44 090	9.97 321	4	5	30 2.5 2.2
56	9.53 266	35	9.55 949	39	0.44 051	9.97 317	4	4	40 3.3 3.0
57	9.53 301	34	9.55 988	39	0.44 011	9.97 312	4	3	50 4.1 3.7
58	9.53 335	35	9.56 028	39	0.43 972	9.97 308	5	2	
59	9.53 370	35	9.56 067	39	0.43 932	9.97 303	4	1	
60	9.53 405	34	9.56 106	39	0.43 893	9.97 298	5	0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.		P. P.
788	58	9·32 747	62	0·67 252	9·99 040	60	
847	59	9·32 809	62	0·67 190	9·99 038	59	
906	59	9·32 871	62	0·67 129	9·99 035	58	
966	59	9·32 933	62	0·67 066	9·99 032	57	
025	59	9·32 995	62	0·67 004	9·99 029	56	
084	59	9·33 057	61	0·66 943	9·99 027	55	
143	59	9·33 118	61	0·66 881	9·99 024	54	
202	58	9·33 180	62	0·66 819	9·99 021	53	
260	59	9·33 242	61	0·66 758	9·99 019	52	
319	58	9·33 303	61	0·66 696	9·99 016	51	
378	58	9·33 364	61	0·66 635	9·99 013	50	
436	58	9·33 426	61	0·66 574	9·99 010	49	
495	58	9·33 487	61	0·66 513	9·99 008	48	
553	58	9·33 548	61	0·66 452	9·99 005	47	
611	58	9·33 609	60	0·66 390	9·99 002	46	
670	58	9·33 670	61	0·66 330	9·98 999	45	
728	58	9·33 731	61	0·66 269	9·98 997	44	
786	58	9·33 792	60	0·66 208	9·98 994	43	
844	58	9·33 852	61	0·66 147	9·98 991	42	
902	58	9·33 913	60	0·66 086	9·98 988	41	
960	57	9·33 974	60	0·66 026	9·98 986	40	
017	58	9·34 034	60	0·65 965	9·98 983	39	
075	57	9·34 095	60	0·65 905	9·98 980	38	
133	57	9·34 155	60	0·65 845	9·98 977	37	
190	57	9·34 215	60	0·65 784	9·98 975	36	
248	57	9·34 275	60	0·65 724	9·98 972	35	
305	57	9·34 336	60	0·65 664	9·98 969	34	
362	57	9·34 396	60	0·65 604	9·98 966	33	
419	57	9·34 456	59	0·65 544	9·98 963	32	
476	57	9·34 515	60	0·65 484	9·98 961	31	
533	57	9·34 575	60	0·65 424	9·98 958	30	
590	57	9·34 635	59	0·65 364	9·98 955	29	
647	57	9·34 695	59	0·65 305	9·98 952	28	
704	56	9·34 754	59	0·65 245	9·98 949	27	
761	56	9·34 814	59	0·65 186	9·98 947	26	
817	56	9·34 873	59	0·65 126	9·98 944	25	
874	56	9·34 933	59	0·65 067	9·98 941	24	
930	56	9·34 992	59	0·65 008	9·98 938	23	
987	56	9·35 051	59	0·64 948	9·98 935	22	
043	56	9·35 110	59	0·64 889	9·98 933	21	
099	56	9·35 169	59	0·64 830	9·98 930	20	
156	56	9·35 228	59	0·64 771	9·98 927	19	
212	56	9·35 287	59	0·64 712	9·98 924	18	
268	56	9·35 346	59	0·64 653	9·98 921	17	
324	56	9·35 405	59	0·64 594	9·98 918	16	
378	55	9·35 464	58	0·64 536	9·98 915	15	
435	56	9·35 522	58	0·64 477	9·98 913	14	
491	55	9·35 581	59	0·64 418	9·98 910	13	
547	56	9·35 640	58	0·64 360	9·98 907	12	
602	55	9·35 698	58	0·64 302	9·98 904	11	
658	55	9·35 756	58	0·64 243	9·98 901	10	
713	55	9·35 815	58	0·64 185	9·98 898	9	
768	55	9·35 873	58	0·64 127	9·98 895	8	
824	55	9·35 931	58	0·64 068	9·98 892	7	
879	55	9·35 989	58	0·64 010	9·98 890	6	
934	55	9·36 047	58	0·63 952	9·98 887	5	
989	55	9·36 105	57	0·63 894	9·98 884	4	
044	54	9·36 163	58	0·63 837	9·98 881	3	
099	55	9·36 221	57	0·63 779	9·98 878	2	
154	55	9·36 278	58	0·63 721	9·98 875	1	
209	55	9·36 336	58	0·63 663	9·98 872	0	
Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′	P. P.

	62	61	61
6	6·2	6·1	6·1
7	7·2	7·2	7·1
8	8·2	8·2	8·1
9	9·3	9·2	9·1
10	10·3	10·2	10·1
20	20·6	20·5	20·3
30	31·0	30·7	30·5
40	41·3	41·0	40·6
50	51·6	51·2	50·8

	60	60	59	59
6	6·0	6·0	5·9	5·9
7	7·0	7·0	6·9	6·9
8	8·0	8·0	7·9	7·8
9	9·1	9·0	8·9	8·8
10	10·1	10·0	9·9	9·8
20	20·1	20·0	19·8	19·6
30	30·2	30·0	29·7	29·5
40	40·3	40·0	39·6	39·3
50	50·4	50·0	49·6	49·1

	58	58	57	57
6	5·8	5·8	5·7	5·7
7	6·8	6·7	6·7	6·6
8	7·8	7·7	7·6	7·6
9	8·8	8·7	8·6	8·5
10	9·7	9·6	9·6	9·5
20	19·5	19·3	19·1	19·0
30	29·2	29·0	28·7	28·5
40	39·0	38·6	38·3	38·0
50	48·7	48·3	47·9	47·5

	56	56	55	55
6	5·6	5·6	5·5	5·5
7	6·6	6·5	6·5	6·4
8	7·5	7·4	7·4	7·3
9	8·5	8·4	8·3	8·2
10	9·4	9·3	9·2	9·1
20	18·8	18·6	18·5	18·3
30	28·2	28·0	27·7	27·5
40	37·6	37·3	37·0	36·6
50	47·1	46·6	46·2	45·8

	54	3	2
6	5·4	0·3	0·2
7	6·3	0·3	0·3
8	7·2	0·4	0·3
9	8·2	0·4	0·4
10	9·1	0·5	0·4
20	18·1	1·0	0·8
30	27·2	1·5	1·2
40	36·3	2·0	1·6
50	45·4	2·5	2·1

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9.55 433		9.58 417		0.41 583	9.97 015		60	
1	9.55 466	33	9.58 455	38	0.41 544	9.97 010	4	59	
2	9.55 498	32	9.58 493	38	0.41 507	9.97 005	5	58	
3	9.55 531	33	9.58 531	38	0.41 469	9.97 000	5	57	
4	9.55 564	33	9.58 569	38	0.41 431	9.96 995	5	56	
5	9.55 597	33	9.58 606	37	0.41 394	9.96 991	4	55	
6	9.55 630	33	9.58 644	38	0.41 356	9.96 986	5	54	
7	9.55 662	32	9.58 681	37	0.41 319	9.96 981	5	53	
8	9.55 695	33	9.58 719	38	0.41 281	9.96 976	5	52	
9	9.55 728	33	9.58 756	37	0.41 245	9.96 971	4	51	
10	9.55 760	32	9.58 794	37	0.41 206	9.96 966	5	50	
11	9.55 793	33	9.58 831	37	0.41 169	9.96 961	5	49	
12	9.55 826	33	9.58 869	38	0.41 131	9.96 956	5	48	
13	9.55 858	32	9.58 906	37	0.41 093	9.96 952	4	47	
14	9.55 891	33	9.58 944	38	0.41 056	9.96 947	5	46	
15	9.55 923	32	9.58 981	37	0.41 018	9.96 942	5	45	
16	9.55 956	33	9.59 019	38	0.40 981	9.96 937	5	44	
17	9.55 988	32	9.59 056	37	0.40 944	9.96 932	5	43	
18	9.56 020	32	9.59 093	37	0.40 907	9.96 927	5	42	
19	9.56 053	33	9.59 131	38	0.40 869	9.96 923	4	41	
20	9.56 085	32	9.59 168	37	0.40 832	9.96 917	5	40	
21	9.56 118	33	9.59 205	37	0.40 795	9.96 912	5	39	
22	9.56 150	32	9.59 242	37	0.40 757	9.96 907	5	38	
23	9.56 182	32	9.59 280	38	0.40 720	9.96 902	5	37	
24	9.56 214	32	9.59 317	37	0.40 683	9.96 897	5	36	
25	9.56 247	33	9.59 354	37	0.40 646	9.96 892	5	35	
26	9.56 279	32	9.59 391	37	0.40 609	9.96 887	5	34	
27	9.56 311	32	9.59 429	38	0.40 571	9.96 882	5	33	
28	9.56 343	32	9.59 465	37	0.40 534	9.96 877	5	32	
29	9.56 375	32	9.59 503	38	0.40 497	9.96 873	4	31	
30	9.56 407	32	9.59 540	37	0.40 460	9.96 868	5	30	
31	9.56 439	32	9.59 577	37	0.40 423	9.96 863	5	29	
32	9.56 471	32	9.59 614	37	0.40 386	9.96 858	5	28	
33	9.56 503	32	9.59 651	37	0.40 349	9.96 853	5	27	
34	9.56 535	32	9.59 688	37	0.40 312	9.96 848	5	26	
35	9.56 567	32	9.59 724	36	0.40 275	9.96 843	5	25	
36	9.56 599	32	9.59 761	37	0.40 239	9.96 838	5	24	
37	9.56 631	32	9.59 798	37	0.40 201	9.96 833	5	23	
38	9.56 663	32	9.59 835	37	0.40 164	9.96 828	5	22	
39	9.56 695	32	9.59 872	36	0.40 128	9.96 823	5	21	
40	9.56 727	32	9.59 909	37	0.40 091	9.96 818	5	20	
41	9.56 758	31	9.59 946	37	0.40 054	9.96 813	5	19	
42	9.56 790	32	9.59 982	36	0.40 017	9.96 808	5	18	
43	9.56 822	32	9.60 019	37	0.39 980	9.96 802	6	17	
44	9.56 854	32	9.60 056	36	0.39 944	9.96 797	5	16	
45	9.56 885	31	9.60 093	37	0.39 907	9.96 792	5	15	
46	9.56 917	32	9.60 129	36	0.39 870	9.96 787	5	14	
47	9.56 949	32	9.60 166	37	0.39 833	9.96 782	5	13	
48	9.56 980	31	9.60 203	37	0.39 797	9.96 777	5	12	
49	9.57 012	32	9.60 239	36	0.39 760	9.96 772	5	11	
50	9.57 043	31	9.60 276	37	0.39 724	9.96 767	5	10	
51	9.57 075	32	9.60 312	36	0.39 687	9.96 762	5	9	
52	9.57 106	31	9.60 349	37	0.39 650	9.96 757	5	8	
53	9.57 138	32	9.60 386	36	0.39 614	9.96 752	5	7	
54	9.57 169	31	9.60 422	36	0.39 577	9.96 747	5	8	
55	9.57 201	32	9.60 459	36	0.39 541	9.96 742	5	5	
56	9.57 232	31	9.60 495	36	0.39 504	9.96 737	5	4	
57	9.57 263	31	9.60 531	36	0.39 468	9.96 732	5	3	
58	9.57 295	32	9.60 568	36	0.39 432	9.96 727	5	2	
59	9.57 326	31	9.60 604	36	0.39 395	9.96 721	6	1	
60	9.57 357	31	9.60 641	36	0.39 359	9.96 716	5	0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

	38	37	37
6	3.8	3.7	3.7
7	4.4	4.3	4.3
8	5.0	4.9	4.9
9	5.7	5.6	5.5
10	6.3	6.2	6.2
20	12.6	12.3	12.3
30	19.0	18.5	18.4
40	25.3	24.6	24.6
50	31.6	30.8	30.8

	36	36
6	3.6	3.6
7	4.2	4.2
8	4.8	4.8
9	5.4	5.4
10	6.0	6.0
20	12.0	12.0
30	18.0	18.0
40	24.0	24.0
50	30.0	30.0

	33	32	32
6	3.3	3.2	3.2
7	3.8	3.8	3.7
8	4.4	4.3	4.3
9	4.9	4.9	4.8
10	5.5	5.4	5.3
20	11.0	10.8	10.7
30	16.5	16.2	16.0
40	22.0	21.6	21.3
50	27.5	27.1	26.7

	31	31
6	3.1	3.1
7	3.7	3.6
8	4.2	4.1
9	4.7	4.6
10	5.2	5.1
20	10.5	10.3
30	15.7	15.5
40	21.0	20.6
50	26.2	25.8

	3	3	3
6	0.3	0.3	0.3
7	0.3	0.4	0.3
8	0.3	0.4	0.4
9	0.3	0.4	0.4
10	0.3	0.3	0.3
20	0.7	0.7	0.7
30	1.0	1.0	1.0
40	1.3	1.3	1.3
50	1.7	1.7	1.7

	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′	P. P.
	9.39 677	54	0.60 323	9.98 690		60	
	9.39 731	53	0.60 269	9.98 687		59	
	9.39 784	54	0.60 216	9.98 684		58	
	9.39 838	53	0.60 161	9.98 681		57	
	9.39 892		0.60 108	9.98 678		56	
	9.39 945	53	0.60 054	9.98 674		55	
	9.39 999	53	0.60 001	9.98 671		54	
	9.40 052	53	0.59 947	9.98 668		53	
	9.40 106	53	0.59 894	9.98 665		52	
	9.40 159		0.59 841	9.98 662		51	
	9.40 212	53	0.59 787	9.98 658		50	
	9.40 265	53	0.59 734	9.98 655		49	
	9.40 318	53	0.59 681	9.98 652		48	
	9.40 372	53	0.59 628	9.98 649		47	
	9.40 425		0.59 575	9.98 646		46	
	9.40 478	53	0.59 522	9.98 642		45	
	9.40 531	53	0.59 469	9.98 639		44	
	9.40 583	52	0.59 416	9.98 636		43	
	9.40 636	53	0.59 363	9.98 633		42	
	9.40 689	52	0.59 311	9.98 630		41	
	9.40 742	53	0.59 258	9.98 626		40	
	9.40 794	52	0.59 205	9.98 623		39	
	9.40 847	53	0.59 153	9.98 620		38	
	9.40 895	52	0.59 101	9.98 617		37	
	9.40 952		0.59 048	9.98 613		36	
	9.41 004	52	0.58 996	9.98 610		35	
	9.41 057	53	0.58 943	9.98 607		34	
	9.41 109	52	0.58 891	9.98 604		33	
	9.41 161	52	0.58 838	9.98 600		32	
	9.41 213		0.58 786	9.98 597		31	
	9.41 266	53	0.58 734	9.98 594		30	
	9.41 318	52	0.58 682	9.98 591		29	
	9.41 370	52	0.58 630	9.98 587		28	
	9.41 422	52	0.58 578	9.98 584		27	
	9.41 474	52	0.58 526	9.98 581		26	
	9.41 525	51	0.58 474	9.98 578		25	
	9.41 577	52	0.58 422	9.98 574		24	
	9.41 629	52	0.58 370	9.98 571		23	
	9.41 681	51	0.58 319	9.98 568		22	
	9.41 732	51	0.58 267	9.98 564		21	
	9.41 784	52	0.58 216	9.98 561		20	
	9.41 836	51	0.58 164	9.98 558		19	
	9.41 887	51	0.58 113	9.98 554		18	
	9.41 938	51	0.58 061	9.98 551		17	
	9.41 990	51	0.58 010	9.98 548		16	
	9.42 041	51	0.57 958	9.98 544		15	
	9.42 092	51	0.57 907	9.98 541		14	
	9.42 144	51	0.57 856	9.98 538		13	
	9.42 195	51	0.57 805	9.98 534		12	
	9.42 246	51	0.57 753	9.98 531		11	
	9.42 297	51	0.57 702	9.98 528		10	
	9.42 348	51	0.57 651	9.98 524		9	
	9.42 399	51	0.57 600	9.98 521		8	
	9.42 450	50	0.57 549	9.98 518		7	
	9.42 501		0.57 499	9.98 514		6	
	9.42 552	51	0.57 448	9.98 511		5	
	9.42 602	50	0.57 397	9.98 508		4	
	9.42 653	51	0.57 346	9.98 504		3	
	9.42 704	50	0.57 296	9.98 501		2	
	9.42 754	50	0.57 245	9.98 498		1	
	9.42 805	50	0.57 195	9.98 494		0	
	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

	54	53	53
6	5.4	5.3	5.3
7	6.3	6.2	6.2
8	7.2	7.1	7.0
9	8.1	8.0	7.9
10	9.0	8.9	8.8
20	18.0	17.8	17.6
30	27.0	26.7	26.5
40	36.0	35.6	35.3
50	45.0	44.6	44.1

	52	52	51	51
6	5.2	5.2	5.1	5.1
7	6.1	6.0	6.0	5.9
8	7.0	6.9	6.8	6.8
9	7.9	7.8	7.7	7.6
10	8.7	8.6	8.6	8.5
20	17.5	17.3	17.1	17.0
30	26.2	26.0	25.7	25.5
40	35.0	34.6	34.3	34.0
50	43.7	43.3	42.9	42.5

	50	50	49	49
6	5.0	5.0	4.9	4.9
7	5.9	5.8	5.8	5.7
8	6.7	6.6	6.6	6.5
9	7.6	7.5	7.4	7.3
10	8.4	8.3	8.2	8.1
20	16.8	16.6	16.5	16.3
30	25.2	25.0	24.7	24.5
40	33.6	33.3	33.0	32.6
50	42.1	41.6	41.2	40.8

	48	48	47	47
6	4.8	4.8	4.7	4.7
7	5.6	5.6	5.5	5.5
8	6.4	6.4	6.3	6.2
9	7.3	7.2	7.1	7.0
10	8.1	8.0	7.9	7.8
20	16.1	16.0	15.8	15.6
30	24.2	24.0	23.7	23.5
40	32.3	32.0	31.6	31.3
50	40.4	40.0	39.6	39.1

	3	3
6	0.3	0.3
7	0.4	0.3
8	0.4	0.4
9	0.5	0.4
10	0.6	0.5
20	1.2	1.0
30	1.7	1.5
40	2.3	2.0
50	2.9	2.5

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9.41 299	47	9.42 805	51	0.57 195	9.98 494	6	60	
1	9.41 346	47	9.42 856	50	0.57 144	9.98 491	5	59	
2	9.41 394	47	9.42 906	50	0.57 094	9.98 487	4	58	
3	9.41 441	47	9.42 956	50	0.57 043	9.98 484	3	57	
4	9.41 488	48	9.43 007	50	0.56 993	9.98 481	3	56	50 50
5	9.41 534	47	9.43 057	50	0.56 942	9.98 477	4	55	6 5.0 5.
6	9.41 581	47	9.43 107	50	0.56 892	9.98 474	3	54	7 5.9 5.
7	9.41 628	47	9.43 157	50	0.56 842	9.98 470	4	53	8 6.7 6.
8	9.41 675	46	9.43 208	50	0.56 792	9.98 467	3	52	9 7.6 7.
9	9.41 721	46	9.43 258	50	0.56 742	9.98 464	3	51	10 8.4 8.
10	9.41 768	47	9.43 308	50	0.56 692	9.98 460	4	50	20 16.8 16.
11	9.41 815	46	9.43 358	50	0.56 642	9.98 457	3	49	30 25.2 25.
12	9.41 861	46	9.43 408	50	0.56 592	9.98 453	4	48	40 33.6 33.
13	9.41 908	46	9.43 458	50	0.56 542	9.98 450	3	47	50 42.1 41.
14	9.41 954	46	9.43 508	50	0.56 492	9.98 446	4	46	
15	9.42 000	46	9.43 557	49	0.56 442	9.98 443	3	45	49 49 4
16	9.42 047	46	9.43 607	50	0.56 392	9.98 439	4	44	6 4.9 4.9 4.
17	9.42 093	46	9.43 657	49	0.56 343	9.98 436	3	43	7 5.8 5.7 5.
18	9.42 139	46	9.43 706	49	0.56 293	9.98 433	4	42	8 6.6 6.5 6.
19	9.42 185	46	9.43 756	50	0.56 243	9.98 429	3	41	9 7.4 7.3 7.
20	9.42 232	46	9.43 806	49	0.56 194	9.98 426	4	40	10 8.2 8.1 8.
21	9.42 278	46	9.43 855	49	0.56 144	9.98 422	3	39	20 16.5 16.3 16.
22	9.42 324	46	9.43 905	49	0.56 095	9.98 419	4	38	30 24.7 24.5 24.
23	9.42 369	45	9.43 954	49	0.56 045	9.98 415	3	37	40 33.0 32.6 32.
24	9.42 415	46	9.44 003	49	0.55 996	9.98 412	4	36	50 41.2 40.8 40.
25	9.42 461	46	9.44 053	49	0.55 947	9.98 408	3	35	
26	9.42 507	46	9.44 102	49	0.55 898	9.98 405	4	34	47 47 4
27	9.42 553	45	9.44 151	49	0.55 848	9.98 401	3	33	6 4.7 4.7 4.
28	9.42 598	45	9.44 200	49	0.55 799	9.98 398	4	32	7 5.5 5.5 5.
29	9.42 644	46	9.44 249	49	0.55 750	9.98 394	3	31	8 6.3 6.2 6.
30	9.42 690	45	9.44 299	49	0.55 701	9.98 391	4	30	9 7.1 7.0 7.
31	9.42 735	45	9.44 348	49	0.55 652	9.98 387	3	29	10 7.9 7.8 7.
32	9.42 781	45	9.44 397	49	0.55 603	9.98 384	4	28	20 15.8 15.6 15.
33	9.42 826	45	9.44 446	49	0.55 554	9.98 380	3	27	30 23.7 23.5 23.
34	9.42 871	45	9.44 494	48	0.55 505	9.98 377	4	26	40 31.6 31.3 31.
35	9.42 917	45	9.44 543	49	0.55 456	9.98 373	3	25	50 39.6 39.1 38.
36	9.42 962	45	9.44 592	49	0.55 407	9.98 370	4	24	
37	9.43 007	45	9.44 641	48	0.55 359	9.98 366	3	23	45 45 4
38	9.43 052	45	9.44 690	49	0.55 310	9.98 363	4	22	6 4.5 4.5 4.
39	9.43 098	45	9.44 738	48	0.55 261	9.98 359	3	21	7 5.3 5.2 5.
40	9.43 143	45	9.44 787	48	0.55 213	9.98 356	4	20	8 6.0 6.0 5.
41	9.43 188	45	9.44 835	48	0.55 164	9.98 352	3	19	9 6.8 6.7 6.
42	9.43 233	45	9.44 884	48	0.55 116	9.98 348	4	18	10 7.6 7.5 7.
43	9.43 278	45	9.44 932	48	0.55 067	9.98 345	3	17	20 15.1 15.0 14.
44	9.43 322	44	9.44 981	48	0.55 019	9.98 341	4	16	30 22.7 22.5 22.
45	9.43 367	45	9.45 029	48	0.54 970	9.98 338	3	15	40 30.3 30.0 29.
46	9.43 412	44	9.45 077	48	0.54 922	9.98 334	4	14	50 37.9 37.5 37.
47	9.43 457	45	9.45 126	48	0.54 874	9.98 331	3	13	
48	9.43 501	44	9.45 174	48	0.54 825	9.98 327	4	12	
49	9.43 546	44	9.45 222	48	0.54 777	9.98 324	3	11	4 3 3
50	9.43 591	45	9.45 270	48	0.54 729	9.98 320	4	10	6 0.4 0.3 0.
51	9.43 635	44	9.45 318	48	0.54 681	9.98 316	3	9	7 0.4 0.4 0.
52	9.43 680	44	9.45 367	48	0.54 633	9.98 313	4	8	8 0.5 0.4 0.
53	9.43 724	44	9.45 415	48	0.54 585	9.98 309	3	7	9 0.6 0.5 0.
54	9.43 768	44	9.45 463	48	0.54 537	9.98 306	4	6	10 0.6 0.6 0.
55	9.43 813	44	9.45 510	47	0.54 489	9.98 302	3	5	20 1.3 1.1 1.
56	9.43 857	44	9.45 558	48	0.54 441	9.98 298	4	4	30 2.0 1.7 1.
57	9.43 901	44	9.45 606	48	0.54 393	9.98 295	3	3	40 2.6 2.3 2.
58	9.43 945	44	9.45 654	47	0.54 346	9.98 291	4	2	50 3.3 2.9 2.
59	9.43 989	44	9.45 702	48	0.54 298	9.98 288	3	1	
	1.034	44	9.45 745	47	0.54 255	9.98 284	4	0	
	Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

og. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
·44 034		9·45 749		0·54 250	9·98 284		60	
·44 078	44	9·45 787	48	0·54 202	9·98 280	4	59	
·44 122	44	9·45 825	47	0·54 155	9·98 277	3	58	
·44 166	44	9·45 892	47	0·54 107	9·98 273	4	57	
·44 209	43	9·45 940	47	0·54 060	9·98 269	4	56	
·44 253	44	9·45 987	47	0·54 012	9·98 266	3	55	
·44 297	44	9·46 035	47	0·53 965	9·98 262	4	54	
·44 341	43	9·46 082	47	0·53 917	9·98 258	4	53	
·44 384	43	9·46 129	47	0·53 870	9·98 255	3	52	
·44 428	44	9·46 177	47	0·53 823	9·98 251	4	51	
·44 472	43	9·46 224	47	0·53 776	9·98 247	4	50	
·44 515	43	9·46 271	47	0·53 728	9·98 244	3	49	
·44 559	43	9·46 318	47	0·53 681	9·98 240	4	48	
·44 602	43	9·46 366	47	0·53 634	9·98 236	4	47	
·44 646	43	9·46 413	47	0·53 587	9·98 233	3	46	
·44 689	43	9·46 460	47	0·53 540	9·98 229	4	45	
·44 732	43	9·46 507	47	0·53 493	9·98 225	4	44	
·44 776	43	9·46 554	47	0·53 446	9·98 222	3	43	
·44 819	43	9·46 601	47	0·53 399	9·98 218	4	42	
·44 862	43	9·46 647	46	0·53 352	9·98 214	3	41	
·44 905	43	9·46 694	47	0·53 305	9·98 211	4	40	
·44 948	43	9·46 741	47	0·53 258	9·98 207	4	39	
·44 991	43	9·46 788	46	0·53 212	9·98 203	3	38	
·45 034	43	9·46 834	46	0·53 165	9·98 200	4	37	
·45 077	43	9·46 881	47	0·53 118	9·98 196	3	36	
·45 120	43	9·46 928	46	0·53 072	9·98 192	4	35	
·45 163	42	9·46 974	46	0·53 025	9·98 188	4	34	
·45 206	43	9·47 021	46	0·52 979	9·98 185	3	33	
·45 249	43	9·47 067	46	0·52 932	9·98 181	4	32	
·45 291	42	9·47 114	46	0·52 886	9·98 177	4	31	
·45 334	43	9·47 160	46	0·52 839	9·98 173	3	30	
·45 377	42	9·47 207	46	0·52 793	9·98 170	4	29	
·45 419	42	9·47 253	46	0·52 747	9·98 166	4	28	
·45 462	42	9·47 299	46	0·52 700	9·98 162	4	27	
·45 504	42	9·47 345	46	0·52 654	9·98 158	4	26	
·45 547	42	9·47 392	46	0·52 608	9·98 155	3	25	
·45 589	42	9·47 438	46	0·52 562	9·98 151	4	24	
·45 631	42	9·47 484	46	0·52 516	9·98 147	4	23	
·45 674	42	9·47 530	46	0·52 469	9·98 143	4	22	
·45 716	42	9·47 576	46	0·52 423	9·98 140	3	21	
·45 758	42	9·47 622	46	0·52 377	9·98 136	4	20	
·45 800	42	9·47 668	46	0·52 331	9·98 132	4	19	
·45 842	42	9·47 714	46	0·52 286	9·98 128	4	18	
·45 885	42	9·47 760	46	0·52 240	9·98 124	4	17	
·45 927	42	9·47 806	46	0·52 194	9·98 121	3	16	
·45 969	42	9·47 851	46	0·52 148	9·98 117	4	15	
·46 011	41	9·47 897	46	0·52 102	9·98 113	4	14	
·46 052	42	9·47 943	46	0·52 057	9·98 109	4	13	
·46 094	42	9·47 989	46	0·52 011	9·98 105	4	12	
·46 136	42	9·48 034	45	0·51 965	9·98 102	3	11	
·46 178	41	9·48 080	46	0·51 920	9·98 098	4	10	
·46 220	42	9·48 125	45	0·51 874	9·98 094	4	9	
·46 261	41	9·48 171	46	0·51 829	9·98 090	4	8	
·46 303	41	9·48 216	45	0·51 783	9·98 086	4	7	
·46 345	42	9·48 262	45	0·51 738	9·98 082	4	6	
·46 386	41	9·48 307	45	0·51 692	9·98 079	3	5	
·46 428	41	9·48 353	45	0·51 647	9·98 075	4	4	
·46 469	41	9·48 398	45	0·51 602	9·98 071	4	3	
·46 511	41	9·48 443	45	0·51 556	9·98 067	4	2	
·46 552	41	9·48 488	45	0·51 511	9·98 063	4	1	
·46 593		9·48 534		0·51 466	9·98 059		0	
og. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

48	47	47
6 4·8	4·7	4·7
7 5·6	5·5	5·5
8 6·4	6·3	6·2
9 7·2	7·1	7·0
10 8·0	7·9	7·8
20 16·0	15·8	15·6
30 24·0	23·7	23·5
40 32·0	31·6	31·3
50 40·0	39·6	39·1

46	46	45	45
6 4·6	4·6	4·5	4·5
7 5·4	5·3	5·3	5·2
8 6·2	6·1	6·0	6·0
9 7·0	6·9	6·8	6·7
10 7·7	7·6	7·6	7·5
20 15·5	15·3	15·1	15·0
30 23·2	23·0	22·7	22·5
40 31·0	30·6	30·3	30·0
50 38·7	38·3	37·9	37·5

44	43	43
6 4·4	4·3	4·3
7 5·1	5·1	5·0
8 5·8	5·8	5·7
9 6·6	6·5	6·4
10 7·3	7·2	7·1
20 14·6	14·5	14·3
30 22·0	21·7	21·5
40 29·3	29·0	28·6
50 36·6	36·2	35·8

42	42	41	41
6 4·2	4·2	4·1	4·1
7 4·9	4·9	4·8	4·8
8 5·6	5·6	5·5	5·4
9 6·4	6·3	6·2	6·1
10 7·1	7·0	6·9	6·8
20 14·1	14·0	13·8	13·6
30 21·2	21·0	20·7	20·5
40 28·3	28·0	27·6	27·3
50 35·4	35·0	34·6	34·1

4	3
6 0·4	0·3
7 0·4	0·4
8 0·5	0·4
9 0·6	0·5
10 0·6	0·6
20 1·3	1·1
30 2·0	1·7
40 2·6	2·3
50 3·3	2·9

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9.51 264	37	9.53 697	41	0.46 303	9.97 567	4	60	
1	9.51 301	36	9.53 738	41	0.46 262	9.97 562	4	59	
2	9.51 337	36	9.53 779	41	0.46 221	9.97 558	4	58	
3	9.51 374	36	9.53 820	41	0.46 180	9.97 554	4	57	
4	9.51 410	36	9.53 861	41	0.46 139	9.97 549	4	56	41 40
5	9.51 447	36	9.53 902	41	0.46 098	9.97 545	4	55	6 4.1 4.0
6	9.51 483	36	9.53 943	41	0.46 057	9.97 541	4	54	7 4.8 4.7
7	9.51 520	36	9.53 983	40	0.46 016	9.97 536	4	53	8 5.4 5.4
8	9.51 556	36	9.54 024	41	0.45 975	9.97 532	4	52	9 6.1 6.1
9	9.51 593	36	9.54 065	41	0.45 934	9.97 527	4	51	10 6.8 6.7
10	9.51 629	36	9.54 106	40	0.45 894	9.97 523	4	50	20 13.6 13.5
11	9.51 665	36	9.54 147	41	0.45 853	9.97 519	4	49	30 20.5 20.2
12	9.51 702	36	9.54 187	40	0.45 813	9.97 514	4	48	40 27.3 27.0
13	9.51 738	36	9.54 228	40	0.45 772	9.97 510	4	47	50 34.1 33.7
14	9.51 774	36	9.54 269	41	0.45 731	9.97 505	4	46	
15	9.51 810	36	9.54 309	40	0.45 690	9.97 501	4	45	39 38
16	9.51 847	36	9.54 350	40	0.45 650	9.97 497	4	44	6 3.9 3.8
17	9.51 883	36	9.54 390	40	0.45 609	9.97 492	4	43	7 4.6 4.4
18	9.51 919	36	9.54 431	40	0.45 569	9.97 488	4	42	8 5.2 5.1
19	9.51 955	36	9.54 471	40	0.45 528	9.97 483	4	41	9 5.9 5.7
20	9.51 991	36	9.54 512	40	0.45 488	9.97 479	4	40	10 6.6 6.4
21	9.52 027	36	9.54 552	40	0.45 447	9.97 475	4	39	20 13.1 12.8
22	9.52 063	36	9.54 593	40	0.45 407	9.97 470	4	38	30 19.7 19.2
23	9.52 099	36	9.54 633	40	0.45 367	9.97 466	4	37	40 26.3 25.6
24	9.52 135	36	9.54 673	40	0.45 326	9.97 461	4	36	50 32.9 32.0
25	9.52 170	35	9.54 714	40	0.45 286	9.97 457	4	35	
26	9.52 206	36	9.54 754	40	0.45 246	9.97 452	4	34	37 36
27	9.52 242	36	9.54 794	40	0.45 205	9.97 448	4	33	6 3.7 3.6
28	9.52 278	35	9.54 834	40	0.45 165	9.97 443	4	32	7 4.3 4.2
29	9.52 314	36	9.54 874	40	0.45 125	9.97 439	4	31	8 4.9 4.8
30	9.52 349	35	9.54 915	40	0.45 085	9.97 434	4	30	9 5.5 5.4
31	9.52 385	35	9.54 955	40	0.45 045	9.97 430	4	29	10 6.1 6.0
32	9.52 421	36	9.54 995	40	0.45 005	9.97 425	4	28	20 12.3 12.1
33	9.52 456	35	9.55 035	40	0.44 965	9.97 421	4	27	30 18.5 18.1
34	9.52 492	35	9.55 075	40	0.44 925	9.97 416	4	26	40 24.6 24.2
35	9.52 527	35	9.55 115	39	0.44 884	9.97 412	4	25	50 30.8 30.2
36	9.52 563	35	9.55 155	40	0.44 845	9.97 407	4	24	
37	9.52 598	35	9.55 195	40	0.44 805	9.97 403	4	23	35 35
38	9.52 634	35	9.55 235	40	0.44 765	9.97 398	4	22	6 3.5 3.5
39	9.52 669	35	9.55 275	40	0.44 725	9.97 394	4	21	7 4.1 4.1
40	9.52 704	35	9.55 315	40	0.44 685	9.97 389	4	20	8 4.7 4.6
41	9.52 740	35	9.55 355	40	0.44 645	9.97 385	4	19	9 5.3 5.2
42	9.52 775	35	9.55 394	39	0.44 605	9.97 380	4	18	10 5.9 5.8
43	9.52 810	35	9.55 434	40	0.44 565	9.97 376	4	17	20 11.8 11.6
44	9.52 846	35	9.55 474	39	0.44 526	9.97 371	4	16	30 17.7 17.5
45	9.52 881	35	9.55 514	40	0.44 486	9.97 367	4	15	40 23.6 23.3
46	9.52 916	35	9.55 553	39	0.44 446	9.97 362	4	14	50 29.6 29.1
47	9.52 951	35	9.55 593	40	0.44 406	9.97 358	4	13	
48	9.52 986	35	9.55 633	39	0.44 367	9.97 353	4	12	
49	9.53 021	35	9.55 672	39	0.44 327	9.97 349	4	11	5 4
50	9.53 056	35	9.55 712	39	0.44 288	9.97 344	4	10	6 0.5 0.4
51	9.53 091	35	9.55 751	39	0.44 248	9.97 340	5	9	7 0.6 0.5
52	9.53 126	35	9.55 791	40	0.44 208	9.97 335	4	8	8 0.7 0.6
53	9.53 161	35	9.55 831	39	0.44 169	9.97 330	5	7	9 0.7 0.7
54	9.53 196	35	9.55 870	39	0.44 129	9.97 326	4	6	10 0.8 0.7
55	9.53 231	34	9.55 909	39	0.44 090	9.97 321	4	5	20 1.7 1.5
56	9.53 266	35	9.55 949	39	0.44 051	9.97 317	4	4	30 2.5 2.2
57	9.53 301	35	9.55 988	39	0.44 011	9.97 312	5	3	40 3.3 3.0
58	9.53 335	34	9.56 028	39	0.43 972	9.97 308	4	2	50 4.1 3.7
59	9.53 370	35	9.56 067	39	0.43 933	9.97 303	5	1	
	9.53 405	34	9.56 106	39	0.43 893	9.97 298	4	0	
	Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	d.		P. P.
35	9·56 106	39	0·43 893	9·97 298	4	60	
34	9·56 146	39	0·43 854	9·97 294	4	59	
34	9·56 185	39	0·43 815	9·97 289	5	58	
35	9·56 224	39	0·43 775	9·97 285	4	57	
34	9·56 263	39	0·43 736	9·97 280	5	56	39 39
34	9·56 303	39	0·43 697	9·97 275	4	55	6 3·9 3·9
34	9·56 342	39	0·43 658	9·97 271	4	54	7 4·6 4·5
34	9·56 381	39	0·43 619	9·97 266	5	53	8 5·2 5·2
34	9·56 420	39	0·43 580	9·97 261	5	52	9 5·9 5·8
34	9·56 459	39	0·43 540	9·97 257	4	51	10 6·6 6·5
34	9·56 498	39	0·43 501	9·97 252	5	50	20 13·1 13·0
34	9·56 537	39	0·43 462	9·97 248	4	49	30 19·7 19·5
34	9·56 576	39	0·43 423	9·97 243	5	48	40 26·3 26·0
34	9·56 615	39	0·43 384	9·97 238	5	47	50 32·9 32·5
34	9·56 654	38	0·43 346	9·97 234	4	46	
34	9·56 693	39	0·43 307	9·97 229	5	45	38 38 37
34	9·56 732	39	0·43 268	9·97 224	4	44	6 3·8 3·2 3·7
34	9·56 771	39	0·43 229	9·97 220	4	43	7 4·5 4·4 4·4
34	9·56 810	39	0·43 190	9·97 215	5	42	8 5·1 5·0 5·0
34	9·56 846	38	0·43 151	9·97 210	4	41	9 5·8 5·7 5·6
34	9·56 887	39	0·43 112	9·97 206	5	40	10 6·4 6·3 6·2
34	9·56 926	39	0·43 074	9·97 201	4	39	20 12·8 12·6 12·5
34	9·56 965	39	0·43 035	9·97 196	5	38	30 19·2 19·0 18·7
34	9·57 003	38	0·42 996	9·97 191	5	37	40 25·6 25·3 25·0
34	9·57 042	38	0·42 958	9·97 187	4	36	50 32·1 31·6 31·2
33	9·57 081	39	0·42 919	9·97 182	5	35	
34	9·57 119	38	0·42 880	9·97 177	5	34	35 34 34
34	9·57 158	38	0·42 842	9·97 173	4	33	6 3·5 3·4 3·4
34	9·57 196	38	0·42 803	9·97 168	5	32	7 4·1 4·0 3·9
33	9·57 235	38	0·42 765	9·97 163	5	31	8 4·6 4·6 4·5
34	9·57 274	39	0·42 726	9·97 159	4	30	9 5·2 5·2 5·1
34	9·57 312	38	0·42 687	9·97 154	5	29	10 5·8 5·7 5·6
33	9·57 350	38	0·42 649	9·97 149	5	28	20 11·6 11·5 11·3
34	9·57 389	39	0·42 611	9·97 144	5	27	30 17·5 17·2 17·0
33	9·57 427	38	0·42 572	9·97 140	4	26	40 23·3 23·0 22·6
33	9·57 466	38	0·42 534	9·97 135	5	25	50 29·1 28·7 28·3
33	9·57 504	38	0·42 496	9·97 130	5	24	
34	9·57 542	38	0·42 457	9·97 125	5	23	33 33
33	9·57 581	39	0·42 419	9·97 121	4	22	6 3·3 3·3
33	9·57 619	38	0·42 380	9·97 116	5	21	7 3·9 3·8
33	9·57 657	38	0·42 342	9·97 111	5	20	8 4·4 4·4
33	9·57 696	39	0·42 304	9·97 106	4	19	9 5·0 4·9
33	9·57 734	38	0·42 266	9·97 102	5	18	10 5·6 5·5
33	9·57 772	38	0·42 227	9·97 097	5	17	20 11·1 11·0
33	9·57 810	38	0·42 189	9·97 092	5	16	30 16·7 16·5
33	9·57 848	38	0·42 151	9·97 087	5	15	40 22·3 22·0
33	9·57 886	38	0·42 113	9·97 082	5	14	50 27·9 27·5
33	9·57 925	38	0·42 075	9·97 078	4	13	
33	9·57 963	38	0·42 037	9·97 073	5	12	
33	9·58 001	38	0·41 999	9·97 068	5	11	5 4
33	9·58 039	38	0·41 961	9·97 063	5	10	6 0·5 0·4
33	9·58 077	38	0·41 923	9·97 058	5	9	7 0·6 0·5
33	9·58 115	38	0·41 885	9·97 054	4	8	8 0·7 0·6
33	9·58 153	38	0·41 847	9·97 049	5	7	9 0·8 0·7
33	9·58 190	37	0·41 809	9·97 044	5	6	10 0·8 0·7
33	9·58 228	38	0·41 771	9·97 039	4	5	20 1·6 1·5
33	9·58 266	38	0·41 733	9·97 034	5	4	30 2·5 2·2
33	9·58 304	38	0·41 695	9·97 029	5	3	40 3·3 3·0
33	9·58 342	38	0·41 658	9·97 025	4	2	50 4·1 3·7
33	9·58 380	38	0·41 620	9·97 020	5	1	
33	9·58 417	37	0·41 582	9·97 015	5	0	
d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	d.	′	P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9.55 433		9.58 417		0.41 582	0.97 015		60	
1	9.55 456	33	9.58 455	37	0.41 544	0.97 010	4	59	
2	9.55 498	32	9.58 493	38	0.41 507	0.97 005	5	58	
3	9.55 531	33	9.58 531	37	0.41 469	0.97 000	5	57	
4	9.55 564	33	9.58 568	37	0.41 431	0.96 995	5	56	
5	9.55 597	32	9.58 606	37	0.41 394	0.96 991	4	55	
6	9.55 630	33	9.58 644	38	0.41 356	0.96 986	5	54	
7	9.55 662	32	9.58 681	37	0.41 318	0.96 981	5	53	
8	9.55 695	33	9.58 719	37	0.41 281	0.96 976	5	52	
9	9.55 728	33	9.58 756	37	0.41 243	0.96 971	4	51	
10	9.55 760	32	9.58 794	37	0.41 207	0.96 966	5	50	
11	9.55 793	33	9.58 831	37	0.41 168	0.96 961	5	49	
12	9.55 826	33	9.58 869	38	0.41 131	0.96 956	5	48	
13	9.55 858	32	9.58 906	37	0.41 093	0.96 952	4	47	
14	9.55 891	33	9.58 944	37	0.41 056	0.96 947	5	46	
15	9.55 923	32	9.58 981	37	0.41 018	0.96 942	5	45	
16	9.55 956	33	9.59 019	38	0.40 981	0.96 937	5	44	
17	9.55 988	32	9.59 056	37	0.40 944	0.96 932	5	43	
18	9.56 020	32	9.59 093	37	0.40 906	0.96 927	5	42	
19	9.56 053	33	9.59 131	37	0.40 869	0.96 922	4	41	
20	9.56 085	32	9.59 168	37	0.40 832	0.96 917	5	40	
21	9.56 118	33	9.59 205	37	0.40 795	0.96 912	5	39	
22	9.56 150	32	9.59 242	37	0.40 757	0.96 907	5	38	
23	9.56 182	32	9.59 280	38	0.40 720	0.96 902	5	37	
24	9.56 214	32	9.59 317	37	0.40 683	0.96 897	5	36	
25	9.56 247	33	9.59 354	37	0.40 646	0.96 892	5	35	
26	9.56 279	32	9.59 391	37	0.40 608	0.96 887	5	34	
27	9.56 311	32	9.59 428	37	0.40 571	0.96 882	5	33	
28	9.56 343	32	9.59 465	37	0.40 534	0.96 877	5	32	
29	9.56 375	32	9.59 502	37	0.40 497	0.96 873	4	31	
30	9.56 407	32	9.59 540	37	0.40 460	0.96 868	5	30	
31	9.56 439	32	9.59 577	37	0.40 423	0.96 863	5	29	
32	9.56 471	32	9.59 614	37	0.40 386	0.96 858	5	28	
33	9.56 503	32	9.59 651	37	0.40 349	0.96 853	5	27	
34	9.56 535	32	9.59 688	37	0.40 312	0.96 848	5	26	
35	9.56 567	32	9.59 724	36	0.40 275	0.96 843	5	25	
36	9.56 599	32	9.59 761	37	0.40 239	0.96 838	5	24	
37	9.56 631	32	9.59 798	37	0.40 201	0.96 833	5	23	
38	9.56 663	31	9.59 835	37	0.40 164	0.96 828	5	22	
39	9.56 695	32	9.59 872	36	0.40 128	0.96 823	5	21	
40	9.56 727	32	9.59 909	37	0.40 091	0.96 818	5	20	
41	9.56 758	31	9.59 946	37	0.40 054	0.96 813	5	19	
42	9.56 790	32	9.59 982	36	0.40 017	0.96 808	5	18	
43	9.56 822	32	9.60 019	37	0.39 980	0.96 802	6	17	
44	9.56 854	31	9.60 056	36	0.39 944	0.96 797	5	16	
45	9.56 885	32	9.60 093	37	0.39 907	0.96 792	5	15	
46	9.56 917	31	9.60 129	36	0.39 870	0.96 787	5	14	
47	9.56 949	32	9.60 166	37	0.39 833	0.96 782	5	13	
48	9.56 980	31	9.60 203	36	0.39 797	0.96 777	5	12	
49	9.57 012	32	9.60 239	36	0.39 760	0.96 772	5	11	
50	9.57 043	31	9.60 276	36	0.39 724	0.96 767	5	10	
51	9.57 075	32	9.60 312	36	0.39 687	0.96 762	5	9	
52	9.57 106	31	9.60 349	37	0.39 650	0.96 757	5	8	
53	9.57 138	32	9.60 386	36	0.39 614	0.96 752	5	7	
54	9.57 169	31	9.60 422	36	0.39 577	0.96 747	5	6	
55	9.57 201	32	9.60 459	36	0.39 541	0.96 742	5	5	
56	9.57 232	31	9.60 495	36	0.39 502	0.96 737	5	4	
57	9.57 263	31	9.60 531	36	0.39 468	0.96 732	5	3	
58	9.57 295	32	9.60 568	36	0.39 432	0.96 727	5	2	
59	9.57 326	31	9.60 604	36	0.39 395	0.96 721	6	1	
60	9.57 357	31	9.60 641	36	0.39 359	0.96 716	5	0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9.57 357	31	9.60 641	36	0.39 359	9.96 716	5	60	
1	9.57 389	31	9.60 677	36	0.39 323	9.96 711	5	59	
2	9.57 420	31	9.60 713	36	0.39 286	9.96 706	5	58	
3	9.57 451	31	9.60 750	36	0.39 250	9.96 701	5	57	
4	9.57 482	31	9.60 786	36	0.39 213	9.96 696	5	56	36 35 6 3.6 3.5 7 4.2 4.1 8 4.8 4.7 9 5.4 5.3 10 6.0 5.9 20 12.0 11.8 30 18.0 17.7 40 24.0 23.6 50 30.0 29.5
5	9.57 513	31	9.60 822	36	0.39 177	9.96 691	5	55	
6	9.57 544	31	9.60 859	36	0.39 141	9.96 686	5	54	
7	9.57 576	31	9.60 895	36	0.39 105	9.96 681	5	53	
8	9.57 607	31	9.60 931	36	0.39 069	9.96 675	5	52	
9	9.57 638	31	9.60 967	36	0.39 032	9.96 670	5	51	
10	9.57 669	31	9.61 003	36	0.38 996	9.96 665	5	50	
11	9.57 700	31	9.61 039	36	0.38 960	9.96 660	5	49	
12	9.57 731	31	9.61 076	36	0.38 924	9.96 655	5	48	
13	9.57 762	31	9.61 112	36	0.38 888	9.96 650	5	47	
14	9.57 792	30	9.61 148	36	0.38 852	9.96 644	5	46	35 35 6 3.5 3.5 7 4.1 4.1 8 4.7 4.6 9 5.3 5.2 10 5.9 5.8 20 11.8 11.6 30 17.7 17.5 40 23.6 23.3 50 29.6 29.1
15	9.57 823	31	9.61 184	36	0.38 816	9.96 639	5	45	
16	9.57 854	31	9.61 220	36	0.38 780	9.96 634	5	44	
17	9.57 885	31	9.61 256	36	0.38 744	9.96 629	5	43	
18	9.57 916	30	9.61 292	36	0.38 708	9.96 624	5	42	
19	9.57 947	31	9.61 328	36	0.38 672	9.96 619	5	41	
20	9.57 977	30	9.61 364	36	0.38 636	9.96 613	5	40	
21	9.58 008	31	9.61 400	36	0.38 600	9.96 608	5	39	
22	9.58 039	30	9.61 436	36	0.38 564	9.96 603	5	38	
23	9.58 070	31	9.61 472	36	0.38 528	9.96 598	5	37	
24	9.58 100	30	9.61 507	35	0.38 492	9.96 593	5	36	
25	9.58 131	30	9.61 543	36	0.38 456	9.96 587	5	35	
26	9.58 162	31	9.61 579	36	0.38 420	9.96 582	5	34	
27	9.58 192	30	9.61 615	35	0.38 385	9.96 577	5	33	
28	9.58 223	30	9.61 651	36	0.38 349	9.96 572	5	32	
29	9.58 253	30	9.61 686	35	0.38 313	9.96 567	5	31	31 30 6 3.1 3.0 7 3.7 3.5 8 4.2 4.0 9 4.7 4.5 10 5.2 5.0 20 10.5 10.0 30 15.7 15.0 40 21.0 20.0 50 26.2 25.0
30	9.58 284	30	9.61 722	36	0.38 277	9.96 561	5	30	
31	9.58 314	30	9.61 758	36	0.38 242	9.96 556	5	29	
32	9.58 345	30	9.61 794	36	0.38 206	9.96 551	5	28	
33	9.58 375	30	9.61 829	35	0.38 170	9.96 546	5	27	
34	9.58 406	30	9.61 865	35	0.38 135	9.96 540	6	26	
35	9.58 436	30	9.61 901	36	0.38 099	9.96 535	5	25	
36	9.58 466	30	9.61 936	35	0.38 063	9.96 530	5	24	
37	9.58 497	30	9.61 972	36	0.38 028	9.96 525	5	23	
38	9.58 527	30	9.62 007	35	0.37 992	9.96 519	6	22	30 30 29 6 3.0 3.0 2.9 7 3.5 3.5 3.4 8 4.0 4.0 3.9 9 4.6 4.5 4.4 10 5.1 5.0 4.9 20 10.1 10.0 9.8 30 15.2 15.0 14.7 40 20.3 20.0 19.6 50 25.4 25.0 24.6
39	9.58 557	30	9.62 043	35	0.37 957	9.96 514	5	21	
40	9.58 587	30	9.62 078	35	0.37 921	9.96 509	5	20	
41	9.58 618	30	9.62 114	35	0.37 886	9.96 503	6	19	
42	9.58 648	30	9.62 149	35	0.37 850	9.96 498	5	18	
43	9.58 678	30	9.62 185	35	0.37 815	9.96 493	5	17	
44	9.58 708	30	9.62 220	35	0.37 779	9.96 488	5	16	
45	9.58 738	30	9.62 256	35	0.37 744	9.96 482	5	15	
46	9.58 769	30	9.62 291	35	0.37 708	9.96 477	6	14	
47	9.58 799	30	9.62 327	35	0.37 673	9.96 472	5	13	
48	9.58 829	30	9.62 362	35	0.37 637	9.96 466	5	12	
49	9.58 859	30	9.62 397	35	0.37 602	9.96 461	5	11	5 5 6 0.5 0.5 7 0.6 0.6 8 0.7 0.6 9 0.8 0.7 10 0.9 0.8 20 1.8 1.6 30 2.7 2.5 40 3.6 3.3 50 4.6 4.1
50	9.58 889	30	9.62 433	35	0.37 567	9.96 456	6	10	
51	9.58 919	30	9.62 468	35	0.37 531	9.96 450	5	9	
52	9.58 949	30	9.62 503	35	0.37 496	9.96 445	5	8	
53	9.58 979	30	9.62 539	35	0.37 461	9.96 440	6	7	
54	9.59 009	30	9.62 574	35	0.37 426	9.96 434	5	6	
55	9.59 038	29	9.62 609	35	0.37 390	9.96 429	5	5	
56	9.59 068	30	9.62 644	35	0.37 355	9.96 424	5	4	
57	9.59 098	30	9.62 679	35	0.37 320	9.96 418	6	3	
58	9.59 128	29	9.62 715	35	0.37 285	9.96 413	5	2	
59	9.59 158	30	9.62 750	35	0.37 250	9.96 408	5	1	
60	9.59 188	30	9.62 785	35	0.37 215	9.96 402	5	0	
′	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

23°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.59 188	29	9.62 785	35	0.37 215	9.96 402	5	60
1	9.59 217	30	9.62 820	35	0.37 180	9.96 397	5	59
2	9.59 247	29	9.62 855	35	0.37 144	9.96 392	5	58
3	9.59 277	29	9.62 890	35	0.37 109	9.96 386	6	57
4	9.59 306	29	9.62 925	35	0.37 073	9.96 381	5	56
5	9.59 336	30	9.62 960	35	0.37 039	9.96 375	6	55
6	9.59 366	29	9.62 995	35	0.37 004	9.96 370	5	54
7	9.59 395	29	9.63 030	35	0.36 969	9.96 365	5	53
8	9.59 425	29	9.63 065	35	0.36 934	9.96 359	6	52
9	9.59 454	29	9.63 100	35	0.36 899	9.96 354	5	51
10	9.59 484	29	9.63 135	35	0.36 864	9.96 349	5	50
11	9.59 513	29	9.63 170	35	0.36 829	9.96 343	6	49
12	9.59 543	29	9.63 205	34	0.36 794	9.96 338	5	48
13	9.59 572	29	9.63 240	35	0.36 760	9.96 332	6	47
14	9.59 602	29	9.63 275	35	0.36 725	9.96 327	5	46
15	9.59 631	29	9.63 310	34	0.36 690	9.96 321	6	45
16	9.59 661	29	9.63 344	35	0.36 655	9.96 316	5	44
17	9.59 690	29	9.63 379	35	0.36 620	9.96 311	5	43
18	9.59 719	29	9.63 414	34	0.36 585	9.96 305	6	42
19	9.59 749	29	9.63 449	34	0.36 551	9.96 300	5	41
20	9.59 778	29	9.63 484	35	0.36 516	9.96 294	6	40
21	9.59 807	29	9.63 518	34	0.36 481	9.96 289	5	39
22	9.59 837	29	9.63 553	34	0.36 447	9.96 283	6	38
23	9.59 866	29	9.63 588	35	0.36 412	9.96 278	5	37
24	9.59 895	29	9.63 622	34	0.36 377	9.96 272	6	36
25	9.59 924	29	9.63 657	34	0.36 343	9.96 267	5	35
26	9.59 953	29	9.63 692	35	0.36 308	9.96 261	6	34
27	9.59 982	29	9.63 726	34	0.36 273	9.96 256	5	33
28	9.60 012	29	9.63 761	34	0.36 239	9.96 251	5	32
29	9.60 041	29	9.63 795	34	0.36 204	9.96 245	6	31
30	9.60 070	29	9.63 830	34	0.36 170	9.96 240	5	30
31	9.60 099	29	9.63 864	34	0.36 135	9.96 234	6	29
32	9.60 128	29	9.63 899	34	0.36 101	9.96 229	5	28
33	9.60 157	29	9.63 933	34	0.36 066	9.96 223	6	27
34	9.60 186	29	9.63 968	34	0.36 032	9.96 218	5	26
35	9.60 215	29	9.64 002	34	0.35 997	9.96 212	6	25
36	9.60 244	29	9.64 037	34	0.35 963	9.96 206	6	24
37	9.60 273	29	9.64 071	34	0.35 928	9.96 201	5	23
38	9.60 301	28	9.64 106	34	0.35 894	9.96 195	6	22
39	9.60 330	29	9.64 140	34	0.35 859	9.96 190	5	21
40	9.60 359	29	9.64 174	34	0.35 825	9.96 184	6	20
41	9.60 388	29	9.64 209	34	0.35 791	9.96 179	5	19
42	9.60 417	29	9.64 243	34	0.35 756	9.96 173	6	18
43	9.60 445	28	9.64 277	34	0.35 722	9.96 168	5	17
44	9.60 474	29	9.64 312	34	0.35 688	9.96 162	6	16
45	9.60 503	29	9.64 346	34	0.35 653	9.96 157	5	15
46	9.60 532	29	9.64 380	34	0.35 619	9.96 151	6	14
47	9.60 560	28	9.64 415	34	0.35 585	9.96 146	5	13
48	9.60 589	29	9.64 449	34	0.35 551	9.96 140	6	12
49	9.60 618	29	9.64 483	34	0.35 517	9.96 134	6	11
50	9.60 646	28	9.64 517	34	0.35 482	9.96 129	5	10
51	9.60 675	29	9.64 551	34	0.35 448	9.96 123	6	9
52	9.60 703	28	9.64 585	34	0.35 414	9.96 118	5	8
53	9.60 732	29	9.64 620	34	0.35 380	9.96 112	6	7
54	9.60 760	28	9.64 654	34	0.35 345	9.96 106	6	6
55	9.60 789	29	9.64 688	34	0.35 312	9.96 101	5	5
56	9.60 817	28	9.64 722	34	0.35 278	9.96 095	6	4
57	9.60 846	29	9.64 756	34	0.35 244	9.96 090	5	3
58	9.60 874	28	9.64 790	34	0.35 209	9.96 084	6	2
59	9.60 903	29	9.64 824	34	0.35 175	9.96 078	6	1
60	9.60 931	28	9.64 858	34	0.35 141	9.96 073	5	0
′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′

P. P.

	35	35
6	3.5	3.5
7	4.1	4.1
8	4.7	4.7
9	5.2	5.2
10	5.8	5.8
20	11.7	11.7
30	17.5	17.5
40	23.3	23.3
50	29.2	29.2

	34	34
6	3.4	3.4
7	4.0	3.9
8	4.5	4.5
9	5.1	5.1
10	5.7	5.7
20	11.3	11.3
30	17.0	17.0
40	22.7	22.7
50	28.3	28.3

	30	
6	3.0	
7	3.5	
8	4.0	
9	4.5	
10	5.0	
20	10.0	
30	15.0	
40	20.0	
50	25.0	

	25	29	1
6	2.5	2.9	2
7	2.9	3.4	2
8	3.3	3.8	3
9	3.8	4.3	4
10	4.2	4.8	4
20	8.3	9.7	8
30	12.5	14.5	12
40	16.7	19.3	16
50	20.8	24.2	20

	6	5
6	0.6	0.5
7	0.7	0.6
8	0.8	0.7
9	0.9	0.8
10	1.0	0.8
20	2.0	1.7
30	3.0	2.5
40	4.0	3.3
50	5.0	4.2

Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
9·60 931		9·64 858		0·35 141	9·96 073		60	
9·60 959	28	9·64 892	34	0·35 107	9·96 067	5	59	
9·60 988	28	9·64 926	34	0·35 073	9·96 062	5	58	
9·61 016	28	9·64 960	33	0·35 040	9·96 056	6	57	
9·61 044	28	9·64 994	34	0·35 006	9·96 050	5	56	
9·61 073	28	9·65 028	34	0·34 972	9·96 045	6	55	34 33 33
9·61 101	28	9·65 062	34	0·34 938	9·96 039	5	54	6 3·4 3·3 3·3
9·61 129	28	9·65 096	34	0·34 904	9·96 033	5	53	7 3·9 3·9 3·8
9·61 157	28	9·65 129	33	0·34 870	9·96 028	5	52	8 4·5 4·4 4·4
9·61 186	28	9·65 163	34	0·34 836	9·96 022	6	51	9 5·1 5·0 4·9
9·61 214	28	9·65 197	34	0·34 802	9·96 016	5	50	10 5·6 5·6 5·5
9·61 242	28	9·65 231	33	0·34 769	9·96 011	5	49	20 11·3 11·1 11·0
9·61 270	28	9·65 265	34	0·34 735	9·96 005	6	48	30 17·0 16·7 16·5
9·61 298	28	9·65 299	34	0·34 701	9·95 999	5	47	40 22·6 22·3 22·0
9·61 326	28	9·65 332	33	0·34 667	9·95 994	5	46	50 28·3 27·9 27·5
9·61 354	28	9·65 366	34	0·34 633	9·95 988	6	45	
9·61 382	28	9·65 400	33	0·34 600	9·95 982	5	44	
9·61 410	28	9·65 433	34	0·34 566	9·95 977	5	43	
9·61 438	28	9·65 467	33	0·34 532	9·95 971	6	42	
9·61 466	28	9·65 501	34	0·34 499	9·95 965	5	41	28 28
9·61 494	28	9·65 535	33	0·34 465	9·95 959	6	40	6 2·8 2·8
9·61 522	28	9·65 568	33	0·34 431	9·95 954	5	39	7 3·3 3·2
9·61 550	27	9·65 602	33	0·34 398	9·95 948	6	38	8 3·8 3·7
9·61 578	28	9·65 635	33	0·34 364	9·95 942	5	37	9 4·3 4·2
9·61 606	28	9·65 669	33	0·34 331	9·95 937	5	36	10 4·7 4·6
9·61 634	28	9·65 703	34	0·34 297	9·95 931	6	35	20 9·5 9·3
9·61 661	27	9·65 736	33	0·34 263	9·95 925	6	34	30 14·2 14·0
9·61 689	28	9·65 770	33	0·34 230	9·95 919	5	33	40 19·0 18·6
9·61 717	27	9·65 803	33	0·34 196	9·95 914	6	32	50 23·7 23·3
9·61 745	28	9·65 837	33	0·34 163	9·95 908	6	31	
9·61 772	27	9·65 870	33	0·34 129	9·95 902	5	30	27 27
9·61 800	28	9·65 904	33	0·34 096	9·95 896	6	29	6 2·7 2·7
9·61 828	27	9·65 937	33	0·34 062	9·95 891	5	28	7 3·2 3·1
9·61 856	28	9·65 971	34	0·34 029	9·95 885	6	27	8 3·6 3·6
9·61 883	27	9·66 004	33	0·33 996	9·95 879	6	26	9 4·1 4·0
9·61 911	28	9·66 037	33	0·33 962	9·95 873	6	25	10 4·6 4·5
9·61 938	27	9·66 071	33	0·33 929	9·95 867	6	24	20 9·1 9·0
9·61 965	27	9·66 104	33	0·33 895	9·95 862	5	23	30 13·7 13·5
9·61 994	28	9·66 137	33	0·33 862	9·95 856	6	22	40 18·3 18·0
9·62 021	27	9·66 171	33	0·33 829	9·95 850	5	21	50 22·9 22·5
9·62 049	27	9·66 204	33	0·33 795	9·95 844	6	20	
9·62 076	27	9·66 237	33	0·33 762	9·95 838	6	19	
9·62 104	27	9·66 271	33	0·33 729	9·95 833	5	18	
9·62 131	27	9·66 304	33	0·33 696	9·95 827	6	17	
9·62 158	27	9·66 337	33	0·33 662	9·95 821	5	16	
9·62 186	27	9·66 370	33	0·33 629	9·95 815	6	15	
9·62 213	27	9·66 404	33	0·33 596	9·95 809	6	14	
9·62 241	27	9·66 437	33	0·33 563	9·95 804	5	13	6 5
9·62 268	27	9·66 470	33	0·33 529	9·95 798	6	12	6 0·6 0·5
9·62 295	27	9·66 503	33	0·33 496	9·95 792	6	11	7 0·7 0·6
9·62 323	27	9·66 536	33	0·33 463	9·95 786	5	10	8 0·8 0·7
9·62 350	27	9·66 57·	33	0·33 430	9·95 780	6	9	9 0·9 0·8
9·62 377	27	9·66 603	33	0·33 397	9·95 774	6	8	10 1·0 0·9
9·62 404	27	9·66 636	33	0·33 364	9·95 768	6	7	20 2·0 1·8
9·62 432	27	9·66 669	33	0·33 331	9·95 763	5	6	30 3·0 2·7
9·62 459	27	9·66 702	33	0·33 298	9·95 757	6	5	40 4·0 3·6
9·62 486	27	9·66 735	33	0·33 265	9·95 751	6	4	50 5·0 4·5
9·62 513	27	9·66 768	33	0·33 232	9·95 745	6	3	
9·62 540	27	9·66 801	33	0·33 198	9·95 739	6	2	
9·62 567	27	9·66 834	33	0·33 165	9·95 733	6	1	
9·62 595		9·66 867		0·33 132	9·95 727	6	0	
Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9·62 595	27	9·66 867	33	0·33 132	9·95 727	6	60	
1	9·62 622	27	9·66 900	33	0·33 100	9·95 721	6	59	
2	9·62 649	27	9·66 933	33	0·33 067	9·95 716	5	58	
3	9·62 676	27	9·66 966	33	0·33 034	9·95 710	6	57	
4	9·62 703	27	9·66 999	33	0·33 001	9·95 704	6	56	
5	9·62 730	27	9·67 032	33	0·32 968	9·95 698	6	55	
6	9·62 757	27	9·67 065	33	0·32 935	9·95 692	6	54	
7	9·62 784	27	9·67 097	32	0·32 902	9·95 686	5	53	
8	9·62 811	27	9·67 130	33	0·32 869	9·95 680	6	52	
9	9·62 838	27	9·67 163	33	0·32 836	9·95 674	6	51	
10	9·62 864	26	9·67 196	33	0·32 803	9·95 668	6	50	
11	9·62 891	27	9·67 229	32	0·32 771	9·95 662	6	49	
12	9·62 918	27	9·67 262	33	0·32 738	9·95 656	6	48	
13	9·62 945	27	9·67 294	32	0·32 706	9·95 650	6	47	
14	9·62 972	26	9·67 327	33	0·32 672	9·95 644	6	46	
15	9·62 999	27	9·67 360	32	0·32 640	9·95 638	6	45	
16	9·63 025	26	9·67 393	33	0·32 607	9·95 632	6	44	
17	9·63 052	27	9·67 425	32	0·32 574	9·95 627	5	43	
18	9·63 079	27	9·67 458	33	0·32 541	9·95 621	6	42	
19	9·63 106	27	9·67 491	32	0·32 509	9·95 615	6	41	
20	9·63 132	26	9·67 523	32	0·32 476	9·95 609	6	40	
21	9·63 159	27	9·67 556	33	0·32 443	9·95 603	6	39	
22	9·63 186	26	9·67 589	32	0·32 411	9·95 597	6	38	
23	9·63 212	26	9·67 621	33	0·32 378	9·95 591	6	37	
24	9·63 239	26	9·67 654	32	0·32 345	9·95 585	6	36	
25	9·63 266	27	9·67 687	32	0·32 313	9·95 579	6	35	
26	9·63 292	26	9·67 719	33	0·32 280	9·95 573	6	34	
27	9·63 319	26	9·67 752	32	0·32 248	9·95 567	6	33	
28	9·63 345	26	9·67 784	32	0·32 215	9·95 561	6	32	
29	9·63 372	26	9·67 817	32	0·32 183	9·95 555	6	31	
30	9·63 398	26	9·67 849	32	0·32 150	9·95 549	6	30	
31	9·63 425	26	9·67 882	32	0·32 118	9·95 543	6	29	
32	9·63 451	26	9·67 914	32	0·32 085	9·95 537	6	28	
33	9·63 478	26	9·67 947	32	0·32 053	9·95 530	6	27	
34	9·63 504	26	9·67 979	32	0·32 020	9·95 524	6	26	
35	9·63 530	26	9·68 012	32	0·31 988	9·95 518	6	25	
36	9·63 557	26	9·68 044	32	0·31 955	9·95 512	6	24	
37	9·63 583	26	9·68 077	32	0·31 923	9·95 506	6	23	
38	9·63 609	26	9·68 109	32	0·31 891	9·95 500	6	22	
39	9·63 636	26	9·68 141	32	0·31 858	9·95 494	6	21	
40	9·63 662	26	9·68 174	32	0·31 826	9·95 488	6	20	
41	9·63 688	26	9·68 206	32	0·31 794	9·95 482	6	19	
42	9·63 715	26	9·68 238	32	0·31 761	9·95 476	6	18	
43	9·63 741	26	9·68 271	32	0·31 729	9·95 470	6	17	
44	9·63 767	26	9·68 303	32	0·31 696	9·95 464	6	16	
45	9·63 793	26	9·68 335	32	0·31 664	9·95 458	6	15	
46	9·63 819	26	9·68 368	32	0·31 632	9·95 452	6	14	
47	9·63 846	26	9·68 400	32	0·31 600	9·95 445	6	13	
48	9·63 872	26	9·68 432	32	0·31 567	9·95 439	6	12	
49	9·63 898	26	9·68 464	32	0·31 535	9·95 433	6	11	
50	9·63 924	26	9·68 497	32	0·31 503	9·95 427	6	10	
51	9·63 950	26	9·68 529	32	0·31 471	9·95 421	6	9	
52	9·63 976	26	9·68 561	32	0·31 439	9·95 415	6	8	
53	9·64 002	26	9·68 593	32	0·31 406	9·95 409	6	7	
54	9·64 028	26	9·68 625	32	0·31 374	9·95 403	6	6	
55	9·64 054	26	9·68 657	32	0·31 342	9·95 397	6	5	
56	9·64 080	26	9·68 690	32	0·31 310	9·95 390	6	4	
57	9·64 106	26	9·68 722	32	0·31 278	9·95 384	6	3	
58	9·64 132	26	9·68 754	32	0·31 246	9·95 378	6	2	
59	9·64 158	26	9·68 786	32	0·31 214	9·95 372	6	1	
60	9·64 184	25	9·68 818	32	0·31 182	9·95 366	6	0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′	P. P.
9.64 184		9.68 818		0.31 182	9.95 366		60	
9.64 210	26	9.68 850	32	0.31 150	9.95 360	6	59	
9.64 236	26	9.68 882	32	0.31 118	9.95 353	6	58	
9.64 262	26	9.68 914	32	0.31 086	9.95 347	6	57	
9.64 287	25	9.68 946	32	0.31 054	9.95 341	6	56	
9.64 313	26	9.68 978	32	0.31 021	9.95 335	6	55	
9.64 339	26	9.69 010	32	0.30 989	9.95 329	6	54	
9.64 365	26	9.69 042	32	0.30 957	9.95 323	6	53	
9.64 391	26	9.69 074	32	0.30 926	9.95 316	6	52	
9.64 416	25	9.69 106	31	0.30 894	9.95 310	6	51	
9.64 442	26	9.69 138	32	0.30 862	9.95 304	6	50	
9.64 468	26	9.69 170	32	0.30 830	9.95 298	6	49	
9.64 493	25	9.69 202	32	0.30 798	9.95 292	6	48	
9.64 519	26	9.69 234	32	0.30 766	9.95 285	6	47	
9.64 545	26	9.69 265	31	0.30 734	9.95 279	6	46	
9.64 570	25	9.69 297	32	0.30 702	9.95 273	6	45	
9.64 596	26	9.69 329	32	0.30 670	9.95 267	6	44	
9.64 622	26	9.69 361	32	0.30 639	9.95 260	6	43	
9.64 647	25	9.69 393	32	0.30 607	9.95 254	6	42	
9.64 673	26	9.69 425	32	0.30 575	9.95 248	6	41	
9.64 698	25	9.69 456	31	0.30 543	9.95 242	6	40	
9.64 724	26	9.69 488	32	0.30 511	9.95 235	6	39	
9.64 749	25	9.69 520	32	0.30 480	9.95 229	6	38	
9.64 775	26	9.69 552	32	0.30 448	9.95 223	6	37	
9.64 800	25	9.69 583	31	0.30 416	9.95 217	6	36	
9.64 823	25	9.69 615	32	0.30 384	9.95 210	6	35	
9.64 851	25	9.69 647	32	0.30 353	9.95 204	6	34	
9.64 876	25	9.69 678	31	0.30 321	9.95 198	6	33	
9.64 902	26	9.69 710	32	0.30 289	9.95 191	6	32	
9.64 927	25	9.69 742	32	0.30 258	9.95 185	6	31	
9.64 952	25	9.69 773	31	0.30 226	9.95 179	6	30	
9.64 978	26	9.69 805	32	0.30 194	9.95 173	6	29	
9.65 003	25	9.69 837	32	0.30 163	9.95 166	6	28	
9.65 028	25	9.69 868	31	0.30 131	9.95 160	6	27	
9.65 054	26	9.69 900	32	0.30 100	9.95 154	6	26	
9.65 079	25	9.69 931	31	0.30 068	9.95 147	6	25	
9.65 104	25	9.69 963	31	0.30 037	9.95 141	6	24	
9.65 129	25	9.69 994	31	0.30 005	9.95 135	6	23	
9.65 155	26	9.70 026	32	0.29 973	9.95 128	6	22	
9.65 180	25	9.70 058	31	0.29 942	9.95 122	6	21	
9.65 205	25	9.70 089	31	0.29 910	9.95 116	6	20	
9.65 230	25	9.70 121	31	0.29 879	9.95 109	6	19	
9.65 255	25	9.70 152	31	0.29 847	9.95 103	6	18	
9.65 280	25	9.70 183	31	0.29 816	9.95 097	6	17	
9.65 305	25	9.70 215	31	0.29 785	9.95 090	6	16	
9.65 331	26	9.70 246	31	0.29 753	9.95 084	6	15	
9.65 356	25	9.70 278	32	0.29 722	9.95 078	6	14	
9.65 381	25	9.70 309	31	0.29 690	9.95 071	6	13	
9.65 408	25	9.70 341	31	0.29 659	9.95 065	6	12	
9.65 431	25	9.70 372	31	0.29 628	9.95 058	6	11	
9.65 456	25	9.70 403	31	0.29 596	9.95 052	6	10	
9.65 481	25	9.70 435	32	0.29 565	9.95 046	6	9	
9.65 505	24	9.70 466	31	0.29 534	9.95 039	6	8	
9.65 530	25	9.70 497	31	0.29 502	9.95 033	6	7	
9.65 555	25	9.70 529	31	0.29 471	9.95 026	6	6	
9.65 580	25	9.70 560	31	0.29 439	9.95 020	6	5	
9.65 605	25	9.70 591	31	0.29 408	9.95 014	6	4	
9.65 630	24	9.70 623	31	0.29 377	9.95 007	6	3	
9.65 655	25	9.70 654	31	0.29 346	9.95 001	6	2	
9.65 680	25	9.70 685	31	0.29 314	9.94 994	6	1	
9.65 704	24	9.70 716	31	0.29 283	9.94 988	6	0	
log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

P. P.

	32	32
6	3.2	3.2
7	3.8	3.7
8	4.3	4.2
9	4.9	4.8
10	5.4	5.3
20	10.8	10.6
30	16.2	16.0
40	21.6	21.3
50	27.1	26.6

	31	31
6	3.1	3.1
7	3.7	3.6
8	4.2	4.1
9	4.7	4.6
10	5.2	5.1
20	10.5	10.3
30	15.7	15.5
40	21.0	20.6
50	26.2	25.8

	26	25	25
6	2.6	2.5	2.5
7	3.0	3.0	2.9
8	3.4	3.4	3.3
9	3.9	3.8	3.7
10	4.3	4.2	4.1
20	8.6	8.5	8.3
30	13.0	12.7	12.5
40	17.3	17.0	16.6
50	21.6	21.2	20.8

	24	6	6
6	2.4	0.6	0.6
7	2.8	0.7	0.7
8	3.2	0.8	0.8
9	3.7	1.0	0.9
10	4.1	1.1	1.0
20	8.1	2.1	2.0
30	12.2	3.2	3.0
40	16.3	4.3	4.0
50	20.4	5.4	5.0

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS AND COTANGENTS.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot	Log. Cos.	d.		P. P.
0	9.65 704	25	9.70 716	31	0.29 283	9.94 988	59	60	
1	9.65 729	24	9.70 748	31	0.29 252	9.94 981	59	59	
2	9.65 754	25	9.70 779	31	0.29 221	9.94 975	58	58	
3	9.65 779	24	9.70 810	31	0.29 190	9.94 969	58	57	
4	9.65 803	24	9.70 841	31	0.29 158	9.94 962	57	56	
5	9.65 828	25	9.70 872	31	0.29 127	9.94 956	57	55	
6	9.65 853	24	9.70 903	31	0.29 096	9.94 949	56	54	
7	9.65 878	25	9.70 935	31	0.29 065	9.94 943	56	53	
8	9.65 902	24	9.70 966	31	0.29 034	9.94 936	55	52	
9	9.65 927	24	9.70 997	31	0.29 003	9.94 930	55	51	
10	9.65 951	24	9.71 028	31	0.28 972	9.94 923	54	50	
11	9.65 976	25	9.71 059	31	0.28 940	9.94 917	54	49	
12	9.66 001	24	9.71 090	31	0.28 909	9.94 910	53	48	
13	9.66 025	24	9.71 121	31	0.28 878	9.94 904	53	47	
14	9.66 050	24	9.71 152	31	0.28 847	9.94 897	52	46	
15	9.66 074	24	9.71 183	31	0.28 816	9.94 891	52	45	
16	9.66 099	24	9.71 214	31	0.28 785	9.94 884	51	44	
17	9.66 123	24	9.71 245	31	0.28 754	9.94 878	51	43	
18	9.66 148	24	9.71 276	31	0.28 723	9.94 871	50	42	
19	9.66 172	24	9.71 307	31	0.28 692	9.94 865	50	41	
20	9.66 197	24	9.71 338	31	0.28 661	9.94 858	49	40	
21	9.66 221	24	9.71 369	31	0.28 630	9.94 852	49	39	
22	9.66 246	24	9.71 400	31	0.28 599	9.94 845	48	38	
23	9.66 270	24	9.71 431	31	0.28 568	9.94 839	48	37	
24	9.66 294	24	9.71 462	31	0.28 537	9.94 832	47	36	
25	9.66 319	24	9.71 493	31	0.28 506	9.94 825	47	35	
26	9.66 343	24	9.71 524	31	0.28 476	9.94 819	46	34	
27	9.66 367	24	9.71 555	31	0.28 445	9.94 812	46	33	
28	9.66 392	24	9.71 586	31	0.28 414	9.94 806	45	32	
29	9.66 416	24	9.71 617	30	0.28 383	9.94 799	45	31	
30	9.66 440	24	9.71 647	31	0.28 352	9.94 793	44	30	
31	9.66 465	24	9.71 678	31	0.28 321	9.94 786	44	29	
32	9.66 489	24	9.71 709	30	0.28 290	9.94 779	43	28	
33	9.66 513	24	9.71 740	31	0.28 260	9.94 773	43	27	
34	9.66 537	24	9.71 771	30	0.28 229	9.94 766	42	26	
35	9.66 561	24	9.71 801	31	0.28 198	9.94 760	42	25	
36	9.66 586	24	9.71 832	31	0.28 167	9.94 753	41	24	
37	9.66 610	24	9.71 863	30	0.28 136	9.94 746	41	23	
38	9.66 634	24	9.71 894	31	0.28 106	9.94 740	40	22	
39	9.66 658	24	9.71 925	30	0.28 075	9.94 733	40	21	
40	9.66 682	24	9.71 955	30	0.28 044	9.94 727	39	20	
41	9.66 706	24	9.71 986	31	0.28 014	9.94 720	39	19	
42	9.66 730	24	9.72 017	30	0.27 983	9.94 713	38	18	
43	9.66 754	24	9.72 047	30	0.27 952	9.94 707	38	17	
44	9.66 778	24	9.72 078	31	0.27 921	9.94 700	37	16	
45	9.66 802	24	9.72 109	30	0.27 891	9.94 693	37	15	
46	9.66 826	24	9.72 139	30	0.27 860	9.94 687	36	14	
47	9.66 850	24	9.72 170	30	0.27 830	9.94 680	36	13	
48	9.66 874	24	9.72 201	31	0.27 799	9.94 674	35	12	
49	9.66 898	24	9.72 231	30	0.27 768	9.94 667	35	11	
50	9.66 922	24	9.72 262	30	0.27 738	9.94 660	34	10	
51	9.66 946	24	9.72 292	30	0.27 707	9.94 654	34	9	
52	9.66 970	24	9.72 323	30	0.27 677	9.94 647	33	8	
53	9.66 994	23	9.72 354	31	0.27 646	9.94 640	33	7	
54	9.67 018	24	9.72 384	30	0.27 615	9.94 633	32	6	
55	9.67 042	24	9.72 415	30	0.27 585	9.94 627	32	5	
56	9.67 066	24	9.72 445	30	0.27 554	9.94 620	31	4	
57	9.67 089	23	9.72 476	30	0.27 524	9.94 613	31	3	
58	9.67 113	24	9.72 506	30	0.27 493	9.94 607	30	2	
59	9.67 137	23	9.72 537	30	0.27 463	9.94 600	30	1	
60	9.67 161	24	9.72 567	30	0.27 432	9.94 593	29	0	
	Log. Cos.	d.	Log. Cot	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

g. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
67 161	23	9·72 567	30	0·27 432	9·94 593	6	60	
67 184	24	9·72 598	30	0·27 402	9·94 587	7	59	
67 208	23	9·72 628	30	0·27 371	9·94 580	6	58	
67 232	24	9·72 659	30	0·27 341	9·94 573	7	57	
67 256	23	9·72 689	30	0·27 311	9·94 566	6	56	
67 279	23	9·72 719	30	0·27 280	9·94 560	7	55	30 20 29
67 303	24	9·72 750	30	0·27 250	9·94 553	7	54	6 3·0 3·0 2·9
67 327	23	9·72 780	30	0·27 219	9·94 546	6	53	7 3·5 3·5 3·4
67 350	23	9·72 811	30	0·27 189	9·94 539	7	52	8 4·0 4·0 3·9
67 374	24	9·72 841	30	0·27 159	9·94 533	6	51	9 4·6 4·5 4·4
67 397	23	9·72 871	30	0·27 125	9·94 526	7	50	10 5·1 5·0 4·9
67 421	24	9·72 902	30	0·27 098	9·94 519	6	49	20 10·1 10·0 9·8
67 445	23	9·72 932	30	0·27 067	9·94 512	7	48	30 15·2 15·0 14·7
67 468	23	9·72 962	30	0·27 037	9·94 506	6	47	40 20·3 20·0 19·6
67 492	24	9·72 993	30	0·27 007	9·94 499	7	46	50 25·4 25·0 24·6
67 515	23	9·73 023	30	0·26 976	9·94 492	7	45	
67 539	23	9·73 053	30	0·26 946	9·94 485	6	44	
67 562	23	9·73 084	30	0·26 916	9·94 478	7	43	
67 586	23	9·73 114	30	0·26 886	9·94 472	6	42	
67 609	23	9·73 144	30	0·26 855	9·94 465	7	41	24
67 633	23	9·73 174	30	0·26 825	9·94 458	7	40	6 2·4
67 656	23	9·73 205	30	0·26 795	9·94 451	6	39	7 2·8
67 679	23	9·73 235	30	0·26 765	9·94 444	7	38	8 3·2
67 703	23	9·73 265	30	0·26 734	9·94 437	7	37	9 3·6
67 726	23	9·73 295	30	0·26 704	9·94 431	6	36	10 4·0
67 750	23	9·73 325	30	0·26 674	9·94 424	7	35	20 8·0
67 773	23	9·73 356	30	0·26 644	9·94 417	7	34	30 12·0
67 796	23	9·73 386	30	0·26 614	9·94 410	6	33	40 16·0
67 819	23	9·73 416	30	0·26 584	9·94 403	7	32	50 20·0
67 843	23	9·73 446	30	0·26 553	9·94 396	7	31	
67 866	23	9·73 476	30	0·26 523	9·94 390	6	30	
67 889	23	9·73 506	30	0·26 493	9·94 383	7	29	
67 913	23	9·73 536	30	0·26 463	9·94 376	7	28	
67 936	23	9·73 567	30	0·26 433	9·94 369	6	27	23 23 22
67 959	23	9·73 597	30	0·26 403	9·94 362	7	26	6 2·3 2·3 2·2
67 982	23	9·73 627	30	0·26 373	9·94 355	7	25	7 2·7 2·7 2·6
68 005	23	9·73 657	30	0·26 343	9·94 348	7	24	8 3·1 3·0 3·0
68 029	23	9·73 687	30	0·26 313	9·94 341	6	23	9 3·5 3·4 3·4
68 052	23	9·73 717	30	0·26 283	9·94 335	7	22	10 3·9 3·8 3·7
68 075	23	9·73 747	30	0·26 253	9·94 328	7	21	20 7·8 7·6 7·5
68 098	23	9·73 777	30	0·26 223	9·94 321	7	20	30 11·7 11·5 11·2
68 121	23	9·73 807	30	0·26 193	9·94 314	7	19	40 15·6 15·3 15·0
68 144	23	9·73 837	30	0·26 163	9·94 307	6	18	50 19·6 19·1 18·7
68 167	23	9·73 867	30	0·26 133	9·94 300	7	17	
68 190	23	9·73 897	30	0·26 103	9·94 293	7	16	
68 213	23	9·73 927	30	0·26 073	9·94 286	7	15	
68 236	23	9·73 957	30	0·26 043	9·94 279	7	14	
68 259	23	9·73 987	30	0·26 013	9·94 272	7	13	7 6
68 282	23	9·74 017	30	0·25 983	9·94 265	7	12	6 0·7 0·6
68 305	23	9·74 047	29	0·25 953	9·94 258	7	11	7 0·8 0·7
68 328	23	9·74 076	30	0·25 923	9·94 251	6	10	8 0·9 0·8
68 351	23	9·74 106	30	0·25 893	9·94 245	7	9	9 1·0 1·0
68 374	23	9·74 136	30	0·25 863	9·94 238	7	8	10 1·1 1·1
68 397	23	9·74 166	30	0·25 833	9·94 231	7	7	20 2·3 2·1
68 420	23	9·74 196	29	0·25 804	9·94 224	7	6	30 3·5 3·2
68 443	23	9·74 226	30	0·25 774	9·94 217	7	5	40 4·6 4·3
68 466	23	9·74 256	30	0·25 744	9·94 210	7	4	50 5·8 5·4
68 489	23	9·74 286	30	0·25 714	9·94 203	7	3	
68 511	23	9·74 315	29	0·25 684	9·94 196	7	2	
68 534	23	9·74 345	30	0·25 654	9·94 189	7	1	
68 557	22	9·74 375	29	0·25 625	9·94 182	7	0	
g. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TA
AND COTANGENTS.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.68 557	23	9.74 370	30	0.25 625	9.94 162	7	60
1	9.68 580	22	9.74 405	30	0.25 595	9.94 175	7	59
2	9.68 602	22	9.74 435	29	0.25 565	9.94 168	7	58
3	9.68 625	23	9.74 464	30	0.25 535	9.94 161	7	57
4	9.68 648	22	9.74 494	30	0.25 505	9.94 154	7	56
5	9.68 671	23	9.74 524	29	0.25 476	9.94 147	7	55
6	9.68 693	22	9.74 554	30	0.25 446	9.94 140	7	54
7	9.68 716	23	9.74 583	29	0.25 416	9.94 133	7	53
8	9.68 739	23	9.74 613	30	0.25 387	9.94 126	7	52
9	9.68 761	22	9.74 643	29	0.25 357	9.94 118	7	51
10	9.68 784	23	9.74 672	30	0.25 327	9.94 111	7	50
11	9.68 807	23	9.74 702	29	0.25 297	9.94 104	7	49
12	9.68 829	22	9.74 732	29	0.25 268	9.94 097	7	48
13	9.68 852	23	9.74 761	30	0.25 238	9.94 090	7	47
14	9.68 874	22	9.74 791	29	0.25 208	9.94 083	7	46
15	9.68 897	23	9.74 821	29	0.25 179	9.94 076	7	45
16	9.68 920	23	9.74 850	29	0.25 149	9.94 069	7	44
17	9.68 942	22	9.74 880	29	0.25 120	9.94 062	7	43
18	9.68 965	23	9.74 909	29	0.25 090	9.94 055	7	42
19	9.68 987	22	9.74 939	30	0.25 060	9.94 048	7	41
20	9.69 010	23	9.74 969	29	0.25 031	9.94 041	7	40
21	9.69 032	22	9.74 998	29	0.25 001	9.94 034	7	39
22	9.69 055	23	9.75 028	29	0.24 972	9.94 026	7	38
23	9.69 077	22	9.75 057	29	0.24 943	9.94 019	7	37
24	9.69 099	22	9.75 087	29	0.24 913	9.94 012	7	36
25	9.69 122	23	9.75 116	29	0.24 885	9.94 005	7	35
26	9.69 144	22	9.75 146	29	0.24 854	9.93 998	7	34
27	9.69 167	23	9.75 175	29	0.24 824	9.93 991	7	33
28	9.69 189	22	9.75 205	29	0.24 795	9.93 984	7	32
29	9.69 211	22	9.75 234	29	0.24 765	9.93 977	7	31
30	9.69 234	23	9.75 264	29	0.24 736	9.93 969	7	30
31	9.69 256	22	9.75 293	29	0.24 706	9.93 962	7	29
32	9.69 278	22	9.75 323	29	0.24 677	9.93 955	7	28
33	9.69 301	23	9.75 352	29	0.24 647	9.93 948	7	27
34	9.69 323	22	9.75 382	29	0.24 618	9.93 941	7	26
35	9.69 345	22	9.75 411	29	0.24 588	9.93 934	7	25
36	9.69 367	22	9.75 441	29	0.24 559	9.93 926	7	24
37	9.69 390	23	9.75 470	29	0.24 529	9.93 919	7	23
38	9.69 412	22	9.75 499	29	0.24 500	9.93 912	7	22
39	9.69 434	22	9.75 529	29	0.24 471	9.93 905	7	21
40	9.69 456	22	9.75 558	29	0.24 441	9.93 898	7	20
41	9.69 478	22	9.75 588	29	0.24 412	9.93 891	7	19
42	9.69 500	22	9.75 617	29	0.24 383	9.93 883	7	18
43	9.69 523	23	9.75 646	29	0.24 353	9.93 876	7	17
44	9.69 545	22	9.75 676	29	0.24 324	9.93 869	7	16
45	9.69 567	22	9.75 705	29	0.24 295	9.93 862	7	15
46	9.69 589	22	9.75 734	29	0.24 265	9.93 854	7	14
47	9.69 611	22	9.75 764	29	0.24 236	9.93 847	7	13
48	9.69 633	22	9.75 793	29	0.24 207	9.93 840	7	12
49	9.69 655	22	9.75 822	29	0.24 177	9.93 833	7	11
50	9.69 677	22	9.75 851	29	0.24 148	9.93 826	7	10
51	9.69 699	22	9.75 881	29	0.24 119	9.93 818	7	9
52	9.69 721	22	9.75 910	29	0.24 090	9.93 811	7	8
53	9.69 743	22	9.75 939	29	0.24 060	9.93 804	7	7
54	9.69 765	22	9.75 968	29	0.24 031	9.93 796	7	6
55	9.69 787	22	9.75 998	29	0.24 002	9.93 789	7	5
56	9.69 809	22	9.76 027	29	0.23 973	9.93 782	7	4
57	9.69 831	22	9.76 056	29	0.23 945	9.93 775	7	3
58	9.69 853	21	9.76 085	29	0.23 914	9.93 767	7	2
59	9.69 875	22	9.76 115	29	0.23 885	9.93 760	7	1
60	9.69 897	22	9.76 144	29	0.23 856	9.93 753	7	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

Log. Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	d.		P. P.
9.69 897	22	9.76 144	29	0.23 856	9.93 753	7	60	
9.69 919	21	9.76 173	29	0.23 827	9.93 746	7	59	
9.69 940	22	9.76 202	29	0.23 797	9.93 738	7	58	
9.69 962	22	9.76 231	29	0.23 768	9.93 731	7	57	
9.69 984	22	9.76 260	29	0.23 739	9.93 724	7	56	
9.70 006	21	9.76 289	29	0.23 710	9.93 716	7	55	
9.70 028	22	9.76 319	29	0.23 681	9.93 709	7	54	
9.70 050	22	9.76 348	29	0.23 652	9.93 702	7	53	
9.70 071	21	9.76 377	29	0.23 623	9.93 694	7	52	
9.70 093	22	9.76 406	29	0.23 594	9.93 687	7	51	
9.70 115	21	9.76 435	29	0.23 565	9.93 680	7	50	
9.70 137	22	9.76 464	29	0.23 535	9.93 672	7	49	
9.70 158	21	9.76 493	29	0.23 506	9.93 665	7	48	
9.70 180	22	9.76 522	29	0.23 477	9.93 658	7	47	
9.70 202	22	9.76 551	29	0.23 448	9.93 650	7	46	
9.70 223	21	9.76 580	29	0.23 419	9.93 643	7	45	
9.70 245	21	9.76 609	29	0.23 390	9.93 635	7	44	
9.70 267	22	9.76 638	29	0.23 361	9.93 628	7	43	
9.70 288	21	9.76 667	29	0.23 332	9.93 621	7	42	
9.70 310	22	9.76 696	29	0.23 303	9.93 613	7	41	
9.70 331	21	9.76 725	29	0.23 274	9.93 606	7	40	
9.70 353	22	9.76 754	29	0.23 245	9.93 599	7	39	
9.70 375	22	9.76 783	29	0.23 216	9.93 591	7	38	
9.70 396	21	9.76 812	29	0.23 187	9.93 584	7	37	
9.70 418	22	9.76 841	29	0.23 158	9.93 576	7	36	
9.70 439	21	9.76 870	29	0.23 129	9.93 569	7	35	
9.70 461	22	9.76 899	29	0.23 101	9.93 562	7	34	
9.70 482	21	9.76 928	29	0.23 072	9.93 554	7	33	
9.70 504	22	9.76 957	29	0.23 043	9.93 547	7	32	
9.70 525	21	9.76 986	29	0.23 014	9.93 539	7	31	
9.70 547	22	9.77 015	29	0.22 985	9.93 532	7	30	
9.70 568	21	9.77 043	28	0.22 956	9.93 524	7	29	
9.70 590	22	9.77 072	29	0.22 927	9.93 517	7	28	
9.70 611	21	9.77 101	29	0.22 898	9.93 509	7	27	
9.70 632	21	9.77 130	29	0.22 869	9.93 502	7	26	
9.70 654	22	9.77 159	28	0.22 841	9.93 495	7	25	
9.70 675	21	9.77 188	29	0.22 812	9.93 487	7	24	
9.70 696	21	9.77 217	29	0.22 783	9.93 480	7	23	
9.70 718	22	9.77 245	28	0.22 754	9.93 472	7	22	
9.70 739	21	9.77 274	29	0.22 725	9.93 465	7	21	
9.70 760	21	9.77 303	29	0.22 696	9.93 457	7	20	
9.70 782	21	9.77 332	28	0.22 668	9.93 450	7	19	
9.70 803	22	9.77 361	29	0.22 639	9.93 442	7	18	
9.70 824	21	9.77 389	28	0.22 610	9.93 435	7	17	
9.70 846	21	9.77 418	29	0.22 581	9.93 427	7	16	
9.70 867	21	9.77 447	29	0.22 553	9.93 420	7	15	
9.70 888	21	9.77 476	29	0.22 524	9.93 412	7	14	
9.70 909	21	9.77 504	28	0.22 495	9.93 405	7	13	
9.70 930	21	9.77 533	29	0.22 466	9.93 397	7	12	
9.70 952	21	9.77 562	29	0.22 438	9.93 390	7	11	
9.70 973	21	9.77 591	29	0.22 409	9.93 382	8	10	
9.70 994	21	9.77 619	28	0.22 380	9.93 374	7	9	
9.71 015	21	9.77 648	29	0.22 352	9.93 367	7	8	
9.71 036	21	9.77 677	29	0.22 323	9.93 359	7	7	
9.71 057	21	9.77 705	28	0.22 294	9.93 352	7	6	
9.71 078	21	9.77 734	29	0.22 266	9.93 344	7	5	
9.71 099	21	9.77 763	29	0.22 237	9.93 337	7	4	
9.71 121	21	9.77 791	28	0.22 208	9.93 329	8	3	
9.71 142	21	9.77 820	29	0.22 180	9.93 321	7	2	
9.71 163	21	9.77 849	29	0.22 151	9.93 314	7	1	
9.71 184	21	9.77 877	28	0.22 122	9.93 306	7	0	
Log. Cos.	d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	d.	′	P. P.

	29	29	28
6	2.9	2.9	2.8
7	3.4	3.4	3.3
8	3.9	3.8	3.8
9	4.4	4.3	4.3
10	4.9	4.8	4.7
20	9.8	9.6	9.5
30	14.7	14.5	14.2
40	19.6	19.3	19.0
50	24.6	24.1	23.7

	22	21	21
6	2.2	2.1	2.1
7	2.5	2.5	2.4
8	2.9	2.8	2.8
9	3.3	3.2	3.1
10	3.6	3.6	3.5
20	7.3	7.1	7.0
30	11.0	10.7	10.5
40	14.6	14.3	14.0
50	18.3	17.9	17.5

	8	7	7
6	0.8	0.7	0.7
7	0.9	0.9	0.8
8	1.0	1.0	0.9
9	1.2	1.1	1.0
10	1.3	1.2	1.1
20	2.6	2.5	2.3
30	4.0	3.7	3.5
40	5.3	5.0	4.6
50	6.6	6.2	5.8

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS AND COTANGENTS.

′	Log. Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9·71 184	21	9·77 877	25	0·22 123	9·93 306	5	60	
1	9·71 205	21	9·77 906	29	0·22 094	9·93 299	7	59	
2	9·71 226	21	9·77 934	28	0·22 065	9·93 291	7	58	
3	9·71 247	21	9·77 963	28	0·22 037	9·93 284	7	57	
4	9·71 268	21	9·77 992	29	0·22 008	9·93 276	8	56	
5	9·71 289	21	9·78 020	28	0·21 979	9·93 268	7	55	
6	9·71 310	21	9·78 049	28	0·21 951	9·93 261	7	54	
7	9·71 331	21	9·78 077	28	0·21 922	9·93 253	8	53	
8	9·71 351	20	9·78 106	28	0·21 894	9·93 245	7	52	
9	9·71 372	21	9·78 134	28	0·21 865	9·93 238	7	51	
10	9·71 393	21	9·78 163	29	0·21 837	9·93 230	7	50	
11	9·71 414	21	9·78 191	28	0·21 808	9·93 223	8	49	
12	9·71 435	20	9·78 220	28	0·21 780	9·93 215	7	48	
13	9·71 456	21	9·78 248	28	0·21 751	9·93 207	7	47	
14	9·71 477	21	9·78 277	28	0·21 723	9·93 200	8	46	
15	9·71 498	21	9·78 305	28	0·21 694	9·93 192	7	45	
16	9·71 518	20	9·78 334	29	0·21 666	9·93 184	7	44	
17	9·71 539	21	9·78 362	28	0·21 637	9·93 177	8	43	
18	9·71 560	20	9·78 391	28	0·21 609	9·93 169	7	42	
19	9·71 581	21	9·78 419	28	0·21 580	9·93 161	8	41	
20	9·71 601	20	9·78 448	28	0·21 552	9·93 153	7	40	
21	9·71 622	21	9·78 476	28	0·21 523	9·93 146	7	39	
22	9·71 643	20	9·78 505	29	0·21 495	9·93 138	8	38	
23	9·71 663	21	9·78 533	28	0·21 467	9·93 130	7	37	
24	9·71 684	20	9·78 561	28	0·21 438	9·93 123	8	36	
25	9·71 705	21	9·78 590	29	0·21 410	9·93 115	7	35	
26	9·71 726	20	9·78 618	28	0·21 381	9·93 107	7	34	
27	9·71 746	20	9·78 647	28	0·21 353	9·93 100	8	33	
28	9·71 767	21	9·78 675	28	0·21 325	9·93 092	7	32	
29	9·71 788	20	9·78 703	28	0·21 296	9·93 084	8	31	
30	9·71 808	20	9·78 732	29	0·21 268	9·93 076	8	30	
31	9·71 829	20	9·78 760	28	0·21 239	9·93 069	7	29	
32	9·71 849	21	9·78 788	28	0·21 211	9·93 061	8	28	
33	9·71 870	20	9·78 817	28	0·21 183	9·93 053	8	27	
34	9·71 891	20	9·78 845	28	0·21 154	9·93 045	7	26	
35	9·71 911	20	9·78 873	28	0·21 126	9·93 038	8	25	
36	9·71 932	20	9·78 902	28	0·21 098	9·93 030	7	24	
37	9·71 953	20	9·78 930	28	0·21 070	9·93 022	8	23	
38	9·71 973	20	9·78 958	28	0·21 041	9·93 014	8	22	
39	9·71 993	20	9·78 987	28	0·21 013	9·93 006	8	21	
40	9·72 014	20	9·79 015	28	0·20 985	9·92 999	7	20	
41	9·72 034	20	9·79 043	28	0·20 956	9·92 991	8	19	
42	9·72 055	20	9·79 071	28	0·20 928	9·92 983	8	18	
43	9·72 075	20	9·79 100	28	0·20 900	9·92 975	8	17	
44	9·72 096	20	9·79 128	28	0·20 872	9·92 967	8	16	
45	9·72 116	20	9·79 156	28	0·20 843	9·92 960	7	15	
46	9·72 136	20	9·79 184	28	0·20 815	9·92 952	8	14	
47	9·72 157	20	9·79 213	28	0·20 787	9·92 944	8	13	
48	9·72 177	20	9·79 241	28	0·20 759	9·92 936	8	12	
49	9·72 198	20	9·79 269	28	0·20 731	9·92 928	8	11	
50	9·72 218	20	9·79 297	28	0·20 703	9·92 920	8	10	
51	9·72 238	20	9·79 325	28	0·20 674	9·92 913	8	9	
52	9·72 259	20	9·79 354	28	0·20 646	9·92 905	8	8	
53	9·72 279	20	9·79 382	28	0·20 618	9·92 897	7	7	
54	9·72 299	20	9·79 410	28	0·20 590	9·92 889	8	6	
55	9·72 319	20	9·79 438	28	0·20 561	9·92 881	8	5	
56	9·72 340	20	9·79 466	28	0·20 533	9·92 873	8	4	
57	9·72 360	20	9·79 494	28	0·20 505	9·92 865	8	3	
58	9·72 380	20	9·79 522	28	0·20 477	9·92 858	7	2	
59	9·72 400	20	9·79 551	28	0·20 449	9·92 850	8	1	
60	9·72 421	20	9·79 579	28	0·20 421	9·92 842	8	0	
	Log. Cos.	d.		d.	Log. Tan.	Log. Sin.	d.	′	P. P.

Log. Tan.	c.d.	Log. Cot.	Log. Cos.	d.		P. P.
9·79 579	28	0·20 421	9·92 842	8	60	
9·79 607	28	0·20 393	9·92 834	7	59	
9·79 635	28	0·20 365	9·92 826	8	58	
9·79 663	28	0·20 337	9·92 818	8	57	
9·79 691	28	0·20 308	9·92 810	8	56	
9·79 719	28	0·20 280	9·92 802	8	55	
9·79 747	28	0·20 252	9·92 794	8	54	
9·79 775	28	0·20 224	9·92 786	8	53	
9·79 803	28	0·20 196	9·92 778	8	52	
9·79 831	28	0·20 168	9·92 771	7	51	28 28 27 6 2·8 2·8 2·7
9·79 859	28	0·20 140	9·92 763	8	50	7 3·3 3·2 3·2
9·79 887	28	0·20 112	9·92 755	8	49	8 3·8 3·7 3·6
9·79 915	28	0·20 084	9·92 747	8	48	9 4·3 4·2 4·1
9·79 943	28	0·20 056	9·92 739	8	47	10 4·7 4·6 4·6
9·79 971	28	0·20 028	9·92 731	8	46	20 9·5 9·3 9·1
9·79 999	28	0·20 000	9·92 723	8	45	30 14·2 14·0 13·7
9·80 027	28	0·19 972	9·92 715	8	44	40 19·0 18·6 18·3
9·80 055	28	0·19 944	9·92 707	8	43	50 23·7 23·3 22·9
9·80 083	28	0·19 916	9·92 699	8	42	
9·80 111	28	0·19 888	9·92 691	8	41	
9·80 139	28	0·19 860	9·92 683	8	40	
9·80 167	28	0·19 832	9·92 675	8	39	
9·80 195	28	0·19 804	9·92 667	8	38	
9·80 223	28	0·19 776	9·92 659	8	37	
9·80 251	28	0·19 748	9·92 651	8	36	
9·80 279	27	0·19 721	9·92 643	8	35	
9·80 307	28	0·19 693	9·92 635	8	34	20 20 19
9·80 335	28	0·19 665	9·92 627	8	33	6 2·0 2·0 1·9
9·80 363	28	0·19 637	9·92 619	8	32	7 2·4 2·3 2·3
9·80 391	28	0·19 609	9·92 611	8	31	8 2·7 2·6 2·6
9·80 418	27	0·19 581	9·92 603	8	30	9 3·1 3·0 2·9
9·80 446	28	0·19 553	9·92 595	8	29	10 3·4 3·3 3·2
9·80 474	28	0·19 525	9·92 587	8	28	20 6·8 6·6 6·5
9·80 502	28	0·19 497	9·92 579	8	27	30 10·2 10·0 9·7
9·80 530	27	0·19 470	9·92 570	9	26	40 13·6 13·3 13·0
9·80 558	28	0·19 442	9·92 562	8	25	50 17·1 16·6 16·2
9·80 586	28	0·19 414	9·92 554	8	24	
9·80 613	27	0·19 386	9·92 546	8	23	
9·80 641	28	0·19 359	9·92 538	8	22	
9·80 669	28	0·19 330	9·92 530	8	21	
9·80 697	27	0·19 303	9·92 522	8	20	
9·80 725	28	0·19 275	9·92 514	8	19	
9·80 752	27	0·19 247	9·92 506	8	18	
9·80 780	28	0·19 219	9·92 498	8	17	
9·80 808	28	0·19 191	9·92 489	9	16	8 8 7
9·80 836	27	0·19 164	9·92 481	8	15	6 0·8 0·8 0·7
9·80 864	28	0·19 136	9·92 473	8	14	7 1·0 0·9 0·9
9·80 891	27	0·19 109	9·92 465	8	13	8 1·1 1·0 1·0
9·80 919	28	0·19 080	9·92 457	8	12	9 1·3 1·2 1·1
9·80 947	27	0·19 053	9·92 449	8	11	10 1·4 1·3 1·2
9·80 975	28	0·19 025	9·92 441	8	10	20 2·8 2·6 2·5
9·81 002	27	0·18 997	9·92 433	8	9	30 4·2 4·0 3·7
9·81 030	28	0·18 970	9·92 424	8	8	40 5·6 5·3 5·0
9·81 058	28	0·18 942	9·92 416	8	7	50 7·1 6·6 6·2
9·81 085	27	0·18 914	9·92 408	8	6	
9·81 113	28	0·18 886	9·92 400	8	5	
9·81 141	27	0·18 859	9·92 392	8	4	
9·81 168	27	0·18 831	9·92 383	8	3	
9·81 196	28	0·18 803	9·92 375	8	2	
9·81 224	27	0·18 776	9·92 367	8	1	
9·81 251	27	0·18 748	9·92 359	8	0	
Log. Cot.	c.d.	Log. Tan.	Log. Sin.	d.	′	P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9·73 611	19	9·81 251	28	0·18 749	9·92 389		60	
1	9·73 630	19	9·81 279	27	0·18 720	9·92 381		59	
2	9·73 650	19	9·81 307	27	0·18 693	9·92 373		58	
3	9·73 669	19	9·81 334	28	0·18 665	9·92 364		57	
4	9·73 688	19	9·81 362	27	0·18 637	9·92 356		56	
5	9·73 708	19	9·81 390	27	0·18 610	9·92 348		55	
6	9·73 727	19	9·81 417	27	0·18 583	9·92 340		54	
7	9·73 746	19	9·81 445	28	0·18 555	9·92 331		53	
8	9·73 766	19	9·81 473	27	0·18 527	9·92 323		52	
9	9·73 785	19	9·81 500	27	0·18 499	9·92 315		51	
10	9·73 805	19	9·81 528	27	0·18 472	9·92 307		50	
11	9·73 824	19	9·81 555	27	0·18 444	9·92 298		49	
12	9·73 843	19	9·81 583	27	0·18 417	9·92 290		48	
13	9·73 862	19	9·81 610	27	0·18 389	9·92 282		47	
14	9·73 882	19	9·81 638	28	0·18 362	9·92 244		46	
15	9·73 901	19	9·81 666	27	0·18 334	9·92 265		45	
16	9·73 920	19	9·81 693	27	0·18 306	9·92 257		44	
17	9·73 940	19	9·81 721	27	0·18 279	9·92 249		43	
18	9·73 959	19	9·81 748	27	0·18 251	9·92 240		42	
19	9·73 978	19	9·81 776	27	0·18 224	9·92 232		41	
20	9·73 997	19	9·81 803	27	0·18 196	9·92 224		40	
21	9·74 016	19	9·81 831	27	0·18 169	9·92 215		39	
22	9·74 036	19	9·81 858	27	0·18 141	9·92 177		38	
23	9·74 055	19	9·81 886	27	0·18 114	9·92 169		37	
24	9·74 074	19	9·81 913	27	0·18 086	9·92 160		36	
25	9·74 093	19	9·81 941	27	0·18 059	9·92 152		35	
26	9·74 112	19	9·81 968	27	0·18 031	9·92 144		34	
27	9·74 131	19	9·81 996	27	0·18 004	9·92 135		33	
28	9·74 151	19	9·82 023	27	0·17 976	9·92 127		32	
29	9·74 170	19	9·82 051	27	0·17 949	9·92 119		31	
30	9·74 189	19	9·82 078	27	0·17 921	9·92 110		30	
31	9·74 208	19	9·82 105	27	0·17 894	9·92 102		29	
32	9·74 227	19	9·82 133	27	0·17 867	9·92 094		28	
33	9·74 246	19	9·82 160	27	0·17 839	9·92 085		27	
34	9·74 265	19	9·82 188	27	0·17 812	9·92 077		26	
35	9·74 284	19	9·82 215	27	0·17 784	9·92 069		25	
36	9·74 303	19	9·82 243	27	0·17 757	9·92 060		24	
37	9·74 322	19	9·82 270	27	0·17 729	9·92 052		23	
38	9·74 341	19	9·82 297	27	0·17 702	9·92 043		22	
39	9·74 360	18	9·82 325	27	0·17 675	9·92 035		21	
40	9·74 379	19	9·82 352	27	0·17 647	9·92 027		20	
41	9·74 398	19	9·82 380	27	0·17 620	9·92 018		19	
42	9·74 417	19	9·82 407	27	0·17 593	9·92 010		18	
43	9·74 436	19	9·82 434	27	0·17 565	9·92 001		17	
44	9·74 455	19	9·82 462	27	0·17 538	9·91 993		16	
45	9·74 474	19	9·82 489	27	0·17 510	9·91 984		15	
46	9·74 493	19	9·82 516	27	0·17 483	9·91 976		14	
47	9·74 511	18	9·82 544	27	0·17 456	9·91 967		13	
48	9·74 530	19	9·82 571	27	0·17 428	9·91 959		12	
49	9·74 549	19	9·82 598	27	0·17 401	9·91 951		11	
50	9·74 568	19	9·82 626	27	0·17 374	9·91 942		10	
51	9·74 587	19	9·82 653	27	0·17 347	9·91 934		9	
52	9·74 606	19	9·82 680	27	0·17 319	9·91 925		8	
53	9·74 625	19	9·82 708	27	0·17 292	9·91 917		7	
54	9·74 643	18	9·82 735	27	0·17 265	9·91 908		6	
55	9·74 662	19	9·82 762	27	0·17 237	9·91 900		5	
56	9·74 681	19	9·82 789	27	0·17 210	9·91 891		4	
57	9·74 700	19	9·82 817	27	0·17 183	9·91 883		3	
58	9·74 718	19	9·82 844	27	0·17 156	9·91 874		2	
59	9·74 737	19	9·82 871	27	0·17 128	9·91 866		1	
60	9·74 756	18	9·82 898	27	0·17 101	9·91 857		0	
	Log. Cos.	d.	Log. Tan.		Log. Cot.	Log. Sin.	d.	′	P. P.

g. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
74 756	19	9·82 898	27	0·17 101	9·91 857	8	60	
74 775	19	9·82 926	27	0·17 074	9·91 849	8	59	
74 793	19	9·82 953	27	0·17 047	9·91 840	9	58	
74 812	18	9·82 980	27	0·17 019	9·91 832	8	57	
74 831	18	9·83 007	27	0·16 992	9·91 823	9	56	
74 849	19	9·83 035	27	0·16 965	9·91 814	8	55	
74 868	18	9·83 062	27	0·16 938	9·91 806	8	54	
74 887	18	9·83 089	27	0·16 910	9·91 797	9	53	
74 905	19	9·83 116	27	0·16 883	9·91 789	8	52	27 27 26
74 924	18	9·83 143	27	0·16 856	9·91 780	9	51	6 2·7 2·7 2·6
74 943	18	9·83 171	27	0·16 828	9·91 772	8	50	7 3·2 3·1 3·1
74 961	18	9·83 198	27	0·16 802	9·91 763	8	49	8 3·6 3·6 3·5
74 980	18	9·83 225	27	0·16 774	9·91 755	9	48	9 4·1 4·0 4·0
74 998	18	9·83 252	27	0·16 747	9·91 746	8	47	10 4·6 4·5 4·4
75 017	18	9·83 279	27	0·16 720	9·91 737	9	46	20 9·1 9·0 8·8
75 036	19	9·83 307	27	0·16 693	9·91 729	8	45	30 13·7 13·5 13·2
75 054	18	9·83 334	27	0·16 666	9·91 720	8	44	40 18·3 18·0 17·6
75 073	19	9·83 361	27	0·16 639	9·91 712	9	43	50 22·9 22·5 22·1
75 091	18	9·83 388	27	0·16 612	9·91 703	8	42	
75 110	18	9·83 415	27	0·16 584	9·91 694	8	41	
75 128	18	9·83 442	27	0·16 557	9·91 686	9	40	
75 147	19	9·83 469	27	0·16 530	9·91 677	8	39	
75 165	18	9·83 496	27	0·16 503	9·91 668	9	38	
75 184	18	9·83 524	27	0·16 476	9·91 660	8	37	
75 202	18	9·83 551	27	0·16 449	9·91 651	9	36	
75 221	18	9·83 578	27	0·16 422	9·91 642	9	35	
75 239	18	9·83 605	27	0·16 395	9·91 634	8	34	19 18 18
75 257	18	9·83 632	27	0·16 368	9·91 625	9	33	6 1·9 1·8 1·8
75 275	18	9·83 659	27	0·16 340	9·91 616	9	32	7 2·2 2·1 2·1
75 294	19	9·83 686	27	0·16 313	9·91 608	8	31	8 2·5 2·4 2·4
75 313	18	9·83 713	27	0·16 286	9·91 599	9	30	9 2·8 2·8 2·7
75 331	18	9·83 740	27	0·16 259	9·91 590	9	29	10 3·1 3·1 3·0
75 349	18	9·83 767	27	0·16 232	9·91 582	8	28	20 6·3 6·1 6·0
75 368	18	9·83 794	27	0·16 205	9·91 573	9	27	30 9·5 9·2 9·0
75 386	18	9·83 821	27	0·16 178	9·91 564	9	26	40 12·6 12·3 12·0
75 404	18	9·83 848	27	0·16 151	9·91 556	8	25	50 15·8 15·4 15·0
75 423	19	9·83 875	27	0·16 124	9·91 547	9	24	
75 441	18	9·83 902	27	0·16 097	9·91 538	9	23	
75 459	18	9·83 929	27	0·16 070	9·91 529	9	22	
75 478	19	9·83 957	27	0·16 043	9·91 521	8	21	
75 496	18	9·83 984	27	0·16 016	9·91 512	9	20	
75 514	18	9·84 011	27	0·15 989	9·91 503	9	19	
75 532	18	9·84 038	27	0·15 962	9·91 495	8	18	
75 551	18	9·84 065	27	0·15 935	9·91 486	9	17	
75 569	18	9·84 091	26	0·15 909	9·91 477	9	16	
75 587	18	9·84 118	27	0·15 881	9·91 468	9	15	9 8
75 605	18	9·84 145	27	0·15 855	9·91 460	8	14	6 0·9 0·8
75 623	18	9·84 172	27	0·15 828	9·91 451	9	13	7 1·0 1·0
75 642	19	9·84 199	27	0·15 801	9·91 442	9	12	8 1·2 1·1
75 660	18	9·84 226	27	0·15 775	9·91 433	9	11	9 1·3 1·3
75 678	18	9·84 253	27	0·15 746	9·91 424	9	10	10 1·5 1·4
75 696	18	9·84 280	27	0·15 719	9·91 416	8	9	20 3·0 2·8
75 714	18	9·84 307	27	0·15 693	9·91 407	9	8	30 4·5 4·2
75 732	18	9·84 334	27	0·15 666	9·91 398	9	7	40 6·0 5·6
75 750	18	9·84 361	26	0·15 639	9·91 389	9	6	50 7·5 7·1
75 769	18	9·84 388	27	0·15 612	9·91 380	9	5	
75 787	18	9·84 415	27	0·15 585	9·91 372	8	4	
75 805	18	9·84 442	27	0·15 558	9·91 363	9	3	
75 823	18	9·84 469	27	0·15 531	9·91 354	9	2	
75 841	18	9·84 496	27	0·15 504	9·91 345	9	1	
75 859	18	9·84 522	26	0·15 477	9·91 336	9	0	
g. Cos.	d.	Log. Cot.	c. d.	og. Tan.	Log. Sin.	d.	′	P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d		
0	9·75 859	18	9·84 522	27	0·15 477	9·91 336	9	60	
1	9·75 877	18	9·84 549	27	0·15 450	9·91 327	9	59	
2	9·75 895	18	9·84 576	27	0·15 423	9·91 318	9	58	
3	9·75 913	18	9·84 603	27	0·15 396	9·91 310	9	57	
4	9·75 931	18	9·84 630	26	0·15 370	9·91 301	9	56	
5	9·75 949	18	9·84 657	27	0·15 343	9·91 292	9	55	
6	9·75 967	18	9·84 684	27	0·15 316	9·91 283	9	54	
7	9·75 985	18	9·84 711	27	0·15 289	9·91 274	9	53	
8	9·76 003	18	9·84 737	26	0·15 262	9·91 265	9	52	
9	9·76 021	18	9·84 764	27	0·15 235	9·91 256	9	51	
10	9·76 039	18	9·84 791	27	0·15 208	9·91 247	9	50	
11	9·76 057	18	9·84 818	27	0·15 182	9·91 239	9	49	
12	9·76 075	18	9·84 845	27	0·15 155	9·91 230	9	48	
13	9·76 092	17	9·84 871	26	0·15 128	9·91 221	9	47	
14	9·76 110	18	9·84 898	27	0·15 101	9·91 212	9	46	
15	9·76 128	18	9·84 925	27	0·15 074	9·91 203	9	45	
16	9·76 146	18	9·84 952	27	0·15 048	9·91 194	9	44	
17	9·76 164	18	9·84 979	27	0·15 021	9·91 185	9	43	
18	9·76 182	18	9·85 005	26	0·14 994	9·91 176	9	42	
19	9·76 200	18	9·85 032	27	0·14 967	9·91 167	9	41	
20	9·76 217	17	9·85 059	27	0·14 940	9·91 158	9	40	
21	9·76 235	18	9·85 086	27	0·14 914	9·91 149	9	39	
22	9·76 253	18	9·85 113	27	0·14 887	9·91 140	9	38	
23	9·76 271	18	9·85 139	26	0·14 860	9·91 131	9	37	
24	9·76 289	18	9·85 166	27	0·14 833	9·91 122	9	36	
25	9·76 306	17	9·85 193	27	0·14 807	9·91 113	9	35	
26	9·76 324	18	9·85 220	27	0·14 780	9·91 104	9	34	
27	9·76 342	18	9·85 246	26	0·14 753	9·91 095	9	33	
28	9·76 360	18	9·85 273	27	0·14 726	9·91 086	9	32	
29	9·76 377	17	9·85 300	27	0·14 700	9·91 077	9	31	
30	9·76 395	18	9·85 327	27	0·14 673	9·91 068	9	30	
31	9·76 413	18	9·85 353	26	0·14 646	9·91 059	9	29	
32	9·76 431	18	9·85 380	27	0·14 620	9·91 050	9	28	
33	9·76 448	17	9·85 407	27	0·14 593	9·91 041	9	27	
34	9·76 466	18	9·85 433	26	0·14 566	9·91 032	9	26	
35	9·76 484	18	9·85 460	27	0·14 539	9·91 023	9	25	
36	9·76 501	17	9·85 487	27	0·14 513	9·91 014	9	24	
37	9·76 519	18	9·85 513	26	0·14 486	9·91 005	9	23	
38	9·76 536	17	9·85 540	27	0·14 459	9·90 995	9	22	
39	9·76 554	18	9·85 567	26	0·14 433	9·90 987	9	21	
40	9·76 572	17	9·85 594	27	0·14 406	9·90 978	9	20	
41	9·76 589	17	9·85 620	26	0·14 379	9·90 969	9	19	
42	9·76 607	17	9·85 647	26	0·14 353	9·90 960	9	18	
43	9·76 624	17	9·85 673	27	0·14 326	9·90 951	9	17	
44	9·76 642	18	9·85 700	26	0·14 299	9·90 942	9	16	
45	9·76 660	17	9·85 727	26	0·14 273	9·90 933	9	15	
46	9·76 677	17	9·85 753	26	0·14 246	9·90 923	9	14	
47	9·76 695	17	9·85 780	27	0·14 219	9·90 914	9	13	
48	9·76 712	17	9·85 807	26	0·14 193	9·90 905	9	12	
49	9·76 730	17	9·85 833	26	0·14 166	9·90 896	9	11	
50	9·76 747	17	9·85 860	27	0·14 140	9·90 887	9	10	
51	9·76 765	17	9·85 887	26	0·14 113	9·90 878	9	9	
52	9·76 782	17	9·85 913	26	0·14 086	9·90 869	9	8	
53	9·76 800	17	9·85 940	26	0·14 060	9·90 860	9	7	
54	9·76 817	17	9·85 966	26	0·14 033	9·90 850	9	6	
55	9·76 835	17	9·85 993	27	0·14 007	9·90 841	9	5	
56	9·76 852	17	9·86 020	26	0·13 980	9·90 832	9	4	
57	9·76 869	17	9·86 046	26	0·13 953	9·90 823	9	3	
58	9·76 887	17	9·86 073	26	0·13 927	9·90 814	9	2	
59	9·76 904	17	9·86 099	26	0·13 900	9·90 805	9	1	
60	9·76 922	17	9·86 126	26	0·13 874	9·90 796	9	0	
	Log. Cos.	d.	Log. Cot.	c d.	Log. Tan.	Log. Sin.	d.	′	

d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′	P. P.
17	9.86 126	26	0.13 874	9.90 796	9	60	
17	9.86 152	26	0.13 847	9.90 786	9	59	
17	9.86 179	27	0.13 821	9.90 777	9	58	
17	9.86 206	26	0.13 794	9.90 768	9	57	
	9.86 232	26	0.13 767	9.90 759	9	56	
17	9.86 259	26	0.13 741	9.90 750	9	55	
17	9.86 285	26	0.13 714	9.90 740	9	54	
17	9.86 312	26	0.13 688	9.90 731	9	53	
17	9.86 338	26	0.13 661	9.90 722	9	52	
	9.86 365	26	0.13 635	9.90 713	9	51	27 26 26
17	9.86 391	26	0.13 608	9.90 703	9		6 2.7 2.6 2.6
17	9.86 418	26	0.13 582	9.90 694	9	50	7 3.1 3.0 3.0
17	9.86 444	26	0.13 555	9.90 685	9	49	8 3.6 3.5 3.4
17	9.86 471	26	0.13 529	9.90 676	9	48	9 4.0 4.0 3.9
17	9.86 497	26	0.13 502	9.90 666	9	47	10 4.5 4.4 4.3
						46	20 9.0 8.8 8.6
17	9.86 524	26	0.13 476	9.90 657	9		30 13.5 13.2 13.0
17	9.86 550	26	0.13 449	9.90 648	9	45	40 18.0 17.6 17.3
17	9.86 577	26	0.13 423	9.90 639	9	44	50 22.5 22.1 21.6
17	9.86 603	26	0.13 396	9.90 629	9	43	
	9.86 630	26	0.13 370	9.90 620	9	42	
17	9.86 656	26	0.13 343	9.90 611	9	41	
17	9.86 683	26	0.13 317	9.90 602	9	40	
17	9.86 709	26	0.13 290	9.90 592	9	39	
17	9.86 736	26	0.13 264	9.90 583	9	38	
	9.86 762	26	0.13 237	9.90 574	9	37	
17	9.86 788	26	0.13 211	9.90 564	9	36	
17	9.86 815	26	0.13 185	9.90 555	9	35	
17	9.86 841	26	0.13 158	9.90 546	9	34	
17	9.86 868	26	0.13 132	9.90 536	9	33	17 17 16
	9.86 894	26	0.13 105	9.90 527	9	32	6 1.7 1.7 1.6
17	9.86 921	26	0.13 079	9.90 518	9	31	7 2.0 2.0 1.9
17	9.86 947	26	0.13 052	9.90 508	9	30	8 2.3 2.2 2.2
17	9.86 973	26	0.13 026	9.90 499	9	29	9 2.6 2.5 2.5
17	9.87 000	26	0.13 000	9.90 490	9	28	10 2.9 2.8 2.7
	9.87 026	26	0.12 973	9.90 480	9	27	20 5.8 5.6 5.5
17	9.87 053	26	0.12 947	9.90 471	9	26	30 8.7 8.5 8.2
17	9.87 079	26	0.12 920	9.90 461	9	25	40 11.6 11.3 11.0
17	9.87 105	26	0.12 894	9.90 452	9	24	50 14.6 14.1 13.7
17	9.87 132	26	0.12 868	9.90 443	9	23	
	9.87 158	26	0.12 841	9.90 433	9	22	
17	9.87 185	26	0.12 815	9.90 424	9	21	
17	9.87 211	26	0.12 789	9.90 414	9	20	
17	9.87 237	26	0.12 762	9.90 405	9	19	
17	9.87 264	26	0.12 736	9.90 396	9	18	
	9.87 290	26	0.12 709	9.90 386	9	17	
16	9.87 316	26	0.12 683	9.90 377	9	16	9 9
17	9.87 343	26	0.12 657	9.90 367	9	15	6 0.9 0.9
17	9.87 369	26	0.12 631	9.90 358	9	14	7 1.1 1.0
17	9.87 395	26	0.12 604	9.90 348	9	13	8 1.2 1.2
	9.87 422	26	0.12 578	9.90 339	9	12	9 1.4 1.3
16	9.87 448	26	0.12 551	9.90 330	9	11	10 1.6 1.5
17	9.87 474	26	0.12 525	9.90 320	9	10	20 3.1 3.0
17	9.87 501	26	0.12 499	9.90 311	9	9	30 4.7 4.5
16	9.87 527	26	0.12 472	9.90 301	9	8	40 6.3 6.0
17	9.87 553	26	0.12 446	9.90 292	9	7	50 7.9 7.5
17	9.87 580	26	0.12 420	9.90 282	9	6	
16	9.87 606	26	0.12 393	9.90 273	9	5	
17	9.87 632	26	0.12 367	9.90 263	9	4	
17	9.87 659	26	0.12 341	9.90 254	9	3	
16	9.87 685	26	0.12 315	9.90 244	9	2	
17	9.87 711	26	0.12 288	9.90 235	9	1	
						0	
d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9.77 946	15	9.87 711	26	0.12 289	9.90 235	9	60	
1	9.77 963	17	9.87 737	26	0.12 263	9.90 225	9	59	
2	9.77 980	16	9.87 764	26	0.12 236	9.90 216	9	58	
3	9.77 996	17	9.87 790	26	0.12 209	9.90 206	10	57	
4	9.78 013	15	9.87 816	26	0.12 183	9.90 196	9	56	
5	9.78 030	16	9.87 843	26	0.12 157	9.90 187	9	55	
6	9.78 046	17	9.87 869	26	0.12 131	9.90 177	9	54	
7	9.78 063	16	9.87 895	26	0.12 104	9.90 168	9	53	
8	9.78 080	17	9.87 921	26	0.12 078	9.90 158	9	52	
9	9.78 097	16	9.87 948	26	0.12 052	9.90 149	9	51	
10	9.78 113	16	9.87 974	26	0.12 026	9.90 139	9	50	
11	9.78 130	17	9.88 000	26	0.11 999	9.90 130	10	49	
12	9.78 147	16	9.88 026	26	0.11 973	9.90 120	9	48	
13	9.78 163	16	9.88 053	26	0.11 947	9.90 110	9	47	
14	9.78 180	16	9.88 079	26	0.11 921	9.90 101	9	46	
15	9.78 196	16	9.88 105	26	0.11 895	9.90 091	9	45	
16	9.78 213	17	9.88 131	26	0.11 868	9.90 082	10	44	
17	9.78 230	16	9.88 157	26	0.11 842	9.90 072	9	43	
18	9.78 246	16	9.88 184	26	0.11 816	9.90 062	10	42	
19	9.78 263	16	9.88 210	26	0.11 790	9.90 053	9	41	
20	9.78 279	16	9.88 236	26	0.11 763	9.90 043	9	40	
21	9.78 296	16	9.88 262	26	0.11 737	9.90 034	10	39	
22	9.78 312	16	9.88 288	26	0.11 711	9.90 024	9	38	
23	9.78 329	17	9.88 315	26	0.11 685	9.90 014	10	37	
24	9.78 346	16	9.88 341	26	0.11 659	9.90 004	9	36	
25	9.78 362	16	9.88 367	26	0.11 633	9.89 995	9	35	
26	9.78 379	16	9.88 393	26	0.11 606	9.89 985	10	34	
27	9.78 395	16	9.88 419	26	0.11 580	9.89 975	9	33	
28	9.78 412	16	9.88 445	26	0.11 554	9.89 966	9	32	
29	9.78 428	16	9.88 472	26	0.11 528	9.89 956	10	31	
30	9.78 444	15	9.88 498	26	0.11 502	9.89 946	9	30	
31	9.78 461	16	9.88 524	26	0.11 476	9.89 937	10	29	
32	9.78 477	16	9.88 550	26	0.11 449	9.89 927	9	28	
33	9.78 494	16	9.88 576	26	0.11 423	9.89 917	10	27	
34	9.78 510	16	9.88 602	26	0.11 397	9.89 908	9	26	
35	9.78 527	16	9.88 629	26	0.11 371	9.89 898	10	25	
36	9.78 543	16	9.88 655	26	0.11 345	9.89 888	9	24	
37	9.78 559	16	9.88 681	26	0.11 319	9.89 878	10	23	
38	9.78 576	16	9.88 707	26	0.11 293	9.89 869	9	22	
39	9.78 592	16	9.88 733	26	0.11 266	9.89 858	10	21	
40	9.78 609	16	9.88 759	26	0.11 240	9.89 849	9	20	
41	9.78 625	16	9.88 785	26	0.11 214	9.89 839	10	19	
42	9.78 641	16	9.88 811	26	0.11 188	9.89 830	9	18	
43	9.78 658	16	9.88 838	26	0.11 162	9.89 820	10	17	
44	9.78 674	16	9.88 864	26	0.11 136	9.89 810	9	16	
45	9.78 690	16	9.88 890	26	0.11 110	9.89 800	10	15	
46	9.78 707	16	9.88 916	26	0.11 084	9.89 791	9	14	
47	9.78 723	16	9.88 942	26	0.11 058	9.89 781	10	13	
48	9.78 739	16	9.88 968	26	0.11 032	9.89 771	10	12	
49	9.78 755	16	9.88 994	26	0.11 005	9.89 761	9	11	
50	9.78 772	15	9.89 020	26	0.10 979	9.89 751	10	10	
51	9.78 788	16	9.89 046	26	0.10 953	9.89 742	9	9	
52	9.78 804	16	9.89 072	26	0.10 927	9.89 732	10	8	
53	9.78 821	16	9.89 098	26	0.10 901	9.89 722	10	7	
54	9.78 837	16	9.89 124	26	0.10 875	9.89 712	9	6	
55	9.78 853	16	9.89 150	26	0.10 849	9.89 702	10	5	
56	9.78 869	16	9.89 177	26	0.10 823	9.89 692	10	4	
57	9.78 885	16	9.89 203	26	0.10 797	9.89 683	9	3	
58	9.78 902	16	9.89 229	26	0.10 771	9.89 673	10	2	
59	9.78 918	16	9.89 255	26	0.10 745	9.89 663	10	1	
60	9.78 934	16	9.89 281	26	0.10 719	9.89 653	10	0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

Min.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	d.		P. P.
34	16	9.89 281	26	0.10 719	9.89 653	9	60	
50	16	9.89 307	26	0.10 693	9.89 643	10	59	
66	16	9.89 333	26	0.10 667	9.89 633	10	58	
82	16	9.89 359	26	0.10 641	9.89 623	10	57	
99	16	9.89 385	26	0.10 615	9.89 613	9	56	
115	16	9.89 411	26	0.10 589	9.89 604	9	55	
131	16	9.89 437	26	0.10 563	9.89 594	10	54	
147	16	9.89 463	26	0.10 537	9.89 584	10	53	
163	16	9.89 489	26	0.10 511	9.89 574	10	52	
179	16	9.89 515	26	0.10 485	9.89 564	10	51	26 25½
195	16	9.89 541	26	0.10 459	9.89 554	9	50	6 2.6 2.5
211	16	9.89 567	26	0.10 433	9.89 544	10	49	7 3.0 3.0
227	16	9.89 593	26	0.10 407	9.89 534	10	48	8 3.4 3.4
243	16	9.89 619	26	0.10 381	9.89 524	10	47	9 3.9 3.8
259	16	9.89 645	26	0.10 355	9.89 514	10	46	10 4.3 4.2
275	16	9.89 671	26	0.10 329	9.89 504	10	45	20 8.6 8.5
291	16	9.89 697	26	0.10 303	9.89 494	10	44	30 13.0 12.7
307	16	9.89 723	26	0.10 277	9.89 484	10	43	40 17.3 17.0
323	16	9.89 749	26	0.10 251	9.89 474	10	42	50 21.6 21.2
339	16	9.89 775	26	0.10 225	9.89 464	10	41	
355	16	9.89 801	26	0.10 199	9.89 454	10	40	
371	16	9.89 827	26	0.10 173	9.89 444	10	39	
387	16	9.89 853	26	0.10 147	9.89 434	10	38	
403	16	9.89 879	26	0.10 121	9.89 424	10	37	
419	16	9.89 905	26	0.10 095	9.89 414	10	36	
435	16	9.89 931	26	0.10 069	9.89 404	10	35	
451	16	9.89 957	26	0.10 043	9.89 394	10	34	16 16 15
467	16	9.89 982	25	0.10 017	9.89 384	10	33	6 1.6 1.6 1.5
483	15	9.90 008	26	0.09 991	9.89 374	10	32	7 1.9 1.8 1.8
499	16	9.90 034	26	0.09 965	9.89 364	10	31	8 2.2 2.1 2.0
515	16	9.90 060	26	0.09 939	9.89 354	10	30	9 2.5 2.4 2.3
531	16	9.90 086	26	0.09 913	9.89 344	10	29	10 2.7 2.6 2.6
546	15	9.90 112	26	0.09 887	9.89 334	10	28	20 5.5 5.3 5.1
562	16	9.90 138	26	0.09 861	9.89 324	10	27	30 8.2 8.0 7.7
578	16	9.90 164	25	0.09 836	9.89 314	10	26	40 11.0 10.6 10.3
594	15	9.90 190	26	0.09 810	9.89 304	10	25	50 13.7 13.3 12.9
610	16	9.90 216	26	0.09 784	9.89 294	10	24	
626	16	9.90 242	26	0.09 758	9.89 284	10	23	
641	15	9.90 268	26	0.09 732	9.89 274	10	22	
657	16	9.90 294	26	0.09 706	9.89 264	10	21	
673	16	9.90 319	25	0.09 680	9.89 253	10	20	
689	15	9.90 345	26	0.09 654	9.89 243	10	19	
705	16	9.90 371	26	0.09 628	9.89 233	10	18	
720	15	9.90 397	26	0.09 602	9.89 223	10	17	
736	16	9.90 423	25	0.09 577	9.89 213	10	16	10 10 9
752	15	9.90 449	26	0.09 551	9.89 203	10	15	6 1.0 1.0 0.9
768	16	9.90 475	26	0.09 525	9.89 193	10	14	7 1.2 1.1 1.1
783	15	9.90 501	26	0.09 499	9.89 182	10	13	8 1.4 1.3 1.2
799	16	9.90 526	25	0.09 473	9.89 172	10	12	9 1.6 1.5 1.4
815	16	9.90 552	26	0.09 447	9.89 162	10	11	10 1.7 1.6 1.6
830	15	9.90 578	26	0.09 421	9.89 152	10	10	20 3.5 3.3 3.1
846	16	9.90 604	26	0.09 395	9.89 142	10	9	30 5.2 5.0 4.7
862	16	9.90 630	25	0.09 370	9.89 132	10	8	40 7.0 6.6 6.3
877	15	9.90 656	26	0.09 344	9.89 121	10	7	50 8.7 8.3 7.9
893	16	9.90 682	26	0.09 318	9.89 111	10	6	
909	16	9.90 707	25	0.09 292	9.89 101	10	5	
924	15	9.90 733	26	0.09 266	9.89 091	10	4	
940	16	9.90 759	26	0.09 240	9.89 081	10	3	
956	16	9.90 785	26	0.09 214	9.89 070	10	2	
971	15	9.90 811	25	0.09 189	9.89 060	10	1	
987	15	9.90 837	26	0.09 163	9.89 050	10	0	
Min.	d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	d.	′	P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		
0	9·79 887	16	9·90 837	26	0·09 163	9·89 050	10	60	
1	9·79 903	15	9·90 863	25	0·09 137	9·89 040	10	59	
2	9·79 918	15	9·90 888	26	0·09 111	9·89 030	10	58	
3	9·79 934	15	9·90 914	25	0·09 085	9·89 019	10	57	
4	9·79 949	15	9·90 940	26	0·09 060	9·89 009	10	56	
5	9·79 965	15	9·90 966	26	0·09 034	9·88 999	10	55	
6	9·79 980	15	9·90 992	25	0·09 008	9·88 989	10	54	
7	9·79 996	15	9·91 017	26	0·08 982	9·88 978	10	53	
8	9·80 011	15	9·91 043	26	0·08 956	9·88 968	10	52	
9	9·80 027	15	9·91 069	25	0·08 930	9·88 958	10	51	
10	9·80 042	15	9·91 095	26	0·08 905	9·88 947	10	50	
11	9·80 058	15	9·91 121	25	0·08 879	9·88 937	10	49	
12	9·80 073	15	9·91 146	26	0·08 853	9·88 927	10	48	
13	9·80 089	15	9·91 172	25	0·08 827	9·88 917	10	47	
14	9·80 104	15	9·91 198	26	0·08 802	9·88 906	10	46	
15	9·80 120	15	9·91 224	26	0·08 776	9·88 896	10	45	
16	9·80 135	15	9·91 250	25	0·08 750	9·88 886	10	44	
17	9·80 151	15	9·91 275	26	0·08 724	9·88 875	10	43	
18	9·80 166	15	9·91 301	25	0·08 698	9·88 865	10	42	
19	9·80 182	15	9·91 327	26	0·08 673	9·88 855	10	41	
20	9·80 197	15	9·91 353	25	0·08 647	9·88 844	10	40	
21	9·80 213	15	9·91 378	26	0·08 621	9·88 834	10	39	
22	9·80 228	15	9·91 404	25	0·08 595	9·88 823	10	38	
23	9·80 243	15	9·91 430	26	0·08 570	9·88 813	10	37	
24	9·80 259	15	9·91 456	25	0·08 544	9·88 803	10	36	
25	9·80 274	15	9·91 481	26	0·08 519	9·88 792	10	35	
26	9·80 289	15	9·91 507	25	0·08 493	9·88 782	10	34	
27	9·80 305	15	9·91 533	26	0·08 467	9·88 772	10	33	
28	9·80 320	15	9·91 559	25	0·08 441	9·88 761	10	32	
29	9·80 335	15	9·91 584	26	0·08 415	9·88 751	10	31	
30	9·80 351	15	9·91 610	25	0·08 389	9·88 740	10	30	
31	9·80 366	15	9·91 636	26	0·08 364	9·88 730	10	29	
32	9·80 381	15	9·91 662	25	0·08 338	9·88 720	10	28	
33	9·80 397	15	9·91 687	26	0·08 313	9·88 709	10	27	
34	9·80 412	15	9·91 713	26	0·08 286	9·88 699	10	26	
35	9·80 427	15	9·91 739	25	0·08 261	9·88 688	10	25	
36	9·80 443	15	9·91 765	26	0·08 235	9·88 678	10	24	
37	9·80 458	15	9·91 790	25	0·08 209	9·88 667	10	23	
38	9·80 473	15	9·91 816	26	0·08 183	9·88 657	10	22	
39	9·80 488	15	9·91 842	25	0·08 158	9·88 646	10	21	
40	9·80 504	15	9·91 867	25	0·08 133	9·88 636	10	20	
41	9·80 519	15	9·91 893	26	0·08 106	9·88 625	10	19	
42	9·80 534	15	9·91 919	25	0·08 081	9·88 615	10	18	
43	9·80 549	15	9·91 945	26	0·08 055	9·88 604	10	17	
44	9·80 564	15	9·91 970	25	0·08 020	9·88 594	10	16	
45	9·80 580	15	9·91 996	25	0·08 004	9·88 583	10	15	
46	9·80 595	15	9·92 022	26	0·07 978	9·88 573	10	14	
47	9·80 610	15	9·92 047	25	0·07 952	9·88 562	10	13	
48	9·80 625	15	9·92 073	26	0·07 926	9·88 552	10	12	
49	9·80 640	15	9·92 099	25	0·07 901	9·88 541	10	11	
50	9·80 655	15	9·92 124	25	0·07 875	9·88 531	10	10	
51	9·80 671	15	9·92 150	26	0·07 849	9·88 520	10	9	
52	9·80 686	15	9·92 176	25	0·07 824	9·88 510	10	8	
53	9·80 701	15	9·92 201	26	0·07 798	9·88 499	10	7	
54	9·80 716	15	9·92 227	25	0·07 775	9·88 488	10	6	
55	9·80 731	15	9·92 253	25	0·07 747	9·88 478	10	5	
56	9·80 746	15	9·92 278	26	0·07 721	9·88 467	11	4	
57	9·80 761	15	9·92 304	25	0·07 695	9·88 457	10	3	
58	9·80 776	15	9·92 330	25	0·07 670	9·88 446	10	2	
59	9·80 791	15	9·92 355	26	0·07 644	9·88 436	10	1	
60	9·80 806	15	9·92 381	26	0·07 619	9·88 425	10	0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	

Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
06	15	9.92 381	25	0.07 618	9.88 425	10	60	
22	15	9.92 407	25	0.07 593	9.88 415	11	59	
37	15	9.92 432	26	0.07 567	9.88 404	10	58	
52	15	9.92 458	25	0.07 541	9.88 393	10	57	
67	15	9.92 484	25	0.07 516	9.88 383	10	56	
82	15	9.92 509	25	0.07 490	9.88 372	11	55	
97	15	9.92 535	26	0.07 465	9.88 361	10	54	
12	15	9.92 561	25	0.07 439	9.88 351	10	53	
27	15	9.92 586	25	0.07 413	9.88 340	11	52	
42	15	9.92 612	26	0.07 388	9.88 329	10	51	26 25
57	15	9.92 638	25	0.07 362	9.88 319	10	50	6 2.6 2.5
72	15	9.92 663	25	0.07 336	9.88 308	11	49	7 3.0 3.0
87	15	9.92 689	25	0.07 311	9.88 297	10	48	8 3.4 3.4
01	14	9.92 714	26	0.07 285	9.88 287	10	47	9 3.9 3.8
16	15	9.92 740	25	0.07 259	9.88 276	11	46	10 4.3 4.2
31	15	9.92 766	25	0.07 234	9.88 265	10	45	20 8.6 8.5
46	15	9.92 791	25	0.07 208	9.88 255	10	44	30 13.0 12.7
61	15	9.92 817	25	0.07 183	9.88 244	11	43	40 17.3 17.0
76	14	9.92 842	25	0.07 157	9.88 233	10	42	50 21.6 21.2
91		9.92 838		0.07 131	9.88 223		41	
06	15	9.92 894	25	0.07 106	9.88 212	11	40	
21	15	9.92 919	25	0.07 080	9.88 201	10	39	
36	15	9.92 945	26	0.07 055	9.88 190	11	38	
50	14	9.92 971	25	0.07 029	9.88 180	10	37	
65	15	9.92 996	25	0.07 003	9.88 169	11	36	
80	15	9.93 022	25	0.06 978	9.88 158	10	35	
95	14	9.93 047	25	0.06 952	9.88 147	11	34	15 15 14
10	15	9.93 073	25	0.06 927	9.88 137	10	33	6 1.5 1.5 1.4
25	15	9.93 098	25	0.06 901	9.88 126	11	32	7 1.8 1.7 1.7
39	14	9.93 124	26	0.06 875	9.88 115	10	31	8 2.0 2.0 1.9
54	15	9.93 150	25	0.06 850	9.88 104	11	30	9 2.3 2.2 2.2
69	14	9.93 175	25	0.06 824	9.88 094	10	29	10 2.6 2.5 2.4
84	15	9.93 201	26	0.06 799	9.88 083	11	28	20 5.1 5.0 4.8
99	15	9.93 226	25	0.06 773	9.88 072	11	27	30 7.7 7.5 7.2
13	14	9.93 252	25	0.06 748	9.88 061	10	26	40 10.3 10.0 9.6
28	15	9.93 278	26	0.06 722	9.88 050	11	25	50 12.9 12.5 12.1
43	14	9.93 303	25	0.06 696	9.88 039	11	24	
58	15	9.93 329	26	0.06 671	9.88 029	10	23	
72	14	9.93 354	25	0.06 645	9.88 018	11	22	
87	14	9.93 380	25	0.06 620	9.88 007	11	21	
02	15	9.93 405	25	0.06 594	9.87 996	10	20	
16	14	9.93 431	26	0.06 569	9.87 985	11	19	
31	15	9.93 456	25	0.06 543	9.87 974	11	18	
46	14	9.93 482	25	0.06 518	9.87 963	11	17	
60	14	9.93 508	26	0.06 492	9.87 953	10	16	11 10
75	15	9.93 533	25	0.06 466	9.87 942	11	15	6 1.1 1.0
90	14	9.93 559	25	0.06 441	9.87 931	11	14	7 1.3 1.2
04	14	9.93 584	25	0.06 415	9.87 920	11	13	8 1.4 1.4
19	15	9.93 610	25	0.06 390	9.87 909	10	12	9 1.6 1.6
34	14	9.93 635	25	0.06 364	9.87 898	11	11	10 1.8 1.7
48	14	9.93 661	25	0.06 339	9.87 887	11	10	20 3.6 3.5
63	14	9.93 686	25	0.06 313	9.87 876	11	9	30 5.5 5.2
78	15	9.93 712	25	0.06 288	9.87 865	11	8	40 7.3 7.0
93	14	9.93 737	25	0.06 262	9.87 854	10	7	50 9.1 8.7
07	14	9.93 763	25	0.06 237	9.87 844	11	6	
21	14	9.93 788	25	0.06 211	9.87 833	11	5	
36	14	9.93 814	26	0.06 186	9.87 822	11	4	
50	14	9.93 840	25	0.06 160	9.87 811	11	3	
65	14	9.93 865	25	0.06 134	9.87 800	11	2	
80	15	9.93 891	25	0.06 109	9.87 789	11	1	
94	14	9.93 916	25	0.06 083	9.87 778	11	0	
os.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS AND COTANGENTS.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9·81 694		9·93 916		0·06 083	9·87 778		60	
1	9·81 709	15	9·93 942	26	0·06 058	9·87 767	11	59	
2	9·81 723	14	9·93 967	25	0·06 032	9·87 756	11	58	
3	9·81 738	15	9·93 993	26	0·06 007	9·87 745	11	57	
4	9·81 752	14	9·94 018	25	0·05 981	9·87 734	11	56	
5	9·81 767	15	9·94 044	26	0·05 956	9·87 723	11	55	
6	9·81 781	14	9·94 069	25	0·05 930	9·87 712	11	54	
7	9·81 796	15	9·94 095	26	0·05 905	9·87 701	11	53	
8	9·81 810	14	9·94 120	25	0·05 879	9·87 690	11	52	
9	9·81 824	14	9·94 146	26	0·05 854	9·87 679	11	51	
10	9·81 839	15	9·94 171	25	0·05 828	9·87 668	11	50	
11	9·81 853	14	9·94 197	26	0·05 803	9·87 657	11	49	
12	9·81 868	15	9·94 222	25	0·05 777	9·87 645	11	48	
13	9·81 882	14	9·94 248	26	0·05 752	9·87 634	11	47	
14	9·81 897	15	9·94 273	25	0·05 726	9·87 623	11	46	
15	9·81 911	14	9·94 299	26	0·05 701	9·87 612	11	45	
16	9·81 926	15	9·94 324	25	0·05 675	9·87 601	11	44	
17	9·81 940	14	9·94 350	26	0·05 650	9·87 590	11	43	
18	9·81 954	14	9·94 375	25	0·05 625	9·87 579	11	42	
19	9·81 969	15	9·94 400	25	0·05 599	9·87 568	11	41	
20	9·81 983	14	9·94 426	26	0·05 574	9·87 557	11	40	
21	9·81 997	14	9·94 451	25	0·05 548	9·87 546	11	39	
22	9·82 012	15	9·94 477	26	0·05 523	9·87 535	11	38	
23	9·82 026	14	9·94 502	25	0·05 497	9·87 523	11	37	
24	9·82 040	14	9·94 528	26	0·05 472	9·87 512	11	36	
25	9·82 055	15	9·94 553	25	0·05 446	9·87 501	11	35	
26	9·82 069	14	9·94 579	26	0·05 421	9·87 490	11	34	
27	9·82 083	14	9·94 604	25	0·05 395	9·87 479	11	33	
28	9·82 098	15	9·94 630	26	0·05 370	9·87 468	11	32	
29	9·82 112	14	9·94 655	25	0·05 344	9·87 457	11	31	
30	9·82 126	14	9·94 681	26	0·05 319	9·87 445	11	30	
31	9·82 140	14	9·94 706	25	0·05 293	9·87 434	11	29	
32	9·82 155	15	9·94 732	26	0·05 268	9·87 423	11	28	
33	9·82 169	14	9·94 757	25	0·05 243	9·87 412	11	27	
34	9·82 183	14	9·94 782	25	0·05 217	9·87 401	11	26	
35	9·82 197	14	9·94 808	26	0·05 192	9·87 389	11	25	
36	9·82 212	15	9·94 833	25	0·05 166	9·87 378	11	24	
37	9·82 226	14	9·94 859	26	0·05 141	9·87 367	11	23	
38	9·82 240	14	9·94 884	25	0·05 115	9·87 356	11	22	
39	9·82 254	14	9·94 910	26	0·05 090	9·87 345	11	21	
40	9·82 269	15	9·94 935	25	0·05 064	9·87 333	11	20	
41	9·82 283	14	9·94 961	26	0·05 039	9·87 322	11	19	
42	9·82 297	14	9·94 986	25	0·05 014	9·87 311	11	18	
43	9·82 311	14	9·95 011	25	0·04 988	9·87 300	11	17	
44	9·82 325	14	9·95 037	26	0·04 963	9·87 288	11	16	
45	9·82 339	14	9·95 062	25	0·04 937	9·87 277	11	15	
46	9·82 354	15	9·95 088	26	0·04 912	9·87 266	11	14	
47	9·82 368	14	9·95 113	25	0·04 886	9·87 254	11	13	
48	9·82 382	14	9·95 139	26	0·04 861	9·87 243	11	12	
49	9·82 396	14	9·95 164	25	0·04 836	9·87 232	11	11	
50	9·82 410	14	9·95 189	25	0·04 810	9·87 221	11	10	
51	9·82 424	14	9·95 215	26	0·04 785	9·87 209	12	9	
52	9·82 438	14	9·95 240	25	0·04 759	9·87 198	11	8	
53	9·82 452	14	9·95 266	26	0·04 734	9·87 187	11	7	
54	9·82 467	15	9·95 291	25	0·04 709	9·87 175	12	6	
55	9·82 481	14	9·95 316	25	0·04 683	9·87 164	11	5	
56	9·82 495	14	9·95 342	26	0·04 658	9·87 153	11	4	
57	9·82 509	14	9·95 367	25	0·04 632	9·87 141	12	3	
58	9·82 523	14	9·95 393	26	0·04 607	9·87 130	11	2	
59	9·82 537	14	9·95 418	25	0·04 581	9·87 118	12	1	
60	9·82 551	14	9·95 443	25	0·04 556	9·87 107	11	0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′	P. P.
14	9.95 443	25	0.04 556	9.87 107	11	60	
14	9.95 469	25	0.04 531	9.87 096	11	59	
14	9.95 494	25	0.04 505	9.87 084	11	58	
14	9.95 520	25	0.04 480	9.87 073	11	57	
14	9.95 545	25	0.04 454	9.87 062	11	56	
14	9.95 571	25	0.04 429	9.87 050	11	55	
14	9.95 596	25	0.04 404	9.87 039	11	54	
14	9.95 621	25	0.04 378	9.87 027	11	53	
14	9.95 647	25	0.04 353	9.87 016	11	52	
14	9.95 672	25	0.04 327	9.87 004	11	51	25 25
14	9.95 697	25	0.04 302	9.86 993	11	50	6 2.5 2.5
14	9.95 723	25	0.04 277	9.86 982	11	49	7 3.0 2.9
14	9.95 748	25	0.04 251	9.86 970	11	48	8 3.4 3.3
14	9.95 774	25	0.04 226	9.86 959	11	47	9 3.8 3.7
13	9.95 799	25	0.04 200	9.86 947	11	46	10 4.2 4.1
14	9 95 824	25	0.04 175	9.86 936	11	45	20 8.5 8.3
14	9.95 850	25	0.04 150	9.86 924	11	44	30 12.7 12.5
14	9.95 875	25	0.04 124	9.86 913	11	43	40 17.0 16.6
14	9.95 901	25	0.04 099	9.86 901	11	42	50 21.2 20.8
13	9.95 926	25	0.04 074	9.86 890	11	41	
14	9.95 951	25	0.04 048	9.86 878	11	40	
14	9.95 977	25	0.04 023	9.86 867	11	39	
14	9.96 002	25	0.03 997	9.86 855	11	38	
13	9.96 027	25	0.03 972	9.86 844	11	37	
14	9.96 053	25	0.03 947	9.86 832	11	36	
14	9.96 078	25	0.03 921	9.86 821	11	35	
13	9.96 104	25	0.03 896	9.86 809	11	34	14 13
14	9.96 129	25	0.03 871	9.86 798	11	33	6 1.4 1.3
13	9.96 154	25	0.03 845	9.86 786	12	32	7 1.6 1.6
14	9.98 180	25	0.03 820	9.86 774	11	31	8 1.8 1.8
14	9.96 205	25	0.03 795	9.86 763	11	30	9 2.1 2.0
13	9.96 230	25	0.03 769	9.86 751	11	29	10 2.3 2.2
14	9.96 256	25	0.03 744	9.86 740	11	28	20 4.6 4.5
13	9.96 281	25	0.03 718	9.86 728	12	27	30 7.0 6.7
14	9.96 306	25	0.03 693	9.86 716	12	26	40 9.3 9.0
13	9.96 332	25	0.03 668	9.86 705	11	25	50 11.6 11.2
14	9.96 357	25	0.03 642	9.86 693	11	24	
13	9.96 383	25	0.03 617	9.86 682	11	23	
13	9.96 408	25	0.03 592	9.86 670	12	22	
13	9.96 433	25	0.03 566	9.86 658	12	21	
14	9.96 459	25	0.03 541	9.86 647	11	20	
13	9.96 484	25	0.03 516	9.86 635	12	19	
14	9.96 509	25	0.03 490	9.86 623	12	18	
13	9.96 535	25	0.03 465	9.86 612	11	17	
13	9.96 560	25	0.03 440	9.86 600	12	16	12 11 11
13	9.96 585	25	0.03 414	9.86 588	12	15	6 1.2 1.1 1.1
14	9.96 611	25	0.03 389	9.86 577	11	14	7 1.4 1.3 1.3
13	9.96 636	25	0.03 364	9.86 565	11	13	8 1.6 1.5 1.4
13	9.96 661	25	0.03 338	9.86 553	12	12	9 1.8 1.7 1.6
14	9.96 687	25	0.03 313	9.86 542	11	11	10 2.0 1.9 1.8
13	9.96 712	25	0.03 287	9.86 530	12	10	20 4.0 3.8 3.6
13	9.96 737	25	0.03 262	9.86 518	11	9	30 6.0 5.7 5.5
13	9.96 763	25	0.03 237	9.86 507	11	8	40 8.0 7.6 7.3
14	9.96 788	25	0.03 211	9.86 495	12	7	50 10.0 9.6 9.1
13	9.96 813	25	0.03 186	9.86 483	11	6	
13	9.96 839	25	0.03 161	9.86 471	12	5	
13	9.96 864	25	0.03 135	9.86 460	11	4	
13	9.96 889	25	0.03 110	9.86 448	12	3	
13	9.96 915	25	0.03 085	9.86 436	11	2	
14	9.96 940	25	0.03 059	9.86 424	12	1	
13	9.96 965	25	0.03 034	9.86 413	12	0	
d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS AND COTANGENTS.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9.83 378	13	9.96 965	25	0.03 034	9.86 412	11	60	
1	9.83 392	13	9.96 991	25	0.03 009	9.86 401	12	59	
2	9.83 405	13	9.97 016	25	0.02 984	9.86 389	11	58	
3	9.83 419	13	9.97 041	25	0.02 958	9.86 377	12	57	
4	9.83 432	13	9.97 067	25	0.02 933	9.86 365	11	56	
5	9.83 446	13	9.97 092	25	0.02 908	9.86 354	12	55	
6	9.83 459	13	9.97 117	25	0.02 882	9.86 342	12	54	
7	9.83 473	13	9.97 143	25	0.02 857	9.86 330	11	53	
8	9.83 486	13	9.97 168	25	0.02 832	9.86 318	12	52	
9	9.83 500	13	9.97 193	25	0.02 806	9.86 306	12	51	25 13
10	9.83 513	13	9.97 219	25	0.02 781	9.86 294	12	50	5 2.1 1.3
11	9.83 527	13	9.97 244	25	0.02 756	9.86 282	12	49	7 2.9 1.8
12	9.83 540	13	9.97 269	25	0.02 730	9.86 271	11	48	8 3.3 2.1
13	9.83 554	13	9.97 295	25	0.02 705	9.86 259	12	47	9 3.8 2.4
14	9.83 567	13	9.97 320	25	0.02 680	9.86 247	12	46	10 4.2 2.7
15	9.83 580	13	9.97 345	25	0.02 654	9.86 235	12	45	20 8.3 5.3
16	9.83 594	13	9.97 370	25	0.02 629	9.86 223	12	44	30 12.5 7.9
17	9.83 607	13	9.97 396	25	0.02 604	9.86 211	12	43	40 17.0 10.6
18	9.83 621	13	9.97 421	25	0.02 578	9.86 199	12	42	50 21.2 13.2
19	9.83 634	13	9.97 446	25	0.02 553	9.86 187	12	41	
20	9.83 647	13	9.97 472	25	0.02 528	9.86 176	12	40	
21	9.83 661	13	9.97 497	25	0.02 502	9.86 164	12	39	
22	9.83 674	13	9.97 522	25	0.02 477	9.86 152	12	38	
23	9.83 688	13	9.97 548	25	0.02 452	9.86 140	12	37	
24	9.83 701	13	9.97 573	25	0.02 427	9.86 128	12	36	
25	9.83 714	13	9.97 598	25	0.02 401	9.86 116	12	35	
26	9.83 728	13	9.97 624	25	0.02 376	9.86 104	12	34	13
27	9.83 741	13	9.97 649	25	0.02 351	9.86 092	12	33	6 1.3
28	9.83 754	13	9.97 674	25	0.02 325	9.86 080	12	32	7 1.5
29	9.83 768	13	9.97 699	25	0.02 300	9.86 068	12	31	8 1.8
30	9.83 781	13	9.97 725	25	0.02 275	9.86 056	12	30	9 2.0
31	9.83 794	13	9.97 750	25	0.02 249	9.86 044	12	29	10 2.2
32	9.83 808	13	9.97 775	25	0.02 224	9.86 032	12	28	20 4.3
33	9.83 821	13	9.97 801	25	0.02 199	9.86 020	12	27	30 6.7
34	9.83 834	13	9.97 826	25	0.02 174	9.86 008	12	26	40 8.9
35	9.83 847	13	9.97 851	25	0.02 148	9.85 996	12	25	50 11.2
36	9.83 861	13	9.97 877	25	0.02 123	9.85 984	12	24	
37	9.83 874	13	9.97 902	25	0.02 098	9.85 972	12	23	
38	9.83 887	13	9.97 927	25	0.02 072	9.85 960	12	22	
39	9.83 900	13	9.97 952	25	0.02 047	9.85 948	12	21	
40	9.83 914	13	9.97 978	25	0.02 022	9.85 936	12	20	
41	9.83 927	13	9.98 003	25	0.01 996	9.85 924	12	19	
42	9.83 940	13	9.98 028	25	0.01 971	9.85 912	12	18	
43	9.83 953	13	9.98 054	25	0.01 946	9.85 900	12	17	
44	9.83 967	13	9.98 079	25	0.01 921	9.85 887	12	16	13 12
45	9.83 980	13	9.98 104	25	0.01 895	9.85 875	12	15	6 1.3 1.2
46	9.83 993	13	9.98 129	25	0.01 870	9.85 863	12	14	7 1.5 1.4
47	9.84 006	13	9.98 155	25	0.01 845	9.85 851	12	13	8 1.7 1.6
48	9.84 019	13	9.98 180	25	0.01 819	9.85 839	12	12	9 1.9 1.8
49	9.84 033	13	9.98 205	25	0.01 794	9.85 827	12	11	10 2.1 2.0
50	9.84 046	13	9.98 231	25	0.01 769	9.85 815	12	10	20 4.3 4.0
51	9.84 059	13	9.98 256	25	0.01 744	9.85 803	12	9	30 6.5 6.0
52	9.84 072	13	9.98 281	25	0.01 718	9.85 791	12	8	40 8.8 8.0
53	9.84 085	13	9.98 306	25	0.01 693	9.85 778	12	7	50 10.9 10.0
54	9.84 098	13	9.98 332	25	0.01 668	9.85 766	12	6	
55	9.84 111	13	9.98 357	25	0.01 642	9.85 754	12	5	
56	9.84 124	13	9.98 382	25	0.01 617	9.85 742	12	4	
57	9.84 138	13	9.98 408	25	0.01 592	9.85 730	12	3	
58	9.84 151	13	9.98 433	25	0.01 567	9.85 718	12	2	
59	9.84 164	13	9.98 458	25	0.01 541	9.85 705	12	1	
60	9.84 177	13	9.98 483	25	0.01 516	9.85 693	12	0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

′.	Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	14 177	13	9·98 483	25	0·01 516	9·85 693	12	60	
1	14 190	13	9·98 509	25	0·01 491	9·85 681	12	59	
2	14 203	13	9·98 534	25	0·01 465	9·85 669	12	58	
3	14 216	13	9·98 559	25	0·01 440	9·85 657	12	57	
4	14 229		9·98 585		0·01 415	9·85 644		56	
5	14 242	13	9·98 610	25	0·01 390	9·85 632	12	55	
6	14 255	13	9·98 635	25	0·01 364	9·85 620	12	54	
7	14 268	13	9·98 660	25	0·01 339	9·85 608	12	53	
8	14 281	13	9·98 686	25	0·01 314	9·85 595	12	52	25 25
9	14 294		9·98 711		0·01 289	9·85 583		51	6 2·5 2·5
10	14 307	13	9·98 736	25	0·01 263	9·85 571	12	50	7 3·0 2·9
11	14 320	13	9·98 762	25	0·01 238	9·85 559	12	49	8 3·4 3·3
12	14 333	13	9·98 787	25	0·01 213	9·85 546	12	48	9 3·8 3·7
13	14 346	13	9·98 812	25	0·01 187	9·85 534	12	47	10 4·2 4·1
14	14 359		9·98 837		0·01 162	9·85 522		46	20 8·5 8·3
15	14 372	13	9·98 863	25	0·01 137	9·85 509	12	45	30 12·7 12·5
16	14 385	13	9·98 888	25	0·01 112	9·85 497	12	44	40 17·0 16·6
17	14 398	13	9·98 913	25	0·01 086	9·85 485	12	43	50 21·2 20·8
18	14 411	13	9·98 938	25	0·01 061	9·85 472	12	42	
19	14 424	12	9·98 964	25	0·01 036	9·85 460	12	41	
20	14 437	13	9·98 989	25	0·01 010	9·85 448	12	40	
21	14 450	13	9·99 014	25	0·00 985	9·85 435	12	39	
22	14 463	13	9·99 040	25	0·00 960	9·85 423	12	38	
23	14 476	13	9·99 065	25	0·00 935	9·85 411	12	37	
24	14 489		9·99 090		0·00 909	9·85 398		36	
25	14 502	13	9·99 115	25	0·00 884	9·85 386	12	35	
26	14 514	12	9·99 141	25	0·00 859	9·85 374	12	34	13 13
27	14 527	13	9·99 166	25	0·00 834	9·85 361	12	33	6 1·3 1·3
28	14 540	13	9·99 191	25	0·00 808	9·85 349	12	32	7 1·6 1·5
29	14 553	13	9·99 216	25	0·00 783	9·85 336	12	31	8 1·8 1·7
30	14 566	12	9·99 242	25	0·00 758	9·85 324	12	30	9 2·0 1·8
31	14 579	13	9·99 267	25	0·00 733	9·85 312	12	29	10 2·2 2·1
32	14 592	13	9·99 292	25	0·00 707	9·85 299	12	28	20 4·5 4·3
33	14 604	12	9·99 318	25	0·00 682	9·85 287	12	27	30 6·7 6·5
34	14 617	13	9·99 343	25	0·00 657	9·85 274	12	26	40 9·0 8·6
35	14 630	13	9·99 368	25	0·00 631	9·85 262	12	25	50 11·2 10·8
36	14 643	13	9·99 393	25	0·00 606	9·85 249	12	24	
37	14 656	13	9·99 419	25	0·00 581	9·85 237	12	23	
38	14 669	13	9·99 444	25	0·00 556	9·85 224	12	22	
39	14 681	12	9·99 469	25	0·00 530	9·85 212	12	21	
40	14 694	13	9·99 494	25	0·00 505	9·85 199	12	20	
41	14 707	12	9·99 520	25	0·00 480	9·85 187	12	19	
42	14 720	13	9·99 545	25	0·00 455	9·85 174	12	18	
43	14 732	12	9·99 570	25	0·00 429	9·85 162	12	17	
44	14 745	13	9·99 595	25	0·00 404	9·85 149	12	16	12 12
45	14 758	12	9·99 621	25	0·00 379	9·85 137	12	15	6 1·2 1·2
46	14 771	13	9·99 646	25	0·00 353	9·85 124	12	14	7 1·4 1·4
47	14 783	12	9·99 671	25	0·00 328	9·85 112	12	13	8 1·6 1·6
48	14 796	13	9·99 697	25	0·00 303	9·85 099	12	12	9 1·9 1·8
49	14 809	12	9·99 722	25	0·00 278	9·85 087	12	11	10 2·1 2·0
50	14 822	13	9·99 747	25	0·00 252	9·85 074	12	10	20 4·1 4·0
51	14 834	12	9·99 772	25	0·00 227	9·85 062	12	9	30 6·2 6·0
52	14 847	13	9·99 798	25	0·00 202	9·85 049	12	8	40 8·3 8·0
53	14 860	12	9·99 823	25	0·00 177	9·85 037	12	7	50 10·4 10·0
54	14 872	12	9·99 848	25	0·00 151	9·85 024	13	6	
55	14 885	13	9·99 873	25	0·00 126	9·85 011	12	5	
56	14 898	13	9·99 899	25	0·00 101	9·84 999	12	4	
57	14 910	12	9·99 924	25	0·00 076	9·84 986	12	3	
58	14 923	13	9·99 949	25	0·00 050	9·84 974	13	2	
59	14 936	13	9·99 974	25	0·00 025	9·84 961	13	1	
60	14 948	12	0·00 000	25	0·00 000	9·84 948	12	0	
′.	Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

′	0° Log. Vers.	D	Log. Exsec.	D	1° Log. Vers.	D	Log. Exsec.	D
0	− ∞		− ∞		6·18271		6·18278	
1	2·62642	60206	2·62642	60206	·19707	1435	·19714	1436
2	3·22848	35218	3·22848	35218	·21142	1412	·21150	1413
3	3·58066	24987	3·58066	24987	·22509	1389	·22516	1388
4	3·83054	19382	3·83054	19382	·23877	1368	·23884	1368
5	4·02436	15836	4·02436	15836	6·25223	1346	6·25231	1347
6	·18272	13389	·18272	13389	·26549	1326	·26557	1326
7	·31662	11598	·31662	11598	·27856	1307	·27864	1307
8	·43260	10230	·43260	10230	·29142	1286	·29151	1287
9	·53490	9151	·53491	9151	·30410	1268	·30418	1267
10	4·62642	8279	4·62642	8279	6·31660	1250	6·31669	1251
11	·70920	7557	·70921	7557	·32892	1232	·32901	1232
12	·78478	6952	·78478	6952	·34107	1214	·34116	1215
13	·85431	6437	·85431	6437	·35305	1198	·35316	1200
14	·91868	5993	·91868	5993	·36487	1182	·36497	1181
15	4·97860	5605	4·97861	5605	6·37653	1166	6·37663	1166
16	5·03466	5266	5·03466	5266	·38803	1150	·38814	1151
17	·08732	4964	·08732	4964	·39938	1135	·39949	1135
18	·13696	4696	·13697	4696	·41059	1121	·41070	1121
19	·18393	4455	·18393	4456	·42165	1106	·42177	1107
20	5·22848	4238	5·22849	4238	6·43258	1093	6·43270	1093
21	·27088	4040	·27087	4040	·44337	1079	·44349	1079
22	·31126	3861	·31127	3861	·45403	1066	·45415	1066
23	·34987	3697	·34988	3697	·46455	1052	·46468	1053
24	·38684	3545	·38685	3545	·47496	1040	·47509	1041
25	5·42230	3406	5·42231	3407	6·48524	1028	6·48537	1028
26	·45636	3278	·45638	3278	·49539	1015	·49553	1016
27	·48915	3158	·48916	3159	·50544	1004	·50557	1004
28	·52073	3048	·52075	3048	·51536	992	·51550	993
29	·55121	2944	·55123	2945	·52518	981	·52532	982
30	5·58066	2848	5·58068	2848	6·53488	970	6·53503	971
31	·60914	2757	·60916	2758	·54448	960	·54463	960
32	·63672	2672	·63674	2672	·55397	949	·55413	950
33	·66344	2593	·66346	2593	·56338	939	·56352	939
34	·68937	2518	·68940	2517	·57265	929	·57281	929
35	5·71455	2447	5·71457	2447	6·58184	919	6·58201	920
36	·73902	2379	·73904	2380	·59093	909	·59110	909
37	·76282	2316	·76284	2316	·59993	900	·60011	901
38	·78598	2256	·78601	2256	·60884	891	·60902	891
39	·80854	2199	·80857	2199	·61766	882	·61784	882
40	5·83053	2145	5·83056	2145	6·62639	873	6·62657	873
41	·85198	2093	·85201	2093	·63503	864	·63522	865
42	·87291	2044	·87295	2043	·64359	856	·64378	856
43	·89335	1996	·89338	1997	·65206	847	·65228	850
44	·91332	1952	·91335	1952	·66045	839	·66065	837
45	5·93284	1909	5·93288	1909	6·66876	831	6·66897	832
46	·95193	1868	·95197	1868	·67700	823	·67720	823
47	·97061	1829	·97065	1829	·68515	815	·68536	816
48	5·98890	1790	5·98894	1791	·69323	808	·69345	809
49	6·00680	1755	6·00686	1755	·70124	800	·70145	800
50	6·02435	1720	6·02440	1720	6·70917	793	6·70938	793
51	·04155	1687	·04160	1687	·71703	786	·71725	787
52	·05842	1654	·05847	1654	·72482	779	·72505	780
53	·07496	1623	·07501	1623	·73254	772	·73277	772
54	·09120	1594	·09125	1594	·74019	765	·74043	766
55	6·10714	1565	6·10719	1565	6·74777	758	6·74802	759
56	·12279	1537	·12284	1537	·75529	752	·75554	752
57	·13816	1511	·13822	1511	·76275	746	·76300	746
58	·15327	1484	·15333	1485	·77014	739	·77040	740
59	·16811	1460	·16818	1460	·77747	733	·77773	733
60	6·18271		6·18278		6·78474	726	6·78500	727
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D

2°				3°			
D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
721	6·78500	721	7·13687	481	7·13746	481	0
714	·79221	715	·14168	478	·14228	479	1
709	·79937	709	·14646	475	·14707	476	2
703	·80646	703	·15122	473	·15183	474	3
697	·81350	698	·15595	470	·15657	471	4
692	6·82048	692	7·16066	468	7·16129	469	5
686	·82740	687	·16534	466	·16598	466	6
681	·83427	682	·17000	463	·17064	464	7
676	·84109	678	·17463	460	·17528	461	8
670	·84785	671	·17923	458	·17989	459	9
665	6·85457	666	7·18382	455	7·18448	456	10
660	·86123	660	·18837	453	·18905	454	11
655	·86783	656	·19291	451	·19359	452	12
650	·87439	651	·19742	448	·19811	449	13
646	·88090	646	·20191	446	·20260	447	14
641	6·88737	641	7·20637	444	7·20707	445	15
636	·89378	636	·21081	442	·21152	442	16
631	·90015	632	·21523	440	·21595	440	17
627	·90647	628	·21963	437	·22035	438	18
622	·91275	623	·22400	435	·22473	436	19
618	6·91898	618	7·22836	433	7·22909	434	20
613	·92516	614	·23269	431	·23343	431	21
609	·93131	610	·23700	429	·23775	429	22
605	·93741	605	·24129	426	·24204	427	23
601	·94346	601	·24555	424	·24632	425	24
597	6·94948	597	7·24980	422	7·25057	423	25
592	·95545	593	·25402	420	·25480	421	26
589	·96139	589	·25823	418	·25902	419	27
584	·96728	585	·26241	416	·26321	417	28
581	·97313	581	·26658	414	·26738	415	29
577	6·97895	577	7·27072	412	7·27153	413	30
573	·98472	574	·27485	410	·27567	411	31
569	·99046	570	·27895	409	·27978	409	32
565	6·99616	566	·28304	406	·28387	407	33
562	7·00182	563	·28711	405	·28795	405	34
558	7·00745	559	7·29116	402	7·29200	404	35
555	·01304	555	·29518	401	·29604	402	36
551	·01860	552	·29919	399	·30006	400	37
548	·02412	548	·30319	397	·30406	398	38
544	·02960	545	·30716	395	·30804	395	39
541	7·03505	541	7·31112	393	7·31201	394	40
537	·04047	538	·31505	392	·31595	393	41
534	·04585	535	·31897	390	·31988	391	42
531	·05120	531	·32288	388	·32375	389	43
527	·05652	528	·32676	386	·32768	388	44
525	7·06180	525	7·33063	385	7·33156	385	45
521	·06706	522	·33448	383	·33542	384	46
518	·07228	519	·33831	382	·33926	382	47
515	·07747	516	·34213	380	·34309	380	48
512	·08263	513	·34593	378	·34689	379	49
509	7·08776	509	7·34971	377	7·35069	377	50
505	·09286	507	·35348	375	·35446	376	51
503	·09793	503	·35723	373	·35822	374	52
500	·10297	501	·36097	371	·36196	373	53
497	·10798	498	·36468	370	·36565	371	54
495	7·11297	495	7·36839	368	7·36940	369	55
492	·11792	493	·37207	367	·37310	368	56
489	·12285	490	·37574	366	·37678	366	57
486	·12775	487	·37940	364	·38044	365	58
484	·13262	484	·38304	362	·38405	363	59
	7·13746		7·38667		7·38773		60
D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

′	4° Lg. Vers.	D	Log. Exs.	D	5° Lg. Vers.	D	Log. Exs.	D	′
0	7·38667	361	7·38773	361	7·58039	289	7·58204	290	0
1	·39028	359	·39134	360	·58328	287	·58494	289	1
2	·39387	358	·39495	359	·58615	287	·58783	288	2
3	·39745	356	·39854	357	·58902	286	·59071	287	3
4	·40102	355	·40211	356	·59188	285	·59358	286	4
5	7·40457	353	7·40567	354	7·59473	284	7·59645	285	5
6	·40810	352	·40922	353	·59758	283	·59930	284	6
7	·41163	350	·41275	352	·60041	282	·60214	283	7
8	·41513	349	·41627	350	·60323	281	·60498	282	8
9	·41863	348	·41977	349	·60604	280	·60780	281	9
10	7·42211	346	7·42326	347	7·60885	279	7·61062	280	10
11	·42557	345	·42673	346	·61164	279	·61342	280	11
12	·42903	343	·43019	345	·61443	277	·61622	278	12
13	·43246	342	·43364	343	·61721	277	·61901	278	13
14	·43589	341	·43708	342	·61998	276	·62176	277	14
15	7·43930	339	7·44050	340	7·62274	275	7·62456	276	15
16	·44270	338	·44390	339	·62549	274	·62733	275	16
17	·44608	337	·44730	338	·62823	273	·63008	274	17
18	·44946	335	·45068	337	·63096	272	·63282	274	18
19	·45281	334	·45405	335	·63369	272	·63556	273	19
20	7·45616	333	7·45740	334	7·63641	270	7·63829	272	20
21	·45949	332	·46075	332	·63911	270	·64101	271	21
22	·46281	330	·46407	332	·64181	269	·64372	270	22
23	·46612	329	·46739	330	·64451	268	·64643	269	23
24	·46941	328	·47070	329	·64719	267	·64912	269	24
25	7·47270	327	7·47399	328	7·64986	266	7·65181	268	25
26	·47597	325	·47727	327	·65253	266	·65449	267	26
27	·47922	324	·48054	325	·65519	265	·65716	266	27
28	·48247	323	·48379	324	·65784	264	·65982	265	28
29	·48570	322	·48703	323	·66048	263	·66247	264	29
30	7·48892	321	7·49026	322	7·66311	263	7·66512	264	30
31	·49213	320	·49346	321	·66574	261	·66776	263	31
32	·49533	318	·49666	319	·66836	261	·67039	262	32
33	·49852	317	·49985	318	·67097	260	·67301	261	33
34	·50169	316	·50307	317	·67357	259	·67562	261	34
35	7·50485	315	7·50624	317	7·67617	258	7·67823	260	35
36	·50800	314	·50941	315	·67875	258	·68083	259	36
37	·51114	313	·51256	313	·68133	257	·68342	258	37
38	·51427	311	·51569	313	·68390	256	·68601	257	38
39	·51739	311	·51882	312	·68647	255	·68858	257	39
40	7·52050	309	7·52194	312	7·68902	255	7·69115	256	40
41	·52359	308	·52504	310	·69157	254	·69371	255	41
42	·52667	307	·52814	309	·69411	253	·69627	254	42
43	·52975	306	·53122	308	·69665	252	·69881	254	43
44	·53281	305	·53429	307	·69917	252	·70135	253	44
45	7·53586	304	7·53735	306	7·70169	251	7·70388	252	45
46	·53890	303	·54041	305	·70421	250	·70641	252	46
47	·54193	302	·54345	304	·70671	250	·70893	251	47
48	·54495	300	·54648	303	·70921	249	·71144	250	48
49	·54796	300	·54950	302	·71170	249	·71394	250	49
50	7·55096	299	7·55251	301	7·71418	248	7·71644	248	50
51	·55395	297	·55550	299	·71666	247	·71892	248	51
52	·55692	297	·55849	299	·71913	246	·72141	247	52
53	·55989	295	·56147	298	·72159	246	·72388	246	53
54	·56285	295	·56444	296	·72404	245	·72635	246	54
55	7·56580	293	7·56740	296	7·72649	244	7·72881	245	55
56	·56873	293	·57035	295	·72893	243	·73126	245	56
57	·57166	292	·57329	294	·73137	242	·73371	244	57
58	·57458	290	·57621	292	·73379	242	·73615	243	58
59	·57749	290	·57913	292	·73621	241	·73859	242	59
60	7·58039		7·58204		7·73863		7·74101		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.			
	360	350	
6	36·0	35·0	
7	42·0	40·8	
8	48·0	46·6	
9	54·0	51·3	
10	60·0	58·1	
20	120·0	116·2	
30	180·0	175·0	
40	240·0	233·1	
50	300·0	291·4	
	330	320	
6	33·0	32·0	
7	38·5	37·3	
8	44·0	42·6	
9	49·5	48·0	
10	55·0	53·3	
20	110·0	106·6	
30	165·0	160·0	
40	220·0	213·3	
50	275·0	266·6	
	300	290	
6	30·0	29·0	
7	35·0	33·8	
8	40·0	38·6	
9	45·0	43·5	
10	50·0	48·3	
20	100·0	96·6	
30	150·0	145·0	
40	200·0	193·3	
50	250·0	241·6	
	270	260	
6	27·0	26·0	
7	31·5	30·3	
8	36·0	34·6	
9	40·5	39·0	
10	45·0	43·3	
20	90·0	86·6	
30	135·0	130·0	
40	180·0	173·3	
50	225·0	216·6	
	240	230	
6	24·0	23·0	
7	28·0	26·8	
8	32·0	30·6	
9	36·0	34·5	
10	40·0	38·3	
20	80·0	76·6	
30	120·0	115·0	
40	160·0	153·3	
50	200·0	191·6	
	210	200	
6	21·0	20·0	
7	24·5	23·3	
8	28·0	26·6	
9	31·5	30·0	
10	35·0	33·3	
20	70·0	66·6	
30	105·0	100·0	
40	140·0	133·3	
50	175·0	166·6	

	6°				7°			
Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
73863	241	7·7410$\bar{1}$	242	7·87238	206	7·87563	208	0
74104	240	·7434$\bar{3}$	241	·87444	205	·87771	207	1
74344	239	·74585	241	·87650	205	·87978	207	2
74583	239	·74826	240	·87855	204	·88185	206	3
74822		·75066		·88060	204	·88391		4
75060	238	7·75305	239	7·88264	204	7·88597	206	5
75297	237	·75544	239	·88468	203	·88803	205	6
75534	236	·75782	238	·88672	203	·89008	205	7
75770	236	·76019	237	·88875	202	·89212	204	8
76006	235	·76256	237	·89077	202	·89416	204	9
76240	234	7·76492	236	7·89279	202	7·89620	203	10
76475	234	·76728	235	·89481	201	·89823	203	11
76708	233	·76963	235	·89682	201	·90025	202	12
76941	233	·77197	234	·89882	200	·90228	202	13
77173	232	·77431	233	·90082	200	·90429	201	14
77405	232	7·77664	233	7·90282	199	7·90630	201	15
77636	231	·77897	232	·90481	199	·90831	201	16
77867	230	·78128	231	·90680	198	·91032	200	17
78097	230	·78360	231	·90878	198	·91231	199	18
78326	229	·78590	230	·91076	197	·91431	199	19
78554	228	7·78820	230	7·91273	197	7·91630	199	20
78783	228	·79050	229	·91470	197	·91828	198	21
79010	227	·79279	229	·91667	196	·92027	198	22
79237	227	·79507	228	·91863	196	·92224	197	23
79463	226	·79735	228	·92058	195	·92421	197	24
79689	225	7·79962	227	7·92253	195	7·92618	197	25
79914	225	·80188	226	·92448	195	·92815	196	26
80138	224	·80414	226	·92642	194	·93010	195	27
80362	224	·80639	225	·92836	194	·93206	195	28
80586	223	·80864	225	·93029	193	·93401	195	29
80808	222	7·81088	224	7·93222	193	7·93596	195	30
81031	222	·81312	224	·93415	192	·93790	194	31
81252	221	·81535	223	·93607	192	·93984	194	32
81473	221	·81758	222	·93799	191	·94177	193	33
81694	220	·81980	222	·93990	191	·94370	193	34
81914	220	7·82201	221	7·94181	190	7·94562	192	35
82133	219	·82422	221	·94371	190	·94754	192	36
82352	219	·82642	220	·94561	190	·94946	192	37
82570	218	·82862	220	·94751	189	·95137	191	38
82788	217	·83081	219	·94940	189	·95328	191	39
83005	217	7·83300	219	7·95129	189	7·95519	190	40
83222	217	·83518	218	·95317	188	·95709	190	41
83438	216	·83735	217	·95505	188	·95898	189	42
83653	215	·83952	217	·95693	188	·96088	189	43
83868	215	·84169	216	·95880	187	·96276	188	44
84083	214	7·84385	216	7·96066	186	7·96465	188	45
84297	214	·84600	215	·96253	186	·96653	188	46
84510	213	·84815	215	·96439	186	·96841	188	47
84723	213	·85030	214	·96624	185	·97028	187	48
84935	212	·85243	213	·96809	185	·97215	187	49
85147	212	7·85457	213	7·96994	184	7·97401	186	50
85359	211	·85670	213	·97178	184	·97587	186	51
85570	211	·85882	212	·97362	184	·97773	185	52
85780	210	·86094	211	·97546	183	·97958	185	53
85990	210	·86305	211	·97729	183	·98143	184	54
86199	209	7·86516	211	7·97912	183	7·98327	184	55
86408	209	·86726	210	·98094	182	·98512	184	56
86616	208	·86936	210	·98276	182	·98695	183	57
86824	208	·87146	209	·98458	182	·98879	183	58
87031	207	·87354	208	·98639	181	·99062	183	59
87238	206	7·87563	208	7·98820	181	7·99244	182	60
Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.			
	180	9	9
6	18·0	0·9	0·9
7	21·0	1·1	1·0
8	24·0	1·2	1·2
9	27·0	1·4	1·3
10	30·0	1·6	1·5
20	60·0	3·1	3·0
30	90·0	4·7	4·5
40	120·0	6·3	6·0
50	150·0	7·9	7·5
	8	8	7
6	0·8	0·8	0·7
7	1·0	0·9	0·9
8	1·1	1·0	1·0
9	1·3	1·2	1·1
10	1·4	1·3	1·2
20	2·8	2·6	2·5
30	4·2	4·0	3·7
40	5·6	5·3	5·0
50	7·1	6·6	6·2
	7	6	6
6	0·7	0·6	0·6
7	0·8	0·7	0·7
8	0·9	0·8	0·8
9	1·0	1·0	0·9
10	1·1	1·1	1·0
20	2·3	2·1	2·0
30	3·5	3·2	3·0
40	4·6	4·3	4·0
50	5·8	5·4	5·0
	5	5	4	4
6	0·5	0·5	0·4	0·4
7	0·6	0·6	0·5	0·4
8	0·7	0·6	0·6	0·5
9	0·8	0·7	0·7	0·6
10	0·9	0·8	0·7	0·6
20	1·8	1·6	1·5	1·3
30	2·7	2·5	2·2	2·0
40	3·6	3·3	3·0	2·6
50	4·6	4·1	3·7	3·3
	3	3	2	2
6	0·3	0·3	0·2	0·2
7	0·4	0·3	0·3	0·2
8	0·4	0·4	0·3	0·2
9	0·5	0·4	0·4	0·3
10	0·6	0·5	0·4	0·3
20	1·1	1·0	0·8	0·6
30	1·7	1·5	1·2	1·0
40	2·3	2·0	1·6	1·3
50	2·9	2·5	2·1	1·6
	1	1	0
6	0·1	0·1	0·0
7	0·2	0·1	0·0
8	0·2	0·1	0·0
9	0·2	0·1	0·1
10	0·2	0·1	0·1
20	0·5	0·3	0·1
30	0·7	0·5	0·2
40	1·0	0·6	0·3
50	1·2	0·8	0·4

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

	8°				9°					
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.
0	7.98820	180	7.99244	183	8.09031	180	8.09569	165	0	180 129 16
1	.99000	180	.99427	182	.09192	160	.09732	162	1	6 18.0 12.9 1.6
2	.99180	179	.99609	181	.09352	160	.09894	162	2	7 21.0 15.1 1.8
3	.99360	179	.99790	181	.09512	160	.10056	161	3	8 24.0 17.2 2.1
4	.99539	179	7.99971	181	.09671	159	.10217	161	4	9 27.0 19.3 2.4
5	7.99718	179	8.00152	180	8.09830	159	8.10378	161	5	10 30.0 21.5 2.7
6	7.99897	178	.00332	180	.09989	159	.10539	161	6	20 60.0 43.0 5.3
7	8.00075	178	.00512	180	.10148	158	.10700	160	7	30 90.0 64.5 8.0
8	.00253	177	.00692	180	.10306	158	.10860	160	8	40 120.0 86.0 10.7
9	.00431	178	.00871	179	.10464	158	.11020	160	9	50 150.0 107.5 13.3
10	8.00608	177	8.01050	179	8.10622	157	8.11180	160	10	
11	.00784	176	.01229	179	.10779	157	.11340	159	11	150 140
12	.00961	176	.01407	178	.10936	157	.11499	159	12	6 15.0 14.0
13	.01137	176	.01585	178	.11093	157	.11658	158	13	7 17.5 16.3
14	.01313	176	.01763	177	.11250	156	.11816	158	14	8 20.0 18.6
15	8.01488	175	8.01940	177	8.11406	156	8.11975	158	15	9 22.5 21.0
16	.01663	175	.02117	177	.11562	156	.12133	158	16	10 25.0 23.3
17	.01838	174	.02293	176	.11718	155	.12291	157	17	20 50.0 46.6
18	.02012	174	.02469	176	.11873	155	.12448	157	18	30 75.0 70.0
19	.02186	174	.02645	175	.12028	155	.12605	157	19	40 100.0 93.3
20	8.02359	173	8.02820	175	8.12184	155	8.12763	157	20	50 125.0 116.6
21	.02533	173	.02995	175	.12338	154	.12919	156	21	
22	.02706	173	.03170	174	.12492	154	.13075	156	22	9 8 7
23	.02878	172	.03345	174	.12647	153	.13232	157	23	6 0.9 0.8 0.7
24	.03050	172	.03519	174	.12800	153	.13387	155	24	7 1.1 1.0 0.9
25	8.03222	172	8.03692	173	8.12954	153	8.13543	156	25	8 1.2 1.1 1.0
26	.03394	171	.03866	173	.13107	153	.13698	155	26	9 1.4 1.3 1.2
27	.03565	171	.04039	173	.13260	153	.13854	156	27	10 1.6 1.4 1.3
28	.03736	171	.04212	173	.13413	153	.14008	154	28	20 3.1 2.8 2.6
29	.03906	170	.04384	172	.13565	152	.14163	154	29	30 4.7 4.3 3.9
30	8.04076	170	8.04556	172	8.13717	152	8.14317	154	30	40 6.3 5.7 5.2
31	.04246	170	.04728	171	.13869	152	.14471	154	31	50 7.9 7.2 6.5
32	.04416	169	.04899	171	.14021	151	.14625	153	32	
33	.04585	169	.05070	171	.14173	151	.14778	153	33	8 7 7
34	.04754	169	.05241	170	.14323	151	.14932	153	34	6 0.8 0.7 0.7
35	8.04922	168	8.05411	170	8.14474	151	8.15085	153	35	7 0.9 0.9 0.8
36	.05090	168	.05581	170	.14625	150	.15237	152	36	8 1.0 1.0 0.9
37	.05258	168	.05751	170	.14775	150	.15390	153	37	9 1.2 1.1 1.0
38	.05426	167	.05921	169	.14925	150	.15542	152	38	10 1.3 1.2 1.1
39	.05593	167	.06090	169	.15075	149	.15694	152	39	20 2.6 2.5 2.3
40	8.05760	167	8.06259	169	8.15225	150	8.15846	152	40	30 4.0 3.7 3.4
41	.05926	166	.06427	168	.15374	149	.15997	151	41	40 5.3 5.0 4.6
42	.06093	166	.06595	168	.15523	149	.16148	151	42	50 6.7 6.2 5.7
43	.06259	166	.06763	168	.15672	149	.16299	151	43	
44	.06424	165	.06931	167	.15820	148	.16450	150	44	6 5
45	8.06589	165	8.07098	167	8.15968	148	8.16600	150	45	6 0.6 0.5
46	.06754	165	.07265	167	.16116	148	.16750	150	46	7 0.7 0.6
47	.06919	164	.07431	166	.16264	147	.16900	149	47	8 0.8 0.7
48	.07083	164	.07598	166	.16412	147	.17050	149	48	9 1.0 0.9
49	.07247	164	.07764	166	.16559	147	.17199	149	49	10 1.1 1.0
50	8.07411	164	8.07929	165	8.16706	147	8.17349	149	50	20 2.2 2.0
51	.07575	163	.08095	165	.16852	146	.17497	148	51	30 3.3 3.0
52	.07738	163	.08260	165	.16999	146	.17646	149	52	40 4.4 4.0
53	.07900	162	.08424	164	.17145	146	.17795	148	53	50 5.4 5.0
54	.08063	162	.08589	164	.17291	146	.17943	148	54	
55	8.08225	162	8.08753	164	8.17437	145	8.18091	148	55	5 4
56	.08387	161	.08917	163	.17583	146	.18238	147	56	6 0.5 0.5
57	.08549	162	.09081	164	.17728	145	.18386	147	57	7 0.6 0.6
58	.08710	161	.09244	163	.17873	145	.18533	147	58	8 0.7 0.6
59	.08871	161	.09407	163	.18017	144	.18680	147	59	9 0.8 0.7
60	8.09031	160	8.09565	162	8.18162	145	8.18827	147	60	10 0.9 0.8
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.

ers.	D	10° Log.Exs.	D	Lg. Vers.	D	11° Log.Exs.	D	′
162		8·18827		8·26417		8·27223		0
306	144	·18973	146	·26563	131	·27356	133	1
450	144	·19120	146	·26679	131	·27490	133	2
594	144	·19266	146	·26810	131	·27623	133	3
738	143	·19411	145	·26941	130	·27756	133	4
881	143	8·19557	145	8·27071	130	8·27889	133	5
024	143	·19702	145	·27201	130	·28021	132	6
167	142	·19847	145	·27331	130	·28153	132	7
309	142	·19992	145	·27461	130	·28286	132	8
452	142	·20137	144	·27590	129	·28418	132	9
594	142	8·20281	144	8·27719	129	8·28550	132	10
736	142	·20425	144	·27849	129	·28681	131	11
878	142	·20569	144	·27977	128	·28813	131	12
019	141	·20713	144	·28106	129	·28944	131	13
160	141	·20857	143	·28235	128	·29075	131	14
301	141	8·21000	143	8·28363	128	8·29206	131	15
442	140	·21143	143	·28491	128	·29336	130	16
582	140	·21286	143	·28619	128	·29467	130	17
723	140	·21428	142	·28747	128	·29597	130	18
863	140	·21571	142	·28875	127	·29727	130	19
003	140	8·21713	142	8·29002	127	8·29857	130	20
142	139	·21855	142	·29129	127	·29987	129	21
282	139	·21996	141	·29256	127	·30117	129	22
421	139	·22138	141	·29383	127	·30246	129	23
560	139	·22279	141	·29510	126	·30375	129	24
698	138	8·22420	141	8·29636	126	8·30504	129	25
837	138	·22561	140	·29763	126	·30633	128	26
975	138	·22701	140	·29889	126	·30762	128	27
113	138	·22842	140	·30015	126	·30890	128	28
251	137	·22982	140	·30140	125	·31019	128	29
389	138	8·23122	140	8·30266	125	8·31147	128	30
526	137	·23262	140	·30391	125	·31275	128	31
663	137	·23401	139	·30516	125	·31402	127	32
800	136	·23540	139	·30642	125	·31530	127	33
937	137	·23679	139	·30766	124	·31657	127	34
073	136	8·23818	139	8·30891	124	8·31785	127	35
209	136	·23957	138	·31015	124	·31912	127	36
346	136	·24095	139	·31140	124	·32039	127	37
481	135	·24234	138	·31264	124	·32165	126	38
617	136	·24372	138	·31388	124	·32292	126	39
752	135	8·24509	137	8·31511	123	8·32418	126	40
888	135	·24647	138	·31635	124	·32544	126	41
023	135	·24784	137	·31758	123	·32670	126	42
158	135	·24922	137	·31882	123	·32796	125	43
292	134	·25059	137	·32005	123	·32922	125	44
426	134	8·25195	136	8·32128	123	8·33047	125	45
561	134	·25332	136	·32250	122	·33173	125	46
695	134	·25468	136	·32373	123	·33298	125	47
828	133	·25604	136	·32495	122	·33423	124	48
962	133	·25740	136	·32617	122	·33547	124	49
095	133	8·25876	135	8·32735	122	8·33672	125	50
228	133	·26012	135	·32861	121	·33797	124	51
361	133	·26147	135	·32983	121	·33921	124	52
494	132	·26282	135	·33104	121	·34045	123	53
627	133	·26417	135	·33225	121	·34169	124	54
759	132	8·26552	134	8·33347	121	8·34293	124	55
891	132	·26686	134	·33468	120	·34417	123	56
023	132	·26821	134	·33588	120	·34540	123	57
155	132	·26955	134	·33709	120	·34663	123	58
286	131	·27089	134	·33829	120	·34786	123	59
417	131	8·27223	134	8·33950	120	8·34906	123	60
rs.	D	Log.Exs.	D	Lg. Vers.	D	Log.Exs.	D	′

P. P.

	130	120
6	13·0	12·0
7	15·1	14·0
8	17·3	16·0
9	19·5	18·0
10	21·6	20·0
20	43·3	40·0
30	65·0	60·0
40	86·6	80·0
50	108·3	100·0

	4	4	3
6	0·4	0·4	0·3
7	0·5	0·4	0·4
8	0·6	0·5	0·4
9	0·7	0·6	0·5
10	0·7	0·6	0·6
20	1·5	1·3	1·1
30	2·2	2·0	1·7
40	3·0	2·6	2·3
50	3·7	3·3	2·9

	3	2
6	0·3	0·2
7	0·3	0·3
8	0·4	0·3
9	0·4	0·4
10	0·5	0·4
20	1·0	0·8
30	1·5	1·2
40	2·0	1·6
50	2·5	2·1

	2	1
6	0·2	0·1
7	0·2	0·2
8	0·2	0·2
9	0·3	0·2
10	0·3	0·2
20	0·6	0·5
30	1·0	0·7
40	1·3	1·0
50	1·6	1·2

	1	0
6	0·1	0·0
7	0·1	0·0
8	0·1	0·0
9	0·1	0·1
10	0·1	0·1
20	0·3	0·1
30	0·5	0·2
40	0·6	0·3
50	0·8	0·4

P. P.

′	12° Lg. Vers.	D	Log. Exs.	D	13° Lg. Vers.	D	Log. Exs.	D	′	P. P.
0	8·33950	120	8·34909	123	8·40875	110	8·42002	113	0	120 119 118
1	·34070	120	·35032	122	·40985	110	·42116	113	1	6 12·0 11·9 11·8
2	·34190	119	·35155	122	·41096	110	·42229	113	2	7 14·0 13·9 13·7
3	·34309	120	·35277	122	·41206	110	·42343	113	3	8 16·0 15·8 15·7
4	·34429	119	·35399	122	·41317	110	·42456	113	4	9 18·0 17·8 17·7
5	8·34549	119	8·35522	122	8·41427	110	8·42569	113	5	10 20·0 19·8 19·6
6	·34668	119	·35644	121	·41537	110	·42682	113	6	20 40·0 39·6 39·3
7	·34787	119	·35765	122	·41647	110	·42795	113	7	30 60·0 59·5 59·0
8	·34906	119	·35887	121	·41757	110	·42908	113	8	40 80·0 79·3 78·7
9	·35025	119	·36009	121	·41867	109	·43021	112	9	50 100·0 99·1 98·3
10	8·35143	118	8·36130	121	8·41976	109	8·43133	112	10	117 116 115
11	·35262	118	·36251	121	·42086	109	·43246	112	11	6 11·7 11·6 11·5
12	·35380	118	·36372	120	·42195	109	·43358	112	12	7 13·6 13·5 13·4
13	·35498	118	·36493	121	·42304	109	·43470	112	13	8 15·6 15·4 15·3
14	·35616	117	·36614	120	·42413	109	·43582	112	14	9 17·5 17·3 17·2
15	8·35734	118	8·36734	120	8·42522	108	8·43694	111	15	10 19·5 19·3 19·1
16	·35852	117	·36855	120	·42630	109	·43805	111	16	20 39·0 38·6 38·3
17	·35969	117	·36975	120	·42739	108	·43917	111	17	30 58·5 58·0 57·5
18	·36086	117	·37095	120	·42847	108	·44028	111	18	40 78·0 77·3 76·6
19	·36204	117	·37215	120	·42956	108	·44139	111	19	50 97·5 96·6 95·8
20	8·36321	116	8·37335	119	8·43064	108	8·44251	111	20	114 113 112
21	·36437	116	·37454	119	·43172	108	·44362	111	21	6 11·4 11·3 11·2
22	·36554	117	·37574	119	·43280	108	·44473	110	22	7 13·3 13·2 13·0
23	·36671	116	·37693	119	·43388	107	·44583	110	23	8 15·2 15·0 14·9
24	·36787	116	·37812	119	·43495	108	·44694	110	24	9 17·1 16·9 16·8
25	8·36903	116	8·37931	118	8·43603	107	8·44804	110	25	10 19·0 18·8 18·6
26	·37019	116	·38050	119	·43710	107	·44915	110	26	20 38·0 37·6 37·3
27	·37135	115	·38169	118	·43817	107	·45025	110	27	30 57·0 56·5 56·0
28	·37251	115	·38287	118	·43924	107	·45135	109	28	40 76·0 75·3 74·6
29	·37366	115	·38406	118	·44031	106	·45245	110	29	50 95·0 94·1 93·3
30	8·37482	115	8·38524	118	8·44138	107	8·45355	110	30	111 110 109
31	·37597	115	·38642	118	·44245	106	·45465	109	31	6 11·1 11·0 10·9
32	·37712	115	·38760	118	·44351	106	·45574	109	32	7 12·9 12·8 12·7
33	·37827	115	·38878	117	·44458	106	·45684	109	33	8 14·8 14·6 14·5
34	·37942	114	·38995	117	·44564	106	·45793	109	34	9 16·6 16·5 16·3
35	8·38057	114	8·39113	117	8·44670	106	8·45902	109	35	10 18·5 18·3 18·1
36	·38171	114	·39230	117	·44776	105	·46011	109	36	20 37·0 36·6 36·3
37	·38286	114	·39347	117	·44882	106	·46120	108	37	30 55·5 55·0 54·6
38	·38400	114	·39464	117	·44988	105	·46229	109	38	40 74·0 73·3 72·6
39	·38514	114	·39581	116	·45093	105	·46338	108	39	50 92·5 91·6 90·8
40	8·38628	113	8·39698	116	8·45199	105	8·46446	108	40	108 107 106
41	·38741	114	·39814	116	·45304	105	·46555	108	41	6 10·8 10·7 10·6
42	·38855	113	·39931	116	·45409	105	·46663	108	42	7 12·6 12·5 12·3
43	·38969	113	·40047	116	·45514	105	·46771	108	43	8 14·4 14·2 14·1
44	·39082	113	·40163	116	·45619	105	·46879	108	44	9 16·2 16·0 15·9
45	8·39195	113	8·40279	116	8·45724	104	8·46987	107	45	10 18·0 17·8 17·6
46	·39308	113	·40395	115	·45829	105	·47095	108	46	20 36·0 35·6 35·3
47	·39421	113	·40511	115	·45934	104	·47203	107	47	30 54·0 53·5 53·0
48	·39534	112	·40626	115	·46038	104	·47310	107	48	40 72·0 71·3 70·6
49	·39646	112	·40742	115	·46142	104	·47417	107	49	50 90·0 89·1 88·3
50	8·39758	112	8·40857	115	8·46247	104	8·47525	107	50	105 104 0
51	·39871	112	·40972	115	·46351	104	·47632	107	51	6 10·5 10·4 0·0
52	·39983	112	·41087	115	·46455	103	·47739	107	52	7 12·2 12·1 0·0
53	·40095	112	·41202	114	·46558	104	·47846	106	53	8 14·0 13·8 0·0
54	·40207	111	·41317	115	·46662	103	·47953	107	54	9 15·7 15·6 0·1
55	8·40318	111	8·41431	114	8·46766	103	8·48060	106	55	10 17·5 17·3 0·1
56	·40430	111	·41546	114	·46869	103	·48166	106	56	20 35·0 34·6 0·2
57	·40541	111	·41660	114	·46972	103	·48273	106	57	30 52·5 52·0 0·2
58	·40652	111	·41774	114	·47076	103	·48379	106	58	40 70·0 69·3 0·3
59	·40764	111	·41888	114	·47179	103	·48485	106	59	50 87·5 86·6 0·4
60	8·40875	111	8·42002	114	8·47282	103	8·48591	106	60	
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.

14°				15°					
Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.
8.47282	102	8.48591	106	8.53242	96	8.54748	95	0	
.47384	103	.48697	106	.53338	95	.54847	99	1	
.47487	102	.48803	106	.53434	96	.54946	99	2	
.47590	102	.48909	105	.53530	95	.55045	99	3	103 102 101
.47692	103	.49014	105	.53625	95	.55144	99	4	6 10.3 10.2 10.1
8.47795	102	8.49120	105	8.53721	95	8.55243	99	5	7 12.0 11.9 11.8
.47897	102	.49225	105	.53816	95	.55342	99	6	8 13.7 13.6 13.4
.47999	102	.49331	105	.53911	95	.55441	98	7	9 15.4 15.3 15.1
.48101	102	.49436	105	.54007	95	.55539	98	8	10 17.1 17.0 16.8
.48203	101	.49541	105	.54102	95	.55638	98	9	20 34.3 34.0 33.6
8.48304	101	8.49646	104	8.54197	94	8.55736	98	10	30 51.5 51.0 50.5
.48406	101	.49750	105	.54291	94	.55834	98	11	40 68.6 68.0 67.3
.48507	101	.49855	104	.54386	95	.55933	98	12	50 85.8 85.0 84.1
.48609	101	.49960	104	.54481	94	.56031	98	13	
.48710	101	.50064	104	.54575	94	.56129	98	14	
8.48811	101	8.50168	104	8.54670	94	8.56226	97	15	100 99 98
.48912	101	.50273	104	.54764	94	.56324	98	16	6 10.0 9.9 9.8
.49013	101	.50377	104	.54858	94	.56422	97	17	7 11.6 11.5 11.4
.49114	100	.50481	104	.54952	94	.56519	97	18	8 13.3 13.2 13.0
.49215	101	.50585	104	.55046	94	.56617	97	19	9 15.0 14.8 14.7
8.49315	100	8.50688	103	8.55140	94	8.56714	97	20	10 16.6 16.5 16.3
.49415	100	.50792	104	.55234	93	.56812	97	21	20 33.3 33.0 32.6
.49516	100	.50896	103	.55328	94	.56909	97	22	30 50.0 49.5 49.0
.49616	100	.50999	103	.55421	93	.57006	97	23	40 66.6 66.0 65.3
.49716	100	.51102	103	.55515	93	.57103	97	24	50 83.3 82.5 82.6
8.49816	100	8.51205	103	8.55608	93	8.57200	97	25	
.49916	100	.51309	103	.55701	93	.57296	97	26	97 96 95
.50015	99	.51412	103	.55795	93	.57393	97	27	6 9.7 9.6 9.5
.50115	100	.51514	102	.55888	93	.57490	96	28	7 11.3 11.2 11.1
.50215	99	.51617	103	.55981	93	.57586	97	29	8 12.9 12.8 12.6
8.50314	99	8.51720	103	8.56074	93	8.57682	96	30	9 14.5 14.4 14.2
.50413	99	.51822	102	.56166	92	.57779	96	31	10 16.1 16.0 15.8
.50512	99	.51925	102	.56259	93	.57875	96	32	20 32.3 32.0 31.6
.50611	99	.52027	102	.56352	92	.57971	96	33	30 48.5 48.0 47.5
.50710	98	.52129	102	.56444	92	.58067	95	34	40 64.6 64.0 63.3
8.50809	99	8.52231	102	8.56536	92	8.58163	96	35	50 80.8 80.0 79.1
.50908	98	.52333	102	.56629	92	.58259	95	36	
.51006	98	.52435	101	.56721	92	.58354	96	37	94 93 92
.51105	98	.52537	101	.56813	92	.58450	95	38	6 9.4 9.3 9.2
.51203	98	.52638	101	.56905	92	.58546	95	39	7 10.9 10.8 10.7
8.51301	98	8.52740	101	8.56997	92	8.58641	95	40	8 12.5 12.4 12.2
.51399	98	.52841	101	.57089	91	.58736	95	41	9 14.1 13.9 13.8
.51497	98	.52943	101	.57180	91	.58832	95	42	10 15.6 15.5 15.3
.51595	97	.53044	101	.57272	91	.58927	95	43	20 31.3 31.0 30.6
.51693	98	.53145	101	.57363	91	.59022	95	44	30 47.0 46.5 46.0
8.51791	97	8.53246	101	8.57455	91	8.59117	95	45	40 62.6 62.0 61.3
.51888	97	.53347	101	.57546	91	.59211	94	46	50 78.3 77.5 76.6
.51986	97	.53448	100	.57637	91	.59306	95	47	
.52083	97	.53548	100	.57728	91	.59401	94	48	
.52180	97	.53649	100	.57819	91	.59495	94	49	91 90 0
8.52277	97	8.53749	100	8.57910	91	8.59590	94	50	6 9.1 9.0 0.0
.52374	97	.53850	100	.58001	90	.59684	94	51	7 10.6 10.5 0.0
.52471	97	.53950	100	.58092	91	.59779	94	52	8 12.1 12.0 0.0
.52568	96	.54050	100	.58182	90	.59873	94	53	9 13.6 13.5 0.1
.52665	97	.54150	100	.58273	90	.59967	94	54	10 15.1 15.0 0.1
8.52761	96	8.54250	100	8.58363	90	8.60061	94	55	20 30.3 30.0 0.1
.52858	96	.54350	99	.58453	90	.60155	94	56	30 45.5 45.0 0.2
.52954	96	.54449	100	.58544	90	.60249	93	57	40 60.6 60.0 0.3
.53050	96	.54549	99	.58634	90	.60342	94	58	50 75.8 75.0 0.4
.53146	96	.54649	99	.58724	90	.60436	93	59	
8.53242		8.54748		8.58814		8.60530		60	
Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.

	16°			17°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	8·58814	90	8·60530	93	8·64043	84	8·65984	85	0
1	.58904	89	.60623	93	.64128	84	.66072	88	1
2	.58993	90	.60716	93	.64212	84	.66160	88	2
3	.59083	89	.60810	93	.64296	84	.66248	88	3
4	.59173	89	.60903	93	.64381	84	.66336	88	4
5	8·59262	89	8·60996	93	8·64465	84	8·66425	87	5
6	.59351	89	.61089	93	.64549	84	.66512	88	6
7	.59441	89	.61182	93	.64633	84	.66600	88	7
8	.59530	89	.61275	93	.64717	84	.66688	87	8
9	.59619	89	.61368	93	.64801	84	.63776	87	9
10	8·59708	89	8·61460	92	8·64884	83	8·66863	88	10
11	.59797	89	.61553	92	.64968	84	.66951	87	11
12	.59886	89	.61645	92	.65052	83	.67039	87	12
13	.59974	88	.61738	92	.65135	83	.67126	87	13
14	.60063	89	.61830	92	.65218	83	.67213	87	14
15	8·60152	88	8·61923	92	8·65302	83	8·67301	87	15
16	.60240	88	.62014	92	.65385	83	.67388	87	16
17	.60328	88	.62106	92	.65468	83	.67475	87	17
18	.60417	88	.62198	92	.65551	83	.67562	87	18
19	.60505	88	.62290	92	.65634	83	.67649	87	19
20	8·60593	88	8·62382	91	8·65717	83	8·67736	86	20
21	.60681	88	.62474	92	.65800	83	.67822	87	21
22	.60769	88	.62565	91	.65883	82	.67909	86	22
23	.60857	88	.62657	92	.65965	83	.67996	86	23
24	.60944	87	.62748	91	.66048	82	.68082	86	24
25	8·61032	88	8·62840	91	8·66131	82	8·68169	86	25
26	.61119	87	.62931	91	.66213	82	.68255	86	26
27	.61207	87	.63022	91	.66295	82	.68341	86	27
28	.61294	87	.63113	91	.66378	82	.68428	86	28
29	.61381	87	.63204	90	.66460	82	.68514	86	29
30	8·61469	87	8·63295	91	8·66542	82	8·68600	86	30
31	.61556	87	.63386	91	.66624	82	.68686	85	31
32	.61643	87	.63477	90	.66706	82	.68772	86	32
33	.61730	86	.63567	90	.66788	82	.68858	86	33
34	.61816	87	.63658	90	.66870	81	.68944	85	34
35	8·61903	86	8·63748	90	8·66951	81	8·69029	86	35
36	.61990	86	.63839	90	.67033	82	.69115	85	36
37	.62076	86	.63929	90	.67115	81	.69201	85	37
38	.62163	86	.64019	90	.67196	81	.69286	85	38
39	.62249	86	.64109	90	.67277	81	.69372	85	39
40	8·62336	86	8·64199	90	8·67359	81	8·69457	85	40
41	.62422	86	.64289	90	.67440	81	.69542	85	41
42	.62508	86	.64379	90	.67521	81	.69627	85	42
43	.62594	86	.64469	90	.67602	81	.69712	85	43
44	.62680	86	.64559	89	.67683	81	.69798	85	44
45	8·62766	86	8·64649	90	8·67764	81	8·69883	85	45
46	.62852	86	.64738	89	.67845	80	.69967	84	46
47	.62937	85	.64828	90	.67926	81	.70052	85	47
48	.63023	86	.64917	89	.68007	81	.70137	85	48
49	.63108	85	.65006	89	.68087	80	.70222	84	49
50	8·63194	86	8·65096	89	8·68168	80	8·70306	84	50
51	.63279	85	.65185	89	.68248	80	.70391	85	51
52	.63364	85	.65274	89	.68329	80	.70475	84	52
53	.63449	85	.65363	89	.68409	80	.70560	84	53
54	.63534	85	.65452	89	.68489	80	.70644	84	54
55	8·63619	85	8·65541	89	8·68569	80	8·70728	84	55
56	.63704	85	.65629	88	.68650	80	.70813	84	56
57	.63789	85	.65718	89	.68730	80	.70897	84	57
58	.63874	84	.65807	89	.68810	80	.70981	84	58
59	.63959	85	.65895	88	.68889	79	.71065	84	59
	43	84	8·65984	88	8·68969	80	8·71149	84	60
	′s.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.			
	93	92	91
6	9·3	9·2	9·1
7	10·8	10·7	10·6
8	12·4	12·2	12·1
9	13·9	13·8	13·6
10	15·5	15·3	15·1
20	31·0	30·6	30·3
30	46·5	46·0	45·5
40	62·0	61·3	60·6
50	77·5	76·6	75·8
	90	89	88
6	9·0	8·9	8·8
7	10·5	10·4	10·3
8	12·0	11·8	11·7
9	13·5	13·3	13·2
10	15·0	14·8	14·6
20	30·0	29·6	29·3
30	45·0	44·5	44·0
40	60·0	59·3	58·6
50	75·0	74·1	73·3
	87	86	85
6	8·7	8·6	8·5
7	10·1	10·0	9·9
8	11·6	11·4	11·3
9	13·0	12·9	12·7
10	14·5	14·3	14·2
20	29·0	28·6	28·3
30	43·5	43·0	42·5
40	58·0	57·3	56·7
50	72·5	71·6	70·8
	84	83	82
6	8·4	8·3	8·2
7	9·8	9·7	9·6
8	11·2	11·0	10·9
9	12·6	12·4	12·3
10	14·0	13·8	13·7
20	28·0	27·6	27·3
30	42·0	41·5	41·0
40	56·0	55·3	54·7
50	70·0	69·1	68·3
	81	80	79
6	8·1	8·0	7·9
7	9·4	9·3	9·2
8	10·8	10·6	10·5
9	12·1	12·0	11·8
10	13·5	13·3	13·2
20	27·0	26·6	26·3
30	40·5	40·0	39·5
40	54·0	53·3	52·7
50	67·5	66·6	65·8
	$\overline{0}$
6	0·0
7	0·0
8	0·0
9	0·1
10	0·1
20	0·1
30	0·2
40	0·3
50	0·4

	18°			19°				
ers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
969	79	8·71149	83	8·73625	75	8·76058	79	0
049	79	·71232	84	·73709	75	·76137	80	1
129	80	·71316	84	·73793	76	·76217	79	2
208	79	·71400	83	·73851	76	·76297	79	3
288	79	·71484	84	·73926	75	·76376	79	4
367	79	8·71567	83	8·74001	75	8·76456	80	5
446	79	·71651	83	·74076	75	·76536	79	6
526	79	·71734	83	·74151	75	·76615	79	7
605	79	·71817	83	·74226	75	·76694	79	8
684	79	·71901	83	·74301	75	·76774	79	9
763	79	8·71984	83	8·74376	74	8·76853	79	10
842	79	·72067	83	·74451	75	·76932	79	11
921	79	·72150	83	·74526	74	·77011	79	12
000	78	·72233	83	·74600	74	·77090	79	13
079	79	·72316	83	·74675	74	·77169	79	14
157	78	8·72399	83	8·74749	74	8·77248	79	15
236	78	·72481	82	·74824	74	·77327	79	16
314	78	·72564	83	·74898	74	·77406	79	17
393	78	·72647	82	·74973	74	·77485	78	18
471	78	·72729	82	·75047	74	·77563	78	19
550	78	8·72812	82	8·75121	74	8·77642	78	20
628	78	·72894	82	·75195	74	·77720	78	21
706	78	·72977	82	·75269	74	·77799	78	22
784	78	·73059	82	·75343	74	·77877	78	23
862	78	·73141	82	·75417	74	·77956	78	24
940	78	8·73223	82	8·75491	73	8·78034	78	25
018	78	·73306	82	·75565	74	·78112	78	26
096	77	·73388	82	·75639	73	·78191	78	27
174	78	·73470	82	·75712	73	·78269	78	28
251	77	·73551	81	·75786	73	·78347	78	29
329	77	8·73633	82	8·75860	74	8·78425	78	30
406	77	·73715	82	·75933	73	·78503	78	31
484	77	·73797	81	·76006	73	·78581	78	32
561	77	·73878	81	·76080	73	·78659	77	33
639	77	·73960	81	·76153	73	·78736	78	34
716	77	8·74041	81	8·76226	73	8·78814	78	35
793	77	·74123	81	·76300	73	·78892	77	36
870	77	·74204	81	·76373	73	·78969	77	37
947	77	·74286	81	·76446	73	·79047	77	38
024	77	·74367	81	·76519	73	·79124	77	39
101	77	8·74446	81	8·76592	73	8·79202	77	40
178	76	·74529	81	·76664	72	·79279	77	41
255	77	·74610	81	·76737	73	·79357	77	42
331	76	·74691	81	·76810	73	·79434	77	43
408	77	·74772	80	·76883	73	·79511	77	44
485	76	8·74853	81	8·76955	72	8·79588	77	45
561	76	·74934	81	·77028	73	·79665	77	46
637	76	·75014	80	·77100	72	·79742	77	47
714	76	·75095	80	·77173	73	·79819	77	48
790	76	·75175	80	·77245	72	·79896	77	49
866	76	8·75256	80	8·77317	72	8·79973	76	50
942	76	·75336	80	·77390	73	·80050	77	51
018	76	·75417	80	·77462	72	·80126	76	52
094	76	·75497	80	·77534	72	·80203	77	53
170	76	·75577	80	·77606	72	·80280	76	54
246	76	8·75658	80	8·77678	72	8·80356	76	55
322	76	·75738	80	·77750	72	·80433	76	56
398	75	·75818	80	·77822	72	·80509	76	57
473	76	·75898	80	·77893	71	·80586	76	58
549	75	·75978	80	·77965	72	·80662	76	59
625		8·76058		8·78037	71	8·80738	76	60
ers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.			
	84	83	82
6	8.4	8.3	8.2
7	9.8	9.7	9.5
8	11.2	11.0	10.9
9	12.6	12.4	12.3
10	14.0	13.8	13.6
20	28.0	27.6	27.3
30	42.0	41.5	41.0
40	56.0	55.3	54.6
50	70.0	69.1	68.3

P. P.			
	81	80	79
6	8.1	8.0	7.9
7	9.4	9.3	9.2
8	10.8	10.6	10.5
9	12.1	12.0	11.8
10	13.5	13.3	13.1
20	27.0	26.6	26.3
30	40.5	40.0	39.5
40	54.0	53.3	52.6
50	67.5	66.6	65.8

P. P.			
	78	77	76
6	7.8	7.7	7.6
7	9.1	9.0	8.9
8	10.4	10.2	10.1
9	11.7	11.5	11.4
10	13.0	12.8	12.6
20	26.0	25.6	25.3
30	39.0	38.5	38.0
40	52.0	51.3	50.6
50	65.0	64.1	63.3

P. P.			
	75	74	73
6	7.5	7.4	7.3
7	8.7	8.6	8.5
8	10.0	9.8	9.7
9	11.2	11.1	10.9
10	12.5	12.3	12.1
20	25.0	24.6	24.3
30	37.5	37.0	36.5
40	50.0	49.3	48.6
50	62.5	61.6	60.8

P. P.			
	72	71	0
6	7.2	7.1	0.6
7	8.4	8.3	0.0
8	9.6	9.4	0.0
9	10.8	10.6	0.1
10	12.0	11.8	0.1
20	24.0	23.6	0.2
30	36.0	35.5	0.2
40	48.0	47.3	0.3
50	60.0	59.1	0.4

	20°				21°					
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.
0	8·78037	71	8·80738	76	8·82229	68	8·85214	73	0	
1	·78108	71	·80814	76	·82297	68	·85287	73	1	
2	·78180	71	·80891	76	·82366	68	·85360	72	2	
3	·78251	71	·80967	76	·82434	68	·85433	73	3	
4	·78323	71	·81043	76	·82502	68	·85506	73	4	76 75 74
5	8·78394	71	8·81119	76	8·82569	67	8·85579	73	5	6 7·6 7·5 7·4
6	·78466	71	·81195	76	·82637	68	·85651	72	6	7 8·3 8·7 8·6
7	·78537	71	·81271	76	·82705	68	·85724	73	7	8 10·1 10·0 9·8
8	·78608	71	·81346	75	·82773	67	·85797	73	8	9 11·4 11·2 11·1
9	·78679	71	·81422	76	·82841	68	·85869	72	9	10 12·6 12·5 12·3
10	8·78750	71	8·81498	75	8·82908	67	8·85942	73	10	20 25·3 25·0 24·6
11	·78821	71	·81573	75	·82976	67	·86014	72	11	30 38·0 37·5 37·0
12	·78892	71	·81649	76	·83043	67	·86087	73	12	40 50·6 50·0 49·3
13	·78963	70	·81725	75	·83111	67	·86159	72	13	50 63·3 62·5 61·6
14	·79034		·81800	75	·83178	67	·86231	72	14	
15	8·79105	71	8·81876	75	8·83246	67	8·86304	73	15	73 72 71
16	·79175	70	·81951	75	·83313	67	·86376	72	16	6 7·3 7·2 7·1
17	·79246	71	·82026	75	·83380	67	·86448	72	17	7 8·5 8·4 8·3
18	·79317	70	·82102	75	·83447	67	·86520	72	18	8 9·7 9·6 9·4
19	·79387	70	·82177	75	·83515	67	·86592	72	19	9 10·9 10·8 10·6
20	8·79458	70	8·82252	75	8·83582	67	8·86664	72	20	10 12·1 12·0 11·8
21	·79528	70	·82327	75	·83649	67	·86736	72	21	20 24·3 24·0 23·6
22	·79598	70	·82402	75	·83716	67	·86808	72	22	30 36·5 36·0 35·5
23	·79669	70	·82477	74	·83783	67	·86880	71	23	40 48·6 48·0 47·3
24	·79739	70	·32552		·83850	67	·86952	72	24	50 60·8 60·0 59·1
25	8·79809	70	8·82627	75	8·83916	66	8·87024	72	25	
26	·79879	70	·82702	75	·83983	67	·87095	71	26	70 69 68
27	·79949	70	·82776	74	·84050	66	·87167	72	27	6 7·0 6·9 6·8
28	·80019	70	·82851	75	·84117	67	·87239	72	28	7 8·1 8·0 7·9
29	·80089	70	·82926	74	·84183	66	·87310	71	29	8 9·3 9·2 9·0
30	8·80159	70	8·83000	74	8·84250	66	8·87382	72	30	9 10·5 10·3 10·2
31	·80229	70	·83075	74	·84316	66	·87453	71	31	10 11·6 11·5 11·3
32	·80299	69	·83149	74	·84383	66	·87525	72	32	20 23·3 23·0 22·6
33	·80369	70	·83224	74	·84449	66	·87596	71	33	30 35·0 34·5 34·0
34	·80438	69	·83298	74	·84515	66	·87668	71	34	40 46·6 46·0 45·3
35	8·80508	69	8·83373	74	8·84582	66	8·87739	71	35	50 58·3 57·5 56·6
36	·80577	69	·83447	74	·84648	66	·87810	71	36	
37	·80647	69	·83521	74	·84714	66	·87881	71	37	67 66 65
38	·80716	69	·83595	74	·84780	66	·87953	71	38	6 6·7 6·6 6·5
39	·80786	69	·83670		·84846	66	·88024	71	39	7 7·8 7·7 7·6
40	8·80855	69	8·83744	74	8·84912	66	8·88095	71	40	8 8·9 8·8 8·6
41	·80924	69	·83818	74	·84978	66	·88166	71	41	9 10·0 9·9 9·7
42	·80993	69	·83892	74	·85044	66	·88237	71	42	10 11·1 11·0 10·8
43	·81063	69	·83966	73	·85110	66	·88308	71	43	20 22·3 22·0 21·6
44	·81132	69	·84039	74	·85176	66	·88378	70	44	30 33·5 33·0 32·5
45	8·81201	69	8·84113	74	8·85242	65	8·88449	71	45	40 44·6 44·0 43·3
46	·81270	69	·84187	73	·85308	66	·88520	71	46	50 55·8 55·0 54·1
47	·81339	69	·84261	74	·85373	65	·88591	70	47	
48	·81407	68	·84334	73	·85439	66	·88661	70	48	
49	·81476	69	·84408	73	·85505	66	·88732	71	49	Ō
50	8·81545	69	8·84481	73	8·85570	65	8·88803	70	50	6 0·0
51	·81614	69	·84555	73	·85636	65	·88873	70	51	7 0·0
52	·81683	68	·84628	73	·85701	65	·88944	70	52	8 0·0
53	·81751	68	·84702	73	·85766	65	·89014	70	53	9 0·1
54	·81819	68	·84775	73	·85832	65	·89085	70	54	10 0·1
55	8·81888	69	8·84848	73	8·85897	65	8·89155	70	55	20 0·2
56	·81956	68	·84922	73	·85962	65	·89225	70	56	30 0·3
57	·82025	68	·84995	73	·86027	65	·89295	70	57	40 0·3
58	·82093	68	·85068	73	·86092	65	·89366	70	58	50 0·4
59	·82161	68	·85141	73	·86158	65	·89436	70	59	
	32									
	D		Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.

	22°			23°				
, Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
86223	64	8·89506	70	8·90034	62	8·93631	67	0
86287	65	·89576	70	·90096	62	·93699	67	1
86352	65	·89646	70	·90158	62	·93766	67	2
86417	65	·89716	69	·90220	62	·93833	67	3
86482	64	·89786	70	·90282	62	·93901	67	4
86547	65	8·89856	70	8·90344	62	8·93968	67	5
86612	64	·89926	69	·90406	61	·94035	67	6
86676	64	·89995	70	·90467	62	·94102	67	7
86741	64	·90065	69	·90529	61	·94170	67	8
86805	64	·90135	70	·90591	61	·94237	67	9
86870	64	8·90205	69	8·90652	62	8·94304	67	10
86934	64	·90274	69	·90714	61	·94371	67	11
86999	64	·90344	69	·90776	61	·94438	67	12
87063	64	·90413	69	·90837	61	·94505	67	13
87127	64	·90483	69	·90899	61	·94572	66	14
87192	64	8·90552	69	8·90960	61	8·94638	66	15
87256	64	·90622	69	·91021	61	·94705	67	16
87320	64	·90691	69	·91083	61	·94772	66	17
87384	64	·90760	69	·91144	61	·94839	67	18
87448	64	·90830	69	·91205	61	·94905	66	19
87512	64	8·90899	69	8·91267	61	8·94972	66	20
87576	64	·90968	69	·91328	61	·95039	67	21
87640	64	·91037	69	·91389	61	·95105	66	22
87704	65	·91106	69	·91450	61	·95172	66	23
87768	64	·91175	69	·91511	61	·95238	66	24
87832	63	8·91244	69	8·91572	61	8·95305	66	25
87895	64	·91313	69	·91633	61	·95371	66	26
87959	63	·91382	69	·91694	61	·95437	66	27
88023	63	·91451	69	·91755	60	·95504	66	28
88086	63	·91520	68	·91815	61	·95570	66	29
88150	63	8·91588	69	8·91876	60	8·95636	66	30
88213	63	·91657	68	·91937	60	·95703	66	31
88277	63	·91726	68	·91997	61	·95769	66	32
88340	63	·91794	68	·92058	60	·95835	66	33
88404	63	·91863	69	·92119	60	·95901	66	34
88467	63	8·91932	68	8·92179	60	8·95967	66	35
88530	63	·92000	68	·92240	60	·96033	66	36
88593	63	·92068	68	·92300	60	·96099	66	37
88656	63	·92137	68	·92361	60	·96165	66	38
88720	63	·92205	68	·92421	60	·96231	66	39
88783	63	8·92274	68	8·92487	60	8·96297	66	40
88846	63	·92342	68	·92542	60	·96363	65	41
88909	62	·92410	68	·92602	60	·96428	66	42
88971	63	·92478	68	·92662	60	·96494	65	43
89034		·92546		·92722		·96560	66	44
89097	63	8·92615	68	8·92782	60	8·96625	65	45
89160	62	·92683	68	·92842	60	·96691	65	46
89223	63	·92751	68	·92902	60	·96757	66	47
89285	62	·92819	68	·92962	60	·96822	65	48
89348	63	·92887	68	·93022	60	·96888	65	49
89411	62	8·92955	67	8·93082	60	8·96953	65	50
89473	62	·93022	68	·93142	59	·97018	65	51
89536	62	·93090	67	·93202	60	·97084	65	52
89598	62	·93158	68	·93261	59	·97149	65	53
89660	62	·93226	67	·93321	60	·97214	65	54
89723	63	8·93293	68	8·93381	59	8·97280	65	55
89785	62	·93361	67	·93440	59	·97345	65	56
89847	62	·93429	67	·93500	60	·97410	65	57
89910	62	·93496	67	·93560	59	·97475	65	58
89972	62	·93564	67	·93619	59	·97540	65	59
00034	62	8·93631	67	8·93679	59	8·97606	65	60
, Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.			
	70	69	68
6	7·0	6·9	6·8
7	8·1/6	8·0	7·9
8	9·3	9·2	9·0
9	10·5	10·3	10·2
10	11·6	11·5	11·3
20	23·3	23·0	22·6
30	35·0	34·5	34·0
40	46·6	46·0	45·3
50	58·3	57·5	56·6

P. P.			
	67	66	65
6	6·7	6·6	6·5
7	7·8	7·7	7·6
8	8·9	8·8	8·6
9	10·0	9·9	9·7
10	11·1/6	11·0	10·8
20	22·3	22·0	21·6
30	33·5	33·0	32·5
40	44·6	44·0	43·3
50	55·8	55·0	54·1

P. P.			
	64	63	62
6	6·4	6·3	6·2
7	7·4/6	7·3	7·2
8	8·5	8·4	8·2
9	9·6	9·4	9·3
10	10·6	10·5	10·3
20	21·3	21·0	20·6
30	32·0	31·5	31·0
40	42·6	42·0	41·3
50	53·3	52·5	51·6

P. P.			
	61	60	59
6	6·1	6·0	5·9
7	7·1/6	7·0	6·9
8	8·1/3	8·0	7·8
9	9·1/6	9·0	8·8
10	10·1/6	10·0	9·8
20	20·3	20·0	19·6
30	30·5	30·0	29·5
40	40·6	40·0	39·3
50	50·8	50·0	49·1

P. P.	
	0
6	0·0
7	0·0
8	0·0
9	0·1
10	0·1
20	0·1/3
30	0·2/3
40	0·3
50	0·4

	24°			25°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	8·93679	59	8·97606	65	8·97170	57	9·01448	62	0
1	·93738	59	·97671	65	·97227	56	·01505	63	1
2	·93797	59	·97736	65	·97284	57	·01568	62	2
3	·93857	59	·97801	65	·97341	57	·01631	63	3
4	·93916	59	·97865	64	·97398	57	·01694	63	4
5	8·93975	59	8·97930	65	8·97455	56	9·01758	62	5
6	·94034	59	·97995	65	·97511	57	·01819	63	6
7	·94094	58	·98060	64	·97568	57	·01882	62	7
8	·94153	59	·98125	65	·97625	56	·01944	62	8
9	·94212	59	·98190	64	·97681	56	·02007	63	9
10	8·94271	59	8·98254	64	8·97738	57	9·02070	62	10
11	·94330	59	·98319	64	·97795	56	·02132	62	11
12	·94389	59	·98383	64	·97851	56	·02195	62	12
13	·94448	59	·98448	65	·97908	56	·02257	62	13
14	·94506	58	·98513	64	·97964	56	·02319	62	14
15	8·94565	59	8·98577	64	8·98020	56	9·02382	62	15
16	·94624	59	·98642	64	·98077	56	·02444	62	16
17	·94683	58	·98706	64	·98133	56	·02506	62	17
18	·94742	59	·98770	64	·98190	56	·02569	62	18
19	·94800	58	·98835	64	·98246	56	·02631	62	19
20	8·94859	58	8·98899	64	8·98302	56	9·02693	62	20
21	·94917	58	·98963	64	·98358	56	·02755	62	21
22	·94976	58	·99028	64	·98414	56	·02817	62	22
23	·95034	58	·99092	64	·98470	56	·02880	62	23
24	·95093	58	·99156	64	·98527	56	·02942	62	24
25	8·95151	58	8·99220	64	8·98583	56	9·03004	62	25
26	·95210	58	·99284	64	·98639	56	·03066	62	26
27	·95268	58	·99348	64	·98695	55	·03128	62	27
28	·95326	58	·99412	64	·98750	56	·03190	62	28
29	·95384	58	·99476	64	·98806	56	·03252	62	29
30	8·95443	58	8·99540	64	8·98862	56	9·03313	61	30
31	·95501	58	·99604	64	·98918	56	·03375	62	31
32	·95559	58	·99668	64	·98974	55	·03437	62	32
33	·95617	58	·99732	64	·99030	56	·03499	61	33
34	·95675	58	·99796	63	·99085	55	·03561	62	34
35	8·95733	58	8·99860	64	8·99141	56	9·03622	61	35
36	·95791	57	·99923	63	·99197	56	·03684	62	36
37	·95849	58	8·99987	64	·99252	55	·03746	62	37
38	·95907	58	9·00051	64	·99308	56	·03807	61	38
39	·95965	58	·00114	63	·99363	55	·03869	62	39
40	8·96023	58	9·00178	64	8·99419	56	9·03930	61	40
41	·96080	57	·00242	64	·99474	55	·03992	62	41
42	·96138	58	·00305	63	·99529	55	·04053	61	42
43	·96196	58	·00369	64	·99585	56	·04115	62	43
44	·96253	57	·00432	63	·99640	55	·04176	61	44
45	8·96311	58	9·00495	63	8·99695	55	9·04238	62	45
46	·96368	57	·00559	64	·99751	56	·04299	61	46
47	·96426	58	·00622	63	·99806	55	·04360	61	47
48	·96483	57	·00686	64	·99861	55	·04421	61	48
49	·96541	58	·00749	63	·99916	55	·04483	62	49
50	8·96598	57	9·00813	64	8·99971	55	9·04544	61	50
51	·96656	58	·00875	62	9·00027	56	·04605	61	51
52	·96713	57	·00938	63	·00081	54	·04666	61	52
53	·96770	57	·01002	64	·00137	56	·04727	61	53
54	·96827	57	·01065	63	·00191	54	·04788	61	54
55	8·96885	58	9·01128	63	9·00246	55	9·04850	62	55
56	·96942	57	·01191	63	·00301	55	·04911	61	56
57	·96999	57	·01254	63	·00356	55	·04972	61	57
58	·97056	57	·01317	63	·00411	54	·05033	61	58
59	·97113	57	·01380	63	·00466	55	·05093	60	59
60	·97170		9·01443	63	9·00520	54	9·05154	61	60
		D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	65	64	63
6	6·5	6·4	6·3
7	7·6	7·4	7·4
8	8·7	8·5	8·4
9	9·7	9·6	9·4
10	10·8	10·7	10·5
20	21·7	21·3	21·0
30	32·5	32·0	31·5
40	43·3	42·7	42·0
50	54·1	53·3	52·5

	62	61	60
6	6·2	6·1	6·0
7	7·2	7·1	7·0
8	8·3	8·1	8·0
9	9·3	9·2	9·0
10	10·3	10·2	10·0
20	20·7	20·3	20·0
30	31·0	30·5	30·0
40	41·3	40·7	40·0
50	51·7	50·8	50·0

	59	58	57
6	5·9	5·8	5·7
7	6·9	6·8	6·6
8	7·9	7·7	7·6
9	8·9	8·7	8·6
10	9·9	9·7	9·5
20	19·8	19·3	19·0
30	29·5	29·0	28·5
40	39·3	38·7	38·0
50	49·1	48·3	47·5

	56	55	54
6	5·6	5·5	5·4
7	6·5	6·4	6·3
8	7·4	7·3	7·2
9	8·4	8·3	8·1
10	9·3	9·2	9·0
20	18·7	18·3	18·0
30	28·0	27·5	27·0
40	37·3	36·7	36·0
50	46·7	45·8	45·0

	53
6	5·3
7	6·2
8	7·1
9	8·0
10	8·9
20	17·8
30	26·7
40	35·6
50	44·4

Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.
		26°				**27°**			
00520	55	9.05154	61	9.03740	52	9.08752	59	0	
00575	54	.05215	61	.03792	52	.08811	59	1	
00630	54	.05276	60	.03845	53	.08870	59	2	
00684	54	.05337	61	.03898	52	.08929	59	3	
00739	55	.05398	60	.03950	52	.08988	59	4	61 60 59
00794	54	9.05458	61	9.04002	53	9.09047	59	5	6 6.1 6.0 5.9
00848	54	.05519	60	.04055	52	.09106	58	6	7 7.1 7.0 6.9
00903	54	.05580	60	.04107	52	.09164	59	7	8 8.1 8.0 7.9
00957	54	.05640	60	.04160	53	.09223	59	8	9 9.1 9.0 8.9
01011	54	.05701	60	.04212	52	.09282	59	9	10 10.1 10.0 9.9
01066	54	9.05762	61	9.04264	52	9.09341	58	10	20 20.3 20.0 19.8
01120	54	.05822	60	.04317	52	.09400	59	11	30 30.5 30.0 29.7
01174	54	.05883	60	.04369	52	.09458	58	12	40 40.6 40.0 39.6
01229	54	.05943	60	.04421	52	.09517	59	13	50 50.8 50.0 49.6
01283	54	.06004	60	.04473	52	.09576	58	14	
01337	54	9.06064	60	9.04525	52	9.09634	58	15	58 57
01391	54	.06124	60	.04577	52	.09693	59	16	6 5.8 5.7
01445	54	.06185	60	.04630	52	.09752	58	17	7 6.7 6.6
01499	54	.06245	60	.04682	52	.09810	58	18	8 7.7 7.6
01554	54	.06305	60	.04734	52	.09869	59	19	9 8.7 8.5
01608	54	9.06366	60	9.04786	52	9.09927	58	20	10 9.6 9.5
01662	54	.06426	60	.04837	51	.09986	58	21	20 19.3 19.0
01715	53	.06486	60	.04889	52	.10044	58	22	30 29.0 28.5
01769	54	.06546	60	.04941	52	.10102	58	23	40 38.6 38.0
01823	54	.06606	60	.04993	52	.10161	58	24	50 48.3 47.5
01877	54	9.06667	60	9.05045	51	9.10219	58	25	
01931	53	.06727	60	.05097	52	.10278	58	26	55 54
01985	54	.06787	60	.05148	51	.10336	58	27	6 5.5 5.4
02038	53	.06847	60	.05200	52	.10394	58	28	7 6.4 6.3
02092	54	.06907	60	.05252	52	.10452	58	29	8 7.3 7.2
02146	53	9.06967	60	9.05303	51	9.10511	58	30	9 8.2 8.1
02199	53	.07027	60	.05355	52	.10569	58	31	10 9.1 9.0
02253	54	.07087	59	.05407	51	.10627	58	32	20 18.3 18.0
02307	53	.07146	60	.05458	51	.10685	58	33	30 27.5 27.0
02360	53	.07206	60	.05510	51	.10743	58	34	40 36.6 36.0
02414	53	9.07266	59	9.05561	51	9.10801	58	35	50 45.8 45.0
02467	53	.07326	60	.05613	51	.10859	58	36	
02521	53	.07386	59	.05664	51	.10917	58	37	53 52
02574	53	.07445	60	.05715	51	.10975	58	38	6 5.3 5.2
02627	53	.07505	59	.05767	51	.11033	58	39	7 6.2 6.0
02681	53	9.07565	59	9.05818	51	9.11091	58	40	8 7.0 6.9
02734	53	.07624	59	.05869	51	.11149	58	41	9 7.9 7.8
02787	53	.07684	59	.05921	51	.11207	57	42	10 8.8 8.6
02840	53	.07743	59	.05972	51	.11265	58	43	20 17.6 17.3
02894	53	.07803	60	.06023	51	.11323	58	44	30 26.5 26.0
02947	53	9.07863	59	9.06074	51	9.11380	57	45	40 35.3 34.6
03000	53	.07922	59	.06125	51	.11438	58	46	50 44.1 43.3
03053	53	.07981	59	.06176	51	.11496	58	47	
03106	53	.08041	59	.06227	51	.11554	57	48	
03159	53	.08100	59	.06279	51	.11611	57	49	51 0
03212	53	9.08160	59	9.06330	51	9.11668	58	50	6 5.1 0.0
03265	53	.08219	59	.06380	50	.11727	57	51	7 5.9 0.0
03318	53	.08278	59	.06431	51	.11784	57	52	8 6.8 0.0
03371	53	.08338	59	.06482	51	.11842	57	53	9 7.6 0.1
03423	52	.08397	59	.06533	51	.11899	57	54	10 8.5 0.1
03475	53	9.08456	59	9.06584	51	9.11957	58	55	20 17.0 0.1
03529	53	.08515	59	.06635	50	.12015	57	56	30 25.8 0.2
03582	53	.08574	59	.06686	51	.12072	57	57	40 34.0 0.3
03634	52	.08634	59	.06736	50	.12129	57	58	50 42.5 0.4
03687	53	.08693	59	.06787	51	.12187	57	59	
03740	53	9.08752	59	9.06838	50	9.12244	57	60	
Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.

	28°				29°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9.06838	50	9.12244	57	9.09823	49	9.15641	56	0
1	.06888	50	.12302	57	.09872	48	.15697	55	1
2	.06939	50	.12359	57	.09920	49	.15752	56	2
3	.06990	50	.12416	57	.09969	48	.15808	55	3
4	.07040	50	.12474	57	.10018	49	.15864	56	4
5	9.07091	50	9.12531	57	9.10067	48	9.15920	55	5
6	.07141	50	.12588	57	.10115	48	.15975	56	6
7	.07192	50	.12645	57	.10164	49	.16031	55	7
8	.07242	50	.12703	57	.10213	48	.16087	56	8
9	.07293	50	.12760	57	.10261	48	.16142	55	9
10	9.07343	50	9.12817	57	9.10310	49	9.16198	56	10
11	.07393	50	.12874	57	.10358	48	.16254	55	11
12	.07444	50	.12931	57	.10407	48	.16309	55	12
13	.07494	50	.12988	57	.10455	48	.16365	55	13
14	.07544	50	.13045	57	.10504	48	.16420	55	14
15	9.07594	50	9.13102	57	9.10552	48	9.16476	55	15
16	.07644	50	.13159	57	.10601	48	.16531	55	16
17	.07695	50	.13216	57	.10649	48	.16587	56	17
18	.07745	50	.13273	56	.10697	48	.16642	55	18
19	.07795	50	.13330	57	.10746	48	.16698	56	19
20	9.07845	50	9.13387	57	9.10794	48	9.16753	55	20
21	.07895	50	.13444	57	.10842	48	.16808	55	21
22	.07945	50	.13500	56	.10890	48	.16864	56	22
23	.07995	50	.13557	57	.10939	48	.16919	55	23
24	.08045	50	.13614	57	.10987	48	.16974	55	24
25	9.08095	50	9.13671	57	9.11035	48	9.17029	55	25
26	.08145	50	.13727	56	.11083	48	.17085	56	26
27	.08195	50	.13784	57	.11131	48	.17140	55	27
28	.08244	49	.13841	57	.11179	48	.17195	55	28
29	.08294	50	.13897	56	.11227	48	.17250	55	29
30	9.08344	49	9.13954	57	9.11275	48	9.17305	55	30
31	.08394	50	.14011	57	.11323	48	.17361	56	31
32	.08443	49	.14067	56	.11371	48	.17416	55	32
33	.08493	50	.14124	57	.11419	48	.17471	55	33
34	.08543	50	.14180	56	.11467	48	.17526	55	34
35	9.08592	49	9.14237	57	9.11515	48	9.17581	55	35
36	.08642	50	.14293	56	.11562	47	.17636	55	36
37	.08691	49	.14350	57	.11610	48	.17691	55	37
38	.08741	50	.14406	56	.11658	48	.17746	55	38
39	.08790	49	.14462	56	.11706	48	.17801	55	39
40	9.08840	49	9.14519	57	9.11754	48	9.17856	55	40
41	.08889	49	.14575	56	.11801	47	.17910	54	41
42	.08939	49	.14631	56	.11849	48	.17965	55	42
43	.08988	49	.14688	57	.11897	48	.18020	55	43
44	.09037	49	.14744	56	.11944	47	.18075	55	44
45	9.09087	50	9.14800	56	9.11992	48	9.18130	55	45
46	.09136	49	.14856	56	.12039	47	.18185	55	46
47	.09185	49	.14913	57	.12087	48	.18239	54	47
48	.09234	49	.14969	56	.12134	47	.18294	55	48
49	.09284	50	.15025	56	.12182	48	.18349	55	49
50	9.09333	49	9.15081	56	9.12229	47	9.18403	54	50
51	.09382	49	.15137	56	.12277	48	.18458	55	51
52	.09431	49	.15193	56	.12324	47	.18513	54	52
53	.09480	49	.15249	56	.12371	47	.18567	54	53
54	.09529	49	.15305	56	.12419	48	.18622	54	54
55	9.09578	49	9.15361	56	9.12466	47	9.18676	54	55
56	.09627	49	.15417	56	.12513	47	.18731	55	56
57	.09676	49	.15473	56	.12560	47	.18786	54	57
58	.09725	49	.15529	56	.12608	48	.18840	54	58
59	.09774	49	.15585	56	.12655	47	.18894	54	59
′	.09823	D	9.15641	D	9.12702	D	9.18949	D	60
			Log. Exs.		Lg. Vers.		Log. Exs.		′

P. P.			
	57	57	56
6	5.7	5.7	5.6
7	6.7	6.6	6.6
8	7.6	7.6	7.5
9	8.6	8.5	8.5
10	9.6	9.5	9.4
20	19.1	19.0	18.9
30	28.7	28.5	28.2
40	38.3	38.0	37.6
50	47.9	47.5	47.1
	56	55	55
6	5.6	5.5	5.5
7	6.5	6.5	6.4
8	7.4	7.4	7.3
9	8.4	8.3	8.2
10	9.3	9.2	9.1
20	18.6	18.5	18.3
30	28.0	27.7	27.5
40	37.3	37.0	36.7
50	46.6	46.2	45.8
	54	54	
6	5.4	5.4	
7	6.3	6.3	
8	7.2	7.2	
9	8.2	8.1	
10	9.1	9.0	
20	18.1	18.0	
30	27.2	27.0	
40	36.3	36.0	
50	45.4	45.0	
	51	50	50
0	5.1	5.0	5.0
7	5.3	5.9	5.8
8	6.8	6.7	6.6
9	7.6	7.5	7.5
10	8.5	8.4	8.3
20	17.0	16.8	16.6
30	25.5	25.2	25.0
40	34.0	33.6	33.3
50	42.5	42.1	41.6
	49	49	48
6	4.9	4.9	4.8
7	5.8	5.7	5.6
8	6.6	6.5	6.4
9	7.4	7.3	7.2
10	8.2	8.1	8.1
20	16.5	16.3	16.1
30	24.7	24.5	24.2
40	33.0	32.6	32.3
50	41.2	40.8	40.4
	48	47	47
6	4.8	4.7	4.7
7	5.6	5.5	5.4
8	6.4	6.3	6.2
9	7.2	7.1	7.0
10	8.0	7.9	7.8
20	16.0	15.8	15.6
30	24.0	23.7	23.5
40	32.0	31.6	31.1
50	40.0	39.6	39.1

′	30° Lg. Vers.	D	Log. Exs.	D	31° Lg. Vers.	D	Log. Exs.	D	′	P. P.
0	9·12702	47	9·18949	54	9·15483	45	9·22176	53	0	
1	·12749	47	·19003	54	·15528	45	·22229	53	1	
2	·12796	47	·19058	54	·15574	45	·22282	53	2	
3	·12843	47	·19112	54	·15619	45	·22335	53	3	
4	·12890	47	·19167	54	·15665	45	·22388	53	4	
5	9·12937	47	9·19221	54	9·15710	45	9·22441	53	5	
6	·12984	47	·19275	54	·15755	45	·22494	53	6	
7	·13031	47	·19329	54	·15801	45	·22547	53	7	
8	·13078	47	·19384	54	·15846	45	·22600	53	8	
9	·13125	47	·19438	54	·15891	45	·22653	53	9	
0	9·13172	47	9·19492	54	9·15937	45	9·22706	53	10	
1	·13219	46	·19546	54	·15982	45	·22759	53	11	
2	·13266	47	·19601	54	·16027	45	·22815	53	12	
3	·13313	47	·19655	54	·16073	45	·22865	53	13	
4	·13359	46	·19709	54	·16118	45	·22918	52	14	
5	9·13406	47	9·19763	54	9·16163	45	9·22971	53	15	
6	·13453	46	·19817	54	·16208	45	·23024	53	16	
7	·13500	47	·19871	54	·16253	45	·23076	52	17	
8	·13546	46	·19925	54	·16298	45	·23129	53	18	
9	·13593	46	·19979	54	·16343	45	·23182	52	19	
0	9·13639	46	9·20033	54	9·16388	45	9·23235	53	20	
1	·13686	47	·20087	54	·16434	45	·23287	52	21	
2	·13733	46	·20141	54	·16479	44	·23340	53	22	
3	·13779	46	·20195	54	·16523	45	·23393	53	23	
4	·13826	46	·20249	54	·16568	45	·23446	53	24	
5	9·13872	46	9·20303	53	9·16613	45	9·23498	52	25	
6	·13919	46	·20357	54	·16658	45	·23551	53	26	
7	·13965	46	·20411	54	·16703	45	·23603	52	27	
8	·14011	46	·20465	54	·16748	44	·23656	53	28	
9	·14058	46	·20518	53	·16793	44	·23709	53	29	
0	9·14104	46	9·20572	54	9·16838	45	9·23761	52	30	
1	·14151	46	·20626	54	·16882	44	·23814	53	31	
2	·14197	46	·20680	54	·16927	45	·23866	52	32	
3	·14243	46	·20733	53	·16972	44	·23919	53	33	
4	·14289	46	·20787	54	·17017	45	·23971	52	34	
5	9·14336	46	9·20841	53	9·17061	44	9·24024	53	35	
6	·14382	46	·20894	53	·17106	44	·24076	52	36	
7	·14428	46	·20948	54	·17151	45	·24128	52	37	
8	·14474	46	·21002	53	·17195	44	·24181	53	38	
9	·14520	46	·21055	53	·17240	44	·24233	52	39	
0	9·14566	46	9·21109	53	9·17284	44	9·24285	52	40	
1	·14612	46	·21162	53	·17329	44	·24338	52	41	
2	·14658	46	·21216	53	·17373	44	·24390	52	42	
3	·14704	46	·21269	53	·17418	44	·24442	52	43	
4	·14750	46	·21323	53	·17462	44	·24495	52	44	
5	9·14795	46	9·21376	53	9·17507	44	9·24547	52	45	
6	·14842	46	·21430	53	·17551	44	·24599	52	46	
7	·14888	46	·21483	53	·17596	44	·24651	52	47	
8	·14934	45	·21537	53	·17640	44	·24704	52	48	
9	·14980	46	·21590	53	·17684	44	·24756	52	49	
0	9·15026	46	9·21643	53	9·17729	44	9·24808	52	50	
1	·15071	45	·21697	53	·17773	44	·24860	52	51	
2	·15117	46	·21750	53	·17817	44	·24912	52	52	
3	·15163	46	·21803	53	·17861	44	·24964	52	53	
4	·15209	45	·21857	53	·17906	44	·25016	52	54	
5	9·15254	45	9·21910	53	9·17950	44	9·25068	52	55	
6	·15300	46	·21963	53	·17994	44	·25120	52	56	
7	·15346	46	·22016	53	·18038	44	·25172	52	57	
8	·15391	45	·22070	53	·18082	44	·25224	52	58	
9	·15437	46	·22123	53	·18126	44	·25276	52	59	
0	9·15483		9·22178		9·18170		9·25328		60	
	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.

P. P.			
	54	54	53
6	5.4	5.4	5.3
7	6.3	6.3	6.2
8	7.2	7.2	7.1
9	8.2	8.1	8.0
10	9.1	9.0	8.9
20	18.1	18.0	17.8
30	27.2	27.0	26.7
40	36.3	36.0	35.6
50	45.4	45.0	44.6
	53	52	52
6	5.3	5.2	5.2
7	6.2	6.1	6.0
8	7.0	7.0	6.9
9	7.9	7.9	7.8
10	8.8	8.7	8.6
20	17.6	17.5	17.3
30	26.5	26.2	26.0
40	35.3	35.0	34.6
50	44.1	43.7	43.3
	47	47	46
6	4.7	4.7	4.6
7	5.5	5.5	5.4
8	6.3	6.2	6.2
9	7.1	7.0	7.0
10	7.9	7.8	7.7
20	15.8	15.6	15.5
30	23.7	23.5	23.2
40	31.6	31.3	31.0
50	39.6	39.1	38.7
	46	45	45
6	4.6	4.5	4.5
7	5.3	5.3	5.2
8	6.1	6.0	6.0
9	6.9	6.8	6.7
10	7.6	7.6	7.5
20	15.3	15.1	15.0
30	23.0	22.7	22.5
40	30.6	30.3	30.0
50	38.3	37.9	37.5
	44	44	
6	4.4	4.4	
7	5.2	5.1	
8	5.9	5.8	
9	6.7	6.6	
10	7.4	7.3	
20	14.8	14.6	
30	22.2	22.0	
40	29.6	29.3	
50	37.1	36.6	

	32°				33°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9.18170	44	9.25328	52	9.20771	42	9.28412	51	0
1	.18214	44	.25380	52	.20814	42	.28463	51	1
2	.18258	44	.25432	52	.20856	42	.28514	50	2
3	.18302	44	.25484	51	.20899	43	.28564	51	3
4	.18346	44	.25536	51	.20942	42	.28615	51	4
5	9.18390	44	9.25588	52	9.20984	42	9.28665	51	5
6	.18434	44	.25640	52	.21027	42	.28717	50	6
7	.18478	44	.25692	52	.21069	42	.28768	51	7
8	.18522	44	.25743	51	.21112	42	.28818	50	8
9	.18566	43	.25795	52	.21154	42	.28869	50	9
10	9.18610	44	9.25847	51	9.21196	42	9.28920	51	10
11	.18654	44	.25899	52	.21239	42	.28970	50	11
12	.18697	43	.25950	51	.21281	42	.29021	51	12
13	.18741	44	.26002	52	.21324	42	.29072	50	13
14	.18785	43	.26054	51	.21366	42	.29122	50	14
15	9.18829	44	9.26105	51	9.21408	42	9.29173	51	15
16	.18872	43	.26157	52	.21451	42	.29223	50	16
17	.18916	44	.26209	51	.21493	42	.29274	50	17
18	.18959	43	.26260	51	.21535	42	.29324	51	18
19	.19003	44	.26312	52	.21577	42	.29375	50	19
20	9.19047	43	9.26364	52	9.21620	42	9.29426	50	20
21	.19090	44	.26415	51	.21662	42	.29476	50	21
22	.19134	43	.26467	52	.21704	42	.29527	50	22
23	.19177	44	.26518	51	.21746	42	.29577	50	23
24	.19221	43	.26570	52	.21788	42	.29627	50	24
25	9.19264	43	9.26621	51	9.21830	42	9.29678	50	25
26	.19308	44	.26673	52	.21872	42	.29728	50	26
27	.19351	43	.26724	51	.21914	42	.29779	50	27
28	.19395	44	.26776	52	.21956	42	.29829	50	28
29	.19438	43	.26827	51	.21998	42	.29879	50	29
30	9.19481	43	9.26878	51	9.22040	42	9.29930	50	30
31	.19525	44	.26930	52	.22082	42	.29980	50	31
32	.19568	43	.26981	51	.22124	42	.30030	50	32
33	.19611	43	.27033	52	.22166	42	.30081	50	33
34	.19654	43	.27084	51	.22208	42	.30131	50	34
35	9.19698	43	9.27135	51	9.22250	42	9.30181	50	35
36	.19741	43	.27186	51	.22292	41	.30231	50	36
37	.19784	43	.27238	52	.22334	42	.30282	50	37
38	.19827	43	.27289	51	.22376	42	.30332	50	38
39	.19870	43	.27340	51	.22417	41	.30382	50	39
40	9.19914	43	9.27391	51	9.22459	42	9.30432	50	40
41	.19957	43	.27443	51	.22501	41	.30482	50	41
42	.20000	43	.27494	51	.22543	42	.30533	50	42
43	.20043	43	.27545	51	.22584	41	.30583	50	43
44	.20086	43	.27596	51	.22626	42	.30633	50	44
45	9.20129	43	9.27647	51	9.22668	42	9.30683	50	45
46	.20172	43	.27698	51	.22709	41	.30733	50	46
47	.20215	43	.27749	51	.22751	41	.30783	50	47
48	.20258	43	.27800	51	.22792	41	.30833	50	48
49	.20301	43	.27852	51	.22834	41	.30883	50	49
50	9.20343	42	9.27903	51	9.22876	41	9.30935	50	50
51	.20386	43	.27954	51	.22917	41	.30983	50	51
52	.20429	43	.28005	51	.22959	41	.31033	50	52
53	.20472	43	.28056	51	.23000	41	.31083	50	53
54	.20515	42	.28107	51	.23042	41	.31133	50	54
55	9.20558	43	9.28157	50	9.23083	41	9.31183	49	55
56	.20600	42	.28208	51	.23124	41	.31233	50	56
57	.20643	43	.28259	51	.23166	41	.31283	50	57
58	.20686	43	.28310	51	.23207	41	.31333	50	58
59	.20728	42	.28361	51	.23248	41	.31383	50	59
60	9.20771	43	9.28412	50	9.23290	41	9.31432	49	60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.			
	52	51	50
6	5.2	5.1	5.0
7	6.0	6.0	5.8
8	6.9	6.8	6.7
9	7.8	7.7	7.5
10	8.6	8.5	8.3
20	17.3	17.1	17.0
30	26.0	25.7	25.5
40	34.6	34.3	34.0
50	43.3	42.8	42.5
	50	50	49
6	5.0	5.0	4.9
7	5.9	5.8	5.8
8	6.7	6.6	6.6
9	7.6	7.5	7.4
10	8.4	8.3	8.2
20	16.8	16.6	16.5
30	25.2	25.0	24.7
40	33.6	33.3	33.0
50	42.1	41.6	41.2
	44	43	43
6	4.4	4.3	4.3
7	5.1	5.1	5.0
8	5.8	5.8	5.7
9	6.6	6.5	6.4
10	7.3	7.2	7.1
20	14.6	14.5	14.3
30	22.0	21.7	21.5
40	29.3	29.0	28.6
50	36.6	36.2	35.8
	42	42	41
6	4.2	4.2	4.1
7	4.9	4.9	4.8
8	5.6	5.6	5.5
9	6.4	6.3	6.2
10	7.1	7.0	6.9
20	14.2	14.0	13.8
30	21.2	21.0	20.7
40	28.3	28.0	27.6
50	35.4	35.0	34.6
	41		
6	4.1		
7	4.8		
8	5.4		
9	6.1		
10	6.8		
20	13.6		
30	20.5		
40	27.3		
50	34.1		

′	34° Lg. Vers.	D	Log. Exs.	D	35° Lg. Vers.	D	Log. Exs.	D	′
0	9·23290	41	9·31432	50	9·25731	40	9·34395	49	0
1	·23331	41	·31482	50	·25771	40	·34444	48	1
2	·23372	41	·31532	50	·25811	40	·34492	49	2
3	·23414	41	·31582	49	·25851	40	·34541	49	3
4	·23455	41	·31632	50	·25891	40	·34590	49	4
5	9·23496	41	9·31681	50	9·25931	40	9·34639	49	5
6	·23537	41	·31731	49	·25971	40	·34688	48	6
7	·23579	41	·31781	50	·26011	40	·34737	49	7
8	·23620	41	·31831	49	·26051	39	·34785	48	8
9	·23661	41	·31880	50	·26091	40	·34834	49	9
0	9·23702	41	9·31930	49	9·26131	40	9·34883	49	10
1	·23743	41	·31980	49	·26171	40	·34932	48	11
2	·23784	41	·32029	49	·26210	39	·34980	48	12
3	·23825	41	·32079	50	·26250	40	·35029	49	13
4	·23866	41	·32129	49	·26290	40	·35078	48	14
5	9·23907	41	9·32178	49	9·26330	39	9·35127	49	15
6	·23948	41	·32228	49	·26370	40	·35175	48	16
7	·23989	41	·32277	49	·26409	39	·35224	49	17
8	·24030	41	·32327	50	·26449	40	·35273	48	18
9	·24071	41	·32377	49	·26489	39	·35321	48	19
10	9·24112	40	9·32426	49	9·26528	39	9·35370	48	20
1	·24153	41	·32476	49	·26568	40	·35419	49	21
2	·24194	41	·32525	49	·26608	39	·35467	48	22
3	·24235	40	·32575	49	·26647	39	·35516	48	23
4	·24275	41	·32624	49	·26687	39	·35564	48	24
5	9·24316	40	9·32673	49	9·26726	40	9·35613	48	25
6	·24357	41	·32723	49	·26766	39	·35661	48	26
7	·24398	40	·32772	49	·26806	39	·35710	48	27
8	·24438	41	·32822	49	·26845	39	·35758	48	28
9	·24479	40	·32871	49	·26885	39	·35807	48	29
10	9·24520	40	9·32920	49	9·26924	39	9·35855	48	30
1	·24561	40	·32970	49	·26964	39	·35904	48	31
2	·24601	40	·33019	49	·27003	39	·35952	48	32
3	·24642	40	·33069	49	·27042	39	·36001	48	33
4	·24682	40	·33118	49	·27082	39	·36049	48	34
5	9·24723	41	9·33167	49	9·27121	39	9·36098	48	35
6	·24764	40	·33216	49	·27161	39	·36146	48	36
7	·24804	40	·33266	49	·27200	39	·36194	48	37
8	·24845	40	·33315	49	·27239	39	·36243	48	38
9	·24885	40	·33364	49	·27278	39	·36291	48	39
10	9·24926	40	9·33413	49	9·27318	39	9·36340	48	40
1	·24966	40	·33463	49	·27357	39	·36388	48	41
2	·25007	40	·33512	49	·27396	39	·36436	48	42
3	·25047	40	·33561	49	·27435	39	·36484	48	43
4	·25087	40	·33610	49	·27475	39	·36533	48	44
5	9·25128	40	9·33659	49	9·27514	39	9·36581	48	45
6	·25168	40	·33708	49	·27553	39	·36629	48	46
7	·25209	40	·33758	49	·27592	39	·36678	48	47
8	·25249	40	·33807	49	·27631	39	·36726	48	48
9	·25289	40	·33856	49	·27670	39	·36774	48	49
0	9·25329	40	9·33905	49	9·27709	39	9·36822	48	50
1	·25370	40	·33954	49	·27749	39	·36870	48	51
2	·25410	40	·34003	49	·27788	39	·36919	48	52
3	·25450	40	·34052	49	·27827	39	·36967	48	53
4	·25490	40	·34101	49	·27866	39	·37015	48	54
5	9·25531	40	9·34150	49	9·27905	39	9·37063	48	55
6	·25571	40	·34199	49	·27944	39	·37111	48	56
7	·25611	40	·34248	49	·27982	39	·37159	48	57
8	·25651	40	·34297	49	·28021	39	·37207	48	58
9	·25691	40	·34346	49	·28060	39	·37255	48	59
0	9·25731	40	9·34395	49	9·28099	39	9·37303	48	60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.			
	50	49	49
6	5·0	4·9	4·9
7	5·8	5·8	5·7
8	6·6	6·6	6·5
9	7·5	7·4	7·3
10	8·3	8·2	8·1
20	16·6	16·5	16·3
30	25·0	24·7	24·5
40	33·3	33·0	32·6
50	41·6	41·2	40·8
	48	48	
6	4·8	4·8	
7	5·6	5·6	
8	6·4	6·4	
9	7·3	7·2	
10	8·1	8·0	
20	16·1	16·0	
30	24·2	24·0	
40	32·3	32·0	
50	40·4	40·0	
	41	41	
6	4·1	4·1	
7	4·8	4·8	
8	5·5	5·4	
9	6·2	6·1	
10	6·9	6·8	
20	13·8	13·6	
30	20·7	20·5	
40	27·6	27·3	
50	34·6	34·1	
	40	40	
6	4·0	4·0	
7	4·7	4·	
8	5·4	5·3	
9	6·1	6·0	
10	6·7	6·6	
20	13·5	13·3	
30	20·2	20·0	
40	27·0	26·6	
50	33·7	33·3	
	39	39	
6	3·9	3·9	
7	4·6	4·5	
8	5·2	5·2	
9	5·9	5·8	
10	6·6	6·5	
20	13·1	13·0	
30	19·7	19·5	
40	26·3	26·0	
50	32·9	32·5	
	38		
6	3·8		
7	4·5		
8	5·1		
9	5·8		
10	6·4		
20	12·8		
30	19·2		
40	25·6		
50	32·1		

′	36° Lg. Vers.	D	Log. Exs.	D	37° Lg. Vers.	D	Log. Exs.	D	′	P. P.
0	9.28099	39	9.37303	48	9.30398	37	9.40163	47	0	48 48
1	.28138	38	.37352	48	.30436	38	.40210	47	1	6 4.8 4.8
2	.28177	38	.37400	48	.30474	37	.40258	47	2	7 5.6 5.6
3	.28816	39	.37448	48	.30511	37	.40305	47	3	8 6.4 6.4
4	.28255	39	.37496	48	.30549	38	.40352	47	4	9 7.3 7.2
5	9.28293	39	9.37544	48	9.30587	37	9.40399	47	5	10 8.1 8.0
6	.28332	38	.37592	48	.30624	37	.40447	47	6	20 16.1 16.0
7	.28371	38	.37640	48	.30662	38	.40494	47	7	30 24.2 24.0
8	.28410	39	.37687	47	.30700	37	.40541	47	8	40 32.3 32.0
9	.28448	38	.37735	48	.30737	37	.40588	47	9	50 40.4 40.0
10	9.28487	39	9.37783	48	9.30775	37	9.40635	47	10	47 47
11	.28526	38	.37831	48	.30812	37	.40683	47	11	6 4.7 4.7
12	.28564	38	.37879	48	.30850	37	.40730	47	12	7 5.5 5.5
13	.28603	39	.37927	48	.30887	37	.40777	47	13	8 6.3 6.2
14	.28642	38	.37975	47	.30925	37	.40824	47	14	9 7.1 7.0
15	9.28680	38	9.38023	48	9.30962	37	9.40871	47	15	10 7.9 7.8
16	.28719	38	.38071	48	.31000	37	.40918	47	16	20 15.8 15.6
17	.28757	38	.38119	48	.31037	37	.40965	47	17	30 23.7 23.5
18	.28796	39	.38166	47	.31075	37	.41012	47	18	40 31.6 31.3
19	.28835	38	.38214	48	.31112	37	.41059	47	19	50 39.6 39.1
20	9.28873	38	9.38262	48	9.31150	37	9.41106	47	20	46
21	.28912	38	.38310	47	.31187	37	.41153	47	21	6 4.6
22	.28950	38	.38357	48	.31224	37	.41200	47	22	7 5.4
23	.28988	38	.38405	48	.31262	37	.41247	47	23	8 6.2
24	.29027	38	.38453	47	.31299	37	.41294	47	24	9 7.0
25	9.29065	38	9.38501	48	9.31336	37	9.41341	47	25	10 7.7
26	.29104	38	.38548	47	.31374	37	.41388	47	26	20 15.5
27	.29142	38	.38596	48	.31411	37	.41435	47	27	30 23.2
28	.29180	38	.38644	48	.31448	37	.41482	47	28	40 31.0
29	.29219	38	.38692	48	.31485	37	.41529	47	29	50 38.7
30	9.29257	38	9.38739	47	9.31523	37	9.41576	46	30	39 38
31	.29295	38	.38787	47	.31560	37	.41623	47	31	6 3.9 3.8
32	.29334	38	.38834	48	.31597	37	.41670	47	32	7 4.5 4.5
33	.29372	38	.38882	47	.31634	37	.41717	47	33	8 5.2 5.1
34	.29410	38	.38930	47	.31671	37	.41763	46	34	9 5.8 5.8
35	9.29448	38	9.38977	47	9.31708	37	9.41810	47	35	10 6.5 6.4
36	.29487	38	.39025	47	.31746	37	.41857	46	36	20 13.0 12.8
37	.29525	38	.39072	47	.31783	37	.41904	47	37	30 19.5 19.2
38	.29563	38	.39120	48	.31820	37	.41951	47	38	40 26.0 25.6
39	.29601	38	.39168	47	.31857	37	.41998	47	39	50 32.5 32.1
40	9.29639	38	9.39215	47	9.31894	37	9.42044	46	40	38 37
41	.29677	38	.39263	48	.31931	37	.42091	47	41	6 3.8 3.7
42	.29715	38	.39310	47	.31968	37	.42138	46	42	7 4.4 4.4
43	.29754	38	.39358	47	.32005	37	.42185	47	43	8 5.0 5.0
44	.29792	38	.39405	47	.32042	37	.42231	46	44	9 5.7 5.6
45	9.29830	38	9.39453	47	9.32079	37	9.42278	47	45	10 6.3 6.2
46	.29868	38	.39500	47	.32116	37	.42325	46	46	20 12.5 12.5
47	.29906	38	.39548	47	.32153	37	.42372	47	47	30 19.0 18.7
48	.29944	38	.39595	47	.32190	37	.42418	46	48	40 25.3 25.0
49	.29982	38	.39642	47	.32227	37	.42465	47	49	50 31.6 31.2
50	9.30020	37	9.39690	47	9.32263	36	9.42512	46	50	37 36
51	.30057	38	.39737	47	.32300	37	.42558	46	51	6 3.7 3.6
52	.30095	38	.39785	47	.32337	37	.42605	47	52	7 4.3 4.2
53	.30133	38	.39832	47	.32374	36	.42652	46	53	8 4.9 4.8
54	.30171	38	.39879	47	.32411	37	.42698	46	54	9 5.5 5.5
55	9.30208	37	9.39927	47	9.32447	36	9.42745	47	55	10 6.1 6.1
56	.30247	38	.39974	47	.32484	37	.42792	46	56	20 12.3 12.1
57	.30285	37	.40021	47	.32521	36	.42838	46	57	30 18.5 18.2
58	.30322	38	.40069	47	.32558	37	.42885	46	58	40 24.6 24.3
59	.30360	38	.40116	47	.32594	36	.42931	46	59	50 30.8 30.4
60	.30398		9.40163		9.32631	37	9.42978	46	60	
′	Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.

	38°			39°				
′	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0		9·42978	46	9·34802	35	9·45752	45	0
1	36	·43024	46	·34837	35	·45797	46	1
2	36	·43071	47	·34873	36	·45843	46	2
3	37	·43118	46	·34909	35	·45889	46	3
4	36	·43164	46	·34944	35	·45935	46	4
5	36	9·43211	46	9·34980	35	9·45981	45	5
6	36	·43257	46	·35016	36	·46027	46	6
7	37	·43304	46	·35051	35	·46073	46	7
8	36	·43350	46	·35087	35	·46118	45	8
9	36	·43396	46	·35122	35	·46164	46	9
10	36	9·43443	46	9·35158	35	9·46210	46	10
11	36	·43489	46	·35193	35	·46256	45	11
12	36	·43536	46	·35229	35	·46302	46	12
13	36	·43582	46	·35264	35	·46347	45	13
14	36	·43629	46	·35300	35	·46393	46	14
15	36	9·43675	46	9·35335	35	9·46439	45	15
16	36	·43721	46	·35370	35	·46485	46	16
17	36	·43768	46	·35406	35	·46530	45	17
18	36	·43814	46	·35441	35	·46576	46	18
19	36	·43861	46	·35477	35	·46622	45	19
20	36	9·43907	46	9·35512	35	9·46668	46	20
21	36	·43953	46	·35547	35	·46713	45	21
22	36	·43999	46	·35583	35	·46759	46	22
23	36	·44046	46	·35618	35	·46805	46	23
24	36	·44092	46	·35653	35	·46850	45	24
25	36	9·44138	46	9·35689	35	9·46896	46	25
26	36	·44185	46	·35724	35	·46942	45	26
27	36	·44231	46	·35759	35	·46987	45	27
28	36	·44277	46	·35794	35	·47033	45	28
29	38	·44323	46	·35829	35	·47078	46	29
30	36	9·44370	46	9·35865	35	9·47124	45	30
31	36	·44416	46	·35900	35	·47170	45	31
32	36	·44462	46	·35935	35	·47215	45	32
33	36	·44508	46	·35970	35	·47261	45	33
34	36	·44554	46	·36005	35	·47306	46	34
35	38	9·44601	46	9·36040	35	9·47352	46	35
36	36	·44647	46	·36076	35	·47398	45	36
37	36	·44693	46	·36111	35	·47443	45	37
38	36	·44739	46	·36146	35	·47489	45	38
39	36	·44785	46	·36181	35	·47534	45	39
40	36	9·44831	46	9·36216	35	9·47580	45	40
41	36	·44877	46	·36251	35	·47625	45	41
42	36	·44924	46	·36286	35	·47671	46	42
43	36	·44970	46	·36321	35	·47716	45	43
44	36	·45016	46	·36356	35	·47762	45	44
45	36	9·45062	46	9·36391	35	9·47807	45	45
46	36	·45108	46	·36426	35	·47852	46	46
47	36	·45154	46	·36461	35	·47898	45	47
48	36	·45200	46	·36495	34	·47943	45	48
49	35	·45246	46	·36530	35	·47989	45	49
50	38	9·45292	46	9·36565	35	9·48034	45	50
51	36	·45338	46	·36600	35	·48080	45	51
52	35	·45384	46	·36635	34	·48125	45	52
53	36	·45430	46	·36670	35	·48170	45	53
54	35	·45476	46	·36705	35	·48216	45	54
55	36	9·45522	46	9·36740	34	9·48261	45	55
56	35	·45568	46	·36774	35	·48306	45	56
57	36	·45614	46	·36809	34	·48352	45	57
58	35	·45660	46	·36844	35	·48397	45	58
59	36	·45706	46	·36878	34	·48442	45	59
60	35	9·45752	46	9·36913	35	9·48488	45	60
′	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.		
	47	46
6	4·7	4·6
7	5·5	5·4
8	6·2	6·2
9	7·0	7·0
10	7·8	7·7
20	15·6	15·5
30	23·5	23·2
40	31·3	31·0
50	39·1	38·7
	46	45
6	4·6	4·5
7	5·3	5·3
8	6·1	6·0
9	6·9	6·8
10	7·6	7·6
20	15·3	15·1
30	23·0	22·7
40	30·6	30·3
50	38·3	37·9
	45	
6	4·5	
7	5·2	
8	6·0	
9	6·7	
10	7·5	
20	15·0	
30	22·5	
40	30·0	
50	37·5	
	37	36
6	3·7	3·6
7	4·3	4·2
8	4·9	4·8
9	5·5	5·5
10	6·1	6·1
20	12·3	12·1
30	18·5	18·2
40	24·6	24·3
50	30·8	30·4
	36	35
6	3·6	3·5
7	4·2	4·1
8	4·8	4·7
9	5·4	5·3
10	6·0	5·9
20	12·0	11·8
30	18·0	17·7
40	24·0	23·6
50	30·0	29·6
	35	34
6	3·5	3·4
7	4·1	4·0
8	4·6	4·6
9	5·2	5·2
10	5·8	5·7
20	11·6	11·5
30	17·5	17·2
40	23·3	23·0
50	29·1	28·7

	40°			41°			
′	Lg. Vers.	D	Log.Exs.	D	Lg. Vers.	D	Log.Exs.	D	′	P. P.
0	9.36913	34	9.48488	45	9.38968	34	9.51190	45	0	
1	.36948	34	.48533	45	.39002	33	.51235	44	1	
2	.36982	35	.48578	45	.39035	34	.51279	45	2	
3	.37017	34	.48624	45	.39069	33	.51324	44	3	
4	.37052		.48669		.39103		.51369		4	
5	9.37086	34	9.48714	45	9.39137	34	9.51414	45	5	45 45
6	.37121	34	.48759	45	.39170	33	.51458	44	6	6 4.5 4.5
7	.37156	34	.48805	45	.39204	34	.51503	45	7	7 5.3 5.2
8	.37190	34	.48850	45	.39238	33	.51548	44	8	8 6.0 6.0
9	.37225		.48895		.39271		.51592		9	9 6.8 6.7
10	9.37259	34	9.48940	45	9.39305	33	9.51637	45	10	10 7.6 7.5
11	.37294	34	.48986	45	.39339	34	.51682	44	11	20 15.1 15.0
12	.37328	34	.49031	45	.39372	33	.51726	45	12	30 22.7 22.5
13	.37363	34	.49076	45	.39406	33	.51771	44	13	40 30.3 30.0
14	.37397		.49121		.39439		.51816		14	50 37.9 37.6
15	9.37432	34	9.49166	45	9.39473	33	9.51860	44	15	
16	.37466	34	.49211	45	.39507	34	.51905	45	16	44 44
17	.37501	34	.49257	45	.39540	33	.51950	44	17	6 4.4 4.4
18	.37535	34	.49302	45	.39574	33	.51994	44	18	7 5.2 5.1
19	.37570		.49347		.39607		.52039		19	8 5.9 5.8
20	9.37604	34	9.49392	45	9.39641	33	9.52084	45	20	9 6.7 6.6
21	.37639	34	.49437	45	.39674	33	.52128	44	21	10 7.4 7.3
22	.37673	34	.49482	45	.39708	33	.52173	44	22	20 14.8 14.6
23	.37707	34	.49527	45	.39741	33	.52217	44	23	30 22.2 22.0
24	.37742		.49572		.39774		.52262		24	40 29.6 29.3
25	9.37776	34	9.49618	45	9.39808	33	9.52306	44	25	50 37.1 36.6
26	.37810	34	.49663	45	.39841	33	.52351	45	26	
27	.37845	34	.49708	45	.39875	33	.52396	44	27	35 34
28	.37879	34	.49753	45	.39908	33	.52440	44	28	6 3.5 3.4
29	.37913		.49798		.39941		.52485		29	7 4.1 4.0
30	9.37947	34	9.49843	45	9.39975	33	9.52529	44	30	8 4.6 4.6
31	.37982	34	.49888	45	.40008	33	.52574	44	31	9 5.2 5.2
32	.38016	34	.49933	45	.40041	33	.52618	44	32	10 5.8 5.7
33	.38050	34	.49978	45	.40075	33	.52663	44	33	20 11.6 11.5
34	.38084		.50023		.40108		.52707		34	30 17.5 17.2
35	9.38118	34	9.50068	45	9.40141	33	9.52752	44	35	40 23.3 23.0
36	.38153	34	.50113	45	.40175	33	.52796	44	36	50 29.1 28.7
37	.38187	34	.50158	45	.40208	33	.52841	44	37	
38	.38221	34	.50203	45	.40241	33	.52885	44	38	34 33
39	.38255		.50248		.40274		.52930		39	6 3.4 3.3
40	9.38289	34	9.50293	45	9.40307	33	9.52974	44	40	7 3.9 3.9
41	.38323	34	.50338	45	.40341	33	.53018	44	41	8 4.5 4.4
42	.38357	34	.50383	44	.40374	33	.53063	44	42	9 5.1 5.0
43	.38391	34	.50427	45	.40407	33	.53107	44	43	10 5.6 5.6
44	.38425		.50472		.40440		.53152		44	20 11.3 11.1
45	9.38459	34	9.50517	45	9.40473	33	9.53196	44	45	30 17.0 16.7
46	.38493	34	.50562	45	.40506	33	.53240	44	46	40 22.6 22.3
47	.38527	34	.50607	44	.40540	33	.53285	44	47	50 28.3 27.9
48	.38561	34	.50652	45	.40573	33	.53329	44	48	
49	.38595		.50697		.40606		.53374		49	33
50	9.38629	34	9.50742	45	9.40639	33	9.53418	44	50	6 3.3
51	.38663	34	.50787	45	.40672	33	.53462	44	51	7 3.8
52	.38697	34	.50831	44	.40705	33	.53507	44	52	8 4.4
53	.38731	33	.50876	45	.40738	33	.53551	44	53	9 4.9
54	.38765	34	.50921	45	.40771	33	.53595	44	54	10 5.5
55	9.38799	34	9.50966	44	9.40804	33	9.53640	44	55	20 11.0
56	.38833	34	.51011	45	.40837	33	.53684	44	56	30 16.5
57	.38866	33	.51056	44	.40870	33	.53728	44	57	40 22.0
58	.38900	34	.51100	45	.40903	33	.53773	44	58	50 27.5
	.38934	33	.51145	45	.40936	33	.53817	44	59	
	[illegible]	34	9.51190	44	9.40969	33	9.53861	44	60	
	s.	D	Log.Exs.	D	Lg. Vers.	D	Log.Exs.	D	′	P. P.

′	42° Lg. Vers.	D	Log. Exs.	D	43° Lg. Vers.	D	Log. Exs.	D	′	P. P.
0	9.40969	32	9.53861	44	9.42918	32	9.56505	43	0	
1	.41001	33	.53906	44	.42950	32	.56549	44	1	
2	.41034	33	.53950	44	.42982	32	.56593	44	2	
3	.41067	33	.53994	44	.43014	32	.56637	43	3	
4	.41100	33	.54038	44	.43046	32	.56680	44	4	
5	9.41133	32	9.54083	44	9.43078	32	9.56724	44	5	
6	.41166	33	.54127	44	.43110	32	.56768	43	6	
7	.41199	33	.54171	44	.43142	31	.56812	44	7	
8	.41231	33	.54215	44	.43174	32	.56856	43	8	
9	.41264	33	.54259	44	.43206	32	.56899	44	9	
10	9.41297	32	9.54304	44	9.43238	32	9.56943	44	10	
11	.41330	33	.54348	44	.43270	32	.56987	44	11	
12	.41362	33	.54392	44	.43302	32	.57031	43	12	
13	.41395	33	.54436	44	.43334	32	.57075	44	13	
14	.41428	32	.54480	44	.43365	31	.57118	43	14	
15	9.41461	33	9.54525	44	9.43397	32	9.57162	44	15	
16	.41493	33	.54569	44	.43429	32	.57206	43	16	
17	.41526	33	.54613	44	.43461	32	.57250	44	17	
18	.41559	33	.54657	44	.43493	31	.57293	43	18	
19	.41591	32	.54701	44	.43525	32	.57337	44	19	
20	9.41624	33	9.54745	44	9.43557	32	9.57381	43	20	
21	.41657	33	.54790	44	.43588	31	.57424	44	21	
22	.41689	33	.54834	44	.43620	32	.57468	43	22	
23	.41722	33	.54878	44	.43652	31	.57512	44	23	
24	.41754	32	.54922	44	.43684	32	.57556	43	24	
25	9.41787	33	9.54966	44	9.43715	31	9.57599	43	25	
26	.41819	32	.55010	44	.43747	32	.57643	44	26	
27	.41852	33	.55054	44	.43779	31	.57687	43	27	
28	.41885	33	.55098	44	.43810	32	.57730	43	28	
29	.41917	32	.55142	44	.43842	32	.57774	44	29	
30	9.41950	33	9.55186	44	9.43874	31	9.57818	44	30	
31	.41982	32	.55230	44	.43906	32	.57861	43	31	
32	.42014	32	.55275	44	.43937	31	.57905	44	32	
33	.42047	33	.55319	44	.43969	31	.57949	43	33	
34	.42079	32	.55363	44	.44000	31	.57992	43	34	
35	9.42112	33	9.55407	44	9.44032	32	9.58036	43	35	
36	.42144	32	.55451	44	.44064	31	.58079	44	36	
37	.42177	33	.55495	44	.44095	31	.58123	43	37	
38	.42209	32	.55539	44	.44127	31	.58167	43	38	
39	.42241	32	.55583	44	.44158	31	.58210	43	39	
40	9.42274	33	9.55627	44	9.44190	31	9.58254	43	40	
41	.42306	32	.55671	44	.44221	31	.58297	43	41	
42	.42338	32	.55715	44	.44253	31	.58341	44	42	
43	.42371	33	.55759	44	.44284	31	.58385	43	43	
44	.42403	32	.55803	44	.44316	31	.58428	43	44	
45	9.42435	32	9.55847	44	9.44347	31	9.58472	43	45	
46	.42467	32	.55890	43	.44379	31	.58515	43	46	
47	.42500	33	.55934	44	.44410	31	.58559	44	47	
48	.42532	32	.55978	44	.44442	31	.58602	43	48	
49	.42564	32	.56022	44	.44473	31	.58646	43	49	
50	9.42596	32	9.56066	44	9.44504	31	9.58689	43	50	
51	.42629	33	.56110	44	.44536	31	.58733	43	51	
52	.42661	32	.56154	43	.44567	31	.58776	44	52	
53	.42693	32	.56198	44	.44599	31	.58820	43	53	
54	.42725	32	.56242	44	.44630	31	.58864	43	54	
55	9.42757	32	9.56286	44	9.44661	31	9.58907	43	55	
56	.42789	32	.56330	44	.44693	31	.58951	43	56	
57	.42822	32	.56374	44	.44724	31	.58994	43	57	
58	.42854	32	.56417	43	.44755	31	.59037	43	58	
59	.42886	32	.56461	44	.44787	31	.59081	43	59	
60	9.42918	32	9.56505	43	9.44818	31	9.59124	43	60	
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.

	44	43
6	4.4	4.3
7	5.2	5.1
8	5.9	5.8
9	6.7	6.6
10	7.4	7.3
20	14.8	14.6
30	22.2	22.0
40	29.6	29.3
50	37.1	36.6

	43	43
6	4.3	4.3
7	5.1	5.0
8	5.8	5.7
9	6.5	6.4
10	7.2	7.1
20	14.5	14.3
30	21.7	21.5
40	29.0	28.6
50	36.2	35.8

	33	32
6	3.3	3.2
7	3.8	3.8
8	4.4	4.3
9	4.9	4.9
10	5.5	5.4
20	11.0	10.8
30	16.5	16.2
40	22.0	21.6
50	27.5	27.1

	32	31
6	3.2	3.1
7	3.7	3.7
8	4.2	4.2
9	4.8	4.7
10	5.3	5.2
20	10.6	10.5
30	16.0	15.7
40	21.3	21.0
50	26.6	26.2

	31
6	3.1
7	3.6
8	4.1
9	4.6
10	5.1
20	10.3
30	15.5
40	20.6
50	25.8

	44°				45°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9·44818	31	9·59124	43	9·46671	30	9·61722	43	0
1	·44849	31	·59168	43	·46701	30	·61765	43	1
2	·44880	31	·59211	43	·46732	30	·61808	43	2
3	·44912	31	·59255	43	·46762	30	·61852	43	3
4	·44943	31	·59298	43	·46793	30	·61895	43	4
5	9·44974	31	9·59342	43	9·46823	30	9·61938	43	5
6	·45005	31	·59385	43	·46853	30	·61981	43	6
7	·45036	31	·59429	43	·46884	30	·62024	43	7
8	·45068	31	·59472	43	·46914	30	·62067	43	8
9	·45099	31	·59515	43	·46945	30	·62110	43	9
10	9·45130	31	9·59559	43	9·46975	30	9·62153	43	10
11	·45161	31	·59602	43	·47005	30	·62196	43	11
12	·45192	31	·59646	43	·47036	30	·62239	43	12
13	·45223	31	·59689	43	·47066	30	·62282	43	13
14	·45254	31	·59732	43	·47096	30	·62326	43	14
15	9·45285	31	9·59776	43	9·47127	30	9·62369	43	15
16	·45316	31	·59819	43	·47157	30	·62412	43	16
17	·45348	31	·59863	43	·47187	30	·62455	43	17
18	·45379	31	·59906	43	·47218	30	·62498	43	18
19	·45410	31	·59949	43	·47248	30	·62541	43	19
20	9·45441	31	9·59993	43	9·47278	30	9·62584	43	20
21	·45472	31	·60036	43	·47308	30	·62627	43	21
22	·45503	31	·60079	43	·47339	30	·62670	43	22
23	·45534	31	·60123	43	·47369	30	·62713	43	23
24	·45565	31	·60166	43	·47399	30	·62756	43	24
25	9·45595	30	9·60209	43	9·47429	30	9·62799	43	25
26	·45626	31	·60253	43	·47459	30	·62842	43	26
27	·45657	31	·60296	43	·47490	30	·62885	43	27
28	·45688	31	·60339	43	·47520	30	·62928	43	28
29	·45719	31	·60383	43	·47550	30	·62971	43	29
30	9·45750	31	9·60426	43	9·47580	30	9·63014	43	30
31	·45781	31	·60469	43	·47610	30	·63057	43	31
32	·45812	31	·60512	43	·47640	30	·63100	43	32
33	·45843	30	·60556	43	·47670	30	·63143	43	33
34	·45873	31	·60599	43	·47700	30	·63186	43	34
35	9·45904	31	9·60642	43	9·47731	30	9·63229	43	35
36	·45935	30	·60685	43	·47761	30	·63272	43	36
37	·45966	31	·60729	43	·47791	30	·63315	43	37
38	·45997	30	·60772	43	·47821	30	·63358	43	38
39	·46027	31	·60815	43	·47851	30	·63401	43	39
40	9·46058	31	9·60858	43	9·47881	30	9·63443	43	40
41	·46089	30	·60902	43	·47911	30	·63486	43	41
42	·46120	31	·60945	43	·47941	30	·63529	43	42
43	·46150	30	·60988	43	·47971	30	·63572	43	43
44	·46181	31	·61031	43	·48001	30	·63615	43	44
45	9·46212	30	9·61075	43	9·48031	30	9·63658	43	45
46	·46242	31	·61118	43	·48061	30	·63701	43	46
47	·46273	30	·61161	43	·48090	29	·63744	43	47
48	·46304	30	·61204	43	·48120	30	·63787	43	48
49	·46334	31	·61247	43	·48150	30	·63830	43	49
50	9·46365	30	9·61291	43	9·48180	30	9·63873	43	50
51	·46396	30	·61334	43	·48210	30	·63915	42	51
52	·46426	30	·61377	43	·48240	29	·63958	43	52
53	·46457	30	·61420	43	·48270	30	·64001	43	53
54	·46487	30	·61463	43	·48300	30	·64044	43	54
55	9·46518	31	9·61506	43	9·48329	29	9·64087	42	55
56	·46549	30	·61550	43	·48359	30	·64130	43	56
57	·46579	30	·61593	43	·48389	30	·64173	43	57
58	·46610	30	·61636	43	·48419	29	·64216	43	58
59	·46640	30	·61679	43	·48449	30	·64258	42	59
60	·46671	30	·61722	43	9·48479	29	9·64301	43	60
	Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	43	43
6	4.5	4.3
7	5.1	5.0
8	5.8	5.7
9	6.5	6.4
10	7.2	7.1
20	14.5	14.3
30	21.7	21.5
40	29.0	28.6
50	36.2	35.8

	42
6	4.2
7	4.9
8	5.6
9	6.4
10	7.1
20	14.1
30	21.2
40	28.3
50	35.4

	31	31
6	3.1	3.1
7	3.7	3.6
8	4.2	4.1
9	4.7	4.6
10	5.2	5.1
20	10.5	10.3
30	15.7	15.5
40	21.0	20.6
50	26.2	25.8

	30	30
6	3.0	3.0
7	3.5	3.5
8	4.0	4.0
9	4.6	4.5
10	5.1	5.0
20	10.1	10.0
30	15.2	15.0
40	20.3	20.0
50	25.4	25.0

	29
6	2.9
7	3.4
8	3.9
9	4.4
10	4.9
20	9.8
30	14.7
40	19.6
50	24.6

P. P.

	46°				47°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9.48478	30	9.64301	43	9.50243	29	9.66864	42	0
1	.48508	29	.64344	42	.50272	29	.66907	43	1
2	.48538	30	.64387	43	.50301	29	.66950	42	2
3	.48568	29	.64430	43	.50330	29	.66992	42	3
4	.48597		.64473		.50359		.67035		4
5	9.48627	30	9.64515	42	9.50388	29	9.67077	42	5
6	.48657	29	.64558	43	.50417	29	.67120	42	6
7	.48686	29	.64601	43	.50446	29	.67162	42	7
8	.48716	30	.64644	42	.50475	29	.67205	43	8
9	.48746	29	.64687	43	.50504	29	.67248	42	9
10	9.48775	29	9.64729	42	9.50533	29	9.67290	42	10
11	.48805	30	.64772	43	.50562	29	.67333	42	11
12	.48835	29	.64815	43	.50591	29	.67375	42	12
13	.48864	29	.64858	43	.50619	28	.67418	42	13
14	.48894	29	.64901		.50648	29	.67460		14
15	9.48923	29	9.64943	42	9.50677	29	9.67503	42	15
16	.48953	30	.64986	43	.50706	29	.67546	43	16
17	.48983	29	.65029	43	.50735	28	.67588	42	17
18	.49012	29	.65072	43	.50764	29	.67631	43	18
19	.49042	29	.65114	42	.50793	29	.67673	42	19
20	9.49071	29	9.65157	43	9.50821	28	9.67716	42	20
21	.49101	29	.65200	43	.50850	29	.67758	42	21
22	.49130	29	.65243	43	.50879	29	.67801	43	22
23	.49160	29	.65285	42	.50908	29	.67843	42	23
24	.49189	29	.65328	43	.50937	29	.67886	43	24
25	9.49219	29	9.65371	43	9.50965	28	9.67928	42	25
26	.49248	29	.65414	43	.50994	29	.67971	43	26
27	.49278	29	.65456	42	.51023	29	.68013	42	27
28	.49307	29	.65499	43	.51052	29	.68056	43	28
29	.49336	29	.65542	43	.51080	28	.68098	42	29
30	9.49300	29	9.65585	43	9.51109	29	9.68141	43	30
31	.49395	29	.65627	42	.51138	29	.68183	42	31
32	.49425	29	.65670	43	.51167	29	.68226	43	32
33	.49454	29	.65713	43	.51195	28	.68268	42	33
34	.49483	29	.65755	42	.51224	29	.68311	43	34
35	9.49513	29	9.65798	43	9.51253	29	9.68353	42	35
36	.49542	29	.65841	43	.51281	28	.68396	43	36
37	.49571	29	.65884	43	.51310	29	.68438	42	37
38	.49601	29	.65926	42	.51338	28	.68481	43	38
39	.49630	29	.65969	43	.51367	29	.68523	42	39
40	9.49059	29	9.66012	43	9.51396	29	9.68566	43	40
41	.49689	29	.66054	42	.51424	28	.68608	42	41
42	.49718	29	.66097	43	.51453	29	.68651	43	42
43	.49747	29	.66140	43	.51481	28	.68693	42	43
44	.49776	29	.66182	42	.51510	29	.68735	42	44
45	9.49806	29	9.66225	43	9.51539	29	9.68778	43	45
46	.49835	29	.66268	43	.51567	28	.68820	42	46
47	.49864	29	.66310	42	.51596	29	.68863	43	47
48	.49893	29	.66353	43	.51624	28	.68905	42	48
49	.49922	29	.66396	43	.51653	29	.68948	43	49
50	9.49952	29	9.66438	42	9.51681	28	9.68990	42	50
51	.49981	29	.66481	43	.51710	29	.69033	43	51
52	.50010	29	.66523	42	.51738	28	.69075	42	52
53	.50039	29	.66566	43	.51767	29	.69117	42	53
54	.50068	29	.66609	43	.51795	28	.69160	43	54
55	9.50097	29	9.66651	42	9.51823	28	9.69202	42	55
56	.50126	29	.66694	43	.51852	29	.69245	43	56
57	.50155	29	.66737	43	.51880	28	.69287	42	57
58	.50185	29	.66779	42	.51909	29	.69330	43	58
59	.50214	29	.66822	43	.51937	28	.69372	42	59
60	9.50243		9.66864	42	9.51965		9.69414	42	60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.		
	43	42
6	4.3	4.2
7	5.0	4.9
8	5.7	5.6
9	6.4	6.4
10	7.1	7.1
20	14.3	14.1
30	21.5	21.2
40	28.6	28.3
50	35.8	35.4

P. P.	
	42
6	4.2
7	4.9
8	5.6
9	6.3
10	7.0
20	14.0
30	21.0
40	28.0
50	35.0

P. P.		
	30	29
6	3.0	2.9
7	3.5	3.4
8	4.0	3.9
9	4.5	4.4
10	5.0	4.9
20	10.0	9.8
30	15.0	14.7
40	20.0	19.6
50	25.0	24.6

P. P.		
	29	28
6	2.9	2.8
7	3.4	3.3
8	3.8	3.8
9	4.3	4.3
10	4.8	4.7
20	9.6	9.5
30	14.5	14.2
40	19.3	19.0
50	24.1	23.7

P. P.	
	28
6	2.8
7	3.2
8	3.7
9	4.2
10	4.6
20	9.3
30	14.0
40	18.6
50	23.3

	48°				49°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9.51965	28	9.69414	43	9.53648	27	9.71954	42	0
1	.51994	28	.69457	42	.53676	28	.71996	42	1
2	.52022	28	.69499	42	.53704	27	.72038	42	2
3	.52050	28	.69542	42	.53731	27	.72081	42	3
4	.52079	28	.69584	42	.53759	28	.72123	42	4
5	9.52107	28	9.69626	42	9.53787	27	9.72165	42	5
6	.52135	28	.69669	43	.53814	27	.72207	42	6
7	.52164	28	.69711	42	.53842	27	.72250	42	7
8	.52192	28	.69753	42	.53870	28	.72292	42	8
9	.52220	28	.69796	43	.53897	27	.72334	42	9
10	9.52249	28	9.69838	42	9.53925	27	9.72376	42	10
11	.52277	28	.69881	42	.53952	27	.72419	42	11
12	.52305	28	.69923	42	.53980	28	.72461	42	12
13	.52333	28	.69965	42	.54008	27	.72503	42	13
14	.52362	28	.70008	43	.54035	27	.72545	42	14
15	9.52390	28	9.70050	42	9.54063	27	9.72587	42	15
16	.52418	28	.70092	42	.54090	27	.72630	42	16
17	.52446	28	.70135	43	.54118	27	.72672	42	17
18	.52474	28	.70177	42	.54145	27	.72714	42	18
19	.52503	28	.70220	43	.54173	27	.72756	42	19
20	9.52531	28	9.70262	42	9.54200	27	9.72799	42	20
21	.52559	28	.70304	43	.54228	27	.72841	42	21
22	.52587	28	.70347	42	.54255	27	.72883	42	22
23	.52615	28	.70389	42	.54283	27	.72925	42	23
24	.52643	28	.70431	43	.54310	27	.72967	42	24
25	9.52671	28	9.70474	42	9.54338	27	9.73010	42	25
26	.52699	28	.70516	43	.54365	27	.73052	42	26
27	.52727	28	.70558	42	.54393	27	.73094	42	27
28	.52756	28	.70601	43	.54420	27	.73136	42	28
29	.52784	28	.70643	42	.54448	27	.73178	42	29
30	9.52812	28	9.70685	43	9.54475	27	9.73221	42	30
31	.52840	28	.70728	43	.54502	27	.73263	42	31
32	.52868	28	.70770	42	.54530	27	.73305	42	32
33	.52896	28	.70812	43	.54557	27	.73347	42	33
34	.52924	28	.70854	42	.54585	27	.73389	42	34
35	9.52952	28	9.70897	43	9.54612	27	9.73431	42	35
36	.52980	28	.70939	42	.54639	27	.73474	42	36
37	.53008	28	.70981	43	.54667	27	.73516	42	37
38	.53036	28	.71024	43	.54694	27	.73558	42	38
39	.53064	28	.71066	42	.54721	27	.73600	42	39
40	9.53092	28	9.71108	42	9.54748	27	9.73642	42	40
41	.53120	28	.71151	43	.54776	27	.73685	42	41
42	.53147	27	.71193	42	.54803	27	.73727	42	42
43	.53175	28	.71235	42	.54830	27	.73769	42	43
44	.53203	28	.71278	43	.54858	27	.73811	42	44
45	9.53231	28	9.71320	42	9.54885	27	9.73853	42	45
46	.53259	27	.71362	42	.54912	27	.73895	42	46
47	.53287	28	.71404	42	.54939	27	.73938	42	47
48	.53315	28	.71447	43	.54967	27	.73980	42	48
49	.53343	28	.71489	42	.54994	27	.74022	42	49
50	9.53370	27	9.71531	42	9.55021	27	9.74064	42	50
51	.53398	28	.71573	43	.55048	27	.74106	42	51
52	.53426	28	.71616	43	.55075	27	.74148	42	52
53	.53454	27	.71658	42	.55103	27	.74191	42	53
54	.53482	28	.71700	42	.55130	27	.74233	42	54
55	9.53509	27	9.71743	43	9.55157	27	9.74275	42	55
56	.53537	28	.71785	42	.55184	27	.74317	42	56
57	.53565	27	.71827	42	.55211	27	.74359	42	57
58	.53593	28	.71869	42	.55238	27	.74401	42	58
59	.53620	27	.71912	43	.55265	27	.74444	42	59
	.53648	28	9.71954	42	9.55292	27	9.74486	42	60
	Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.		
	42	42
6	4.2	4.2
7	4.9	4.9
8	5.6	5.6
9	6.4	6.3
10	7.1	7.0
20	14.1	14.0
30	21.2	21.0
40	28.3	28.0
50	35.4	35.0
	28	28
6	2.8	2.8
7	3.3	3.3
8	3.8	3.7
9	4.3	4.2
10	4.7	4.7
20	9.5	9.3
30	14.2	14.0
40	19.0	18.6
50	23.7	23.3
	27	27
6	2.7	2.7
7	3.2	3.1
8	3.6	3.6
9	4.1	4.0
10	4.6	4.5
20	9.2	9.0
30	13.7	13.5
40	18.3	18.0
50	22.9	22.5

Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs	D	′	P. P.
		50°				51°			
65292	27	9·74486	42	9·56900	26	9·77012	42	0	
65319	27	·74528	42	·56926	26	·77055	42	1	
65347	27	·74570	42	·56953	26	·77097	42	2	
65374	27	·74612	42	·56979	26	·77139	42	3	
65401	27	·74654	42	·57005	26	·77181	42	4	
65428	27	9·74696	42	9·57032	26	9·77223	42	5	42 42
65455	27	·74739	42	·57058	26	·77265	42	6	6 4·2 4·2
65482	27	·74781	42	·57085	26	·77307	42	7	7 4·9 4·9
65509	27	·74823	42	·57111	26	·77349	42	8	8 5·6 5·6
65538	27	·74865	42	·57138	26	·77391	42	9	9 6·4 6·3
65563	27	9·74907	42	9·57164	26	9·77433	42	10	10 7·1 7·0
65590	27	·74949	42	·57190	26	·77475	42	11	20 14·2 14·0
65617	27	·74991	42	·57217	26	·77517	42	12	30 21·2 21·0
65644	27	·75033	42	·57243	26	·77560	42	13	40 28·3 28·0
65671	27	·75076	42	·57269	26	·77602	42	14	50 35·4 35·0
65698	27	9·75118	42	9·57296	26	9·77644	42	15	
65725	27	·75160	42	·57322	26	·77686	42	16	
65751	26	·75202	42	·57348	26	·77728	42	17	
65778	27	·75244	42	·57375	26	·77770	42	18	
65805	27	·75286	42	·57401	26	·77812	42	19	
65832	27	9·75328	42	9·57427	26	9·77854	42	20	27 27
65859	27	·75370	42	·57454	26	·77896	42	21	6 2·7 2·7
65886	27	·75413	42	·57480	26	·77938	42	22	7 3·2 3·1
65913	27	·75455	42	·57506	26	·77980	42	23	8 3·6 3·6
65940	27	·75497	42	·57532	26	·78022	42	24	9 4·1 4·0
65966	26	9·75539	42	9·57559	26	9·78064	42	25	10 4·6 4·5
65993	27	·75581	42	·57585	26	·78107	42	26	20 9·1 9·0
66020	27	·75623	42	·57611	26	·78149	42	27	30 13·7 13·5
66047	26	·75665	42	·57637	26	·78191	42	28	40 18·3 18·0
66074	27	·75707	42	·57664	26	·78233	42	29	50 22·9 22·5
66101	27	9·75750	42	9·57690	26	9·78275	42	30	
66127	26	·75792	42	·57716	26	·78317	42	31	
66154	27	·75834	42	·57742	26	·78359	42	32	
66181	27	·75876	42	·57768	26	·78401	42	33	
66208	27	·75918	42	·57794	26	·78443	42	34	26 26
66234	26	9·75960	42	9·57821	26	9·78485	42	35	6 2·6 2·6
66261	27	·76002	42	·57847	26	·78527	42	36	7 3·1 3·0
66288	27	·76044	42	·57873	26	·78569	42	37	8 3·5 3·4
66315	27	·76086	42	·57899	26	·78611	42	38	9 4·0 3·9
66341	26	·76128	42	·57925	26	·78653	42	39	10 4·4 4·3
66368	27	9·76171	42	9·57951	26	9·78696	42	40	20 8·8 8·6
66395	27	·76213	42	·57977	26	·78738	42	41	30 13·2 13·0
66421	26	·76255	42	·58003	26	·78780	42	42	40 17·6 17·3
66448	27	·76297	42	·58029	26	·78822	42	43	50 22·1 21·6
66475	27	·76339	42	·58055	26	·78864	42	44	
66501	26	9·76381	42	9·58082	26	9·78906	42	45	
66528	27	·76423	42	·58108	26	·78948	42	46	
66554	26	·76465	42	·58134	26	·78990	42	47	
66581	27	·76507	42	·58160	26	·79032	42	48	25
66608	27	·76548	42	·58186	26	·79074	42	49	6 2·5
66634	26	9·76592	42	9·58212	26	9·79116	42	50	7 3·0
66661	27	·76634	42	·58238	26	·79158	42	51	8 3·4
66687	26	·76676	42	·58264	26	·79200	42	52	9 3·9
66714	27	·76718	42	·58290	26	·79242	42	53	10 4·3
66741	27	·76760	42	·58316	26	·79285	42	54	20 8·5
66767	26	9·76802	42	9·58342	26	9·79327	42	55	30 12·7
66794	27	·76844	42	·58367	25	·79369	42	56	40 17·0
66820	26	·76886	42	·58393	26	·79411	42	57	50 21·2
66847	26	·76928	42	·58419	26	·79453	42	58	
66873	26	·76970	42	·58445	26	·79495	42	59	
66900	26	9·77012	42	9·58471	26	9·79537	42	60	
Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.

	52°				53°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9.58471	26	9.79537	42	9.60008	25	9.82062	42	0
1	.58497	25	.79579	42	.60034	26	.82104	42	1
2	.58523	26	.79621	42	.60059	25	.82146	42	2
3	.58549	26	.79663	42	.60084	25	.82188	42	3
4	.58575	26	.79705	42	.60110	25	.82230	42	4
5	9.58601	26	9.79747	42	9.60135	25	9.82272	42	5
6	.58626	25	.79789	42	.60160	25	.82315	42	6
7	.58652	26	.79831	42	.60185	25	.82357	42	7
8	.58678	26	.79874	42	.60211	25	.82399	42	8
9	.58704	25	.79916	42	.60236	25	.82441	42	9
10	9.58730	26	9.79958	42	9.60261	25	9.82483	42	10
11	.58755	25	.80000	42	.60286	25	.82525	42	11
12	.58781	26	.80042	42	.60312	25	.82567	42	12
13	.58807	26	.80084	42	.60337	25	.82609	42	13
14	.58833	25	.80126	42	.60362	25	.82651	42	14
15	9.58859	26	9.80168	42	9.60387	25	9.82694	42	15
16	.58884	25	.80210	42	.60412	25	.82736	42	16
17	.58910	26	.80252	42	.60438	25	.82778	42	17
18	.58936	26	.80294	42	.60463	25	.82820	42	18
19	.58962	26	.80336	42	.60488	25	.82862	42	19
20	9.58987	25	9.80378	42	9.60513	25	9.82904	42	20
21	.59013	26	.80420	42	.60538	25	.82946	42	21
22	.59039	26	.80463	42	.60563	25	.82988	42	22
23	.59064	25	.80505	42	.60589	26	.83031	42	23
24	.59090	26	.80547	42	.60614	25	.83073	42	24
25	9.59116	26	9.80589	42	9.60639	25	9.83115	42	25
26	.59141	25	.80631	42	.60664	25	.83157	42	26
27	.59167	26	.80673	42	.60689	25	.83199	42	27
28	.59193	26	.80715	42	.60714	25	.83241	42	28
29	.59218	25	.80757	42	.60739	25	.83283	42	29
30	9.59244	26	9.80799	42	9.60764	25	9.83325	42	30
31	.59270	26	.80841	42	.60789	25	.83368	42	31
32	.59295	25	.80883	42	.60814	25	.83410	42	32
33	.59321	26	.80925	42	.60839	25	.83452	42	33
34	.59346	25	.80968	42	.60864	25	.83494	42	34
35	9.59372	26	9.81010	42	9.60889	25	9.83536	42	35
36	.59397	25	.81052	42	.60914	25	.83578	42	36
37	.59423	26	.81094	42	.60939	25	.83620	42	37
38	.59449	26	.81136	42	.60964	25	.83663	42	38
39	.59474	25	.81178	42	.60989	25	.83705	42	39
40	9.59500	26	9.81220	42	9.61014	25	9.83747	42	40
41	.59525	25	.81262	42	.61039	25	.83789	42	41
42	.59551	26	.81304	42	.61064	25	.83831	42	42
43	.59576	25	.81346	42	.61089	25	.83873	42	43
44	.59602	26	.81388	42	.61114	25	.83916	42	44
45	9.59627	25	9.81430	42	9.61139	25	9.83958	42	45
46	.59653	26	.81473	42	.61164	25	.84000	42	46
47	.59678	25	.81515	42	.61189	24	.84042	42	47
48	.59704	25	.81557	42	.61214	25	.84084	42	48
49	.59729	25	.81599	42	.61239	25	.84126	42	49
50	9.59754	25	9.81641	42	9.61264	25	9.84168	42	50
51	.59780	26	.81683	42	.61289	25	.84211	42	51
52	.59805	25	.81725	42	.61313	24	.84253	42	52
53	.59831	26	.81767	42	.61338	25	.84295	42	53
54	.59856	25	.81809	42	.61363	25	.84337	42	54
55	9.59881	25	9.81851	42	9.61388	25	9.84379	42	55
56	.59907	26	.81894	42	.61413	24	.84422	42	56
57	.59932	25	.81936	42	.61438	25	.84464	42	57
58	.59958	26	.81978	42	.61462	24	.84506	42	58
59	.59983	25	.82020	42	.61487	25	.84548	42	59
		25	9.82062	42	9.61512	25	9.84590	42	60
		D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	42	42
6	4.2	4.2
7	4.9	4.9
8	5.6	5.6
9	6.4	6.3
10	7.1	7.0
20	14.1	14.0
30	21.2	21.0
40	28.3	28.0
50	35.4	35.0

	26	25
6	2.6	2.5
7	3.0	3.0
8	3.4	3.4
9	3.9	3.7
10	4.3	4.2
20	8.6	8.5
30	13.0	12.7
40	17.3	17.0
50	21.6	21.2

	25	24
6	2.5	2.4
7	2.9	2.8
8	3.3	3.2
9	3.7	3.7
10	4.1	4.1
20	8.3	8.2
30	12.5	12.3
40	16.6	16.3
50	20.8	20.4

′	54° Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	55° Log. Exs.	D	′	P. P.
0	9·61512	24	9·84590	42	9·62984	24	9·87125	42	0	
1	·61537	25	·84632	42	·63008	24	·87167	42	1	
2	·61562	24	·84675	42	·63032	24	·87209	42	2	
3	·61586	25	·84717	42	·63057	24	·87252	42	3	
4	·61611	24	·84759	42	·63081	24	·87294	42	4	
5	9·61636	25	9·84801	42	9·63105	24	9·87336	42	5	
6	·61661	24	·84843	42	·63129	24	·87379	42	6	
7	·61685	25	·84886	42	·63154	24	·87421	42	7	
8	·61710	24	·84928	42	·63178	24	·87463	42	8	
9	·61735	24	·84970	42	·63202	24	·87506	42	9	42 42
10	9·61760	25	9·85012	42	9·63226	24	9·87548	42	10	6 4·2 4·2
11	·61784	24	·85054	42	·63250	24	·87590	42	11	7 4·9 4·9
12	·61809	24	·85097	42	·63274	24	·87633	42	12	8 5·6 5·6
13	·61834	25	·85139	42	·63298	24	·87675	42	13	9 6·4 6·3
14	·61858	24	·85181	42	·63323	24	·87717	42	14	10 7·1 7·0
15	9·61883	25	9·85223	42	9·63347	24	9·87760	42	15	20 14·2 14·0
16	·61908	24	·85265	42	·63371	24	·87802	42	16	30 21·2 21·0
17	·61932	24	·85308	42	·63395	24	·87844	42	17	40 28·3 28·0
18	·61957	25	·85350	42	·63419	24	·87887	42	18	50 35·4 35·0
19	·61982	24	·85392	42	·63443	24	·87929	42	19	
20	9·62006	24	9·85434	42	9·63468	24	9·87971	42	20	
21	·62031	24	·85476	42	·63492	24	·88014	42	21	
22	·62055	25	·85519	42	·63516	24	·88056	42	22	
23	·62080	24	·85561	42	·63540	24	·88099	42	23	
24	·62105	24	·85603	42	·63564	24	·88141	42	24	
25	9·62129	24	9·85645	42	9·63588	24	9·88183	42	25	
26	·62154	24	·85688	42	·63612	24	·88226	42	26	
27	·62178	24	·85730	42	·63636	24	·88268	42	27	25 24
28	·62203	24	·85772	42	·63660	24	·88310	42	28	6 2·5 2·4
29	·62227	24	·85814	42	·63684	24	·88353	42	29	7 2·9 2·8
30	9·62252	24	9·85857	42	9·63709	24	9·88395	42	30	8 3·3 3·2
31	·62276	24	·85899	42	·63732	24	·88438	42	31	9 3·7 3·7
32	·62301	24	·85941	42	·63756	24	·88480	42	32	10 4·1 4·1
33	·62325	24	·85983	42	·63780	24	·88522	42	33	20 8·3 8·1
34	·62350	24	·86026	42	·63804	24	·88565	42	34	30 12·5 12·2
35	9·62374	24	9·86068	42	9·63828	24	9·88607	42	35	40 16·6 16·3
36	·62399	24	·86110	42	·63852	24	·88650	42	36	50 20·8 20·4
37	·62423	24	·86152	42	·63876	24	·88692	42	37	
38	·62448	24	·86195	42	·63900	24	·88734	42	38	
39	·62472	24	·86237	42	·63924	24	·88777	42	39	
40	9·62497	24	9·86279	42	9·63948	24	9·88819	42	40	
41	·62521	24	·86321	42	·63972	24	·88862	42	41	
42	·62546	24	·86364	42	·63996	24	·88904	42	42	
43	·62570	24	·86406	42	·64019	23	·88947	42	43	
44	·62594	24	·86448	42	·64043	24	·88989	42	44	
45	9·62619	24	9·86490	42	9·64067	24	9·89031	42	45	24 23
46	·62643	24	·86533	42	·64091	24	·89074	42	46	6 2·4 2·3
47	·62668	24	·86575	42	·64115	24	·89116	42	47	7 2·8 2·7
48	·62692	24	·86617	42	·64139	24	·89159	42	48	8 3·2 3·1
49	·62716	24	·86659	42	·64163	24	·89201	42	49	9 3·6 3·5
50	9·62741	24	9·86702	42	9·64187	24	9·89244	42	50	10 4·0 3·9
51	·62765	24	·86744	42	·64210	23	·89286	42	51	20 8·0 7·8
52	·62789	24	·86786	42	·64234	24	·89329	42	52	30 12·0 11·7
53	·62814	24	·86829	42	·64258	24	·89371	42	53	40 16·0 15·6
54	·62838	24	·86871	42	·64282	24	·89414	42	54	50 20·0 19·6
55	9·62862	24	9·86913	42	9·64306	24	9·89456	42	55	
56	·62887	24	·86956	42	·64330	24	·89499	42	56	
57	·62911	24	·86998	42	·64353	23	·89541	42	57	
58	·62935	24	·87040	42	·64377	24	·89583	42	58	
59	·62960	24	·87082	42	·64401	24	·89626	42	59	
60	9·62984		9·87125		9·64425		9·89668		60	
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.

′	68° Lg. Vers.	D	Log. Exs.	D	69° Lg. Vers.	D	Log. Exs.	D	′	P. P.
0	9·79615	18	10·22258	50	9·80728	18	10·25295	51	0	53 52
1	·79634	19	·22308	50	·80747	18	·25347	51	1	6 5·3 5·2
2	·79653	18	·22358	50	·80765	18	·25398	51	2	7 6·2 6·1
3	·79671	18	·22408	50	·80783	18	·25449	51	3	8 7·0 7·0
4	·79690	19	·22458	50	·80802	18	·25501	51	4	9 7·9 7·9
5	9·79709	18	10·22508	50	9·80820	18	10·25552	51	5	10 8·8 8·7
6	·79727	19	·22558	50	·80839	18	·25604	51	6	20 17·6 17·5
7	·79746	18	·22608	50	·80857	18	·25655	51	7	30 26·5 26·2
8	·79765	18	·22658	50	·80875	18	·25707	51	8	40 35·3 35·0
9	·79783	18	·22708	50	·80894	18	·25758	51	9	50 44·1 43·7
10	9·79802	19	10·22759	50	9·80912	18	10·25810	51	10	52 51
11	·79821	18	·22809	50	·80930	18	·25861	51	11	6 5·2 5·1
12	·79839	19	·22859	50	·80949	18	·25913	51	12	7 6·0 6·0
13	·79858	18	·22909	50	·80967	18	·25964	51	13	8 6·9 6·8
14	·79877	18	·22960	50	·80985	18	·26016	52	14	9 7·8 7·7
15	9·79895	18	10·23010	50	9·81003	18	10·26067	51	15	10 8·6 8·6
16	·79914	19	·23060	50	·81022	18	·26119	52	16	20 17·3 17·1
17	·79933	18	·23110	50	·81040	18	·26171	52	17	30 26·0 25·7
18	·79951	18	·23161	50	·81058	18	·26222	52	18	40 34·6 34·3
19	·79970	18	·23211	50	·81077	18	·26274	52	19	50 43·3 42·9
20	9·79988	18	10·23262	50	9·81095	18	10·26326	52	20	51 50
21	·80007	19	·23312	50	·81113	18	·26378	52	21	6 5·1 5·0
22	·80026	18	·23362	50	·81131	18	·26429	52	22	7 5·9 5·9
23	·80044	18	·23413	50	·81150	18	·26481	52	23	8 6·8 6·7
24	·80063	18	·23463	50	·81168	18	·26533	52	24	9 7·6 7·6
25	9·80081	18	10·23514	50	9·81186	18	10·26585	52	25	10 8·5 8·4
26	·80100	19	·23564	50	·81204	18	·26637	52	26	20 17·0 16·8
27	·80119	18	·23615	50	·81223	18	·26689	52	27	30 25·5 25·2
28	·80137	18	·23666	50	·81241	18	·26741	52	28	40 34·0 33·6
29	·80156	18	·23716	50	·81259	18	·26793	52	29	50 42·5 42·1
30	9·80174	18	10·23767	50	9·81277	18	10·26845	52	30	50
31	·80193	19	·23817	50	·81295	18	·26897	52	31	6 5·0
32	·80211	18	·23868	51	·81314	18	·26949	52	32	7 5·8
33	·80230	18	·23919	50	·81332	18	·27001	52	33	8 6·6
34	·80248	18	·23969	50	·81350	18	·27053	52	34	9 7·5
35	9·80267	19	10·24020	51	9·81368	18	10·27105	52	35	10 8·3
36	·80286	18	·24071	50	·81386	18	·27157	52	36	20 16·6
37	·80304	18	·24122	51	·81405	18	·27209	52	37	30 25·0
38	·80323	18	·24172	50	·81423	18	·27261	52	38	40 33·3
39	·80341	18	·24223	51	·81441	18	·27314	53	39	50 41·6
40	9·80360	18	10·24274	51	9·81459	18	10·27366	52	40	19 18
41	·80378	18	·24325	50	·81477	18	·27418	52	41	6 1·9 1·8
42	·80397	18	·24376	51	·81495	18	·27470	52	42	7 2·2 2·1
43	·80415	18	·24427	51	·81513	18	·27523	52	43	8 2·5 2·4
44	·80434	18	·24478	51	·81532	18	·27575	52	44	9 2·8 2·7
45	9·80452	18	10·24529	51	9·81550	18	10·27627	52	45	10 3·1 3·0
46	·80470	18	·24580	51	·81568	18	·27680	53	46	20 6·3 6·0
47	·80489	18	·24631	51	·81586	18	·27732	52	47	30 9·5 9·0
48	·80507	18	·24682	51	·81604	18	·27785	52	48	40 12·6 12·0
49	·80526	18	·24733	51	·81622	18	·27837	52	49	50 15·8 15·0
50	9·80544	18	10·24784	51	9·81640	18	10·27890	53	50	18
51	·80563	18	·24835	51	·81658	18	·27942	52	51	6 1·8
52	·80587	18	·24886	51	·81676	18	·27995	53	52	7 2·1
53	·80600	18	·24937	51	·81695	18	·28047	52	53	8 2·4
54	·80618	18	·24988	51	·81713	18	·28100	53	54	9 2·7
55	9·80636	18	10·25039	51	9·81731	18	10·28152	52	55	10 3·0
56	·80655	18	·25090	51	·81749	18	·28205	53	56	20 6·0
57	·80673	18	·25142	51	·81767	18	·28258	53	57	30 9·0
58	·80692	18	·25193	51	·81785	18	·28310	52	58	40 12·0
59	·80710	18	·25244	51	·81803	18	·28363	53	59	50 15·0
60	9·80728	18	10·25295	51	9·81821	18	10·28416	53	60	P. P.
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	

58°			59°					
D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.
33	9.94796	43	9.68571	22	9.97387	43	0	
33	.94839	43	.68593	22	.97430	43	1	
33	.94882	43	.68615	22	.97473	43	2	
33	.94925	43	.68637	22	.97517	43	3	
33	.94968	43	.68660	22	.97560	43	4	44 43
33	9.95011	43	9.68682	22	9.97603	43	5	6 4.4 4.3
33	.95054	43	.68704	22	.97647	43	6	7 5.1 5.1
33	.95097	43	.68727	22	.97690	43	7	8 5.8 5.8
33	.95140	43	.68749	22	.97734	43	8	9 6.6 6.5
33	.95183	43	.68771	22	.97777	43	9	10 7.3 7.2
33	9.95226	43	9.68793	22	9.97820	43	10	20 14.6 14.5
33	.95269	43	.68816	22	.97864	43	11	30 22.0 21.7
33	.95313	43	.68838	22	.97907	43	12	40 29.3 29.0
33	.95356	43	.68860	22	.97951	43	13	50 36.6 36.2
33	.95399	43	.68882	22	.97994	43	14	
33	9.95442	43	9.68905	22	9.98038	43	15	
33	.95485	43	.68927	22	.98081	43	16	
33	.95528	43	.68949	22	.98125	43	17	
33	.95571	43	.68971	22	.98168	43	18	43
33	.95614	43	.68993	22	.98211	43	19	6 4.3
33	9.95657	43	9.69016	22	9.98255	43	20	7 5.0
33	.95700	43	.69038	22	.98298	43	21	8 5.7
33	.95744	43	.69060	22	.98342	43	22	9 6.4
33	.95787	43	.69082	22	.98385	43	23	10 7.1
33	.95830	43	.69104	22	.98429	43	24	20 14.3
33	9.95873	43	9.69126	22	9.98472	43	25	30 21.5
33	.95916	43	.69149	22	.98516	43	26	40 28.6
33	.95959	43	.69171	22	.98559	43	27	50 35.8
33	.96002	43	.69193	22	.98603	44	28	
33	.96046	43	.69215	22	.98647	43	29	
33	9.96089	43	9.69237	22	9.98690	43	30	
33	.96132	43	.69259	22	.98734	43	31	
33	.96175	43	.69281	22	.98777	43	32	
33	.96218	43	.69303	22	.98821	43	33	23 22
33	.96261	43	.69325	22	.98864	43	34	6 2.3 2.2
33	9.96305	43	9.69347	22	9.98908	44	35	7 2.7 2.6
33	.96348	43	.69369	22	.98952	43	36	8 3.0 3.0
33	.96391	43	.69392	22	.98995	43	37	9 3.4 3.4
33	.96434	43	.69414	22	.99039	43	38	10 3.8 3.7
33	.96478	43	.69436	22	.99082	43	39	20 7.6 7.5
33	9.96521	43	9.69458	22	9.99126	44	40	30 11.5 11.2
33	.96564	43	.69480	22	.99170	43	41	40 15.3 15.0
33	.96607	43	.69502	22	.99213	43	42	50 19.1 18.7
33	.96650	43	.69524	22	.99257	43	43	
33	.96694	43	.69546	22	.99300	44	44	
33	9.96737	43	9.69568	22	9.99344	44	45	
33	.96780	43	.69590	22	.99388	43	46	
33	.96824	43	.69612	22	.99431	44	47	
33	.96867	43	.69634	22	.99475	43	48	22 21
33	.96910	43	.69656	22	.99519	43	49	6 2.2 2.1
33	9.96953	43	9.69673	22	9.99562	43	50	7 2.5 2.5
33	.96997	43	.69700	21	.99606	44	51	8 2.9 2.8
33	.97040	43	.69721	22	.99650	43	52	9 3.3 3.2
33	.97083	43	.69743	22	.99694	44	53	10 3.6 3.6
33	.97127	43	.69765	22	.99737	43	54	20 7.3 7.1
33	9.97170	43	9.69787	22	9.99781	44	55	30 11.0 10.7
33	.97213	43	.69809	22	.99825	43	56	40 14.6 14.3
33	.97257	43	.69831	21	.99868	44	57	50 18.3 17.9
33	.97300	43	.69853	22	.99912	43	58	
33	.97343	43	.69875	22	9.99956	44	59	
	9.97387		.69897		10.00000		60	
D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.

′	72° Lg. Vers.	D	Log. Exs.	D	73° Lg. Vers.	D	Log. Exs.	D	′	P. P.
0	9.83946	17	10.34948	56	9.84980	17	10.38387	53	0	61 60
1	.83964	17	.35005	56	.84997	17	.38445	58	1	6 6.1 6.0
2	.83981	17	.35061	56	.85014	17	.38504	58	2	7 7.1 7.0
3	.83999	17	.35117	56	.85031	17	.38562	58	3	8 8.1 8.0
4	.84016	17	.35174	56	.85049	17	.38621	58	4	9 9.1 9.0
5	9.84033	17	10.35230	56	9.85066	17	10.38679	58	5	10 10.1 10.0
6	.84051	17	.35286	56	.85083	17	.38738	58	6	20 20.3 20.1
7	.84068	17	.35343	56	.85100	17	.38796	58	7	30 30.5 30.2
8	.84085	17	.35399	56	.85117	17	.38855	59	8	40 40.6 40.3
9	.84103	17	.35456	57	.85134	17	.38914	58	9	50 50.8 50.4
10	9.84120	17	10.35513	56	9.85151	17	10.38973	59	10	60 59
11	.84137	17	.35569	56	.85168	17	.39031	58	11	6 6.0 5.9
12	.84155	17	.35626	56	.85185	17	.39090	59	12	7 7.0 6.9
13	.84172	17	.35683	57	.85202	17	.39149	59	13	8 8.0 7.9
14	.84189	17	.35739	56	.85219	17	.39208	58	14	9 9.0 8.9
15	9.84207	17	10.35796	57	9.85236	17	10.39267	59	15	10 10.0 9.9
16	.84224	17	.35853	56	.85253	17	.39326	59	16	20 20.0 19.8
17	.84241	17	.35910	57	.85270	17	.39385	59	17	30 30.0 29.7
18	.84259	17	.35967	57	.85287	17	.39444	59	18	40 40.0 39.6
19	.84276	17	.36023	56	.85304	17	.39503	59	19	50 50.0 49.6
20	9.84293	17	10.36080	57	9.85321	17	10.39562	59	20	
21	.84310	17	.36137	57	.85338	17	.39621	59	21	59 58
22	.84328	17	.36194	57	.85355	17	.39681	59	22	6 5.9 5.8
23	.84345	17	.36251	57	.85372	17	.39740	59	23	7 6.9 6.8
24	.84362	17	.36308	57	.85389	17	.39799	59	24	8 7.8 7.8
25	9.84380	17	10.36366	57	9.85405	16	10.39859	59	25	9 8.8 8.8
26	.84397	17	.36423	57	.85422	17	.39918	59	26	10 9.8 9.7
27	.84414	17	.36480	57	.85439	17	.39977	59	27	20 19.6 19.5
28	.84431	17	.36537	57	.85456	17	.40037	59	28	30 29.5 29.2
29	.84449	17	.36594	57	.85473	17	.40096	59	29	40 39.3 39.0
30	9.84466	17	10.36652	57	9.85490	17	10.40156	60	30	50 49.1 48.7
31	.84483	17	.36709	57	.85507	16	.40216	59	31	58 57
32	.84500	17	.36766	57	.85524	17	.40275	59	32	6 5.8 5.7
33	.84517	17	.36824	57	.85541	17	.40335	60	33	7 6.7 6.7
34	.84535	17	.36881	57	.85558	17	.40395	60	34	8 7.7 7.6
35	9.84552	17	10.36938	57	9.85575	17	10.40454	59	35	9 8.7 8.6
36	.84569	17	.36996	58	.85592	16	.40514	60	36	10 9.6 9.5
37	.84586	17	.37054	58	.85608	17	.40574	60	37	20 19.3 19.1
38	.84603	17	.37111	57	.85625	17	.40634	60	38	30 29.0 28.7
39	.84620	17	.37169	58	.85642	17	.40694	60	39	40 38.6 38.3
40	9.84638	17	10.37226	57	9.85659	16	10.40754	60	40	50 48.3 47.9
41	.84655	17	.37284	57	.85676	17	.40814	60	41	57 56
42	.84672	17	.37342	58	.85693	17	.40874	60	42	6 5.7 5.6
43	.84689	17	.37399	57	.85710	16	.40934	60	43	7 6.6 6.6
44	.84706	17	.37457	58	.85726	17	.40994	60	44	9 7.6 7.5
45	9.84724	17	10.37515	58	9.85743	17	10.41054	60	45	9 8.5 8.5
46	.84741	17	.37573	57	.85760	17	.41114	60	46	10 9.5 9.4
47	.84758	17	.37631	58	.85777	17	.41174	60	47	20 19.0 18.8
48	.84775	17	.37689	58	.85794	17	.41235	60	48	30 28.5 28.2
49	.84792	17	.37747	58	.85811	16	.41295	60	49	40 38.0 37.6
50	9.84809	17	10.37805	58	9.85827	17	10.41355	60	50	50 47.6 47.1
51	.84826	17	.37863	58	.85844	17	.41416	60	51	
52	.84844	17	.37921	58	.85861	16	.41476	60	52	17 17 16
53	.84861	17	.37979	58	.85878	17	.41537	60	53	6 1.7 1.7 1.6
54	.84878	17	.38037	58	.85895	17	.41597	60	54	7 2.0 2.0 1.8
55	9.84895	17	10.38095	58	9.85911	16	10.41658	61	55	8 2.3 2.2 2.1
56	.84912	17	.38153	58	.85928	17	.41719	60	56	9 2.6 2.5 2.4
57	.84929	17	.38212	58	.85945	17	.41779	60	57	10 2.9 2.8 2.7
58	.84946	17	.38270	58	.85962	17	.41840	61	58	20 5.8 5.6 5.4
59	.84963	17	.38328	58	.85979	17	.41901	61	59	30 8.7 8.5 8.1
60	9.84980		10.38387		9.85995	16	10.41962	61	60	40 11.6 11.3 11.1
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	50 14.6 14.1 13.8
										P. P.

62° 63°

D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.
21	10.05310	44	9.73720	20	10.08015	45	0	
21	.05354	45	.73740	20	.08061	45	1	
21	.05399	45	.73761	21	.08106	45	2	
21	.05444	44	.73782	20	.08151	45	3	
	.05489		.73802		.08197		4	46 46
21	10.05534	45	9.73823	20	10.08242	45		6 4.6 4.6
21	.05579	45	.73843	20	.08288	45	5	7 5.4 5.3
21	.05623	44	.73864	20	.08333	45	6	8 6.2 6.1
21	.05668	45	.73884	21	.08379	45	7	9 7.0 6.9
21	.05713	45	.73905	20	.08424	45	8	10 7.7 7.6
21	10.05758	45	9.73926	20	10.08470	45	9	20 15.5 15.3
20	.05803	44	.73946	20	.08515	45	10	30 23.2 23.0
21	.05848	45	.73967	20	.08561	45	11	40 31.0 30.6
21	.05893	45	.73987	20	.08606	45	12	50 38.7 38.3
21	.05938	45	.74008	20	.08652	45	13	
21	10.05983	45	9.74028	20	10.08697	45	14	
21	.06028	45	.74049	20	.08743	46	15	45 45
21	.06072	44	.74069	20	.08789	45	16	6 4.5 4.5
20	.06117	45	.74090	20	.08834	45	17	7 5.3 5.2
21	.06162	45	.74110	20	.08880	45	18	8 6.0 6.0
21	10.06207	45	9.74131	20	10.08926	46	19	9 6.8 6.7
21	.06252	45	.74151	20	.08971	45	20	10 7.6 7.5
20	.06297	45	.74172	20	.09017	46	21	20 15.1 15.0
21	.06342	45	.74192	20	.09062	45	22	30 22.7 22.5
21	.06387	45	.74213	20	.09108	46	23	40 30.3 30.0
21	10.06432	45	9.74233	20	10.09154	46	24	50 37.9 37.5
20	.06477	45	.74254	20	.09200	46	25	
21	.06522	45	.74274	20	.09245	45	26	
21	.06568	45	.74294	20	.09291	46	27	44
20	.06613	45	.74315	20	.09337	46	28	6 4.4
21	10.06658	45	9.74335	20	10.09382	45	29	7 5.2
21	.06703	45	.74356	20	.09428	46	30	8 5.9
20	.06748	45	.74376	20	.09474	46	31	9 6.7
21	.06793	45	.74396	20	.09520	45	32	10 7.4
20	.06838	45	.74417	20	.09566	46	33	20 14.8
21	10.06883	45	9.74437	20	10.09611	45	34	30 22.2
21	.06928	45	.74458	20	.09657	46	35	40 29.6
20	.06974	45	.74478	20	.09703	46	36	50 37.1
21	.07019	45	.74498	20	.09749	45	37	
20	.07064	45	.74519	20	.09795	46	38	21 20
2	10.07109	45	9.74539	20	10.09841	46	39	6 2.1 2.0
21	.07154	45	.74559	20	.09886	45	40	7 2.4 2.4
21	.07200	45	.74580	20	.09932	46	41	8 2.8 2.7
20	.07245	45	.74600	20	.09978	46	42	9 3.1 3.1
21	.07290	45	.74620	20	.10024	46	43	10 3.5 3.4
20	10.07335	45	9.74641	20	10.10070	46	44	20 7.0 6.8
20	.07380	45	.74661	20	.10116	46	45	30 10.5 10.2
21	.07426	45	.74681	20	.10162	46	46	40 14.0 13.6
20	.07471	45	.74702	20	.10208	45	47	50 17.5 17.1
21	.07516	45	.74722	20	.10254	46	48	
20	10.07562	45	9.74742	20	10.10300	46	49	20
20	.07607	45	.74762	20	.10346	46	50	6 2.0
21	.07652	45	.74783	20	.10392	46	51	7 2.3
20	.07697	45	.74803	20	.10438	46	52	8 2.6
20	.07743	45	.74823	20	.10484	46	53	9 3.0
21	10.07788	45	9.74844	20	10.10530	46	54	10 3.3
20	.07834	45	.74864	20	.10576	46	55	20 6.6
20	.07879	45	.74884	20	.10622	46	56	30 10.0
21	.07924	45	.74904	20	.10668	46	57	40 13.3
20	.07970	45	.74924	20	.10714	46	58	50 16.6
20	10.08015	45	9.74945	20	10.10760	46	59	
							60	
D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.

	76°				77°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9.87971	16	10.49604	66	9.88933	16	10.53724	70	0
1	.87987	16	.49670	67	.88949	15	.53794	71	1
2	.88003	16	.49737	67	.88964	16	.53865	70	2
3	.88020	16	.49804	67	.88980	16	.53936	71	3
4	.88036	16	.49871	67	.88996	16	.54007	71	4
5	9.88052	16	10.49939	67	9.89012	16	10.54078	71	5
6	.88068	16	.50006	67	.89028	15	.54149	71	6
7	.88084	16	.50073	67	.89044	16	.54220	71	7
8	.88100	16	.50140	67	.89060	16	.54291	71	8
9	.88116	16	.50208	68	.89075	15	.54362	71	9
10	9.88133	16	10.50275	67	9.89091	16	10.54433	71	10
11	.88149	16	.50342	67	.89107	16	.54505	71	11
12	.88165	16	.50410	67	.89123	16	.54576	71	12
13	.88181	16	.50477	68	.89139	16	.54647	72	13
14	.88197	16	.50545	68	.89155	16	.54719	71	14
15	9.88213	16	10.50613	68	9.89170	15	10.54791	72	15
16	.88229	16	.50681	67	.89186	16	.54862	71	16
17	.88245	16	.50748	68	.89202	16	.54934	72	17
18	.88261	16	.50816	68	.89218	16	.55006	71	18
19	.88277	16	.50884	68	.89234	16	.55078	72	19
20	9.88294	16	10.50952	68	9.89249	15	10.55150	72	20
21	.88310	16	.51020	68	.89265	16	.55222	72	21
22	.88326	16	.51088	68	.89281	16	.55294	72	22
23	.88342	16	.51157	68	.89297	16	.55366	72	23
24	.88358	16	.51225	68	.89312	15	.55438	72	24
25	9.88374	16	10.51293	68	9.89328	16	10.55511	73	25
26	.88390	16	.51361	68	.89344	16	.55583	72	26
27	.88406	16	.51430	68	.89360	16	.55655	72	27
28	.88422	16	.51498	68	.89376	16	.55728	73	28
29	.88438	16	.51567	69	.89391	15	.55801	72	29
30	9.88454	16	10.51636	68	9.89407	16	10.55873	73	30
31	.88470	16	.51704	68	.89423	16	.55946	72	31
32	.88486	16	.51773	69	.89439	16	.56019	73	32
33	.88502	16	.51842	69	.89454	15	.56092	73	33
34	.88518	16	.51911	69	.89470	16	.56165	73	34
35	9.88534	16	10.51980	69	9.89486	16	10.56238	73	35
36	.88550	16	.52049	69	.89501	15	.56311	73	36
37	.88566	16	.52118	69	.89517	16	.56384	73	37
38	.88582	16	.52187	69	.89533	16	.56457	73	38
39	.88598	16	.52256	69	.89548	15	.56531	73	39
40	9.88614	16	10.52325	69	9.89564	16	10.56604	73	40
41	.88630	16	.52394	69	.89580	16	.56678	73	41
42	.88646	15	.52464	69	.89596	15	.56751	74	42
43	.88662	16	.52533	69	.89611	15	.56825	73	43
44	.88678	16	.52603	69	.89627	16	.56899	74	44
45	9.88694	16	10.52672	69	9.89643	16	10.56973	74	45
46	.88710	16	.52742	69	.89658	15	.57047	74	46
47	.88726	16	.52812	69	.89674	16	.57120	74	47
48	.88742	16	.52881	70	.89690	15	.57195	74	48
49	.88758	16	.52951	70	.89705	15	.57269	74	49
50	9.88774	16	10.53021	70	9.89721	16	10.57343	74	50
51	.88790	16	.53091	70	.89737	16	.57417	74	51
52	.88805	15	.53161	70	.89752	15	.57491	74	52
53	.88821	16	.53231	70	.89768	15	.57566	74	53
54	.88837	16	.53301	70	.89783	15	.57640	74	54
55	9.88853	16	10.53372	70	9.89799	16	10.57715	75	55
56	.88869	16	.53442	70	.89815	16	.57790	75	56
57	.88885	15	.53513	70	.89830	15	.57864	74	57
58	.88901	16	.53583	70	.89846	15	.57939	75	58
59	.88917	16	.53653	70	.89862	16	.58014	75	59
60	9.88933	16	10.53724	70	9.89877	15	10.58089	75	60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.			
	75	74	73
6	7.5	7.4	7.3
7	8.7	8.6	8.5
8	10.0	9.8	9.7
9	11.2	11.1	10.9
10	12.5	12.3	12.1
20	25.0	24.6	24.3
30	37.5	37.0	36.4
40	50.0	49.3	48.6
50	62.5	61.6	60.8
	72	71	70
6	7.2	7.1	7.0
7	8.4	8.3	8.2
8	9.6	9.4	9.4
9	10.8	10.6	10.5
10	12.0	11.8	11.7
20	24.0	23.6	23.3
30	36.0	35.5	35.0
40	48.0	47.3	47.0
50	60.0	59.1	58.7
	69	68	67
6	6.9	6.8	6.7
7	8.0	7.9	7.8
8	9.2	9.0	8.9
9	10.3	10.2	10.0
10	11.5	11.3	11.1
20	23.0	22.6	22.3
30	34.5	34.0	33.4
40	46.0	45.3	44.6
50	57.5	56.6	55.8
	66	65	
6	6.6	6.5	
7	7.7	7.6	
8	8.8	8.6	
9	9.9	9.8	
10	11.0	10.8	
20	22.0	21.7	
30	33.0	32.5	
40	44.0	43.3	
50	55.0	54.2	
	16	16	15
6	1.6	1.6	1.5
7	1.9	1.8	1.7
8	2.2	2.1	2.0
9	2.5	2.4	2.3
10	2.7	2.7	2.5
20	5.5	5.3	5.1
30	8.2	8.0	7.7
40	11.0	10.7	10.2
50	13.7	13.3	12.8

66° 67°

D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.
19	10.16393	48	9.78481	19	10.19293	49	0	
19	.16441	47	.78500	19	.19342	49	1	
19	.16489	48	.78519	19	.19391	48	2	
19	.16537	48	.78538	19	.19439	49	3	
19	.16585	48	.78557	19	.19488	49	4	50 49
19	10.16633	48	9.78576	19	10.19537	49	5	6 5.0 4.9
19	.16680	47	.78595	19	.19586	49	6	7 5.8 5.8
19	.16728	48	.78614	19	.19635	49	7	8 6.6 6.6
19	.16776	48	.78633	19	.19684	49	8	9 7.5 7.4
19	.16824	48	.78652	19	.19733	49	9	10 8.3 8.2
19	10.16872	48	9.78671	19	10.19782	49	10	20 16.6 16.5
19	.16920	48	.78690	19	.19831	49	11	30 25.0 24.7
19	.16968	48	.78709	19	.19880	49	12	40 33.3 33.0
19	.17016	48	.78728	19	.19929	49	13	50 41.6 41.2
19	.17064	48	.78747	19	.19979	49	14	
19	10.17112	48	9.78766	19	10.20028	49	15	49 48
19	.17160	48	.78785	19	.20077	49	16	6 4.9 4.8
19	.17209	48	.78804	19	.20126	49	17	7 5.7 5.6
19	.17257	48	.78823	19	.20175	49	18	8 6.5 6.4
19	.17305	48	.78842	19	.20224	49	19	9 7.3 7.3
19	10.17353	48	9.78861	19	10.20273	49	20	10 8.1 8.1
19	.17401	48	.78880	19	.20323	49	21	20 16.3 16.1
19	.17449	48	.78899	19	.20372	49	22	30 24.5 24.2
19	.17498	48	.78918	18	.20421	49	23	40 32.6 32.3
19	.17546	48	.78937	19	.20470	49	24	50 40.8 40.4
19	10.17594	48	9.78956	19	10.20520	49	25	
19	.17642	48	.78975	19	.20569	49	26	48 47
19	.17690	48	.78994	19	.20618	49	27	6 4.8 4.7
19	.17739	48	.79013	19	.20668	49	28	7 5.6 5.5
19	.17787	48	.79032	19	.20717	49	29	8 6.4 6.3
19	10.17835	48	9.79051	19	10.20767	49	30	9 7.2 7.1
19	.17884	48	.79069	18	.20816	49	31	10 8.0 7.9
19	.17932	48	.79088	19	.20865	49	32	20 16.0 15.8
19	.17980	48	.79107	19	.20915	49	33	30 24.0 23.7
19	.18029	48	.79126	19	.20964	49	34	40 32.0 31.6
19	10.18077	48	9.79145	19	10.21014	49	35	50 40.0 39.6
19	.18126	48	.79164	18	.21063	49	36	
19	.18174	48	.79183	19	.21113	49	37	19 19
19	.18222	48	.79202	19	.21162	49	38	6 1.9 1.9
19	.18271	48	.79220	18	.21212	50	39	7 2.3 2.2
19	10.18319	48	9.79239	19	10.21262	49	40	8 2.6 2.5
19	.18368	48	.79258	19	.21311	49	41	9 2.9 2.8
19	.18416	48	.79277	18	.21361	49	42	10 3.2 3.1
19	.18465	49	.79296	19	.21410	49	43	20 6.5 6.3
19	.18514	48	.79315	19	.21460	50	44	30 9.7 9.5
19	10.18562	48	9.79333	18	10.21510	49	45	40 13.0 12.6
19	.18611	48	.79352	19	.21560	50	46	50 16.2 15.8
19	.18659	48	.79371	19	.21609	49	47	
19	.18708	48	.79390	18	.21659	50	48	
19	.18757	49	.79409	19	.21709	49	49	18
19	10.18805	48	9.79427	18	10.21759	50	50	6 1.8
19	.18854	48	.79446	19	.21808	49	51	7 2.1
19	.18903	49	.79465	19	.21858	50	52	8 2.4
19	.18951	48	.79484	18	.21908	50	53	9 2.8
19	.19000	49	.79503	19	.21958	49	54	10 3.1
19	10.19049	48	9.79521	18	10.22008	50	55	20 6.1
19	.19098	49	.79540	19	.22058	50	56	30 9.2
19	.19146	48	.79559	18	.22108	50	57	40 12.3
19	.19195	49	.79578	19	.22158	50	58	50 15.4
19	.19244	49	.79596	18	.22208	50	59	
19	10.19293	48	9.79615	19	10.22258	50	60	
D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.

	80°				81°					
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.
0	9.91716		10.67745		9.92612		10.73178		0	90 80
1	.91731	15	.67836	86	.92626	14	.73275	95	1	6 9.0 8.0
2	.91746	15	.67923	87	.92641	15	.73368	94	2	7 10.5 9.3
3	.91761	15	.68010	87	.92656	14	.73463	95	3	8 12.0 10.6
4	.91776	15	.68097	87	.92671	15	.73558	95	4	9 13.5 12.0
5	9.91791	15	10.68184	87	9.92686	15	10.73653	95	5	10 15.0 13.3
6	.91807	15	.68272	87	.92700	14	.73748	95	6	20 30.0 26.6
7	.91822	15	.68359	87	.92715	15	.73844	96	7	30 45.0 40.0
8	.91837	15	.68447	87	.92730	15	.73940	95	8	40 60.0 53.3
9	.91852	15	.68534	87	.92745	14	.74035	96	9	50 75.0 66.6
10	9.91867	15	10.68622	88	9.92759	15	10.74131	96	10	9 8
11	.91882	15	.68710	88	.92774	14	.74227	96	11	6 0.9 0.8
12	.91897	15	.68798	88	.92789	15	.74324	96	12	7 1.0 0.9
13	.91912	15	.68886	88	.92804	14	.74420	96	13	8 1.2 1.0
14	.91927	15	.68975	88	.92818	15	.74517	96	14	9 1.3 1.2
15	9.91942	15	10.69063	88	9.92833	15	10.74613	97	15	10 1.5 1.3
16	.91957	15	.69152	89	.92848	14	.74710	97	16	20 3.0 2.6
17	.91972	15	.69240	88	.92862	15	.74807	97	17	30 4.5 4.0
18	.91987	15	.69329	89	.92877	14	.74905	97	18	40 6.0 5.3
19	.92002	14	.69418	89	.92892	15	.75002	97	19	50 7.5 6.6
20	9.92016	15	10.69507	89	9.92907	14	10.75099	98	20	7 6
21	.92031	15	.69596	89	.92921	15	.75197	98	21	6 0.7 0.6
22	.92046	15	.69686	90	.92936	14	.75295	98	22	7 0.8 0.7
23	.92061	15	.69775	89	.92951	15	.75393	98	23	8 0.9 0.8
24	.92076	15	.69865	90	.92966	14	.75491	98	24	9 1.0 0.9
25	9.92091	15	10.69955	90	9.92980	15	10.75589	98	25	10 1.1 1.0
26	.92106	15	.70044	89	.92995	14	.75688	99	26	20 2.3 2.0
27	.92121	15	.70134	90	.93009	15	.75786	98	27	30 3.5 3.0
28	.92136	15	.70224	90	.93024	14	.75885	99	28	40 4.6 4.0
29	.92151	14	.70315	90	.93039	14	.75984	99	29	50 5.8 5.0
30	9.92166	15	10.70405	90	9.93053	15	10.76083	99	30	5 4
31	.92181	15	.70495	90	.93068	14	.76182	99	31	6 0.5 0.4
32	.92196	15	.70586	91	.93083	14	.76282	100	32	7 0.6 0.4
33	.92211	15	.70677	91	.93097	15	.76382	99	33	8 0.6 0.5
34	.92226	14	.70768	91	.93112	14	.76481	99	34	9 0.7 0.6
35	9.92240	15	10.70859	91	9.93127	14	10.76581	100	35	10 0.8 0.6
36	.92255	15	.70950	91	.93141	14	.76681	100	36	20 1.6 1.3
37	.92270	15	.71041	91	.93156	15	.76782	100	37	30 2.5 2.0
38	.92285	15	.71133	91	.93171	14	.76882	100	38	40 3.3 2.6
39	.92300	14	.71224	91	.93185	14	.76983	100	39	50 4.1 3.3
40	9.92315	15	10.71316	92	9.93200	14	10.77083	101	40	15 15
41	.92330	15	.71408	92	.93214	15	.77184	101	41	6 1.5 1.5
42	.92345	15	.71500	92	.93229	14	.77286	101	42	7 1.8 1.7
43	.92360	14	.71592	92	.93244	14	.77387	101	43	8 2.0 2.0
44	.92374	15	.71684	92	.93258	14	.77488	101	44	9 2.3 2.3
45	9.92389	15	10.71776	92	9.93273	14	10.77590	102	45	10 2.6 2.5
46	.92404	14	.71869	93	.93287	15	.77692	102	46	20 5.1 5.0
47	.92419	15	.71961	92	.93302	14	.77794	102	47	30 7.7 7.5
48	.92434	15	.72054	93	.93317	14	.77896	102	48	40 10.3 10.0
49	.92449	14	.72147	92	.93331	14	.77998	102	49	50 12.9 12.5
50	9.92463	15	10.72240	93	9.93346	14	10.78101	102	50	14
51	.92478	15	.72333	93	.93360	14	.78203	103	51	6 1.4
52	.92493	14	.72427	93	.93375	14	.78306	103	52	7 1.7
53	.92508	15	.72520	93	.93389	15	.78409	103	53	8 1.9
54	.92523	15	.72614	93	.93404	14	.78513	103	54	9 2.2
55	9.92538	15	10.72707	94	9.93419	14	10.78616	104	55	10 2.4
56	.92553	14	.72801	94	.93433	14	.78720	103	56	20 4.8
57	.92567	15	.72895	94	.93448	14	.78823	104	57	30 7.2
58		15	.72990	94	.93462	14	.78927	104	58	40 9.6
		14	.73084	94	.93477	14	.79031	104	59	50 12.1
		15	10.73178	94	9.93491	14	10.79136	104	60	
		D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.

	82°				83°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9·93191	14	10·79136	104	9·94356	14	10·85766	117	0
1	·93506	14	·79240	105	·94370	14	·85884	117	1
2	·93520	14	·79345	104	·94384	14	·86001	117	2
3	·93535	14	·79450	105	·94398	14	·86119	118	3
4	·93549	14	·79555	105	·94413	14	·86237	118	4
5	9·93564	14	10·79660	105	9·94427	14	10·86355	118	5
6	·93578	14	·79766	105	·94441	14	·86474	118	6
7	·93593	14	·79871	106	·94456	14	·86592	119	7
8	·93607	14	·79977	106	·94470	14	·86711	119	8
9	·93622	14	·80083	106	·94484	14	·86831	119	9
10	9·93636	14	10·80189	106	9·94498	14	10·86950	120	10
11	·93651	14	·80296	106	·94512	14	·87070	120	11
12	·93665	14	·80402	106	·94527	14	·87190	120	12
13	·93680	14	·80509	107	·94541	14	·87310	120	13
14	·93694	14	·80616	107	·94555	14	·87431	121	14
15	9·93709	14	10·80723	107	9·94569	14	10·87552	121	15
16	·93723	14	·80831	107	·94584	14	·87673	121	16
17	·93738	14	·80938	107	·94598	14	·87794	121	17
18	·93752	14	·81046	108	·94612	14	·87916	122	18
19	·93767	14	·81154	108	·94626	14	·88038	122	19
20	9·93781	14	10·81262	108	9·94640	14	10·88160	122	20
21	·93796	14	·81371	108	·94655	14	·88282	122	21
22	·93810	14	·81479	109	·94669	14	·88405	123	22
23	·93824	14	·81588	109	·94683	14	·88528	123	23
24	·93839	14	·81697	109	·94697	14	·88651	123	24
25	9·93853	14	10·81806	109	9·94711	14	10·88775	124	25
26	·93868	14	·81916	109	·94726	14	·88898	123	26
27	·93882	14	·82025	110	·94740	14	·89022	124	27
28	·93897	14	·82135	110	·94754	14	·89147	124	28
29	·93911	14	·82245	110	·94768	14	·89271	124	29
30	9·93925	14	10·82356	110	9·94782	14	10·89396	125	30
31	·93940	14	·82466	110	·94796	14	·89521	125	31
32	·93954	14	·82577	111	·94810	14	·89647	126	32
33	·93969	14	·82688	111	·94825	14	·89773	126	33
34	·93983	14	·82799	111	·94839	14	·89899	126	34
35	9·93997	14	10·82910	111	9·94853	14	10·90025	126	35
36	·94012	14	·83022	111	·94867	14	·90152	127	36
37	·94026	14	·83133	112	·94881	14	·90279	127	37
38	·94041	14	·83245	112	·94895	14	·90406	127	38
39	·94055	14	·83358	112	·94909	14	·90533	127	39
40	9·94069	14	10·83470	112	9·94923	14	10·90661	128	40
41	·94084	14	·83583	113	·94938	14	·90789	128	41
42	·94098	14	·83695	113	·94952	14	·90917	128	42
43	·94112	14	·83809	113	·94966	14	·91046	129	43
44	·94127	14	·83922	113	·94980	14	·91175	129	44
45	9·94141	14	10·84035	114	9·94994	14	10·91304	129	45
46	·94155	14	·84149	114	·95008	14	·91434	130	46
47	·94170	14	·84263	114	·95022	14	·91564	130	47
48	·94184	14	·84377	114	·95036	14	·91694	130	48
49	·94198	14	·84492	115	·95050	14	·91825	131	49
50	9·94213	14	10·84607	115	9·95064	14	10·91956	131	50
51	·94227	14	·84721	115	·95078	14	·92087	131	51
52	·94241	14	·84837	115	·95093	14	·92218	131	52
53	·94256	14	·84952	116	·95107	14	·92350	132	53
54	·94270	14	·85068	116	·95121	14	·92482	132	54
55	9·94284	14	10·85183	115	9·95135	14	10·92614	132	55
56	·94299	14	·85299	116	·95149	14	·92747	133	56
57	·94313	14	·85416	116	·95163	14	·92880	133	57
58	·94327	14	·85532	116	·95177	14	·93014	133	58
59	·94341	14	·85649	117	·95191	14	·93147	133	59
60	9·94356	14	10·85766	117	9·95205	14	10·93281	134	60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	130	120
6	13·0	12·0
7	15·1	14·0
8	17·3	16·0
9	19·5	18·0
10	21·6	20·0
20	43·3	40·0
30	65·0	60·0
40	86·6	80·0
50	108·3	100·0

	110	100
6	11·0	10·0
7	12·8	11·6
8	14·6	13·3
9	16·5	15·0
10	18·3	16·6
20	36·6	33·3
30	55·0	50·0
40	73·3	66·6
50	91·6	83·3

	3	2
6	0·3	0·2
7	0·3	0·2
8	0·4	0·2
9	0·4	0·3
10	0·5	0·3
20	1·0	0·6
30	1·5	1·0
40	2·0	1·3
50	2·5	1·6

	1	0
6	0·1	0·0
7	0·1	0·0
8	0·1	0·0
9	0·1	0·1
10	0·1	0·1
20	0·3	0·1
30	0·5	0·2
40	0·6	0·3
50	0·8	0·4

	14	14
6	1·4	1·4
7	1·7	1·6
8	1·9	1·8
9	2·2	2·1
10	2·4	2·3
20	4·8	4·6
30	7·2	7·0
40	9·6	9·3
50	12·1	11·6

P. P.

	84°				85°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9·95205	14	10·93281	134	9·96039	14	11·02010	158	0
1	·95219	14	·93416	135	·96053	13	·02168	159	1
2	·95233	14	·93551	135	·96067	14	·02327	159	2
3	·95247	14	·93686	135	·96081	14	·02487	159	3
4	·95261	14	·93821	135	·96095	13	·02646	160	4
5	9·95275	14	10·93957	136	9·96108	14	11·02807	161	5
6	·95289	14	·94093	136	·96122	13	·02968	161	6
7	·95303	14	·94229	137	·96136	14	·03129	161	7
8	·95317	14	·94366	137	·96150	13	·03291	162	8
9	·95331	14	·94503	137	·96163	14	·03453	163	9
10	9·95345	14	10·94641	137	9·96177	13	11·03616	163	10
11	·95359	14	·94778	138	·96191	14	·03780	164	11
12	·95373	13	·94917	138	·96205	13	·03944	164	12
13	·95387	14	·95055	139	·96218	14	·04108	165	13
14	·95401	14	·95194	139	·96232	13	·04273	165	14
15	9·95415	14	10·95333	139	9·96246	13	11·04438	166	15
16	·95429	14	·95473	140	·96259	14	·04604	167	16
17	·95443	14	·95613	140	·96273	13	·04771	167	17
18	·95457	14	·95753	140	·96287	14	·04938	167	18
19	·95471	14	·95894	141	·96301	13	·05106	168	19
20	9·95485	14	10·96035	141	9·96314	13	11·05274	169	20
21	·95499	14	·96176	142	·96328	14	·05443	169	21
22	·95513	14	·96318	142	·96342	13	·05612	169	22
23	·95527	13	·96461	142	·96355	14	·05782	170	23
24	·95540	14	·96603	143	·96369	13	·05952	171	24
25	9·95554	14	10·96746	143	9·96383	14	11·06123	171	25
26	·95568	14	·96889	144	·96397	13	·06295	172	26
27	·95582	14	·97033	144	·96410	13	·06467	173	27
28	·95596	14	·97177	144	·96424	14	·06640	173	28
29	·95610	14	·97322	145	·96438	13	·06813	174	29
30	9·95624	13	10·97467	145	9·96451	13	11·06987	174	30
31	·95638	14	·97612	145	·96465	14	·07161	175	31
32	·95652	14	·97758	146	·96479	13	·07336	176	32
33	·95666	14	·97904	146	·96492	13	·07512	176	33
34	·95680	13	·98050	147	·96506	13	·07688	177	34
35	9·95693	14	10·98197	147	9·96519	14	11·07865	177	35
36	·95707	14	·98345	147	·96533	13	·08043	178	36
37	·95721	14	·98492	148	·96547	13	·08221	179	37
38	·95735	14	·98640	149	·96560	14	·08400	179	38
39	·95749	14	·98789	149	·96574	13	·08579	180	39
40	9·95763	13	10·98938	149	9·96588	13	11·08759	180	40
41	·95777	14	·99087	150	·96601	13	·08940	181	41
42	·95791	14	·99237	150	·96615	14	·09121	182	42
43	·95804	13	·99387	151	·96629	13	·09303	182	43
44	·95818	14	·99538	151	·96642	13	·09486	183	44
45	9·95832	14	10·99689	151	9·96656	13	11·09669	184	45
46	·95846	13	·99841	152	·96669	14	·09853	185	46
47	·95860	14	10·99993	152	·96683	14	·10038	185	47
48	·95874	14	11·00145	153	·96697	13	·10223	186	48
49	·95888	13	·00298	153	·96710	13	·10409	186	49
50	9·95901	14	11·00451	154	9·96724	13	11·10595	187	50
51	·95915	13	·00605	154	·96737	13	·10783	188	51
52	·95929	14	·00759	155	·96751	13	·10971	189	52
53	·95943	14	·00914	155	·96764	14	·11160	189	53
54	·95957	13	·01069	155	·96778	13	·11349	190	54
55	9·95970	14	11·01225	156	9·96792	13	11·11539	191	55
56	·95984	14	·01381	156	·96805	13	·11730	191	56
		13	·01537	157	·96819	13	·11922	192	57
		14	·01694	157	·96832	13	·12114	193	58
		13	·01852	158	·96846	13	·12307	193	59
			11·02010		9·96859		11·12501		60
		D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	190	180
6	19·0	18·0
7	22·1	21·0
8	25·3	24·0
9	28·5	27·0
10	31·6	30·0
20	63·3	60·0
30	95·0	90·0
40	126·6	120·0
50	158·3	150·0

	170	160
6	17·0	16·0
7	19·8	18·6
8	22·6	21·3
9	25·5	24·0
10	28·3	26·6
20	56·6	53·3
30	85·0	80·0
40	113·3	106·6
50	141·6	133·3

	150	140
6	15·0	14·0
7	17·5	16·3
8	20·0	18·6
9	22·5	21·0
10	25·0	23·3
20	50·0	46·6
30	75·0	70·0
40	100·0	93·3
50	125·0	116·6

	130	9	8
6	13·0	0·9	0·8
7	15·1	1·0	0·9
8	17·3	1·2	1·0
9	19·5	1·3	1·2
10	21·6	1·5	1·3
20	43·3	3·0	2·6
30	65·0	4·5	4·0
40	86·6	6·0	5·3
50	108·3	7·5	6·6

	7	6	5
6	0·7	0·6	0·5
7	0·8	0·7	0·6
8	0·9	0·8	0·6
9	1·0	0·9	0·7
10	1·1	1·0	0·8
20	2·3	2·0	1·6
30	3·5	3·0	2·5
40	4·6	4·0	3·3
50	5·8	5·0	4·1

	14	14	13
6	1·4	1·4	1·3
7	1·7	1·6	1·5
8	1·9	1·8	1·7
9	2·2	2·1	2·0
10	2·4	2·3	2·2
20	4·8	4·6	4·3
30	7·3	7·0	6·5
40	9·6	9·3	8·7
50	12·1	11·6	10·8

P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

	86°				87°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9.96859	13	11.12501	195	9.97665	13	11.25785	255	0
1	.96837	14	.12696	195	.97679	13	.26040	256	1
2	.96887	13	.12891	196	.97692	13	.26297	257	2
3	.96900	13	.13087	196	.97705	13	.26554	259	3
4	.96914	13	.13284	198	.97718	13	.26814	260	4
5	9.96927	13	11.13482	198	9.97732	13	11.27074	262	5
6	.96941	13	.13680	199	.97745	13	.27336	263	6
7	.96954	14	.13879	200	.97758	13	.27599	265	7
8	.96968	13	.14079	201	.97772	13	.27864	266	8
9	.96981	13	.14280	201	.97785	13	.28131	267	9
10	9.96995	13	11.14482	202	9.97798	13	11.28398	268	10
11	.97008	13	.14684	203	.97811	13	.28668	270	11
12	.97022	13	.14887	204	.97825	13	.28938	272	12
13	.97035	13	.15092	205	.97838	13	.29211	274	13
14	.97049	13	.15297	205	.97851	13	.29485	275	14
15	9.97062	13	11.15502	206	9.97864	13	11.29760	277	15
16	.97076	13	.15709	208	.97878	13	.30037	278	16
17	.97089	13	.15917	208	.97891	13	.30316	280	17
18	.97103	13	.16125	209	.97904	13	.30596	282	18
19	.97116	13	.16334		.97917	13	.30878	283	19
20	9.97130	13	11.16544	210	9.97931	13	11.31162	285	20
21	.97143	13	.16755	211	.97944	13	.31447	287	21
22	.97157	13	.16967	212	.97957	13	.31734	288	22
23	.97170	13	.17180	213	.97970	13	.32023	290	23
24	.97183	13	.17394	214	.97984	13	.32313	292	24
25	9.97197	13	11.17609	214	9.97997	13	11.32606	294	25
26	.97210	13	.17824	215	.98010	13	.32900	296	26
27	.97224	13	.18041	216	.98023	13	.33196	298	27
28	.97237	13	.18259	218	.98036	13	.33494	299	28
29	.97251	13	.18477	218	.98050	13	.33793	301	29
30	9.97264	13	11.18697	219	9.98063	13	11.34095	303	30
31	.97277	13	.18917	220	.98076	13	.34398	305	31
32	.97291	13	.19138	221	.98089	13	.34704	307	32
33	.97304	14	.19361	222	.98102	13	.35011	309	33
34	.97318	13	.19584	223	.98116	13	.35321	311	34
35	9.97331	13	11.19809	224	9.98129	13	11.35632	313	35
36	.97345	13	.20034	225	.98142	13	.35946	315	36
37	.97358	14	.20261	227	.98155	13	.36261	316	37
38	.97371	13	.20489	227	.98168	13	.36579	318	38
39	.97385	13	.20717	228	.98181	13	.36899	320	39
40	9.97398	13	11.20947	230	9.98195	13	11.37221	322	40
41	.97412	13	.21178	230	.98208	13	.37546	324	41
42	.97425	13	.21410	232	.98221	13	.37872	326	42
43	.97438	13	.21643	233	.98234	13	.38201	328	43
44	.97452	13	.21877	234	.98247	13	.38532	331	44
45	9.97465	13	11.22112	235	9.98260	13	11.38866	333	45
46	.97478	13	.22349	236	.98273	13	.39201	335	46
47	.97492	13	.22585	237	.98287	13	.39540	338	47
48	.97505	13	.22825	239	.98300	13	.39880	340	48
49	.97519	13	.23065	239	.98313	13	.40224	343	49
50	9.97532	13	11.23308	241	9.98326	13	11.40569	345	50
51	.97545	13	.23548	242	.98339	13	.40918	348	51
52	.97559	13	.23792	243	.98352	13	.41269	351	52
53	.97572	13	.24037	245	.98365	13	.41622	353	53
54	.97585	13	.24283	246	.98378	13	.41979	356	54
55	9.97599	13	11.24530	247	9.98392	13	11.42338	359	55
56	.97612	13	.24778	248	.98405	13	.42699	361	56
57	.97625	13	.25028	250	.98418	13	.43064	364	57
58	.97639	13	.25279	251	.98431	13	.43431	367	58
59	.97652	13	.25531	252	.98444	13	.43802	370	59
60	9.97665	13	11.25785	254	9.98457	13	11.44175	373	60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.			
	250	240	
6	25.0	24.0	
7	29.1/6	28.0	
8	33.3	32.0	
9	37.5	36.0	
10	41.6	40.0	
20	83.3	80.0	
30	125.0	120.0	
40	166.6	160.0	
50	208.3	200.0	
	230	220	
6	23.0	22.0	
7	26.8	25.6	
8	30.6	29.3	
9	34.5	33.0	
10	38.3	36.6	
20	76.6	73.3	
30	115.0	110.0	
40	153.3	146.6	
50	191.6	183.3	
	210	200	
6	21.0	20.0	
7	24.5	23.3	
8	28.0	26.6	
9	31.5	30.0	
10	35.0	33.3	
20	70.0	66.6	
30	105.0	100.0	
40	140.0	133.3	
50	175.0	166.6	
	190	4	3
6	19.0	0.4	0.3
7	22.1/6	0.4	0.3
8	25.3	0.5	0.4
9	28.5	0.6	0.4
10	31.6	0.6	0.5
20	63.3	1.3	1.0
30	95.0	2.0	1.5
40	126.6	2.6	2.0
50	158.3	3.3	2.5
	2	1	0
6	0.2	0.1	0.0
7	0.2	0.1	0.0
8	0.2	0.1	0.0
9	0.3	0.1	0.1
10	0.3	0.1	0.1
20	0.6	0.3	0.1
30	1.0	0.5	0.2
40	1.3	0.6	0.3
50	1.6	0.8	0.4
	14	13	13
6	1.4	1.3	1.3
7	1.6	1.6	1.5
8	1.8	1.8	1.7
9	2.1	2.0	1.9
10	2.3	2.2	2.1
20	4.6	4.5	4.3
30	7.0	6.7	6.5
40	9.3	9.0	8.6
50	11.6	11.2	10.8

P. P.

′	88° Lg. Vers.	D	Log. Exs.	D	89° Lg. Vers.	D	Log. Exs.	D	′	P. P.
0	9.98457	13	11.44175	376	9.99235	13	11.75050	742	0	
1	.98470	13	.44551	379	.99248	13	.75792	755	1	
2	.98483	13	.44931	382	.99261	13	.76547	766	2	
3	.98496	13	.45313	386	.99274	13	.77316	781	3	
4	.98509	13	.45699	389	.99287	13	.78097	795	4	
5	9.98522	13	11.46088	392	9.99299	12	11.78892	805	5	
6	.98535	13	.46480	395	.99312	13	.79702	825	6	
7	.98548	13	.46876	399	.99325	13	.80527	840	7	
8	.98562	13	.47275	402	.99338	13	.81367	856	8	
9	.98575	13	.47677	406	.99351	13	.82223	872	9	
10	9.98588	13	11.48083	409	9.99363	12	11.83095	890	10	
11	.98601	13	.48493	413	.99376	13	.83986	908	11	
12	.98614	13	.48906	417	.99389	13	.84894	927	12	
13	.98627	13	.49323	420	.99402	13	.85821	947	13	
14	.98640	13	.49743	425	.99415	13	.86768	967	14	
15	9.98653	13	11.50168	426	9.99428	13	11.87735	989	15	
16	.98666	13	.50597	432	.99440	12	.88724	1009	16	
17	.98679	13	.51029	436	.99453	13	.89735	1034	17	
18	.98692	13	.51466	440	.99466	13	.90769	1059	18	
19	.98705	13	.51906	445	.99479	13	.91829	1086	19	
20	9.98718	13	11.52351	449	9.99491	12	11.92914	1112	20	
21	.98731	13	.52801	454	.99504	13	.94026	1140	21	
22	.98744	13	.53255	458	.99517	13	.95167	1171	22	
23	.98757	13	.53713	463	.99530	13	.96338	1203	23	
24	.98770	13	.54176	467	.99543	13	.97541	1236	24	
25	9.98783	13	11.54643	472	9.99555	12	11.98777	1271	25	
26	.98796	13	.55115	477	.99568	13	12.00048	1309	26	
27	.98809	13	.55593	482	.99581	13	.01358	1349	27	
28	.98822	13	.56076	487	.99594	13	.02707	1391	28	
29	.98835	13	.56563	492	.99606	12	.04098	1436	29	
30	9.98848	13	11.57056	498	9.99619	13	12.05535	1485	30	
31	.98861	13	.57554	504	.99632	13	.07020	1537	31	
32	.98874	13	.58058	509	.99645	13	.08557	1592	32	
33	.98887	13	.58567	515	.99657	12	.10149	1652	33	
34	.98900	13	.59082	520	.99670	13	.11801	1716	34	
35	9.98913	13	11.59602	527	9.99683	13	12.13517	1785	35	
36	.98925	12	.60129	533	.99695	12	.15302	1861	36	
37	.98938	13	.60662	539	.99708	13	.17163	1943	37	
38	.98951	13	.61202	545	.99721	13	.19106	2033	38	
39	.98964	13	.61747	552	.99734	13	.21135	2131	39	
40	9.98977	13	11.62300	559	9.99746	12	12.23271	2240	40	
41	.98990	13	.62859	566	.99759	13	.25511	2361	41	
42	.99003	13	.63425	573	.99772	13	.27872	2495	42	
43	.99016	13	.63998	581	.99784	12	.30367	2645	43	
44	.99029	12	.64579	588	.99797	13	.33013	2815	44	
45	9.99042	13	11.65167	595	9.99810	13	12.35828	3009	45	
46	.99055	13	.65762	604	.99823	13	.38837	3231	46	
47	.99068	13	.66366	611	.99835	12	.42068	3489	47	
48	.99081	13	.66978	620	.99848	13	.45557	3791	48	
49	.99093	12	.67598	628	.99861	13	.49349	4152	49	
50	9.99106	13	11.68227	638	9.99873	12	12.53501	4588	50	
51	.99119	13	.68865	646	.99886	13	.58089	5127	51	
52	.99132	13	.69511	656	.99899	13	.63217	5812	52	
53	.99145	13	.70168	666	.99911	12	.69029	6707	53	
54	.99158	13	.70834	675	.99924	13	.75736	7931	54	
55	9.99171	13	11.71509	686	9.99937	13	12.83667	9704	55	
56	.99184	13	.72196	696	.99949	12	.93371	12505	56	
57	.99197	13	.72892	707	.99962	13	13.05877	17621	57	
58	.99210	13	.73600	719	.99974	12	.23499	30116	58	
59	.99223	13	.74319	730	.99987	13	.53615		59	
60	.99236	13	11.75050		10.00000	13	Infinity		60	
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	P. P.

	0°				1°				
	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0	.00000	One	.00000	Infinite	.01745	.99985	.01746	57.2900	60
1	.00029	One	.00029	3437.75	.01774	.99984	.01775	56.3506	59
2	.00058	One	.00058	1718.87	.01803	.99984	.01804	55.4415	58
3	.00087	One	.00087	1145.92	.01832	.99983	.01833	54.5613	57
4	.00116	One	.00116	859.436	.01862	.99983	.01862	53.7086	56
5	.00145	One	.00145	687.549	.01891	.99982	.01891	52.8821	55
6	.00175	One	.00175	572.957	.01920	.99982	.01920	52.0807	54
7	.00204	One	.00204	491.106	.01949	.99981	.01949	51.3032	53
8	.00233	One	.00233	429.718	.01978	.99980	.01978	50.5485	52
9	.00262	One	.00262	381.971	.02007	.99980	.02007	49.8157	51
10	.00291	One	.00291	343.774	.02036	.99979	.02036	49.1039	50
11	.00320	.99999	.00320	312.521	.02065	.99979	.02066	48.4121	49
12	.00349	.99999	.00349	286.478	.02094	.99978	.02095	47.7395	48
13	.00378	.99999	.00378	264.441	.02123	.99977	.02124	47.0853	47
14	.00407	.99999	.00407	245.552	.02152	.99977	.02153	46.4489	46
15	.00436	.99999	.00436	229.182	.02181	.99976	.02182	45.8294	45
16	.00465	.99999	.00465	214.858	.02211	.99976	.02211	45.2261	44
17	.00495	.99999	.00495	202.219	.02240	.99975	.02240	44.6386	43
18	.00524	.99999	.00524	190.984	.02269	.99974	.02269	44.0661	42
19	.00553	.99998	.00553	180.932	.02298	.99974	.02298	43.5081	41
20	.00582	.99998	.00582	171.885	.02327	.99973	.02328	42.9641	40
21	.00611	.99998	.00611	163.700	.02356	.99972	.02357	42.4335	39
22	.00640	.99998	.00640	156.259	.02385	.99972	.02386	41.9158	38
23	.00669	.99998	.00669	149.465	.02414	.99971	.02415	41.4106	37
24	.00698	.99998	.00698	143.237	.02443	.99970	.02444	40.9174	36
25	.00727	.99997	.00727	137.507	.02472	.99969	.02473	40.4358	35
26	.00756	.99997	.00756	132.219	.02501	.99969	.02502	39.9655	34
27	.00785	.99997	.00785	127.321	.02530	.99968	.02531	39.5059	33
28	.00814	.99997	.00815	122.774	.02560	.99967	.02560	39.0568	32
29	.00844	.99996	.00844	118.540	.02589	.99966	.02589	38.6177	31
30	.00873	.99996	.00873	114.589	.02618	.99966	.02619	38.1885	30
31	.00902	.99996	.00902	110.892	.02647	.99965	.02648	37.7686	29
32	.00931	.99996	.00931	107.426	.02676	.99964	.02677	37.3579	28
33	.00960	.99995	.00960	104.171	.02705	.99963	.02706	36.9560	27
34	.00989	.99995	.00989	101.107	.02734	.99963	.02735	36.5627	26
35	.01018	.99995	.01018	98.2179	.02763	.99962	.02764	36.1776	25
36	.01047	.99995	.01047	95.4895	.02792	.99961	.02793	35.8006	24
37	.01076	.99994	.01076	92.9085	.02821	.99960	.02822	35.4313	23
38	.01105	.99994	.01105	90.4633	.02850	.99959	.02851	35.0695	22
39	.01134	.99994	.01135	88.1436	.02879	.99959	.02881	34.7151	21
40	.01164	.99993	.01164	85.9398	.02908	.99958	.02910	34.3678	20
41	.01193	.99993	.01193	83.8435	.02938	.99957	.02939	34.0273	19
42	.01222	.99993	.01222	81.8470	.02967	.99956	.02968	33.6935	18
43	.01251	.99992	.01251	79.9434	.02996	.99955	.02997	33.3662	17
44	.01280	.99992	.01280	78.1263	.03025	.99954	.03026	33.0452	16
45	.01309	.99991	.01309	76.3900	.03054	.99953	.03055	32.7303	15
46	.01338	.99991	.01338	74.7292	.03083	.99952	.03084	32.4213	14
47	.01367	.99991	.01367	73.1390	.03112	.99952	.03114	32.1181	13
48	.01396	.99990	.01396	71.6151	.03141	.99951	.03143	31.8205	12
49	.01425	.99990	.01425	70.1533	.03170	.99950	.03172	31.5284	11
50	.01454	.99989	.01455	68.7501	.03199	.99949	.03201	31.2416	10
51	.01483	.99989	.01484	67.4019	.03228	.99948	.03230	30.9599	9
52	.01513	.99989	.01513	66.1055	.03257	.99947	.03259	30.6833	8
53	.01542	.99988	.01542	64.8580	.03286	.99946	.03288	30.4116	7
54	.01571	.99988	.01571	63.6567	.03316	.99945	.03317	30.1446	6
55	.01600	.99987	.01600	62.4992	.03345	.99944	.03346	29.8823	5
56	.01629	.99987	.01629	61.3829	.03374	.99943	.03376	29.6245	4
57	.01658	.99986	.01658	60.3058	.03403	.99942	.03405	29.3711	3
58	.01687	.99986	.01687	59.2659	.03432	.99941	.03434	29.1220	2
59	.01716	.99985	.01716	58.2612	.03461	.99940	.03463	28.8771	1
60	.01745	.99985	.01746	57.2900	.03490	.99939	.03492	28.6363	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	
	89°						88°		

	2°				3°			
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.
0	.03490	.99939	.03492	28.6363	.05234	.99863	.05241	19.0811
1	.03519	.99938	.03521	28.3994	.05263	.99861	.05270	18.9755
2	.03548	.99937	.03550	28.1664	.05292	.99860	.05299	18.8711
3	.03577	.99936	.03579	27.9372	.05321	.99858	.05328	18.7678
4	.03806	.99935	.03609	27.7117	.05350	.99857	.05357	18.6656
5	.03635	.99934	.03638	27.4899	.05379	.99855	.05387	18.5645
6	.03664	.99933	.03667	27.2715	.05408	.99854	.05416	18.4645
7	.03693	.99932	.03696	27.0566	.05437	.99852	.05445	18.3655
8	.03723	.99931	.03725	26.8450	.05466	.99851	.05474	18.2677
9	.03752	.99930	.03754	26.6367	.05495	.99849	.05503	18.1708
10	.03781	.99929	.03783	26.4316	.05524	.99847	.05533	18.0750
11	.03810	.99927	.03812	26.2296	.05553	.99846	.05562	17.9802
12	.03839	.99926	.03842	26.0307	.05582	.99844	.05591	17.8863
13	.03868	.99925	.03871	25.8348	.05611	.99842	.05620	17.7934
14	.03897	.99924	.03900	25.6418	.05640	.99841	.05649	17.7015
15	.03926	.99923	.03929	25.4517	.05669	.99839	.05678	17.6106
16	.03955	.99922	.03958	25.2644	.05698	.99838	.05708	17.5205
17	.03984	.99921	.03987	25.0798	.05727	.99836	.05737	17.4314
18	.04013	.99919	.04016	24.8978	.05756	.99834	.05766	17.3432
19	.04042	.99918	.04046	24.7185	.05785	.99833	.05795	17.2558
20	.04071	.99917	.04075	24.5418	.05814	.99831	.05824	17.1693
21	.04100	.99916	.04104	24.3675	.05844	.99829	.05854	17.0837
22	.04129	.99915	.04133	24.1957	.05873	.99827	.05883	16.9990
23	.04159	.99913	.04162	24.0263	.05902	.99826	.05912	16.9150
24	.04188	.99912	.04191	23.8593	.05931	.99824	.05941	16.8319
25	.04217	.99911	.04220	23.6945	.05960	.99822	.05970	16.7496
26	.04246	.99910	.04250	23.5321	.05989	.99821	.05999	16.6681
27	.04275	.99909	.04279	23.3718	.06018	.99819	.06029	16.5874
28	.04304	.99907	.04308	23.2137	.06047	.99817	.06058	16.5075
29	.04333	.99906	.04337	23.0577	.06076	.99815	.06087	16.4283
30	.04362	.99905	.04366	22.9038	.06105	.99813	.06116	16.3499
31	.04391	.99904	.04395	22.7519	.06134	.99812	.06145	16.2722
32	.04420	.99902	.04424	22.6020	.06163	.99810	.06175	16.1952
33	.04449	.99901	.04454	22.4541	.06192	.99808	.06204	16.1190
34	.04478	.99900	.04483	22.3081	.06221	.99806	.06233	16.0435
35	.04507	.99898	.04512	22.1640	.06250	.99804	.06262	15.9687
36	.04536	.99897	.04541	22.0217	.06279	.99803	.06291	15.8946
37	.04565	.99896	.04570	21.8813	.06308	.99801	.06321	15.8211
38	.04594	.99894	.04599	21.7426	.06337	.99799	.06350	15.7483
39	.04623	.99893	.04628	21.6056	.06366	.99797	.06379	15.6762
40	.04653	.99892	.04658	21.4704	.06395	.99795	.06408	15.6048
41	.04682	.99890	.04687	21.3369	.06424	.99793	.06437	15.5340
42	.04711	.99889	.04716	21.2049	.06453	.99792	.06467	15.4638
43	.04740	.99888	.04745	21.0747	.06482	.99790	.06496	15.3943
44	.04769	.99886	.04774	20.9460	.06511	.99788	.06525	15.3254
45	.04798	.99885	.04803	20.8188	.06540	.99786	.06554	15.2571
46	.04827	.99883	.04833	20.6932	.06569	.99784	.06584	15.1893
47	.04856	.99882	.04862	20.5691	.06598	.99782	.06613	15.1222
48	.04885	.99881	.04891	20.4465	.06627	.99780	.06642	15.0557
49	.04914	.99879	.04920	20.3253	.06656	.99778	.06671	14.9898
50	.04943	.99878	.04949	20.2056	.06685	.99776	.06700	14.9244
51	.04972	.99876	.04978	20.0872	.06714	.99774	.06730	14.8596
52	.05001	.99875	.05007	19.9702	.06743	.99772	.06759	14.7954
53	.05030	.99873	.05037	19.8546	.06773	.99770	.06788	14.7317
54	.05059	.99872	.05066	19.7403	.06802	.99768	.06817	14.6685
55	.05088	.99870	.05095	19.6273	.06831	.99766	.06847	14.6059
56	.05117	.99869	.05124	19.5156	.06860	.99764	.06876	14.5438
57	.05146	.99867	.05153	19.4051	.06889	.99762	.06905	14.4823
58	.05175	.99866	.05182	19.2959	.06918	.99760	.06934	14.4212
59	.05205	.99864	.05212	19.1879	.06947	.99758	.06963	14.3607
60	.05234	.99863	.05241	19.0811	.06976	.99756	.06993	14.3007
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.
	87°				86°			

′	4° Sin.	Cos.	Tan.	Cot.	5° Sin.	Cos.	Tan.	Cot.	′
0	.06976	.99756	.06993	14.3007	.08716	.99619	.08749	11.4301	60
1	.07005	.99754	.07022	14.2411	.08745	.99617	.08778	11.3919	59
2	.07034	.99752	.07051	14.1821	.08774	.99614	.08807	11.3540	58
3	.07063	.99750	.07080	14.1285	.08803	.99612	.08837	11.3163	57
4	.07092	.99748	.07110	14.0655	.08831	.99609	.08866	11.2789	56
5	.07121	.99746	.07139	14.0079	.08860	.99607	.08895	11.2417	55
6	.07150	.99744	.07168	13.9507	.08889	.99604	.08925	11.2048	54
7	.07179	.99742	.07197	13.8940	.08918	.99602	.08954	11.1681	53
8	.07208	.99740	.07227	13.8378	.08947	.99599	.08983	11.1316	52
9	.07237	.99738	.07256	13.7821	.08976	.99596	.09013	11.0954	51
10	.07266	.99736	.07285	13.7267	.09005	.99594	.09042	11.0594	50
11	.07295	.99734	.07314	13.6719	.09034	.99591	.09071	11.0237	49
12	.07324	.99731	.07344	13.6174	.09063	.99588	.09101	10.9882	48
13	.07353	.99729	.07373	13.5634	.09092	.99586	.09130	10.9529	47
14	.07382	.99727	.07402	13.5098	.09121	.99583	.09159	10.9178	46
15	.07411	.99725	.07431	13.4566	.09150	.99580	.09189	10.8829	45
16	.07440	.99723	.07461	13.4039	.09179	.99578	.09218	10.8483	44
17	.07469	.99721	.07490	13.3515	.09208	.99575	.09247	10.8139	43
18	.07498	.99719	.07519	13.2996	.09237	.99572	.09277	10.7797	42
19	.07527	.99716	.07548	13.2480	.09266	.99570	.09306	10.7457	41
20	.07556	.99714	.07578	13.1969	.09295	.99567	.09335	10.7119	40
21	.07585	.99712	.07607	13.1461	.09324	.99564	.09365	10.6783	39
22	.07614	.99710	.07636	13.0958	.09353	.99562	.09394	10.6450	38
23	.07643	.99708	.07665	13.0458	.09382	.99559	.09423	10.6118	37
24	.07672	.99705	.07695	12.9962	.09411	.99556	.09453	10.5789	36
25	.07701	.99703	.07724	12.9469	.09440	.99553	.09482	10.5462	35
26	.07730	.99701	.07753	12.8981	.09469	.99551	.09511	10.5136	34
27	.07759	.99699	.07782	12.8496	.09498	.99548	.09541	10.4813	33
28	.07788	.99696	.07812	12.8014	.09527	.99545	.09570	10.4491	32
29	.07817	.99694	.07841	12.7536	.09556	.99542	.09600	10.4172	31
30	.07846	.99692	.07870	12.7062	.09585	.99540	.09629	10.3854	30
31	.07875	.99689	.07899	12.6591	.09614	.99537	.09658	10.3538	29
32	.07904	.99687	.07929	12.6124	.09642	.99534	.09688	10.3224	28
33	.07933	.99685	.07958	12.5660	.09671	.99531	.09717	10.2913	27
34	.07962	.99683	.07987	12.5199	.09700	.99528	.09746	10.2602	26
35	.07991	.99680	.08017	12.4742	.09729	.99526	.09776	10.2294	25
36	.08020	.99678	.08046	12.4288	.09758	.99523	.09805	10.1988	24
37	.08049	.99676	.08075	12.3838	.09787	.99520	.09834	10.1683	23
38	.08078	.99673	.08104	12.3390	.09816	.99517	.09864	10.1381	22
39	.08107	.99671	.08134	12.2946	.09845	.99514	.09893	10.1080	21
40	.08136	.99668	.08163	12.2505	.09874	.99511	.09923	10.0780	20
41	.08165	.99666	.08192	12.2067	.09903	.99508	.09952	10.0483	19
42	.08194	.99664	.08221	12.1632	.09932	.99506	.09981	10.0187	18
43	.08223	.99661	.08251	12.1201	.09961	.99503	.10011	9.98931	17
44	.08252	.99659	.08280	12.0772	.09990	.99500	.10040	9.96007	16
45	.08281	.99657	.08309	12.0346	.10019	.99497	.10069	9.93101	15
46	.08310	.99654	.08339	11.9923	.10048	.99494	.10099	9.90211	14
47	.08339	.99652	.08368	11.9504	.10077	.99491	.10128	9.87338	13
48	.08368	.99649	.08397	11.9087	.10106	.99488	.10158	9.84482	12
49	.08397	.99647	.08427	11.8673	.10135	.99485	.10187	9.81641	11
50	.08426	.99644	.08456	11.8262	.10164	.99482	.10216	9.78817	10
51	.08455	.99642	.08485	11.7853	.10192	.99479	.10246	9.76009	9
52	.08484	.99639	.08514	11.7448	.10221	.99476	.10275	9.73217	8
53	.08513	.99637	.08544	11.7045	.10250	.99473	.10305	9.70441	7
54	.08542	.99635	.08573	11.6645	.10279	.99470	.10334	9.67680	6
55	.08571	.99632	.08602	11.6248	.10308	.99467	.10363	9.64935	5
56	.08600	.99630	.08632	11.5853	.10337	.99464	.10393	9.62205	4
57	.08629	.99627	.08661	11.5461	.10366	.99461	.10422	9.59490	3
58	.08658	.99625	.08690	11.5072	.10395	.99458	.10452	9.56791	2
59	.08687	.99622	.08720	11.4685	.10424	.99455	.10481	9.54106	1
60	.08716	.99619	.08749	11.4301	.10453	.99452	.10510	9.51436	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

TABLE IX. – NATURAL SINES, COSINES, TANGENTS, AND COTANGE

′	6° Sin.	6° Cos.	6° Tan.	6° Cot.	7° Sin.	7° Cos.	7° Tan.	7° Cot.
0	.10453	.99452	.10510	9.51436	.12187	.99255	.12278	8.14435
1	.10482	.99449	.10540	9.48781	.12216	.99251	.12308	8.12481
2	.10511	.99446	.10569	9.46141	.12245	.99248	.12338	8.10538
3	.10540	.99443	.10599	9.43515	.12274	.99244	.12367	8.08600
4	.10569	.99440	.10628	9.40904	.12302	.99240	.12397	8.06674
5	.10597	.99437	.10657	9.38307	.12331	.99237	.12426	8.04756
6	.10626	.99434	.10687	9.35724	.12360	.99233	.12156	8.02848
7	.10655	.99431	.10716	9.33155	.12389	.99230	.12485	8.00948
8	.10684	.99428	.10746	9.30599	.12418	.99226	.12515	7.99058
9	.10713	.99424	.10775	9.28058	.12447	.99222	.12544	7.97178
10	.10742	.99421	.10805	9.25530	.12476	.99218	.12574	7.95309
11	.10771	.99418	.10834	9 23016	.12504	.99215	.12603	7.93448
12	.10800	.99415	.10863	9.20516	.12533	.99211	.12633	7.91598
13	.10829	.99412	.10893	9.18028	.12562	.99208	.12662	7.89758
14	.10858	.99409	.10922	9.15554	.12591	.99204	.12692	7.87929
15	.10887	.99406	.10952	9.13093	.12620	.99200	.12722	7.86109
16	.10916	.99402	.10981	9.10646	.12649	.99197	.12751	7.84242
17	.10945	.99399	.11011	9.08211	.12678	.99193	.12781	7.82421
18	.10973	.99396	.11040	9.05789	.12706	.99189	.12810	7.80622
19	.11002	.99393	.11070	9.03379	.12735	.99186	.12840	7.78832
20	.11031	.99390	.11099	9.00983	.12764	.99182	.12869	7.77053
21	.11060	.99386	.11128	8.98598	.12793	.99178	.12899	7.75285
22	.11089	.99383	.11158	8.96227	.12822	.99175	.12929	7.73486
23	.11118	.99380	.11187	8.93867	.12851	.99171	.12958	7.71717
24	.11147	.99377	.11217	8.91520	.12880	.99167	.12988	7.69958
25	.11176	.99374	.11246	8.89185	.12908	.99163	.13017	7.68209
26	.11205	.99370	.11276	8.86862	.12937	.99160	.13047	7.66469
27	.11234	.99367	.11305	8.84551	.12966	.99156	.13076	7.64739
28	.11263	.99364	.11335	8.82252	.12995	.99152	.13108	7.63020
29	.11291	.99360	.11364	8.79964	.13024	.99148	.13136	7.61298
30	.11320	.99357	.11394	8.77689	.13053	.99144	.13165	7.59587
31	.11349	.99354	.11423	8.75425	.13081	.99141	.13195	7.57887
32	.11378	.99351	.11452	8.73172	.13110	.99137	.13224	7.56177
33	.11407	.99347	.11482	8.70931	.13139	.99133	.13254	7.54488
34	.11436	.99344	.11511	8.68701	.13168	.99129	.13284	7.52800
35	.11465	.99341	.11541	8.66482	.13197	.99125	.13313	7.51113
36	.11494	.99337	.11570	8.64275	.13226	.99122	.13343	7.49436
37	.11523	.99334	.11600	8.62078	.13254	.99118	.13372	7.47760
38	.11552	.99331	.11629	8.59893	.13283	.99114	.13402	7.46115
39	.11580	.99327	.11659	8.57718	.13312	.99110	.13432	7.44450
40	.11609	.99324	.11688	8.55555	.13341	.99106	.13461	7.42817
41	.11638	.99320	.11718	8.53402	.13370	.99102	.13491	7.41194
42	.11667	.99317	.11747	8.51259	.13399	.99098	.13521	7.39581
43	.11696	.99314	.11777	8.49128	.13427	.99094	.13550	7.37979
44	.11725	.99310	.11806	8.47007	.13456	.99091	.13580	7.36388
45	.11754	.99307	.11836	8.44896	.13485	.99087	.13609	7.34798
46	.11783	.99303	.11865	8.42795	.13514	.99083	.13639	7.33219
47	.11812	.99300	.11895	8.40705	.13543	.99079	.13669	7.31650
48	.11840	.99297	.11924	8.38625	.13572	.99075	.13698	7.30091
49	.11869	.99293	.11954	8.36555	.13600	.99071	.13728	7.28544
50	.11898	.99290	.11983	8.34496	.13629	.99067	.13758	7.26987
51	.11927	.99286	.12013	8.32446	.13658	.99063	.13787	7.25451
52	.11956	.99283	.12042	8.30406	.13687	.99059	.13817	7.23914
53	.11985	.99279	.12072	8.28376	.13716	.99055	.13846	7.22390
54	.12014	.99276	.12101	8.26355	.13744	.99051	.13876	7.20866
55	.12043	.99272	.12131	8.24345	.13773	.99047	.13906	7.19352
56	.12071	.99269	.12160	8.22344	.13802	.99043	.13935	7.17850
57	.12100	.99265	.12190	8.20352	.13831	.99039	.13965	7.16357
58	.12129	.99262	.12219	8.18370	.13860	.99035	.13995	7.14855
59	.12158	.99258	.12249	8.16398	.13889	.99031	.14024	7.13364
60	.12187	.99255	.12278	8.14435	.13917	.99027	.14054	7.11882
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.

′	8° Sin.	Cos.	Tan.	Cot.	9° Sin.	Cos.	Tan.	Cot.	′
0	.13917	.99027	.14054	7.11537	.15643	.98769	.15838	6.31375	60
1	.13946	.99023	.14084	7.10038	.15672	.98764	.15868	6.30189	59
2	.13975	.99019	.14113	7.08546	.15701	.98760	.15898	6.29007	58
3	.14004	.99015	.14143	7.07059	.15730	.98755	.15928	6.27829	57
4	.14033	.99011	.14173	7.05579	.15758	.98751	.15958	6.26655	56
5	.14061	.99006	.14202	7.04105	.15787	.98746	.15988	6.25486	55
6	.14090	.99002	.14232	7.02637	.15816	.98741	.16017	6.24321	54
7	.14119	.98998	.14262	7.01174	.15845	.98737	.16047	6.23160	53
8	.14148	.98994	.14291	6.99718	.15873	.98732	.16077	6.22003	52
9	.14177	.98990	.14321	6.98268	.15902	.98728	.16107	6.20851	51
10	.14205	.98986	.14351	6.96823	.15931	.98723	.16137	6.19703	50
11	.14234	.98982	.14381	6.95385	.15959	.98718	.16167	6.18559	49
12	.14263	.98978	.14410	6.93952	.15988	.98714	.16196	6.17419	48
13	.14292	.98973	.14440	6.92525	.16017	.98709	.16226	6.16283	47
14	.14320	.98969	.14470	6.91104	.16046	.98704	.16256	6.15151	46
15	.14349	.98965	.14499	6.89688	.16074	.98700	.16286	6.14023	45
16	.14378	.98961	.14529	6.88278	.16103	.98695	.16316	6.12899	44
17	.14407	.98957	.14559	6.86874	.16132	.98690	.16346	6.11779	43
18	.14436	.98953	.14588	6.85475	.16160	.98686	.16376	6.10664	42
19	.14464	.98948	.14618	6.84082	.16189	.98681	.16405	6.09552	41
20	.14493	.98944	.14648	6.82694	.16218	.98676	.16435	6.08444	40
21	.14522	.98940	.14678	6.81312	.16246	.98671	.16465	6.07340	39
22	.14551	.98936	.14707	6.79936	.16275	.98667	.16495	6.06240	38
23	.14580	.98931	.14737	6.78564	.16304	.98662	.16525	6.05143	37
24	.14608	.98927	.14767	6.77199	.16333	.98657	.16555	6.04051	36
25	.14637	.98923	.14796	6.75838	.16361	.98652	.16585	6.02962	35
26	.14666	.98919	.14826	6.74483	.16390	.98648	.16615	6.01878	34
27	.14695	.98914	.14856	6.73133	.16419	.98643	.16645	6.00797	33
28	.14723	.98910	.14886	6.71789	.16447	.98638	.16674	5.99720	32
29	.14752	.98906	.14915	6.70450	.16476	.98633	.16704	5.98646	31
30	.14781	.98902	.14945	6.69116	.16505	.98629	.16734	5.97576	30
31	.14810	.98897	.14975	6.67787	.16533	.98624	.16764	5.96510	29
32	.14838	.98893	.15005	6.66463	.16562	.98619	.16794	5.95448	28
33	.14867	.98889	.15034	6.65144	.16591	.98614	.16824	5.94390	27
34	.14896	.98884	.15064	6.63831	.16620	.98609	.16854	5.93335	26
35	.14925	.98880	.15094	6.62523	.16648	.98604	.16884	5.92283	25
36	.14954	.98876	.15124	6.61219	.16677	.98600	.16914	5.91236	24
37	.14982	.98871	.15153	6.59921	.16706	.98595	.16944	5.90191	23
38	.15011	.98867	.15183	6.58627	.16734	.98590	.16974	5.89151	22
39	.15040	.98863	.15213	6.57339	.16763	.98585	.17004	5.88114	21
40	.15069	.98858	.15243	6.56055	.16792	.98580	.17033	5.87080	20
41	.15097	.98854	.15272	6.54777	.16820	.98575	.17063	5.86051	19
42	.15126	.98849	.15302	6.53503	.16849	.98570	.17093	5.85024	18
43	.15155	.98845	.15332	6.52234	.16878	.98565	.17123	5.84001	17
44	.15184	.98841	.15362	6.50970	.16906	.98561	.17153	5.82982	16
45	.15212	.98836	.15391	6.49710	.16935	.98556	.17183	5.81966	15
46	.15241	.98832	.15421	6.48456	.16964	.98551	.17213	5.80953	14
47	.15270	.98827	.15451	6.47206	.16992	.98546	.17243	5.79944	13
48	.15299	.98823	.15481	6.45961	.17021	.98541	.17273	5.78938	12
49	.15327	.98818	.15511	6.44720	.17050	.98536	.17303	5.77936	11
50	.15356	.98814	.15540	6.43484	.17078	.98531	.17333	5.76937	10
51	.15385	.98809	.15570	6.42253	.17107	.98526	.17363	5.75941	9
52	.15414	.98805	.15600	6.41026	.17136	.98521	.17393	5.74949	8
53	.15442	.98800	.15630	6.39804	.17164	.98516	.17423	5.73960	7
54	.15471	.98796	.15660	6.38587	.17193	.98511	.17453	5.72974	6
55	.15500	.98791	.15689	6.37374	.17222	.98506	.17483	5.71992	5
56	.15529	.98787	.15719	6.36165	.17250	.98501	.17513	5.71013	4
57	.15557	.98782	.15749	6.34961	.17279	.98496	.17543	5.70037	3
58	.15586	.98778	.15779	6.33761	.17308	.98491	.17573	5.69064	2
59	.15615	.98773	.15809	6.32566	.17336	.98486	.17603	5.68094	1
60	.15643	.98769	.15838	6.31375	.17365	.98481	.17633	5.67128	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS

	10°				11°				
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0	.17365	.98481	.17633	5.67128	.19081	.98163	.19438	5.14455	60
1	.17393	.98476	.17663	5.66165	.19109	.98157	.19468	5.13858	59
2	.17422	.98471	.17693	5.65205	.19138	.98152	.19498	5.12862	58
3	.17451	.98466	.17723	5.64248	.19167	.98146	.19529	5.12069	57
4	.17479	.98461	.17753	5.63295	.19195	.98140	.19559	5.11279	56
5	.17508	.98455	.17783	5.62344	.19224	.98135	.19589	5.10490	55
6	.17537	.98450	.17813	5.61397	.19252	.98129	.19619	5.09704	54
7	.17565	.98445	.17843	5.60452	.19281	.98124	.19649	5.08921	53
8	.17594	.98440	.17873	5.59511	.19309	.98118	.19680	5.08139	52
9	.17623	.98435	.17903	5.58573	.19338	.98112	.19710	5.07360	51
10	.17651	.98430	.17933	5.57638	.19366	.98107	.19740	5.06584	50
11	.17680	.98425	.17963	5.56706	.19395	.98101	.19770	5.05809	49
12	.17708	.98420	.17993	5.55777	.19423	.98096	.19801	5.05037	48
13	.17737	.98414	.18023	5.54851	.19452	.98000	.19831	5.04267	47
14	.17766	.98409	.18053	5.53927	.19481	.98084	.19861	5.03499	46
15	.17794	.98404	.18083	5.53007	.19509	.98079	.19891	5.02734	45
16	.17823	.98399	.18113	5.52090	.19538	.98073	.19921	5.01971	44
17	.17852	.98394	.18143	5.51176	.19566	.98067	.19952	5.01210	43
18	.17880	.98389	.18173	5.50264	.19595	.98061	.19982	5.00451	42
19	.17909	.98383	.18203	5.49356	.19623	.98056	.20012	4.99695	41
20	.17937	.98378	.18233	5.48451	.19652	.98050	.20042	4.98940	40
21	.17966	.98373	.18263	5.47548	.19680	.98044	.20073	4.98188	39
22	.17995	.98368	.18293	5.46648	.19709	.98039	.20103	4.97438	38
23	.18023	.98362	.18323	5.45751	.19737	.98033	.20133	4.96690	37
24	.18052	.98357	.18353	5.44857	.19766	.98027	.20164	4.95945	36
25	.18081	.98352	.18384	5.43966	.19794	.98021	.20194	4.95201	35
26	.18109	.98347	.18414	5.43077	.19823	.98016	.20224	4.94460	34
27	.18138	.98341	.18444	5.42192	.19851	.98010	.20254	4.93721	33
28	.18166	.98336	.18474	5.41309	.19880	.98004	.20285	4.92984	32
29	.18195	.98331	.18504	5.40429	.19908	.97998	.20315	4.92249	31
30	.18224	.98325	.18534	5.39552	.19937	.97992	.20345	4.91516	30
31	.18252	.98320	.18564	5.38677	.19965	.97987	.20378	4.90785	29
32	.18281	.98315	.18594	5.37805	.19994	.97981	.20406	4.90056	28
33	.18309	.98310	.18624	5.36936	.20022	.97975	.20436	4.89330	27
34	.18338	.98304	.18654	5.36070	.20051	.97969	.20466	4.88605	26
35	.18367	.98299	.18684	5.35206	.20079	.97963	.20497	4.87882	25
36	.18395	.98294	.18714	5.34345	.20108	.97958	.20527	4.87162	24
37	.18424	.98288	.18745	5.33487	.20136	.97952	.20557	4.86444	23
38	.18452	.98283	.18775	5.32631	.20165	.97946	.20588	4.85727	22
39	.18481	.98277	.18805	5.31778	.20193	.97940	.20618	4.85013	21
40	.18509	.98272	.18835	5.30928	.20222	.97934	.20648	4.84300	20
41	.18538	.98267	.18865	5.30080	.20250	.97928	.20679	4.83589	19
42	.18567	.98261	.18895	5.29235	.20279	.97922	.20709	4.82882	18
43	.18595	.98256	.18925	5.28393	.20307	.97916	.20739	4.82175	17
44	.18624	.98250	.18955	5.27553	.20336	.97910	.20770	4.81471	16
45	.18652	.98245	.18986	5.26715	.20364	.97905	.20800	4.80769	15
46	.18681	.98240	.19016	5.25880	.20393	.97899	.20830	4.80068	14
47	.18710	.98234	.19046	5.25048	.20421	.97893	.20861	4.79370	13
48	.18738	.98229	.19076	5.24218	.20450	.97887	.20891	4.78673	12
49	.18767	.98223	.19106	5.23391	.20478	.97881	.20921	4.77978	11
50	.18795	.98218	.19136	5.22566	.20507	.97875	.20952	4.77288	10
51	.18824	.98212	.19166	5.21744	.20535	.97869	.20982	4.76595	9
52	.18852	.98207	.19197	5.20925	.20563	.97863	.21013	4.75906	8
53	.18881	.98201	.19227	5.20107	.20592	.97857	.21043	4.75219	7
54	.18910	.98196	.19257	5.19293	.20620	.97851	.21073	4.74534	6
55	.18938	.98190	.19287	5.18480	.20649	.97845	.21104	4.73851	5
56	.18967	.98185	.19317	5.17671	.20677	.97839	.21134	4.73170	4
57	.18995	.98179	.19347	5.16863	.20706	.97833	.21164	4.72490	3
58	.19024	.98174	.19378	5.16058	.20734	.97827	.21195	4.71813	2
59	.19052	.98168	.19408	5.15256	.20763	.97821	.21225	4.71137	1
60	.19081	.98163	.19438	5.14455	.20791	.97815	.21256	4.70463	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

	12°				13°			
Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0791	.97815	.21256	4.70463	.22495	.97437	.23087	4.33148	60
0820	.97809	.21286	4.69791	.22523	.97430	.23117	4.32573	59
0848	.97803	.21316	4.69121	.22552	.97424	.23148	4.32001	58
0877	.97797	.21347	4.68452	.22580	.97417	.23179	4.31430	57
0905	.97791	.21377	4.67786	.22608	.97411	.23209	4.30860	56
0933	.97784	.21408	4.67121	.22637	.97404	.23240	4.30291	55
0962	.97778	.21438	4.66458	.22665	.97398	.23271	4.29724	54
0990	.97772	.21469	4.65797	.22693	.97391	.23301	4.29159	53
1019	.97766	.21499	4.65138	.22722	.97384	.23332	4.28595	52
1047	.97760	.21529	4.64480	.22750	.97378	.23363	4.28032	51
1076	.97754	.21560	4.63825	.22778	.97371	.23393	4.27471	50
1104	.97748	.21590	4.63171	.22807	.97365	.23424	4.26911	49
1132	.97742	.21621	4.62518	.22835	.97358	.23455	4.26352	48
1161	.97735	.21651	4.61868	.22863	.97351	.23485	4.25795	47
1189	.97729	.21682	4.61219	.22892	.97345	.23516	4.25239	46
1218	.97723	.21712	4.60572	.22920	.97338	.23547	4.24685	45
1246	.97717	.21743	4.59927	.22948	.97331	.23578	4.24132	44
1275	.97711	.21773	4.59283	.22977	.97325	.23608	4.23580	43
1303	.97705	.21804	4.58641	.23005	.97318	.23639	4.23030	42
1331	.97698	.21834	4.58001	.23033	.97311	.23670	4.22481	41
1360	.97692	.21864	4.57363	.23062	.97304	.23700	4.21933	40
1388	.97686	.21895	4.56726	.23090	.97298	.23731	4.21387	39
1417	.97680	.21925	4.56091	.23118	.97291	.23762	4.20842	38
1445	.97673	.21956	4.55458	.23146	.97284	.23793	4.20298	37
1474	.97667	.21986	4.54826	.23175	.97278	.23823	4.19756	36
1502	.97661	.22017	4.54196	.23203	.97271	.23854	4.19215	35
1530	.97655	.22047	4.53568	.23231	.97264	.23885	4.18675	34
1559	.97648	.22078	4.52941	.23260	.97257	.23916	4.18137	33
1587	.97642	.22108	4.52316	.23288	.97251	.23946	4.17600	32
1616	.97636	.22139	4.51693	.23316	.97244	.23977	4.17064	31
1644	.97630	.22169	4.51071	.23345	.97237	.24008	4.16530	30
1672	.97623	.22200	4.50451	.23373	.97230	.24039	4.15997	29
1701	.97617	.22231	4.49832	.23401	.97223	.24069	4.15465	28
1729	.97611	.22261	4.49215	.23429	.97217	.24100	4.14934	27
1758	.97604	.22292	4.48600	.23458	.97210	.24131	4.14405	26
1786	.97598	.22322	4.47986	.23486	.97203	.24162	4.13877	25
1814	.97592	.22353	4.47374	.23514	.97196	.24193	4.13350	24
1843	.97585	.22383	4.46764	.23542	.97189	.24223	4.12825	23
1871	.97579	.22414	4.46155	.23571	.97182	.24254	4.12301	22
1899	.97573	.22444	4.45548	.23599	.97176	.24285	4.11778	21
1928	.97566	.22475	4.44942	.23627	.97169	.24316	4.11256	20
1956	.97560	.22505	4.44338	.23656	.97162	.24347	4.10736	19
1985	.97553	.22536	4.43735	.23684	.97155	.24377	4.10216	18
2013	.97547	.22567	4.43134	.23712	.97148	.24408	4.09699	17
2041	.97541	.22597	4.42534	.23740	.97141	.24439	4.09182	16
2070	.97534	.22628	4.41936	.23769	.97134	.24470	4.08666	15
2098	.97528	.22658	4.41340	.23797	.97127	.24501	4.08152	14
2126	.97521	.22689	4.40745	.23825	.97120	.24532	4.07639	13
2155	.97515	.22719	4.40152	.23853	.97113	.24562	4.07127	12
2183	.97508	.22750	4.39560	.23882	.97106	.24593	4.06616	11
2212	.97502	.22781	4.38969	.23910	.97100	.24624	4.06107	10
2240	.97496	.22811	4.38381	.23938	.97093	.24655	4.05599	9
2268	.97489	.22842	4.37793	.23966	.97086	.24686	4.05092	8
2297	.97483	.22872	4.37207	.23995	.97079	.24717	4.04586	7
2325	.97476	.22905	4.36623	.24023	.97072	.24747	4.04081	6
2353	.97470	.22934	4.36040	.24051	.97065	.24778	4.03578	5
2382	.97463	.22964	4.35459	.24079	.97058	.24809	4.03076	4
2410	.97457	.22995	4.34879	.24108	.97051	.24840	4.02574	3
2438	.97450	.23026	4.34300	.24136	.97044	.24871	4.02074	2
2467	.97444	.23056	4.33723	.24164	.97037	.24902	4.01576	1
2495	.97437	.23087	4.33148	.24192	.97030	.24933	4.01078	0
Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.

	14°				15°				
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0	.24192	.97030	.24933	4.01078	.25882	.96593	.26795	3.73205	60
1	.24220	.97023	.24964	4.00582	.25910	.96585	.26826	3.72771	59
2	.24249	.97015	.24995	4.00086	.25938	.96578	.26857	3.72338	58
3	.24277	.97008	.25026	3.99592	.25966	.96570	.26888	3.71907	57
4	.24305	.97001	.25056	3.99099	.25994	.96562	.26920	3.71476	56
5	.24333	.96994	.25087	3.98607	.26022	.96555	.26951	3.71046	55
6	.24362	.96987	.25118	3.98117	.26050	.96547	.26982	3.70616	54
7	.24390	.96980	.25149	3.97627	.26079	.96540	.27013	3.70188	53
8	.24418	.96973	.25180	3.97139	.26107	.96532	.27044	3.69761	52
9	.24446	.96966	.25211	3.96651	.26135	.96524	.27076	3.69335	51
10	.24474	.96959	.25242	3.96165	.26163	.96517	.27107	3.68909	50
11	.24503	.96952	.25273	3.95680	.26191	.96509	.27138	3.68485	49
12	.24531	.96945	.25304	3.95196	.26219	.96502	.27169	3.68061	48
13	.24559	.96937	.25335	3.94713	.26247	.96494	.27201	3.67638	47
14	.24587	.96930	.25366	3.94232	.26275	.96486	.27232	3.67217	46
15	.24615	.96923	.25397	3.93751	.26303	.96479	.27263	3.66796	45
16	.24644	.96916	.25428	3.93271	.26331	.96471	.27294	3.66376	44
17	.24672	.96909	.25459	3.92793	.26359	.96463	.27326	3.65957	43
18	.24700	.96902	.25490	3.92316	.26387	.96456	.27357	3.65538	42
19	.24728	.96894	.25521	3.91839	.26415	.96448	.27388	3.65121	41
20	.24756	.96887	.25552	3.91364	.26443	.96440	.27419	3.64705	40
21	.24784	.96880	.25583	3.90890	.26471	.96433	.27451	3.64289	39
22	.24813	.96873	.25614	3.90417	.26500	.96425	.27482	3.63874	38
23	.24841	.96866	.25645	3.89945	.26528	.96417	.27513	3.63461	37
24	.24869	.96858	.25676	3.89474	.26556	.96410	.27545	3.63048	36
25	.24897	.96851	.25707	3.89004	.26584	.96402	.27576	3.62636	35
26	.24925	.96844	.25738	3.88536	.26612	.96394	.27607	3.62224	34
27	.24954	.96837	.25769	3.88068	.26640	.96386	.27638	3.61814	33
28	.24982	.96829	.25800	3.87601	.26668	.96379	.27670	3.61405	32
29	.25010	.96822	.25831	3.87136	.26696	.96371	.27701	3.60996	31
30	.25038	.96815	.25862	3.86671	.26724	.96363	.27732	3.60588	30
31	.25066	.96807	.25893	3.86208	.26752	.96355	.27764	3.60181	29
32	.25094	.96800	.25924	3.85745	.26780	.96347	.27795	3.59775	28
33	.25122	.96793	.25955	3.85284	.26808	.96340	.27826	3.59370	27
34	.25151	.96786	.25983	3.84824	.26836	.96332	.27858	3.58966	26
35	.25179	.96778	.26017	3.84364	.26864	.96324	.27889	3.58562	25
36	.25207	.96771	.26048	3.83906	.26892	.96316	.27921	3.58160	24
37	.25235	.96764	.26079	3.83449	.26920	.96308	.27952	3.57758	23
38	.25263	.96756	.26110	3.82992	.26948	.96301	.27983	3.57357	22
39	.25291	.96749	.26141	3.82537	.26976	.96293	.28015	3.56957	21
40	.25320	.96742	.26172	3.82083	.27004	.96285	.28046	3.56557	20
41	.25348	.96734	.26203	3.81630	.27032	.96277	.28077	3.56159	19
42	.25376	.96727	.26235	3.81177	.27060	.96269	.28109	3.55761	18
43	.25404	.96719	.26266	3.80726	.27088	.96261	.28140	3.55364	17
44	.25432	.96712	.26297	3.80276	.27116	.96253	.28172	3.54968	16
45	.25460	.96705	.26328	3.79827	.27144	.96246	.28203	3.54573	15
46	.25488	.96697	.26359	3.79378	.27172	.96238	.28234	3.54179	14
47	.25516	.96690	.26390	3.78931	.27200	.96230	.28266	3.53785	13
48	.25545	.96682	.26421	3.78485	.27228	.96222	.28297	3.53393	12
49	.25573	.96675	.26452	3.78040	.27256	.96214	.28329	3.53001	11
50	.25601	.96667	.26483	3.77595	.27284	.96206	.28360	3.52609	10
51	.25629	.96660	.26515	3.77152	.27312	.96198	.28391	3.52219	9
52	.25657	.96653	.26546	3.76709	.27340	.96190	.28423	3.51829	8
53	.25685	.96645	.26577	3.76268	.27368	.96182	.28454	3.51441	7
54	.25713	.96638	.26608	3.75828	.27396	.96174	.28486	3.51053	6
55	.25741	.96630	.26639	3.75388	.27424	.96166	.28517	3.50666	5
56	.25769	.96623	.26670	3.74950	.27452	.96158	.28549	3.50279	4
57	.25798	.96615	.26701	3.74512	.27480	.96150	.28580	3.49894	3
58	.25826	.96608	.26733	3.74075	.27508	.96142	.28612	3.49509	2
59	.25854	.96600	.26764	3.73640	.27536	.96134	.28643	3.49125	1
60	.25882	.96593	.26795	3.73205	.27564	.96126	.28675	3.48741	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

75°	76°	74°

	16°				17°				
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0	.27564	.96126	.28675	3.48741	.29237	.95630	.30573	3.27085	60
1	.27592	.96118	.28706	3.48359	.29265	.95622	.30605	3.26745	59
2	.27620	.96110	.28738	3.47977	.29293	.95613	.30637	3.26406	58
3	.27648	.96102	.28769	3.47596	.29321	.95605	.30669	3.26067	57
4	.27676	.96094	.28800	3.47216	.29348	.95596	.30700	3.25729	56
5	.27704	.96086	.28832	3.46837	.29376	.95588	.30732	3.25392	55
6	.27731	.96078	.28864	3.46458	.29404	.95579	.30764	3.25055	54
7	.27759	.96070	.28895	3.46080	.29432	.95571	.30796	3.24719	53
8	.27787	.96062	.28927	3.45703	.29460	.95562	.30828	3.24383	52
9	.27815	.96054	.28958	3.45327	.29487	.95554	.30860	3.24049	51
10	.27843	.96046	.28990	3.44951	.29515	.95545	.30891	3.23714	50
11	.27871	.96037	.29021	3.44576	.29543	.95536	.30923	3.23381	49
12	.27899	.96029	.29053	3.44202	.29571	.95528	.30955	3.23048	48
13	.27927	.96021	.29084	3.43829	.29599	.95519	.30987	3.22715	47
14	.27955	.96013	.29116	3.43456	.29626	.95511	.31019	3.22384	46
15	.27983	.96005	.29147	3.43084	.29654	.95502	.31051	3.22053	45
16	.28011	.95997	.29179	3.42713	.29682	.95493	.31083	3 21722	44
17	.28039	.95989	.29210	3.42343	.29710	.95485	.31115	3.21392	43
18	.28067	.95981	.29242	3.41973	.29737	.95476	.31147	3.21063	42
19	.28095	.95972	.29274	3.41604	.29765	.95467	.31178	3.20734	41
20	.28123	.95964	.29305	3.41236	.29793	.95459	.31210	3.20406	40
21	.28150	.95956	.29337	3.40869	.29821	.95450	.31242	3.20079	39
22	.28178	.95948	.29368	3.40502	.29849	.95441	.31274	3.19752	38
23	.28206	.95940	.29400	3.40136	.29876	.95433	.31306	3.19426	37
24	.28234	.95931	.29432	3.39771	.29904	.95424	.31338	3.19100	36
25	.28262	.95923	.29463	3.39406	.29932	.95415	.31370	3.18775	35
26	.28290	.95915	.29495	3.39042	.29960	.95407	.31402	3.18451	34
27	.28318	.95907	.29526	3.38679	.29987	.95398	.31434	3.18127	33
28	.28346	.95898	.29558	3.38317	.30015	.95389	.31466	3.17804	32
29	.28374	.95890	.29590	3.37955	.30043	.95380	.31498	3.17481	31
30	.28402	.95882	.29621	3.37594	.30071	.95372	.31530	3.17159	30
31	.28429	.95874	.29653	3.37234	.30098	.95363	.31562	3.16838	29
32	.28457	.95865	.29685	3.36875	.30126	.95354	.31594	3.16517	28
33	.28485	.95857	.29716	3.36516	.30154	.95345	.31626	3.16197	27
34	.28513	.95849	.29748	3.36158	.30182	.95337	.31658	3.15877	26
35	.28541	.95841	.29780	3.35800	.30209	.95328	.31690	3.15558	25
36	.28569	.95832	.29811	3.35443	.30237	.95319	.31722	3.15240	24
37	.28597	.95824	.29843	3.35087	.30265	.95310	.31754	3.14922	23
38	.28625	.95816	.29875	3.34732	.30292	.95301	.31786	3.14605	22
39	.28652	.95807	.29906	3.34377	.30320	.95293	.31818	3.14288	21
40	.28680	.95799	.29938	3.34023	.30348	.95284	.31850	3.13972	20
41	.28708	.95791	.29970	3.33670	.30376	.95275	.31882	3.13656	19
42	.28736	.95782	.30001	3.33317	.30403	.95266	.31914	3.13341	18
43	.28764	.95774	.30033	3.32965	.30431	.95257	.31946	3.13027	17
44	.28792	.95766	.30065	3.32614	.30459	.95248	.31978	3.12713	16
45	.28820	.95757	.30097	3.32264	.30486	.95240	.32010	3.12400	15
46	.28847	.95749	.30128	3.31914	.30514	.95231	.32042	3.12087	14
47	.28875	.95740	.30160	3.31565	.30542	.95222	.32074	3.11775	13
48	.28903	.95732	.30192	3.31216	.30570	.95213	.32106	3.11464	12
49	.28931	.95724	.30224	3.30868	.30597	.95204	.32139	3.11153	11
50	.28959	.95715	.30255	3.30521	.30625	.95195	.32171	3.10842	10
51	.28987	.95707	.30287	3.30174	.30653	.95186	.32203	3.10532	9
52	.29015	.95698	.30319	3.29829	.30680	.95177	.32235	3.10223	8
53	.29042	.95690	.30351	3.29483	.30708	.95168	.32267	3.09914	7
54	.29070	.95681	.30382	3.29139	.30736	.95159	.32299	3.09606	6
55	.29098	.95673	.30414	3.28795	.30763	.95150	.32331	3.09298	5
56	.29126	.95664	.30446	3.28452	.30791	.95142	.32363	3.08991	4
57	.29154	.95656	.30478	3.28109	.30819	.95133	.32396	3.08685	3
58	.29182	.95647	.30509	3.27767	.30846	.95124	.32428	3.08379	2
59	.29209	.95639	.30541	3.27426	.30874	.95115	.32460	3.08073	1
60	.29237	.95630	.30573	3.27085	.30902	.95106	.32492	3.07768	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.

	18°				19°				
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0	.30902	.95106	.32492	3.07768	.32557	.94552	.34433	2.90421	60
1	.30929	.95097	.32524	3.07464	.32584	.94542	.34465	2.90147	59
2	.30957	.95088	.32556	3.07160	.32612	.94533	.34498	2.89873	58
3	.30985	.95079	.32588	3.06857	.32639	.94523	.34530	2.89600	57
4	.31012	.95070	.32621	3.06554	.32667	.94514	.34563	2.89327	56
5	.31040	.95061	.32653	3.06252	.32694	.94504	.34596	2.89055	55
6	.31068	.95052	.32685	3.05950	.32722	.94495	.34628	2.88783	54
7	.31095	.95043	.32717	3.05649	.32749	.94485	.34661	2.88511	53
8	.31123	.95033	.32749	3.05349	.32777	.94476	.34693	2.88240	52
9	.31151	.95024	.32782	3.05049	.32804	.94466	.34726	2.87970	51
10	.31178	.95015	.32814	3.04749	.32832	.94457	.34758	2.87700	50
11	.31206	.95006	.32846	3.04450	.32859	.94447	.34791	2.87430	49
12	.31233	.94997	.32878	3.04152	.32887	.94438	.34824	2.87161	48
13	.31261	.94988	.32911	3.03854	.32914	.94428	.34856	2.86892	47
14	.31289	.94979	.32943	3.03556	.32942	.94418	.34889	2.86624	46
15	.31316	.94970	.32975	3.03260	.32969	.94409	.34922	2.86356	45
16	.31344	.94961	.33007	3.02963	.32997	.94399	.34954	2.86089	44
17	.31372	.94952	.33040	3.02667	.33024	.94390	.34987	2.85822	43
18	.31399	.94943	.33072	3.02372	.33051	.94380	.35020	2.85555	42
19	.31427	.94933	.33104	3.02077	.33079	.94370	.35052	2.85289	41
20	.31454	.94924	.33136	3.01783	.33106	.94361	.35085	2.85023	40
21	.31482	.94915	.33169	3.01489	.33134	.94351	.35118	2.84758	39
22	.31510	.94906	.33201	3.01196	.33161	.94342	.35150	2.84494	38
23	.31537	.94897	.33233	3.00903	.33189	.94332	.35183	2.84229	37
24	.31565	.94888	.33266	3.00611	.33216	.94322	.35216	2.83965	36
25	.31593	.94878	.33298	3.00319	.33244	.94313	.35248	2.83702	35
26	.31620	.94869	.33330	3.00028	.33271	.94303	.35281	2.83439	34
27	.31648	.94860	.33363	2.99738	.33298	.94293	.35314	2.83176	33
28	.31675	.94851	.33395	2.99447	.33326	.94284	.35346	2.82914	32
29	.31703	.94842	.33427	2.99158	.33353	.94274	.35379	2.82653	31
30	.31730	.94832	.33460	2.98868	.33381	.94264	.35412	2.82391	30
31	.31758	.94823	.33492	2.98580	.33408	.94254	.35445	2.82130	29
32	.31763	.94814	.33524	2.98292	.33436	.94245	.35477	2.81870	28
33	.31813	.94805	.33557	2.98004	.33463	.94235	.35510	2.81610	27
34	.31841	.94795	.33589	2.97717	.33490	.94225	.35543	2.81350	26
35	.31868	.94786	.33621	2.97430	.33518	.94215	.35576	2.81091	25
36	.31896	.94777	.33654	2.97144	.33545	.94206	.35608	2.80833	24
37	.31923	.94768	.33686	2.96858	.33573	.94196	.35641	2.80574	23
38	.31951	.94758	.33718	2.96573	.33600	.94186	.35674	2.80316	22
39	.31979	.94749	.33751	2.96288	.33627	.94176	.35707	2.80059	21
40	.32006	.94740	.33783	2.96004	.33655	.94167	.35740	2.79802	20
41	.32034	.94730	.33816	2.95721	.33682	.94157	.35772	2.79545	19
42	.32061	.94721	.33848	2.95437	.33710	.94147	.35805	2.79289	18
43	.32089	.94712	.33881	2.95155	.33737	.94137	.35838	2.79033	17
44	.32116	.94702	.33913	2.94872	.33764	.94127	.35871	2.78778	16
45	.32144	.94693	.33945	2.94591	.33792	.94118	.35904	2.78523	15
46	.32171	.94684	.33978	2.94309	.33819	.94108	.35937	2.78269	14
47	.32199	.94674	.34010	2.94028	.33846	.94098	.35969	2.78014	13
48	.32227	.94665	.34043	2.93748	.33874	.94088	.36002	2.77761	12
49	.32254	.94656	.34075	2.93468	.33901	.94078	.36035	2.77507	11
50	.32282	.94646	.34108	2.93189	.33929	.94068	.36068	2.77254	10
51	.32309	.94637	.34140	2.92910	.33956	.94058	.36101	2.77002	9
52	.32337	.94627	.34173	2.92632	.33983	.94049	.36134	2.76750	8
53	.32364	.94618	.34205	2.92354	.34011	.94039	.36167	2.76498	7
54	.32392	.94609	.34238	2.92076	.34038	.94029	.36199	2.76247	6
55	.32419	.94599	.34270	2.91799	.34065	.94019	.36232	2.75996	5
56	.32447	.94590	.34303	2.91523	.34093	.94009	.36265	2.75746	4
57	.32474	.94580	.34335	2.91246	.34120	.93999	.36298	2.75496	3
58	.32502	.94571	.34368	2.90971	.34147	.93989	.36331	2.75246	2
59	.32529	.94561	.34400	2.90696	.34175	.93979	.36364	2.74997	1
60	.32557	.94552	.34433	2.90421	.34202	.93969	.36397	2.74748	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

71°	762	70°

	20°				21°				
	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
	.34202	.93969	.36397	2.74748	.35837	.93358	.38386	2.60509	60
	.34229	.93959	.36430	2.74499	.35864	.93348	.38420	2.60284	59
	.34257	.93949	.36463	2.74251	.35891	.93337	.38453	2.60057	58
	.34284	.93939	.36496	2.74004	.35918	.93327	.38487	2.59831	57
	.34311	.93929	.36529	2.73756	.35945	.93316	.38520	2.59606	56
	.34339	.93919	.36562	2.73509	.35973	.93306	.38553	2.59381	55
	.34366	.93909	.36595	2.73263	.36000	.93295	.38587	2.59156	54
	.34393	.93899	.36628	2.73017	.36027	.93285	.38620	2.58932	53
	.34421	.93889	.36661	2.72771	.36054	.93274	.38654	2.58708	52
	.34448	.93879	.36694	2.72526	.36081	.93264	.38687	2.58484	51
0	.34475	.93869	.36727	2.72281	.36108	.93253	.38721	2.58261	50
1	.34503	.93859	.36760	2.72036	.36135	.93243	.38754	2.58038	49
2	.34530	.93849	.36793	2.71792	.36162	.93232	.38787	2.57815	48
3	.34557	.93839	.36826	2.71548	.36190	.93222	.38821	2.57593	47
14	.34584	.93829	.36859	2.71305	.36217	.93211	.38854	2.57371	46
15	.34612	.93819	.36892	2.71062	.36244	.93201	.38888	2.57150	45
16	.34639	.93809	.36925	2.70819	.36271	.93190	.38921	2.56928	44
17	.34666	.93799	.36958	2.70577	.36298	.93180	.38955	2.56707	43
18	.34694	.93789	.36991	2.70335	.36325	.93169	.38988	2.56487	42
19	.34721	.93779	.37024	2.70094	.36352	.93159	.39022	2.56266	41
20	.34748	.93769	.37057	2.69853	.36379	.93148	.39055	2.56046	40
21	.34775	.93759	.37090	2.69612	.36406	.93137	.39089	2.55827	39
22	.34803	.93748	.37123	2.69371	.36434	.93127	.39122	2.55608	38
23	.34830	.93738	.37157	2.69131	.36461	.93116	.39156	2.55389	37
24	.34857	.93728	.37190	2.68892	.36488	.93106	.39190	2.55170	36
25	.34884	.93718	.37223	2.68653	.36515	.93095	.39223	2.54952	35
26	.34912	.93708	.37256	2.68414	.36542	.93084	.39257	2.54734	34
27	.34939	.93698	.37289	2.68175	.36569	.93074	.39290	2.54516	33
28	.34966	.93688	.37322	2.67937	.36596	.93063	.39324	2.54299	32
29	.34993	.93677	.37355	2.67700	.36623	.93052	.39357	2.54082	31
30	.35021	.93667	.37388	2.67462	.36650	.93042	.39391	2.53865	30
31	.35048	.93657	.37422	2.67225	.36677	.93031	.39425	2.53648	29
32	.35075	.93647	.37455	2.66989	.36704	.93020	.39458	2.53432	28
33	.35102	.93637	.37488	2.66752	.36731	.93010	.39492	2.53217	27
34	.35130	.93626	.37521	2.66516	.36758	.92999	.39526	2.53001	26
35	.35157	.93616	.37554	2.66281	.36785	.92988	.39559	2.52786	25
36	.35184	.93606	.37588	2.66046	.36812	.92978	.39593	2.52571	24
37	.35211	.93596	.37621	2.65811	.36839	.92967	.39626	2.52357	23
38	.35239	.93585	.37654	2.65576	.36867	.92956	.39660	2.52142	22
39	.35266	.93575	.37687	2.65342	.36894	.92945	.39694	2.51929	21
40	.35293	.93565	.37720	2.65109	.36921	.92935	.39727	2.51715	20
41	.35320	.93555	.37754	2.64875	.36948	.92924	.39761	2.51502	19
42	.35347	.93544	.37787	2.64642	.36975	.92913	.39795	2.51289	18
43	.35375	.93534	.37820	2.64410	.37002	.92902	.39829	2.51076	17
44	.35402	.93524	.37853	2.64177	.37029	.92892	.39862	2.50864	16
45	.35429	.93514	.37887	2.63945	.37056	.92881	.39896	2.50652	15
46	.35456	.93503	.37920	2.63714	.37083	.92870	.39930	2.50440	14
47	.35484	.93493	.37953	2.63483	.37110	.92859	.39963	2.50229	13
48	.35511	.93483	.37986	2.63252	.37137	.92849	.39997	2.50018	12
49	.35538	.93472	.38020	2.63021	.37164	.92838	.40031	2.49807	11
50	.35565	.93462	.38053	2.62791	.37191	.92827	.40065	2.49597	10
51	.35592	.93452	.38086	2.62561	.37218	.92816	.40098	2.49386	9
52	.35619	.93441	.38120	2.62332	.37245	.92805	.40132	2.49177	8
53	.35647	.93431	.38153	2.62103	.37272	.92794	.40166	2.48967	7
54	.35674	.93420	.38186	2.61874	.37299	.92784	.40200	2.48758	6
55	.35701	.93410	.38220	2.61646	.37326	.92773	.40234	2.48549	5
56	.35728	.93400	.38253	2.61418	.37353	.92762	.40267	2.48340	4
57	.35755	.93389	.38286	2.61190	.37380	.92751	.40301	2.48132	3
58	.35782	.93379	.38320	2.60963	.37407	.92740	.40335	2.47924	2
59	.35810	.93368	.38353	2.60736	.37434	.92729	.40369	2.47716	1
60	.35837	.93358	.38386	2.60509	.37461	.92718	.40403	2.47509	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.

	18°				19°				
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0	.30902	.95106	.32492	3.07768	.32557	.94552	.34433	2.90421	60
1	.30929	.95097	.32524	3.07464	.32584	.94542	.34465	2.90147	59
2	.30957	.95088	.32556	3.07160	.32612	.94533	.34498	2.89873	58
3	.30985	.95079	.32588	3.06857	.32639	.94523	.34530	2.89600	57
4	.31012	.95070	.32621	3.06554	.32667	.94514	.34563	2.89327	56
5	.31040	.95061	.32653	3.06252	.32694	.94504	.34596	2.89055	55
6	.31068	.95052	.32685	3.05950	.32722	.94495	.34628	2.88783	54
7	.31095	.95043	.32717	3.05649	.32749	.94485	.34661	2.88511	53
8	.31123	.95033	.32749	3.05349	.32777	.94476	.34693	2.88240	52
9	.31151	.95024	.32782	3.05049	.32804	.94466	.34726	2.87970	51
10	.31178	.95015	.32814	3.04749	.32832	.94457	.34758	2.87700	50
11	.31206	.95006	.32846	3.04450	.32859	.94447	.34791	2.87430	49
12	.31233	.94997	.32878	3.04152	.32887	.94438	.34824	2.87161	48
13	.31261	.94988	.32911	3.03854	.32914	.94428	.34856	2.86892	47
14	.31289	.94979	.32943	3.03556	.32942	.94418	.34889	2.86624	46
15	.31316	.94970	.32975	3.03260	.32969	.94409	.34922	2.86356	45
16	.31344	.94961	.33007	3.02963	.32997	.94399	.34954	2.86089	44
17	.31372	.94952	.33040	3.02667	.33024	.94390	.34987	2.85822	43
18	.31399	.94943	.33072	3.02372	.33051	.94380	.35020	2.85555	42
19	.31427	.94933	.33104	3.02077	.33079	.94370	.35052	2.85289	41
20	.31454	.94924	.33136	3.01783	.33106	.94361	.35085	2.85023	40
21	.31482	.94915	.33169	3.01489	.33134	.94351	.35118	2.84758	39
22	.31510	.94906	.33201	3.01196	.33161	.94342	.35150	2.84494	38
23	.31537	.94897	.33233	3.00903	.33189	.94332	.35183	2.84229	37
24	.31565	.94888	.33266	3.00611	.33216	.94322	.35216	2.83965	36
25	.31593	.94878	.33298	3.00319	.33244	.94313	.35248	2.83702	35
26	.31620	.94869	.33330	3.00028	.33271	.94303	.35281	2.83439	34
27	.31648	.94860	.33363	2.99738	.33298	.94293	.35314	2.83176	33
28	.31675	.94851	.33395	2.99447	.33326	.94284	.35346	2.82914	32
29	.31703	.94842	.33427	2.99158	.33353	.94274	.35379	2.82653	31
30	.31730	.94832	.33460	2.98868	.33381	.94264	.35412	2.82391	30
31	.31758	.94823	.33492	2.98580	.33408	.94254	.35445	2.82130	29
32	.31785	.94814	.33524	2.98292	.33436	.94245	.35477	2.81870	28
33	.31813	.94805	.33557	2.98004	.33463	.94235	.35510	2.81610	27
34	.31841	.94795	.33589	2.97717	.33490	.94225	.35543	2.81350	26
35	.31868	.94786	.33621	2.97430	.33518	.94215	.35576	2.81091	25
36	.31896	.94777	.33654	2.97144	.33545	.94206	.35608	2.80833	24
37	.31923	.94768	.33686	2.96858	.33573	.94196	.35641	2.80574	23
38	.31951	.94758	.33718	2.96573	.33600	.94186	.35674	2.80316	22
39	.31979	.94749	.33751	2.96288	.33627	.94176	.35707	2.80059	21
40	.32006	.94740	.33783	2.96004	.33655	.94167	.35740	2.79802	20
41	.32034	.94730	.33816	2.95721	.33682	.94157	.35772	2.79545	19
42	.32061	.94721	.33848	2.95437	.33710	.94147	.35805	2.79289	18
43	.32089	.94712	.33881	2.95155	.33737	.94137	.35838	2.79033	17
44	.32116	.94702	.33913	2.94872	.33764	.94127	.35871	2.78778	16
45	.32144	.94693	.33945	2.94591	.33792	.94118	.35904	2.78523	15
46	.32171	.94684	.33978	2.94309	.33819	.94108	.35937	2.78268	14
47	.32199	.94674	.34010	2.94028	.33846	.94098	.35969	2.78014	13
48	.32227	.94665	.34043	2.93748	.33874	.94088	.36002	2.77761	12
49	.32254	.94656	.34075	2.93468	.33901	.94078	.36035	2.77507	11
50	.32282	.94646	.34108	2.93189	.33929	.94068	.36068	2.77254	10
51	.32309	.94637	.34140	2.92910	.33956	.94058	.36101	2.77002	9
52	.32337	.94627	.34173	2.92632	.33983	.94049	.36134	2.76750	8
53	.32364	.94618	.34205	2.92354	.34011	.94039	.36167	2.76498	7
54	.32392	.94609	.34238	2.92076	.34038	.94029	.36199	2.76247	6
55	.32419	.94599	.34270	2.91799	.34065	.94019	.36232	2.75996	5
56	.32447	.94590	.34303	2.91523	.34093	.94009	.36265	2.75746	4
57	.32474	.94580	.34335	2.91246	.34120	.93999	.36298	2.75496	3
58	.32502	.94571	.34368	2.90971	.34147	.93989	.36331	2.75246	2
59	.32529	.94561	.34400	2.90696	.34175	.93979	.36364	2.74997	1
60	.32557	.94552	.34433	2.90421	.34202	.93969	.36397	2.74748	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′
	71°				70°				

′	20° Sin.	Cos.	Tan.	Cot.	21° Sin.	Cos.	Tan.	Cot.	′
0	.34202	.93969	.36397	2.74748	.35837	.93358	.38386	2.60509	60
1	.34229	.93959	.36430	2.74499	.35864	.93348	.38420	2.60283	59
2	.34257	.93949	.36463	2.74251	.35891	.93337	.38453	2.60057	58
3	.34284	.93939	.36496	2.74004	.35918	.93327	.38487	2.59831	57
4	.34311	.93929	.36529	2.73756	.35945	.93316	.38520	2.59606	56
5	.34339	.93919	.36562	2.73509	.35973	.93306	.38553	2.59381	55
6	.34366	.93909	.36595	2.73263	.36000	.93295	.38587	2.59156	54
7	.34393	.93899	.36628	2.73017	.36027	.93285	.38620	2.58932	53
8	.34421	.93889	.36661	2.72771	.36054	.93274	.38654	2.58708	52
9	.34448	.93879	.36694	2.72526	.36081	.93264	.38687	2.58484	51
10	.34475	.93869	.36727	2.72281	.36108	.93253	.38721	2.58261	50
1	.34503	.93859	.36760	2.72036	.36135	.93243	.38754	2.58038	49
2	.34530	.93849	.36793	2.71792	.36162	.93232	.38787	2.57815	48
3	.34557	.93839	.36826	2.71548	.36190	.93222	.38821	2.57593	47
4	.34584	.93829	.36859	2.71305	.36217	.93211	.38854	2.57371	46
5	.34612	.93819	.36892	2.71062	.36244	.93201	.38888	2.57150	45
6	.34639	.93809	.36925	2.70819	.36271	.93190	.38921	2.56928	44
7	.34666	.93799	.36958	2.70577	.36298	.93180	.38955	2.56707	43
8	.34694	.93789	.36991	2.70335	.36325	.93169	.38988	2.56487	42
9	.34721	.93779	.37024	2.70094	.36352	.93159	.39022	2.56266	41
20	.34748	.93769	.37057	2.69853	.36379	.93148	.39055	2.56046	40
1	.34775	.93759	.37090	2.69612	.36406	.93137	.39089	2.55827	39
2	.34803	.93748	.37123	2.69371	.36434	.93127	.39122	2.55608	38
3	.34830	.93738	.37157	2.69131	.36461	.93116	.39156	2.55389	37
4	.34857	.93728	.37190	2.68892	.36488	.93106	.39190	2.55170	36
5	.34884	.93718	.37223	2.68653	.36515	.93095	.39223	2.54952	35
6	.34912	.93708	.37256	2.68414	.36542	.93084	.39257	2.54734	34
7	.34939	.93698	.37289	2.68175	.36569	.93074	.39290	2.54516	33
8	.34966	.93688	.37322	2.67937	.36596	.93063	.39324	2.54299	32
9	.34993	.93677	.37355	2.67700	.36623	.93052	.39357	2.54082	31
30	.35021	.93667	.37388	2.67462	.36650	.93042	.39391	2.53865	30
1	.35048	.93657	.37422	2.67225	.36677	.93031	.39425	2.53648	29
2	.35075	.93647	.37455	2.66989	.36704	.93020	.39458	2.53432	28
3	.35102	.93637	.37488	2.66752	.36731	.93010	.39492	2.53217	27
4	.35130	.93626	.37521	2.66516	.36758	.92999	.39526	2.53001	26
5	.35157	.93616	.37554	2.66281	.36785	.92988	.39559	2.52786	25
6	.35184	.93606	.37588	2.66046	.36812	.92978	.39593	2.52571	24
7	.35211	.93596	.37621	2.65811	.36839	.92967	.39626	2.52357	23
8	.35239	.93585	.37654	2.65576	.36867	.92956	.39660	2.52142	22
9	.35266	.93575	.37687	2.65342	.36894	.92945	.39694	2.51929	21
0	.35293	.93565	.37720	2.65109	.36921	.92935	.39727	2.51715	20
1	.35320	.93555	.37754	2.64875	.36948	.92924	.39761	2.51502	19
2	.35347	.93544	.37787	2.64642	.36975	.92913	.39795	2.51289	18
3	.35375	.93534	.37820	2.64410	.37002	.92902	.39829	2.51076	17
4	.35402	.93524	.37853	2.64177	.37029	.92892	.39862	2.50864	16
5	.35429	.93514	.37887	2.63945	.37056	.92881	.39896	2.50652	15
6	.35456	.93503	.37920	2.63714	.37083	.92870	.39930	2.50440	14
7	.35484	.93493	.37953	2.63483	.37110	.92859	.39963	2.50229	13
8	.35511	.93483	.37986	2.63252	.37137	.92849	.39997	2.50018	12
9	.35538	.93472	.38020	2.63021	.37164	.92838	.40031	2.49807	11
30	.35565	.93462	.38053	2.62791	.37191	.92827	.40065	2.49597	10
1	.35592	.93452	.38086	2.62561	.37218	.92816	.40098	2.49386	9
2	.35619	.93441	.38120	2.62332	.37245	.92805	.40132	2.49177	8
3	.35647	.93431	.38153	2.62103	.37272	.92794	.40166	2.48967	7
4	.35674	.93420	.38186	2.61874	.37299	.92784	.40200	2.48758	6
5	.35701	.93410	.38220	2.61646	.37326	.92773	.40234	2.48549	5
6	.35728	.93400	.38253	2.61418	.37353	.92762	.40267	2.48340	4
7	.35755	.93389	.38286	2.61190	.37380	.92751	.40301	2.48132	3
8	.35782	.93379	.38320	2.60963	.37407	.92740	.40335	2.47924	2
9	.35810	.93368	.38353	2.60736	.37434	.92729	.40369	2.47716	1
40	.35837	.93358	.38386	2.60509	.37461	.92718	.40403	2.47509	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

	22°				23°			
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.
0	.37461	.92718	.40403	2.47509	.39073	.92050	.42447	2.35585
1	.37488	.92707	.40436	2.47302	.39100	.92039	.42482	2.35395
2	.37515	.92697	.40470	2.47095	.39127	.92028	.42516	2.35205
3	.37542	.92686	.40504	2.46888	.39153	.92016	.42551	2.35015
4	.37569	.92675	.40538	2.46682	.39180	.92005	.42585	2.34825
5	.37595	.92664	.40572	2.46476	.39207	.91994	.42619	2.34636
6	.37622	.92653	.40606	2.46270	.39234	.91982	.42654	2.34447
7	.37649	.92642	.40640	2.46065	.39260	.91971	.42688	2.34258
8	.37676	.92631	.40674	2.45860	.39287	.91959	.42722	2.34069
9	.37703	.92620	.40707	2.45655	.39314	.91948	.42757	2.33881
10	.37730	.92609	.40741	2.45451	.39341	.91936	.42791	2.33693
11	.37757	.92598	.40775	2.45246	.39367	.91925	.42826	2.33505
12	.37784	.92587	.40809	2.45043	.39394	.91914	.42860	2.33317
13	.37811	.92576	.40843	2.44839	.39421	.91902	.42894	2.33130
14	.37838	.92565	.40877	2.44636	.39448	.91891	.42929	2.32943
15	.37865	.92554	.40911	2.44433	.39474	.91879	.42963	2.32756
16	.37892	.92543	.40945	2.44230	.39501	.91868	.42998	2.32570
17	.37919	.92532	.40979	2.44027	.39528	.91856	.43032	2.32383
18	.37946	.92521	.41013	2.43825	.39555	.91845	.43067	2.32197
19	.37973	.92510	.41047	2.43623	.39581	.91833	.43101	2.32012
20	.37999	.92499	.41081	2.43422	.39608	.91822	.43136	2.31826
21	.38026	.92488	.41115	2.43220	.39635	.91810	.43170	2.31641
22	.38053	.92477	.41149	2.43019	.39661	.91799	.43205	2.31456
23	.38080	.92466	.41183	2.42819	.39688	.91787	.43239	2.31271
24	.38107	.92455	.41217	2.42618	.39715	.91775	.43274	2.31086
25	.38134	.92444	.41251	2.42418	.39741	.91764	.43308	2.30902
26	.38161	.92432	.41285	2.42218	.39768	.91752	.43343	2.30718
27	.38188	.92421	.41319	2.42019	.39795	.91741	.43378	2.30534
28	.38215	.92410	.41353	2.41819	.39822	.91729	.43412	2.30351
29	.38241	.92399	.41387	2.41620	.39848	.91718	.43447	2.30167
30	.38268	.92388	.41421	2.41421	.39875	.91706	.43481	2.29984
31	.38295	.92377	.41455	2.41223	.39902	.91694	.43516	2.29801
32	.38322	.92366	.41490	2.41025	.39928	.91683	.43550	2.29619
33	.38349	.92355	.41524	2.40827	.39955	.91671	.43585	2.29437
34	.38376	.92343	.41558	2.40629	.39982	.91660	.43620	2.29254
35	.38403	.92332	.41592	2.40432	.40008	.91648	.43654	2.29073
36	.38430	.92321	.41626	2.40235	.40035	.91636	.43689	2.28891
37	.38456	.92310	.41660	2.40038	.40062	.91625	.43724	2.28710
38	.38483	.92299	.41694	2.39841	.40088	.91613	.43758	2.28528
39	.38510	.92287	.41728	2.39645	.40115	.91601	.43793	2.28348
40	.38537	.92276	.41763	2.39449	.40141	.91590	.43828	2.28167
41	.38564	.92265	.41797	2.39253	.40168	.91578	.43862	2.27987
42	.38591	.92254	.41831	2.39058	.40195	.91566	.43897	2.27806
43	.38617	.92243	.41865	2.38863	.40221	.91555	.43932	2.27626
44	.38644	.92231	.41899	2.38668	.40248	.91543	.43966	2.27447
45	.38671	.92220	.41933	2.38473	.40275	.91531	.44001	2.27267
46	.38698	.92209	.41968	2.38279	.40301	.91519	.44036	2.27088
47	.38725	.92198	.42002	2.38084	.40328	.91508	.44071	2.26909
48	.38752	.92186	.42036	2.37891	.40355	.91496	.44105	2.26730
49	.38778	.92175	.42070	2.37697	.40381	.91484	.44140	2.26552
50	.38805	.92164	.42105	2.37504	.40408	.91472	.44175	2.26374
51	.38832	.92152	.42139	2.37311	.40434	.91461	.44210	2.26196
52	.38859	.92141	.42173	2.37118	.40461	.91449	.44244	2.26018
53	.38886	.92130	.42207	2.36925	.40488	.91437	.44279	2.25840
54	.38912	.92119	.42242	2.36733	.40514	.91425	.44314	2.25663
55	.38939	.92107	.42276	2.36541	.40541	.91414	.44349	2.25486
56	.38966	.92096	.42310	2.36349	.40567	.91402	.44384	2.25309
57	.38993	.92085	.42345	2.36158	.40594	.91390	.44418	2.25132
58	.39020	.92073	.42379	2.35967	.40621	.91378	.44453	2.24956
	1046	.92062	.42413	2.35776	.40647	.91366	.44488	2.24780
	73	.92050	.42447	2.35585	.40674	.91355	.44523	2.24604
	s.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.
		67°					66°	

	24°				25°				
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0	.40674	.91355	.44523	2.24604	.42262	.90631	.46631	2.14451	60
1	.40700	.91343	.44558	2.24428	.42288	.90618	.46666	2.14288	59
2	.40727	.91331	.44593	2.24252	.42315	.90606	.46702	2.14125	58
3	.40753	.91319	.44627	2.24077	.42341	.90594	.46737	2.13963	57
4	.40780	.91307	.44662	2.23902	.42367	.90582	.46772	2.13801	56
5	.40806	.91295	.44697	2.23727	.42394	.90569	.46808	2.13639	55
6	.40833	.91283	.44732	2.23553	.42420	.90557	.46843	2.13477	54
7	.40860	.91272	.44767	2.23378	.42446	.90545	.46879	2.13316	53
8	.40886	.91260	.44802	2.23204	.42473	.90532	.46914	2.13154	52
9	.40913	.91248	.44837	2.23030	.42499	.90520	.46950	2.12993	51
10	.40939	.91236	.44872	2.22857	.42525	.90507	.46985	2.12832	50
11	.40966	.91224	.44907	2.22683	.42552	.90495	.47021	2.12671	49
12	.40992	.91212	.44942	2.22510	.42578	.90483	.47056	2.12511	48
13	.41019	.91200	.44977	2.22337	.42604	.90470	.47092	2.12350	47
14	.41045	.91188	.45012	2.22164	.42631	.90458	.47128	2.12190	46
15	.41072	.91176	.45047	2.21992	.42657	.90446	.47163	2.12030	45
16	.41098	.91164	.45082	2.21819	.42683	.90433	.47199	2.11871	44
17	.41125	.91152	.45117	2.21647	.42709	.90421	.47234	2.11711	43
18	.41151	.91140	.45152	2.21475	.42736	.90408	.47270	2.11552	42
19	.41178	.91128	.45187	2.21304	.42762	.90396	.47305	2.11392	41
20	.41204	.91116	.45222	2.21132	.42788	.90383	.47341	2.11233	40
21	.41231	.91104	.45257	2.20961	.42815	.90371	.47377	2.11075	39
22	.41257	.91092	.45292	2.20790	.42841	.90358	.47412	2.10916	38
23	.41284	.91080	.45327	2.20619	.42867	.90346	.47448	2.10758	37
24	.41310	.91068	.45362	2.20449	.42894	.90334	.47483	2.10600	36
25	.41337	.91056	.45397	2.20278	.42920	.90321	.47519	2.10442	35
26	.41363	.91044	.45432	2.20108	.42946	.90309	.47555	2.10284	34
27	.41390	.91032	.45467	2.19938	.42972	.90296	.47590	2.10126	33
28	.41416	.91020	.45502	2.19769	.42999	.90284	.47626	2.09969	32
29	.41443	.91008	.45538	2.19599	.43025	.90271	.47662	2.09811	31
30	.41469	.90996	.45573	2.19430	.43051	.90259	.47698	2.09654	30
31	.41496	.90984	.45608	2.19261	.43077	.90246	.7733	2.09498	29
32	.41522	.90972	.45643	2.19092	.43104	.90233	.47769	2.09341	28
33	.41549	.90960	.45678	2.18923	.43130	.90221	.47805	2.09184	27
34	.41575	.90948	.45713	2.18755	.43156	.90208	.47840	2.09028	26
35	.41602	.90936	.45748	2.18587	.43182	.90196	.47876	2.08872	25
36	.41628	.90924	.45784	2.18419	.43209	.90183	.47912	2.08716	24
37	.41655	.90911	.45819	2.18251	.43235	.90171	.47948	2.08560	23
38	.41681	.90899	.45854	2.18084	.43261	.90158	.47984	2.08405	22
39	.41707	.90887	.45889	2.17916	.43287	.90146	.48019	2.08250	21
40	.41734	.90875	.45924	2.17749	.43313	.90133	.48055	2.08094	20
41	.41760	.90863	.45960	2.17582	.43340	.90120	.48091	2.07939	19
42	.41787	.90851	.45995	2.17416	.43366	.90108	.48127	2.07785	18
43	.41813	.90839	.46030	2.17249	.43392	.90095	.48163	2.07630	17
44	.41840	.90826	.46065	2.17083	.43418	.90082	.48198	2.07476	16
45	.41866	.90814	.46101	2.16917	.43445	.90070	.48234	2.07321	15
46	.41892	.90802	.46136	2.16751	.43471	.90057	.48270	2.07167	14
47	.41919	.90790	.46171	2.16585	.43497	.90045	.48306	2.07014	13
48	.41945	.90778	.46206	2.16420	.43523	.90032	.48342	2.06860	12
49	.41972	.90766	.46242	2.16255	.43549	.90019	.48378	2.06706	11
50	.41998	.90753	.46277	2.16090	.43575	.90007	.48414	2.06553	10
51	.42024	.90741	.46312	2.15925	.43602	.89994	.48450	2.06400	9
52	.42051	.90729	.46348	2.15760	.43628	.89981	.48486	2.06247	8
53	.42077	.90717	.46383	2.15596	.43654	.89968	.48521	2.06094	7
54	.42104	.90704	.46418	2.15432	.43680	.89956	.48557	2.05942	6
55	.42130	.90692	.46454	2.15268	.43706	.89943	.48593	2.05790	5
56	.42156	.90680	.46489	2.15104	.43733	.89930	.48629	2.05637	4
57	.42183	.90668	.46525	2.14940	.43759	.89918	.48665	2.05485	3
58	.42209	.90655	.46560	2.14777	.43785	.89905	.48701	2.05333	2
59	.42235	.90643	.46595	2.14614	.43811	.89892	.48737	2.05182	1
60	.42262	.90631	.46631	2.14451	.43837	.89879	.48773	2.05030	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

	30°				31°			
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.
0	.50000	.86603	.57735	1.73205	.51504	.85717	.60086	1.66428
1	.50025	.86588	.57774	1.73089	.51529	.85702	.60126	1.66318
2	.50050	.86573	.57813	1.72973	.51554	.85687	.60165	1.66209
3	.50076	.86559	.57851	1.72857	.51579	.85672	.60205	1.66099
4	.50101	.86544	.57890	1.72741	.51604	.85657	.60245	1.65990
5	.50126	.86530	.57929	1.72625	.51628	.85642	.60284	1.65881
6	.50151	.86515	.57968	1.72509	.51653	.85627	.60324	1.65772
7	.50176	.86501	.58007	1.72393	.51678	.85612	.60364	1.65663
8	.50201	.86486	.58046	1.72278	.51703	.85597	.60403	1.65554
9	.50227	.86471	.58085	1.72163	.51728	.85582	.60443	1.65445
10	.50252	.86457	.58124	1.72047	.51753	.85567	.60483	1.65337
11	.50277	.86442	.58162	1.71932	.51778	.85551	.60523	1.65228
12	.50302	.86427	.58201	1.71817	.51803	.85536	.60562	1.65120
13	.50327	.86413	.58240	1.71702	.51828	.85521	.60602	1.65011
14	.50352	.86398	.58279	1.71588	.51852	.85506	.60642	1.64903
15	.50377	.86384	.58318	1.71473	.51877	.85491	.60681	1.64795
16	.50403	.86369	.58357	1.71358	.51902	.85476	.60721	1.64687
17	.50428	.86354	.58396	1.71244	.51.927	.85461	.60761	1.64578
18	.50453	.86340	.58435	1.71129	.51952	.85446	.60801	1.64471
19	.50478	.86325	.58474	1.71015	.51977	.85431	.60841	1.64363
20	.50503	.86310	.58513	1.70901	.52002	.85416	.60821	1.64256
21	.50528	.86295	.58552	1.70787	.52026	.85401	.60921	1.64148
22	.50553	.86281	.58591	1.70673	.52051	.85385	.60960	1.64041
23	.50578	.86266	.58631	1.70560	.52076	.85370	.61000	1.63934
24	.50603	.86251	.58670	1.70446	.52101	.85355	.61040	1.63826
25	.50628	.86237	.58709	1.70332	.52126	.85340	.61080	1.63719
26	.50654	.86222	.58748	1.70219	.52151	.85325	.61120	1.63612
27	.50679	.86207	.58787	1.70106	.52175	.85310	.61160	1.63505
28	.50704	.86192	.58826	1.69992	.52200	.85294	.61200	1.63398
29	.50729	.86178	.58865	1.69879	.52225	.85279	.61240	1.63292
30	.50754	.86163	.58905	1.69766	.52250	.85264	.61280	1.63185
31	.50779	.86148	.58944	1.69653	.52275	.85249	.61320	1.63078
32	.50804	.86133	.58983	1.69541	.52299	.85234	.61360	1.62972
33	.50829	.86119	.59022	1.69428	.52324	.85218	.61400	1.62866
34	.50854	.86104	.59061	1.69316	.52349	.85203	.61440	1.62760
35	.50879	.86089	.59101	1.69203	.52374	.85188	.61480	1.62654
36	.50904	.86074	.59140	1.69091	.52399	.85173	.61520	1.62548
37	.50929	.86059	.59179	1.68979	.52423	.85157	.61561	1.62442
38	.50954	.86045	.59218	1.68866	.52448	.85142	.61601	1.62336
39	.50979	.86030	.59258	1.68754	.52473	.85127	.61641	1.62230
40	.51004	.86015	.59297	1.68643	.52498	.85112	.61681	1.62125
41	.51029	.86000	.59336	1.68531	.52522	.85096	.61721	1.62019
42	.51054	.85985	.59376	1.68419	.52547	.85081	.61761	1.61914
43	.51079	.85970	.59415	1.68308	.52572	.85066	.61801	1.61808
44	.51104	.85956	.59454	1.68196	.52597	.85051	.61842	1.61703
45	.51129	.85941	.59494	1.68085	.52621	.85035	.61882	1.61598
46	.51154	.85926	.59533	1.67974	.52646	.85020	.61922	1.61493
47	.51179	.85911	.59573	1.67863	.52671	.85005	.61962	1.61388
48	.51204	.85896	.59612	1.67752	.52696	.84989	.62003	1.61283
49	.51229	.85881	.59651	1.67641	.52720	.84974	.62043	1.61178
50	.51254	.85866	.59691	1.67530	.52745	.84959	.62083	1.61074
51	.51279	.85851	.59730	1.67419	.52770	.84943	.62124	1.60970
52	.51304	.85836	.59770	1.67309	.52794	.84928	.62164	1.60865
53	.51329	.85821	.59809	1.67198	.52819	.84913	.62204	1.60761
54	.51354	.85806	.59849	1.67088	.52844	.84897	.62245	1.60657
55	.51379	.85792	.59888	1.66978	.52869	.84882	.62285	1.60553
56	.51404	.85777	.59928	1.66867	.52893	.84866	.62325	1.60449
57	.51429	.85762	.59967	1.66757	.52918	.84851	.62366	1.60345
58	.51454	.85747	.60007	1.66647	.52943	.84836	.62406	1.60241
59	.51479	.85732	.60046	1.66538	.52967	.84820	.62446	1.60137
60	.51504	.85717	.60086	1.66428	.52992	.84805	.62487	1.60033
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.
	59°			708	58°			

	28°				29°				
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0	.46947	.88295	.53171	1.88073	.48481	.87462	.55431	1.80405	60
1	.46973	.88281	.53208	1.87941	.48506	.87448	.55469	1.80281	59
2	.46999	.88267	.53246	1.87809	.48532	.87434	.55507	1.80158	58
3	.47024	.88254	.53283	1.87677	.48557	.87420	.55545	1.80034	57
4	.47050	.88240	.53320	1.87546	.48583	.87406	.55583	1.79911	56
5	.47076	.88226	.53358	1.87415	.48608	.87391	.55621	1.79788	55
6	.47101	.88213	.53395	1.87283	.48634	.87377	.55659	1.79665	54
7	.47127	.88199	.53432	1.87152	.48659	.87363	.55697	1.79542	53
8	.47153	.88185	.53470	1.87021	.48684	.87349	.55736	1.79419	52
9	.47178	.88172	.53507	1.86891	.48710	.87335	.55774	1.79296	51
10	.47204	.88158	.53545	1.86760	.48735	.87321	.55812	1.79174	50
11	.47229	.88144	.53582	1.86630	.48761	.87306	.55850	1.79051	49
12	.47255	.88130	.53620	1.86499	.48786	.87292	.55888	1.78929	48
13	.47281	.88117	.53657	1.86369	.48811	.87278	.55926	1.78807	47
14	.47306	.88103	.53694	1.86239	.48837	.87264	.55964	1.78685	46
15	.47332	.88089	.53732	1.86109	.48862	.87250	.56003	1.78563	45
16	.47358	.88075	.53769	1.85979	.48888	.87235	.56041	1.78441	44
17	.47383	.88062	.53807	1.85850	.48913	.87221	.56079	1.78319	43
18	.47409	.88048	.53844	1.85720	.48938	.87207	.56117	1.78198	42
19	.47434	.88034	.53882	1.85591	.48964	.87193	.56156	1.78077	41
20	.47460	.88020	.53920	1.85462	.48989	.87178	.56194	1.77955	40
21	.47486	.88006	.53957	1.85333	.49014	.87164	.56232	1.77834	39
22	.47511	.87993	.53995	1.85204	.49040	.87150	.56270	1.77713	38
23	.47537	.87979	.54032	1.85075	.49065	.87136	.56309	1.77592	37
24	.47562	.87965	.54070	1.84946	.49090	.87121	.56347	1.77471	36
25	.47588	.87951	.54107	1.84818	.49116	.87107	.56385	1.77351	35
26	.47614	.87937	.54145	1.84689	.49141	.87093	.56424	1.77230	34
27	.47639	.87923	.54183	1.84561	.49166	.87079	.56462	1.77110	33
28	.47665	.87909	.54220	1.84433	.49192	.87064	.56501	1.76990	32
29	.47690	.87896	.54258	1.84305	.49217	.87050	.56539	1.76869	31
30	.47716	.87882	.54296	1.84177	.49242	.87036	.56577	1.76749	30
31	.47741	.87868	.54333	1.84049	.49268	.87021	.56616	1.76629	29
32	.47767	.87854	.54371	1.83922	.49293	.87007	.56654	1.76510	28
33	.47793	.87840	.54409	1.83794	.49318	.86993	.56693	1.76390	27
34	.47818	.87826	.54446	1.83667	.49344	.86978	.56731	1.76271	26
35	.47844	.87812	.54484	1.83540	.49369	.86964	.56769	1.76151	25
36	.47869	.87798	.54522	1.83413	.49394	.86949	.56808	1.76032	24
37	.47895	.87784	.54560	1.83286	.49419	.86935	.56846	1.75913	23
38	.47920	.87770	.54597	1.83159	.49445	.86921	.56885	1.75794	22
39	.47946	.87756	.54635	1.83033	.49470	.86906	.56923	1.75675	21
40	.47971	.87743	.54673	1.82906	.49495	.86892	.56962	1.75556	20
41	.47997	.87729	.54711	1.82780	.49521	.86878	.57000	1.75437	19
42	.48022	.87715	.54748	1.82654	.49546	.86863	.57039	1.75319	18
43	.48048	.87701	.54786	1.82528	.49571	.86849	.57078	1.75200	17
44	.48073	.87687	.54824	1.82402	.49596	.86834	.57116	1.75082	16
45	.48099	.87673	.54862	1.82276	.49622	.86820	.57155	1.74964	15
46	.48124	.87659	.54900	1.82150	.49647	.86805	.57193	1.74846	14
47	.48150	.87645	.54938	1.82025	.49672	.86791	.57232	1.74728	13
48	.48175	.87631	.54975	1.81899	.49697	.86777	.57271	1.74610	12
49	.48201	.87617	.55013	1.81774	.49723	.86762	.57309	1.74492	11
50	.48226	.87603	.55051	1.81649	.49748	.86748	.57348	1.74375	10
51	.48252	.87589	.55089	1.81524	.49773	.86733	.57386	1..4257	9
52	.48277	.87575	.55127	1.81399	.49798	.86719	.57425	1.74140	8
53	.48303	.87561	.55165	1.81274	.49824	.86704	.57464	1.74022	7
54	.48328	.87546	.55203	1.81150	.49849	.86690	.57503	1.73905	6
55	.48354	.87532	.55241	1.81025	.49874	.86675	.57541	1.73788	5
56	.48379	.87518	.55279	1.80901	.49899	.86661	.57580	1.73671	4
57	.48405	.87504	.55317	1.80777	.49924	.86646	.57619	1.73555	3
58	.48430	.87490	.55355	1.80653	.49950	.86632	.57657	1.73438	2
59	.48456	.87476	.55393	1.80529	.49975	.86617	.57696	1.73321	1
60	.48481	.87462	.55431	1.80405	.50000	.86603	.57735	1.73205	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′
	61°						60°		

11	.50277	.86442	.58162	1.71982	.51778	.85551	.60522	1.652
12	.50302	.86427	.58201	1.71817	.51803	.85536	.60562	1.651
13	.50327	.86413	.58240	1.71702	.51828	.85521	.60602	1.650
14	.50352	.86398	.58279	1.71588	.51852	.85506	.60642	1.649
15	.50377	.86384	.58318	1.71473	.51877	.85491	.60681	1.647
16	.50403	.86369	.58357	1.71358	.51902	.85476	.60721	1.646
17	.50428	.86354	.58396	1.71244	.5.927	.85461	.60761	1.645
18	.50453	.86340	.58435	1.71129	.51952	.85446	.60801	1.644
19	.50478	.86325	.58474	1.71015	.51977	.85431	.60841	1.643
20	.50503	.86310	.58513	1.70901	.52002	.85416	.60881	1.642
21	.50528	.86295	.58552	1.70787	.52026	.85401	.60921	1.641
22	.50553	.86281	.58591	1.70673	.52051	.85385	.60960	1.640
23	.50578	.86266	.58631	1.70560	.52076	.85370	.61000	1.639
24	.50603	.86251	.58670	1.70446	.52101	.85355	.61040	1.638
25	.50628	.86237	.58709	1.70332	.52126	.85340	.61080	1.637
26	.50654	.86222	.58748	1.70219	.52151	.85325	.61120	1.636
27	.50679	.86207	.58787	1.70106	.52175	.85310	.61160	1.635
28	.50704	.86192	.58826	1.69992	.52200	.85294	.61200	1.633
29	.50729	.86178	.58865	1.69879	.52225	.85279	.61240	1.632
30	.50754	.86163	.58905	1.69766	.52250	.85264	.61280	1.631
31	.50779	.86148	.58944	1.69653	.52275	.85249	.61320	1.630
32	.50804	.86133	.58983	1.69541	.52299	.85234	.61360	1.629
33	.50829	.86119	.59022	1.69428	.52324	.85218	.61400	1.628
34	.50854	.86104	.59061	1.69316	.52349	.85203	.61440	1.627
35	.50879	.86089	.59101	1.69203	.52374	.85188	.61480	1.626
36	.50904	.86074	.59140	1.69091	.52399	.85173	.61520	1.625
37	.50929	.86059	.59179	1.68979	.52423	.85157	.61561	1.624
38	.50954	.86045	.59218	1.68866	.52448	.85142	.61601	1.623
39	.50979	.86030	.59258	1.68754	.52473	.85127	.61641	1.622
40	.51004	.86015	.59297	1.68643	.52498	.85112	.61681	1.621
41	.51029	.86000	.59336	1.68531	.52522	.85096	.61721	1.620
42	.51054	.85985	.59376	1.68419	.52547	.85081	.61761	1.619
43	.51079	.85970	.59415	1.68308	.52572	.85066	.61801	1.618
44	.51104	.85956	.59454	1.68196	.52597	.85051	.61842	1.617
45	.51129	.85941	.59494	1.68085	.52621	.85035	.61882	1.616
46	.51154	.85926	.59533	1.67974	.52646	.85020	.61922	1.614
47	.51179	.85911	.59573	1.67863	.52671	.85005	.61962	1.613

	32°				33°				
′	Sin.	Cos	Tan.	Cot	Sin.	Cos.	Tan.	Cot.	′
0	.52992	.84805	.62487	1.60033	.54464	.83867	.64941	1.53986	60
1	.53017	.84789	.62527	1.59930	.54488	.83851	.64982	1.53888	59
2	.53041	.84774	.62568	1.59826	.54513	.83835	.65024	1.53791	58
3	.53066	.84759	.62608	1.59723	.54537	.83819	.65065	1.53693	57
4	.53091	.84743	.62649	1.59620	.54561	.83804	.65106	1.53595	56
5	.53115	.84728	.62689	1.59517	.54585	.83788	.65148	1.53497	55
6	.53140	.84712	.62730	1.59414	.54610	.83772	.65189	1.53400	54
7	.53164	.84697	.62770	1.59311	.54635	.83756	.65231	1.53302	53
8	.53189	.84681	.62811	1.59208	.54659	.83740	.65272	1.53205	52
9	.53214	.84666	.62852	1.59105	.54683	.83724	.65314	1.53107	51
10	.53238	.84650	.62892	1.59002	.54708	.83708	.65355	1.53010	50
1	.53263	.84635	.62933	1.58900	.54732	.83692	.65397	1.52913	49
2	.53288	.84619	.62973	1.58797	.54756	.83676	.65438	1.52816	48
3	.53312	.84604	.63014	1.58695	.54781	.83660	.65480	1.52719	47
4	.53337	.84588	.63055	1.58593	.54805	.83645	.65521	1.52622	46
5	.53361	.84573	.63095	1.58490	.54829	.83629	.65563	1.52525	45
6	.53386	.84557	.63136	1.58388	.54854	.83613	.65604	1.52429	44
7	.53411	.84542	.63177	1.58286	.54878	.83597	.65646	1.52332	43
8	.53435	.84526	.63217	1.58184	.54902	.83581	.65688	1.52235	42
9	.53460	.84511	.63258	1.58083	.54927	.83565	.65729	1.52139	41
0	.53484	.84495	.63299	1.57981	.54951	.83549	.65771	1.52043	40
1	.53509	.84480	.63340	1.57879	.54975	.83533	.65813	1.51946	39
2	.53534	.84464	.63380	1.57778	.54999	.83517	.65854	1.51850	38
3	.53558	.84448	.63421	1.57676	.55024	.83501	.65896	1.51754	37
4	.53583	.84433	.63462	1.57575	.55048	.83485	.65938	1.51658	36
5	.53607	.84417	.63503	1.57474	.55072	.83469	.65980	1.51562	35
6	.53632	.84402	.63544	1.57372	.55097	.83453	.66021	1.51466	34
7	.53656	.84386	.63584	1.57271	.55121	.83437	.66063	1.51370	33
8	.53681	.84370	.63625	1.57170	.55145	.83421	.66105	1.51275	32
9	.53705	.84355	.63666	1.57069	.55169	.83405	.66147	1.51179	31
0	.53730	.84339	.63707	1.56969	.55194	.83389	.66189	1.51084	30
1	.53754	.84324	.63748	1.56868	.55218	.83373	.66230	1.50988	29
2	.53779	.84308	.63789	1.56767	.55242	.83356	.66272	1.50893	28
3	.53804	.84292	.63830	1.56667	.55266	.83340	.66314	1.50797	27
4	.53828	.84277	.63871	1.56566	.55291	.83324	.66356	1.50702	26
5	.53853	.84261	.63912	1.56466	.55315	.83308	.66398	1.50607	25
6	.53877	.84245	.63953	1.56366	.55339	.83292	.66440	1.50512	24
7	.53902	.84230	.63994	1.56265	.55363	.83276	.66482	1.50417	23
8	.53926	.84214	.64035	1.56165	.55388	.83260	.66524	1.50322	22
9	.53951	.84198	.64076	1.56065	.55412	.83244	.66566	1.50228	21
0	.53975	.84182	.64117	1.55966	.55436	.83228	.66608	1.50133	20
1	.54000	.84167	.64158	1.55866	.55460	.83212	.66650	1.50038	19
2	.54024	.84151	.64199	1.55766	.55484	.83195	.66692	1.49944	18
3	.54049	.84135	.64240	1.55666	.55509	.83179	.66734	1.49849	17
4	.54073	.84120	.64281	1.55567	.55533	.83163	.66776	1.49755	16
5	.54097	.84104	.64322	1.55467	.55557	.83147	.66818	1.49661	15
6	.54122	.84088	.64363	1.55368	.55581	.83131	.66860	1.49566	14
7	.54146	.84072	.64404	1.55269	.55605	.83115	.66902	1.49472	13
8	.54171	.84057	.64446	1.55170	.55630	.83098	.66944	1.49378	12
9	.54195	.84041	.64487	1.55071	.55654	.83082	.66986	1.49284	11
10	.54220	.84025	.64528	1.54972	.55678	.83066	.67028	1.49190	10
1	.54244	.84009	.64569	1.54873	.55702	.83050	.67071	1.49097	9
2	.54269	.83994	.64610	1.54774	.55726	.83034	.67113	1.49003	8
3	.54293	.83978	.64652	1.54675	.55750	.83017	.67155	1.48909	7
4	.54317	.83962	.64693	1.54576	.55775	.83001	.67197	1.48816	6
5	.54342	.83946	.64734	1.54478	.55799	.82985	.67239	1.48722	5
6	.54366	.83930	.64775	1.54379	.55823	.82969	.67282	1.48629	4
7	.54391	.83915	.64817	1.54281	.55847	.82953	.67324	1.48536	3
8	.54415	.83899	.64858	1.54183	.55871	.82936	.67366	1.48442	2
9	.54440	.83883	.64899	1.54085	.55895	.82920	.67409	1.48349	1
0	.54464	.83867	.64941	1.53986	.55919	.82904	.67451	1.48256	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′

	38°				39°				
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0	·61566	·78801	·78129	1·27994	·62932	·77715	·80978	1·23490	60
1	·61589	·78783	·78175	1·27917	·62955	·77696	·81027	1·23416	59
2	·61612	·78765	·78222	1·27841	·62977	·77678	·81075	1·23343	58
3	·61635	·78747	·78269	1·27764	·63000	·77660	·81123	1·23270	57
4	·61658	·78729	·78316	1·27688	·63022	·77641	·81171	1·23196	56
5	·61681	·78711	·78363	1·27611	·63045	·77623	·81220	1·23123	55
6	·61704	·78694	·78410	1·27535	·63068	·77605	·81268	1·23050	54
7	·61726	·78676	·78457	1·27458	·63090	·77586	·81316	1·22977	53
8	·61749	·78658	·78504	1·27382	·63113	·77568	·81364	1·22904	52
9	·61772	·78640	·78551	1·27306	·63135	·77550	·81413	1·22831	51
10	·61795	·78622	·78598	1·27230	·63158	·77531	·81461	1·22758	50
11	·61818	·78604	·78645	1·27153	·63180	·77513	·81510	1·22685	49
12	·61841	·78586	·78692	1·27077	·63203	·77494	·81558	1·22612	48
13	·61864	·78568	·78739	1·27001	·63225	·77476	·81606	1·22539	47
14	·61887	·78550	·78786	1·26925	·63248	·77458	·81655	1·22467	46
15	·61909	·78532	·78834	1·26849	·63271	·77439	·81703	1·22394	45
16	·61932	·78514	·78881	1·26774	·63293	·77421	·81752	1·22321	44
17	·61955	·78496	·78928	1·26698	·63316	·77402	·81800	1·22249	43
18	·61978	·78478	·78975	1·26622	·63338	·77384	·81849	1·22176	42
19	·62001	·78460	·79022	1·26546	·63361	·77366	·81898	1·22104	41
20	·62024	·78442	·79070	1·26471	·63383	·77347	·81946	1·22031	40
21	·62046	·78424	·79117	1·26395	·63406	·77329	·81995	1·21959	39
22	·62069	·78405	·79164	1·26319	·63428	·77310	·82044	1·21886	38
23	·62092	·78387	·79212	1·26244	·63451	·77292	·82093	1·21814	37
24	·62115	·78369	·79259	1·26169	·63473	·77273	·82141	1·21742	36
25	·62138	·78351	·79306	1·26093	·63496	·77255	·82190	1·21670	35
26	·62160	·78333	·79354	1·26018	·63518	·77236	·82238	1·21598	34
27	·62183	·78315	·79401	1·25943	·63540	·77218	·82287	1·21526	33
28	·62206	·78297	·79449	1·25867	·63563	·77199	·82336	1·21454	32
29	·62229	·78279	·79496	1·25792	·63585	·77181	·82385	1·21382	31
30	·62251	·78261	·79544	1·25717	·63608	·77162	·82434	1·21310	30
31	·62274	·78243	·79591	1·25642	·63630	·77144	·82483	1·21238	29
32	·62297	·78225	·79639	1·25567	·63653	·77125	·82531	1·21166	28
33	·62320	·78206	·79686	1·25492	·63675	·77107	·82580	1·21094	27
34	·62342	·78188	·79734	1·25417	·63698	·77088	·82629	1·21023	26
35	·62365	·78170	·79781	1·25343	·63720	·77070	·82678	1·20951	25
36	·62388	·78152	·79829	1·25268	·63742	·77051	·82727	1·20879	24
37	·62411	·78134	·79877	1·25193	·63765	·77033	·82776	1·20808	23
38	·62433	·78116	·79924	1·25118	·63787	·77014	·82825	1·20736	22
39	·62456	·78098	·79972	1·25044	·63810	·76996	·82874	1·20665	21
40	·62479	·78079	·80020	1·24969	·63832	·76977	·82923	1·20593	20
41	·62502	·78061	·80067	1·24895	·63854	·76959	·82972	1·20522	19
42	·62524	·78043	·80115	1·24820	·63877	·76940	·83022	1·20451	18
43	·62547	·78025	·80163	1·24746	·63899	·76921	·83071	1·20379	17
44	·62570	·78007	·80211	1·24672	·63922	·76903	·83120	1·20308	16
45	·62592	·77988	·80259	1·24597	·63944	·76884	·83169	1·20237	15
46	·62615	·77970	·80306	1·24523	·63966	·76866	·83218	1·20166	14
47	·62638	·77952	·80354	1·24449	·63989	·76847	·83268	1·20095	13
48	·62660	·77934	·80402	1·24375	·64011	·76828	·83317	1·20024	12
49	·62683	·77916	·80450	1·24301	·64033	·76810	·83366	1·19953	11
50	·62706	·77897	·80498	1·24227	·64056	·76791	·83415	1·19882	10
51	·62728	·77879	·80546	1·24153	·64078	·76772	·83465	1·19811	9
52	·62751	·77861	·80594	1·24079	·64100	·76754	·83514	1·19740	8
53	·62774	·77843	·80642	1·24005	·64123	·76735	·83564	1·19669	7
54	·62796	·77824	·80690	1·23931	·64145	·76717	·83613	1·19599	6
55	·62819	·77806	·80738	1·23858	·64167	·76698	·83662	1·19528	5
56	·62842	·77788	·80786	1·23784	·64190	·76679	·83712	1·19457	4
57	·62864	·77769	·80834	1·23710	·64212	·76661	·83761	1·19387	3
58	·62887	·77751	·80882	1·23637	·64234	·76642	·83811	1·19316	2
59	·62909	·77733	·80930	1·23563	·64256	·76623	·83860	1·19246	1
60	·62932	·77715	·80978	1·23490	·64279	·76604	·83910	1·19175	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′
	51°						50°		

	36°			37°			
	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
	·80902	·72654	1·37638	·60182	·79864	·75855	1·32704	60
	·80885	·72699	1·37554	·60205	·79846	·75401	1·32624	59
	·80867	·72743	1·37470	·60228	·79829	·75447	1·32544	58
	·80350	·72788	1·37386	·60251	·79811	·75492	1·32464	57
	·80833	·72832	1·37302	·60274	·79793	·75538	1·32384	56
	·80816	·72877	1·37218	·60298	·79776	·75584	1·32304	55
	·80799	·72921	1·37134	·60321	·79758	·75629	1·32224	54
	·80782	·72966	1·37050	·60344	·79741	·75675	1·32144	53
	·80765	·73010	1·36967	·60367	·79723	·75721	1·32064	52
	·80748	·73055	1·36883	·60390	·79706	·75767	1·31984	51
	·80730	·73100	1·36800	·60414	·79688	·75812	1·31904	50
	·80713	·73144	1·36716	·60437	·79671	·75858	1·31825	49
	·80696	·73189	1·36633	·60460	·79653	·75904	1·31745	48
	·80679	·73234	1·36549	·60483	·79635	·75950	1·31666	47
	·80662	·73278	1·36466	·60506	·79618	·75996	1·31586	46
	·80644	·73323	1·36383	·60529	·79600	·76042	1·31507	45
	·80627	·73368	1·36300	·60553	·79583	·76088	1·31427	44
	·80610	·73413	1·36217	·60576	·79565	·76134	1·31348	43
	·80593	·73457	1·36134	·60599	·79547	·76180	1·31269	42
	·80576	·73502	1·36051	·60622	·79530	·76226	1·31190	41
	·80558	·73547	1·35968	·60645	·79512	·76272	1·31110	40
	·80541	·73592	1·35885	·60668	·79494	·76318	1·31031	39
	·80524	·73637	1·35802	·60691	·79477	·76364	1·30952	38
	·80507	·73681	1·35719	·60714	·79459	·76410	1·30873	37
	·80489	·73726	1·35637	·60738	·79441	·76456	1·30795	36
	·80472	·73771	1·35554	·60761	·79424	·76502	1·30716	35
	·80455	·73816	1·35472	·60784	·79406	·76548	1·30637	34
	·80438	·73861	1·35389	·60807	·79388	·76594	1·30558	33
	·80420	·73906	1·35307	·60830	·79371	·76640	1·30480	32
	·80403	·73951	1·35224	·60853	·79353	·76686	1·30401	31
	·80386	·73996	1·35142	·60876	·79335	·76733	1·30323	30
	·80368	·74041	1·35060	·60899	·79318	·76779	1·30244	29
	·80351	·74086	1·34978	·60922	·79300	·76825	1·30166	28
	·80334	·74131	1·34896	·60945	·79282	·76871	1·30087	27
	·80316	·74176	1·34814	·60968	·79264	·76918	1·30009	26
	·80299	·74221	1·34732	·60991	79247	·76964	1·29931	25
	·80282	·74267	1·34650	·61015	·79229	·77010	1·29853	24
	·80264	·74312	1·34568	·61038	·79211	·77057	1·29775	23
	·80247	·74357	1·34487	·61061	·79193	·77103	1·29696	22
	·80230	·74402	1·34405	·61084	·79176	·77149	1·29618	21
	·80212	·74447	1·34323	·61107	·79158	·77196	1·29541	20
	·80195	·74492	1·34242	·61130	·79140	·77242	1·29463	19
	·80178	·74538	1·34160	·61153	·79122	·77289	1·29385	18
	·80160	·74583	1·34079	·61176	·79105	·77335	1·29307	17
	·80143	·74628	1·33998	·61199	·79087	·77382	1·29229	16
	·80125	·74674	1·33916	·61222	·79069	·77428	1·29152	15
	·80108	·74719	1·33835	·61245	·79051	·77475	1·29074	14
	·80091	·74764	1·33754	·61268	·79033	·77521	1·28997	13
	·80073	·74810	1·33673	·61291	·79016	·77568	1·28919	12
	·80056	·74855	1·33592	·61314	·78998	·77615	1·28842	11
	·80038	·74900	1·33511	·61337	·78980	·77661	1·28764	10
	·80021	·74946	1·33430	·61360	·78962	·77708	1·28687	9
	·80003	·74991	1·33349	·61383	·78944	·77754	1·28610	8
	·79986	·75037	1·33268	·61406	·78926	·77801	1·28533	7
	·79968	·75082	1·33187	·61429	·78908	·77848	1·28456	6
	·79951	·75128	1·33107	·61451	·78891	·77895	1·28379	5
	·79934	·75173	1·33026	·61474	·78873	·77941	1·28302	4
	·79916	·75219	1·32946	·61497	·78855	·77988	1·28225	3
	·79899	·75264	1·32865	·61520	·78837	·78035	1·28148	2
	·79881	·75310	1·32785	·61543	78819	·78082	1·28071	1
	·79864	·75355	1·32704	·61566	·78801	·78129	1·27994	0
	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	
	53°					52°		

9	61772	.78640	.78551	1.27308	63135	.77550	.81413	1.228
10	.61795	.78622	.78598	1.27230	.63158	.77531	.81481	1.227
11	.61818	.78604	.78645	1.27153	.63180	.77513	.81510	1.226
12	.61841	.78586	.78692	1.27077	.63203	.77494	.81558	1.226
13	.61864	.78568	.78739	1.27001	.63225	.77476	.81606	1.225
14	.61887	.78550	.78786	1.26925	.63248	.77458	.81655	1.224
15	.61909	.78532	.78834	1.26849	.63271	.77439	.81703	1.223
16	.61932	.78514	.78881	1.26774	.63293	.77421	.81752	1.223
17	.61955	.78496	.78928	1.26698	.63316	.77402	.81800	1.222
18	.61978	.78478	.78975	1.26622	.63338	.77384	.81849	1.221
19	.62001	.78460	.79022	1.26546	.63361	.77366	.81898	1.221
20	.62024	.78442	.79070	1.26471	.63383	.77347	.81946	1.220
21	.62046	.78424	.79117	1.26395	.63406	.77329	.81995	1.219
22	.62069	.78405	.79164	1.26319	.63428	.77310	.82044	1.218
23	.62092	.78387	.79212	1.26244	.63451	.77292	.82093	1.218
24	.62115	.78369	.79259	1.26169	.63473	.77273	.82141	1.217
25	.62138	.78351	.79306	1.26093	.63496	.77255	.82190	1.216
26	.62160	.78333	.79354	1.26018	.63518	.77236	.82238	1.216
27	.62183	.78315	.79401	1.25943	.63540	.77218	.82287	1.215
28	.62206	.78297	.79449	1.25867	.63563	.77199	.82336	1.214
29	.62229	.78279	.79496	1.25792	.63585	.77181	.82385	1.213
30	.62251	.78261	.79544	1.25717	.63608	.77162	.82434	1.213
31	.62274	.78243	.79591	1.25642	.63630	.77144	.82483	1.212
32	.62297	.78225	.79639	1.25567	.63653	.77125	.82531	1.211
33	.62320	.78206	.79686	1.25492	.63675	.77107	.82580	1.210
34	.62342	.78188	.79734	1.25417	.63698	.77088	.82629	1.210
35	.62365	.78170	.79781	1.25343	.63720	.77070	.82678	1.209
36	.62388	.78152	.79829	1.25268	.63742	.77051	.82727	1.208
37	.62411	.78134	.79877	1.25193	.63765	.77033	.82776	1.208
38	.62433	.78116	.79924	1.25118	.63787	.77014	.82825	1.207
39	.62456	.78098	.79972	1.25044	.63810	.76996	.82874	1.206
40	.62479	.78079	.80020	1.24969	.63832	.76977	.82923	1.205
41	.62502	.78061	.80067	1.24895	.63854	.76959	.82972	1.205
42	.62524	.78043	.80115	1.24820	.63877	.76940	.83022	1.204
43	.62547	.78025	.80163	1.24746	.63899	.76921	.83071	1.203
44	.62570	.78007	.80211	1.24672	.63922	.76903	.83120	1.203
45	.62592	.77988	.80258	1.24597	.63944	.76884	.83169	1.202

	40°			41°			
	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
)	·76604	·83910	1·19175	·65606	·75471	·86929	1·15037	60
.	·76586	·83960	1·19105	·65628	·75452	·86980	1·14969	59
i	·76567	·84009	1·19035	·65650	·75433	·87031	1·14902	58
3	·76548	·84059	1·18964	·65672	·75414	·87082	1·14834	57
1	·76530	·84108	1·18894	·65694	·75395	·87133	1·14767	56
)	·76511	·84158	1·18824	·65716	·75375	·87184	1·14699	55
3	·76492	·84208	1·18754	·65738	·75356	·87236	1·14632	54
5	·76473	·84258	1·18684	·65759	·75337	·87287	1·14565	53
7	·76455	·84307	1·18614	·65781	·75318	·87338	1·14498	52
)	·76436	·84357	1·18544	·65803	·75299	·87389	1·14430	51
l	·76417	·84407	1·18474	·65825	·75280	·87441	1·14363	50
4	·76398	·84457	1·18404	·65847	·75261	·87492	1·14296	49
3	·76380	·84507	1·18334	·65869	·75241	·87543	1·14229	48
3	·76361	·84556	1·18264	·65891	·75222	·87595	1·14162	47
)	·76342	·84606	1·18194	·65913	·75203	·87646	1·14095	46
3	·76323	·84656	1·18125	·65935	·75184	·87698	1·14028	45
5	·76304	·84706	1·18055	·65956	·75165	·87749	1·13961	44
7	·76286	·84756	1·17986	·65978	·75146	·87801	1·13894	43
)	·76267	·84806	1·17916	·66000	·75126	·87852	1·13828	42
l	·76248	·84856	1·17846	·66022	·75107	·87904	1·13761	41
3	·76229	·84906	1·17777	·66044	·75088	·87955	1·13694	40
3	·76210	·84958	1·17708	·66066	·75069	·88007	1·13627	39
3	·76192	·85006	1·17638	·66088	·75050	·88059	1·13561	38
)	·76173	·85057	1·17569	·66109	·75030	·88110	1·13494	37
2	·76154	·85107	1·17500	·66131	·75011	·88162	1·13428	36
4	·76135	·85157	1·17430	·66153	·74992	·88204	1·13361	35
3	·76116	·85207	1·17361	·66175	·74973	·88265	1·13295	34
3	·76097	·85257	1·17292	·66197	·74953	·88317	1·13228	33
l	·76078	·85308	1·17223	·66218	·74934	·88369	1·13162	32
3	·76059	·85358	1·17154	·66240	·74915	·88421	1·13096	31
5	·76041	·85408	1·17085	·66262	·74896	·88473	1·13029	30
7	·76022	·85458	1·17016	·66284	·74876	·88524	1·12963	29
9	·76003	·85509	1·16947	·66306	·74857	·88576	1·12897	28
l	·75984	·85559	1·16878	·66327	·74838	·88628	1·12831	27
3	·75965	·85609	1·16809	·66349	·74818	·88680	1·12765	26
5	·75946	·85660	1·16741	·66371	·74799	·88732	1·12699	25
7	·75927	·85710	1·16672	·66393	·74780	·88784	1·12633	24
)	·75908	·85761	1·16603	·66414	·74760	·88836	1·12567	23
2	·75889	·85811	1·16535	·66436	·74741	·88888	1·12501	22
4	·75870	·85862	1·16466	·66458	·74722	·88940	1·12435	21
6	·75851	·85912	1·16398	·66480	·74703	·88992	1·12369	20
8	·75832	·85963	1·16329	·66501	·74683	·89045	1·12303	19
)	·75813	·86014	1·16261	·66523	·74664	·89097	1·12238	18
2	·75794	·86064	1·16192	·66545	·74644	·89149	1·12172	17
4	·75775	·86115	1·16124	·66566	·74625	·89201	1·12106	16
6	·75756	·86166	1·16056	·66588	·74606	·89253	1·12041	15
8	·75738	·86216	1·15987	·66610	·74586	·89306	1·11975	14
)	·75719	·86267	1·15919	·66632	·74567	·89358	1·11909	13
2	·75700	·86318	1·15851	·66653	·74548	·89410	1·11844	12
4	·75680	·86368	1·15783	·66675	·74528	·89463	1·11778	11
6	·75661	·86419	1·15715	·66697	·74509	·89515	1·11713	10
8	·75642	·86470	1·15647	·66718	·74489	·89567	1·11648	9
0	·75623	·86521	1·15579	·66740	·74470	·89620	1·11582	8
2	·75604	·86572	1·15511	·66762	·74451	·89672	1·11517	7
4	·75585	·86623	1·15443	·66783	·74431	·89725	1·11452	6
6	·75566	·86674	1·15375	·66805	·74412	·89777	1·11387	5
8	·75547	·86725	1·15308	·66827	·74392	·89830	1·11321	4
0	·75528	·86776	1·15240	·66848	·74373	·89883	1·11256	3
2	·75509	·86827	1·15172	·66870	·74353	·89935	1·11191	2
4	·75490	·86878	1·15104	·66891	·74334	·89988	1·11126	1
6	·75471	·86929	1·15037	·66913	·74314	·90040	1·11061	0
	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′
	49°					48°		

8	.67086	.74	.90263	1.10928	.68 70	.72976	.93939	1
9	.67107	.74	.90516	1.10478	.68 91	.72957	.93742	1
10	.67129	.74120	.90569	1.10414	.68412	.72937	.93797	1
11	.67151	.74100	.90621	1.10349	.68434	.72917	.93852	1
12	.67172	.74080	.90674	1.10285	.68455	.72897	.93906	1
13	.67194	.74061	.90727	1.10220	.68476	.72877	.93961	1
14	.67215	.74041	.90781	1.10156	.68497	.72857	.94016	1
15	.67237	.74022	.90834	1.10091	.68518	.72837	.94071	1
16	.67258	.74002	.90887	1.10027	.68539	.72817	.94125	1
17	.67280	.73983	.90940	1.09963	.68561	.72797	.94180	1
18	.67301	.73963	.90993	1.09899	.68582	.72777	.94235	1
19	.67323	.73944	.91046	1.09834	.68603	.72757	.94290	1
20	.67344	.73924	.91099	1.09770	.68624	.72737	.94345	1
21	.67366	.73904	.91153	1.09706	.68645	.72717	.94400	1
22	.67387	.73885	.91206	1.09642	.68666	.72697	.94455	1
23	.67409	.73865	.91259	1.09578	.68688	.72677	.94510	1
24	.67430	.73846	.91313	1.09514	.68709	.72657	.94565	1
25	.67452	.73826	.91366	1.09450	.68730	.72637	.94620	1
26	.67473	.73806	.91419	1.09386	.68751	.72617	.94676	1
27	.67495	.73787	.91473	1.09322	.68772	.72597	.94731	1
28	.67516	.73767	.91526	1.09258	.68793	.72577	.94786	1
29	.67538	.73747	.91580	1.09195	.68814	.72557	.94841	1
30	.67559	.73728	.91633	1.09131	.68835	.72537	.94896	1
31	.67580	.73708	.91687	1.09067	.68857	.72517	.94952	1
32	.67602	.73688	.91740	1.09003	.68878	.72497	.95007	1
33	.67623	.73669	.91794	1.08940	.68899	.72477	.95062	1
34	.67645	.73649	.91847	1.08876	.68920	.72457	.95118	1
35	.67666	.73629	.91901	1.08813	.68941	.72437	.95173	1
36	.67688	.73610	.91955	1.08749	.68962	.72417	.95229	1
37	.67709	.73590	.92008	1.08686	.68983	.72397	.95284	1
38	.67730	.73570	.92062	1.08622	.69004	.72377	.95340	1
39	.67752	.73551	.92116	1.08559	.69025	.72357	.95395	1
40	.67773	.73531	.92170	1.08496	.69046	.72337	.95451	1
41	.67795	.73511	.92224	1.08432	.69067	.72317	.95506	1
42	.67816	.73491	.92277	1.08369	.69088	.72297	.95562	1
43	.67837	.73472	.92331	1.08306	.69109	.72277	.95618	1
44	.67859	.73452	.92385	1.08243	.69130	.72257	.95673	1

44° 44°

Cos.	Tan.	Cot.	′	′	Sin.	Cos.	Tan.	Cot.	′
.71934	.96569	1.03553	60	30	.70091	.71825	.98270	1.01761	30
.71914	.96625	1.03493	59	31	.70112	.71805	.98327	1.01702	29
.71894	.96681	1.03433	58	32	.70132	.71784	.98384	1.01642	28
.71873	.96738	1.03372	57	33	.70153	.71764	.98441	1.01583	27
.71853	.96794	1.03312	56	34	.70174	.71743	.98499	1.01524	26
.71833	.96850	1.03252	55	35	.70195	.71723	.98556	1.01465	25
.71813	.96907	1.03192	54	36	.70215	.71703	.98613	1.01406	24
.71792	.96963	1.03132	53	37	.70236	.71682	.98671	1.01347	23
.71772	.97020	1.03072	52	38	.70257	.71662	.98728	1.01288	22
.71752	.97076	1.03012	51	39	.70277	.71641	.98786	1.01229	21
.71732	.97133	1.02952	50	40	.70298	.71621	.98843	1.01170	20
.71711	.97189	1.02892	49	41	.70319	.71600	.98901	1.01112	19
.71691	.97246	1.02832	48	42	.70339	.71580	.98958	1.01053	18
.71671	.97302	1.02772	47	43	.70360	.71559	.99016	1.00994	17
.71650	.97359	1.02713	46	44	.70381	.71539	.99073	1.00935	16
.71630	.97416	1.02653	45	45	.70401	.71519	.99131	1.00876	15
.71610	.97472	1.02593	44	46	.70422	.70998	.99189	1.00818	14
.71590	.97529	1.02533	43	47	70443	.70978	.99247	1.00759	13
.71569	.97586	1.02474	42	48	.70463	.70957	.99304	1.00701	12
.71549	.97643	1.02414	41	49	.70484	.70937	.99362	1.00642	11
.71529	.97700	1.02355	40	50	.70505	.70916	.99420	1.00583	10
.71508	.97756	1.02295	39	51	.70525	.70896	.99478	1.00525	9
.71488	.97813	1.02236	38	52	.70546	.70875	.99536	1.00467	8
.71468	.97870	1.02176	37	53	.70567	.70855	.99594	1.00408	7
.71447	.97927	1.02117	36	54	.70587	.70834	.99652	1.00350	6
.71427	.97984	1.02057	35	55	.70608	.70813	.99710	1.00291	5
.71407	.98041	1.01998	34	56	.70628	.70793	.99768	1.00233	4
.71386	.98098	1.01939	33	57	.70649	.70772	.99826	1.00175	3
.71366	.98155	1.01879	32	58	.70670	.70752	.99884	1.00116	2
.71345	.98213	1.01820	31	59	.70690	.70731	.99942	1.00058	1
.71325	.98270	1.01761	30	60	.70711	.70711	1.00000	1.00000	0
Sin.	Cot.	Tan.	′	′	Cos.	Sin.	Cot.	Tan.	′

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SI

	0°		1°		2°		3°	
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex.
0	.00000	.00000	.00015	.00015	.00061	.00061	.00137	.00
1	.00000	.00000	.00016	.00016	.00062	.00062	.00139	.00
2	.00000	.00000	.00016	.00016	.00063	.00063	.00140	.00
3	.00000	.00000	.00017	.00017	.00064	.00064	.00142	.00
4	.00000	.00000	.00017	.00017	.00065	.00065	.00143	.00
5	.00000	.00000	.00018	.00018	.00066	.00066	.00145	.00
6	.00000	.00000	.00018	.00018	.00067	.00067	.00146	.00
7	.00000	.00000	.00019	.00019	.00068	.00068	.00148	.00
8	.00000	.00000	.00020	.00020	.00069	.00069	.00150	.00
9	.00000	.00000	.00020	.00020	.00070	.00070	.00151	.00
10	.00000	.00000	.00021	.00021	.00071	.00072	.00153	.00
11	.00001	.00001	.00021	.00021	.00073	.00073	.00154	.00
12	.00001	.00001	.00022	.00022	.00074	.00074	.00156	.00
13	.00001	.00001	.00023	.00023	.00075	.00075	.00158	.00
14	.00001	.00001	.00023	.00023	.00076	.00076	.00159	.00
15	.00001	.00001	.00024	.00024	.00077	.00077	.00161	.00
16	.00001	.00001	.00024	.00024	.00078	.00078	.00162	.00
17	.00001	.00001	.00025	.00025	.00079	.00079	.00164	.00
18	.00001	.00001	.00026	.00026	.00081	.00081	.00166	.00
19	.00002	.00002	.00026	.00026	.00082	.00082	.00168	.00
20	.00002	.00002	.00027	.00027	.00083	.00083	.00169	.00
21	.00002	.00002	.00028	.00028	.00084	.00084	.00171	.00
22	.00002	.00002	.00028	.00028	.00085	.00085	.00173	.00
23	.00002	.00002	.00029	.00029	.00087	.00087	.00174	.00
24	.00002	.00002	.00030	.00030	.00088	.00088	.00176	.00
25	.00003	.00003	.00031	.00031	.00089	.00089	.00178	.00
26	.00003	.00003	.00031	.00031	.00090	.00090	.00179	.00
27	.00003	.00003	.00032	.00032	.00091	.00091	.00181	.00
28	.00003	.00003	.00033	.00033	.00093	.00093	.00183	.00
29	.00004	.00004	.00034	.00034	.00094	.00094	.00185	.00
30	.00004	.00004	.00034	.00034	.00095	.00095	.00187	.00
31	.00004	.00004	.00035	.00035	.00096	.00097	.00188	.00
32	.00004	.00004	.00036	.00036	.00098	.00098	.00190	.00
33	.00005	.00005	.00037	.00037	.00099	.00099	.00192	.00
34	.00005	.00005	.00037	.00037	.00100	.00100	.00194	.00
35	.00005	.00005	.00038	.00038	.00102	.00102	.00196	.00
36	.00005	.00005	.00039	.00039	.00103	.00103	.00197	.00
37	.00006	.00006	.00040	.00040	.00104	.00104	.00199	.00
38	.00006	.00006	.00041	.00041	.00106	.00106	.00201	.00
39	.00006	.00006	.00041	.00041	.00107	.00107	.00203	.00
40	.00007	.00007	.00042	.00042	.00108	.00108	.00205	.00
41	.00007	.00007	.00043	.00043	.00110	.00110	.00207	.00
42	.00007	.00007	.00044	.00044	.00111	.00111	.00208	.00
43	.00008	.00008	.00045	.00045	.00112	.00113	.00210	.00
44	.00008	.00008	.00046	.00046	.00114	.00114	.00212	.00
45	.00009	.00009	.00047	.00047	.00115	.00115	.00214	.00
46	.00009	.00009	.00048	.00048	.00117	.00117	.00216	.00
47	.00009	.00009	.00048	.00048	.00118	.00118	.00218	.00
47	.00010	.00010	.00049	.00049	.00119	.00120	.00220	.00
49	.00010	.00010	.00050	.00050	.00121	.00121	.00222	.00
50	.00011	.00011	.00051	.00051	.00122	.00122	.00224	.00
51	.00011	.00011	.00052	.00052	.00124	.00124	.00226	.00
52	.00011	.00011	.00053	.00053	.00125	.00125	.00228	.00
53	.00012	.00012	.00054	.00054	.00127	.00127	.00230	.00
54	.00012	.00012	.00055	.00055	.00128	.00128	.00232	.00
55	.00013	.00013	.00056	.00056	.00130	.00130	.00234	.00
56	.00013	.00013	.00057	.00057	.00131	.00131	.00236	.00
57	.00014	.00014	.00058	.00058	.00133	.00133	.00238	.00
58	.00014	.00014	.00059	.00059	.00134	.00134	.00240	.00
59	.00015	.00015	.00060	.00060	.00136	.00136	.00242	.00
60	.00015	.00015	.00061	.00061	.00137	.00137	.00244	.00

4°		5°		6°		7°	
Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
.00244	.00381	.00382	.00548	.00551	.00745	.00751	0
.00246	.00383	.00385	.00551	.00554	.00749	.00755	1
.00248	.00386	.00387	.00554	.00557	.00752	.00758	2
.00250	.00388	.00390	.00557	.00560	.00756	.00762	3
.00252	.00391	.00392	.00560	.00563	.00760	.00765	4
.00254	.00393	.00395	.00563	.00566	.00763	.00769	5
.00257	.00396	.00397	.00566	.00569	.00767	.00773	6
.00259	.00398	.00400	.00569	.00573	.00770	.00776	7
.00261	.00401	.00403	.00572	.00576	.00774	.00780	8
.00263	.00404	.00405	.00576	.00579	.00778	.00784	9
.00265	.00406	.00408	.00579	.00582	.00781	.00787	10
.00267	.00409	.00411	.00582	.00585	.00785	.00791	11
.00269	.00412	.00413	.00585	.00588	.00789	.00795	12
.00271	.00114	.00416	.00588	.00592	.00792	.00799	13
.00274	.00417	.00419	.00591	.00595	.00796	.00802	14
.00276	.00420	.00421	.00594	.00598	.00800	.00806	15
.00278	.00422	.00424	.00598	.00601	.00803	.00810	16
.00280	.00425	.00427	.00601	.00604	.00807	.00813	17
.00282	.00428	.00429	.00604	.00608	.00811	.00817	18
.00284	.00430	.00432	.00607	.00611	.00814	.00821	19
.00287	.00433	.00435	.00610	.00614	.00818	.00825	20
.00289	.00436	.00438	.00614	.00617	.00822	.00828	21
.00291	.00438	.00440	.00617	.00621	.00825	.00832	22
.00293	.00441	.00443	.00620	.00624	.00829	.00836	23
.00296	.00444	.00446	.00623	.00627	.00833	.00840	24
.00298	.00447	.00449	.00626	.00630	.00837	.00844	25
.00300	.00449	.00451	.00630	.00634	.00840	.00848	26
.00302	.00452	.00454	.00633	.00637	.00844	.00851	27
.00305	.00455	.00457	.00636	.00640	.00848	.00855	28
.00307	.00458	.00460	.00640	.00644	.00852	.00859	29
.00309	.00460	.00463	.00643	.00647	.00856	.00863	30
.00312	.00463	.00465	.00646	.00650	.00859	.00867	31
.00314	.00466	.00468	.00649	.00654	.00863	.00871	32
.00316	.00469	.00471	.00653	.00657	.00867	.00875	33
.00318	.00472	.00474	.00656	.00660	.00871	.00878	34
.00321	.00474	.00477	.00659	.00664	.00875	.00882	35
.00323	.00477	.00480	.00663	.00667	.00878	.00886	36
.00326	.00480	.00482	.00666	.00671	.00882	.00890	37
.00328	.00483	.00485	.00669	.00674	.00886	.00894	38
.00330	.00486	.00488	.00673	.00677	.00890	.00898	39
.00333	.00489	.00491	.00676	.00681	.00894	.00902	40
.00335	.00492	.00494	.00680	.00684	.00898	.00906	41
.00337	.00494	.00497	.00683	.00688	.00902	.00910	42
.00340	.00497	.00500	.00686	.00691	.00906	.00914	43
.00342	.00500	.00503	.00690	.00695	.00909	.00918	44
.00345	.00503	.00506	.00693	.00698	.00913	.00922	45
.00347	.00506	.00509	.00697	.00701	.00917	.00926	46
.00350	.00509	.00512	.00700	.00705	.00921	.00930	47
.00352	.00512	.00515	.00703	.00708	.00925	.00934	48
.00354	.00515	.00518	.00707	.00712	.00929	.00938	49
.00357	.00518	.00521	.00710	.00715	.00933	.00942	50
.00359	.00521	.00524	.00714	.00719	.00937	.00946	51
.00362	.00524	.00527	.00717	.00722	.00941	.00950	52
.00364	.00527	.00530	.00721	.00726	.00945	.00954	53
.00367	.00530	.00533	.00724	.00730	.00949	.00958	54
.00369	.00533	.00536	.00728	.00733	.00953	.00962	55
.00372	.00536	.00539	.00731	.00737	.00957	.00966	56
.00374	.00539	.00542	.00735	.00740	.00961	.00970	57
.00377	.00542	.00545	.00738	.00744	.00965	.00975	58
.00379	.00545	.00548	.00742	.00747	.00969	.00979	59
.00382	.00548	.00551	.00745	.00751	.00973	.00983	60

8	.01006	.01016	.01268	.01284	.01560	.01585	.01882	.1
9	.01010	.01020	.01272	.01289	.01565	.01590	.01888	.1
10	.01014	.01024	.01277	.01294	.01570	.01595	.01893	.1
11	.01018	.01029	.01282	.01298	.01575	.01601	.01899	.1
12	.01022	.01033	.01286	.01303	.01580	.01606	.01904	.1
13	.01027	.01037	.01291	.01308	.01586	.01611	.01910	.1
14	.01031	.01041	.01296	.01313	.01591	.01616	.01916	.1
15	.01035	.01046	.01300	.01318	.01596	.01622	.01921	.1
16	.01039	.01050	.01305	.01322	.01601	.01627	.01927	.1
17	.01043	.01054	.01310	.01327	.01606	.01633	.01933	.1
18	.01047	.01059	.01314	.01332	.01612	.01638	.01939	.1
19	.01052	.01063	.01319	.01337	.01617	.01643	.01944	.1
20	.01056	.01067	.01324	.01342	.01622	.01649	.01950	.1
21	.01060	.01071	.01329	.01346	.01627	.01654	.01956	.1
22	.01064	.01076	.01333	.01351	.01632	.01659	.01961	.1
23	.01069	.01080	.01338	.01356	.01638	.01665	.01967	.1
24	.01073	.01084	.01343	.01361	.01643	.01670	.01973	.1
25	.01077	.01089	.01348	.01366	.01648	.01676	.01979	.1
26	.01081	.01093	.01352	.01371	.01653	.01681	.01984	.1
27	.01086	.01097	.01357	.01376	.01659	.01687	.01990	.1
28	.01090	.01102	.01362	.01381	.01664	.01692	.01996	.1
29	.01094	.01106	.01367	.01386	.01669	.01698	.02002	.1
30	.01098	.01111	.01371	.01391	.01675	.01703	.02008	.1
31	.01103	.01115	.01376	.01395	.01680	.01709	.02013	.1
32	.01107	.01119	.01381	.01400	.01685	.01714	.02019	.1
33	.01111	.01124	.01386	.01405	.01690	.01720	.02025	.1
34	.01116	.01128	.01391	.01410	.01696	.01725	.02031	.1
35	.01120	.01133	.01396	.01415	.01701	.01731	.02037	.1
36	.01124	.01137	.01400	.01420	.01706	.01736	.02042	.1
37	.01129	.01142	.01405	.01425	.01712	.01742	.02048	.1
38	.01133	.01146	.01410	.01430	.01717	.01747	.02054	.1
39	.01137	.01151	.01415	.01435	.01723	.01753	.02060	.1
40	.01142	.01155	.01420	.01440	.01728	.01758	.02066	.1
41	.01146	.01160	.01425	.01445	.01733	.01764	.02072	.1
42	.01151	.01164	.01430	.01450	.01739	.01769	.02078	.1
43	.01155	.01169	.01435	.01455	.01744	.01775	.02084	.1
44	.01159	.01173	.01439	.01461	.01750	.01781	.02090	.1

12°		13°		14°		15°		
Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
2185	.02234	.02563	.02630	.02970	.03061	.03407	.03528	0
2191	.02240	.02570	.02637	.02977	.03069	.03415	.03536	1
2197	.02247	.02576	.02644	.02985	.03076	.03422	.03544	2
2203	.02253	.02583	.02651	.02992	.03084	.03430	.03552	3
2210	.02259	.02589	.02658	.02999	.03091	.03438	.03560	4
2216	.02266	.02596	.02665	.03006	.03099	.03445	.03568	5
2222	.02272	.02602	.02672	.03013	.03106	.03453	.03576	6
2228	.02279	.02609	.02679	.03020	.03114	.03460	.03584	7
2234	.02285	.02616	.02686	.03027	.03121	.03468	.03592	8
2240	.02291	.02622	.02693	.03034	.03129	.03476	.03601	9
2246	.02298	.02629	.02700	.03041	.03137	.03483	.03609	10
2252	.02304	.02635	.02707	.03048	.03144	.03491	.03617	11
2258	.02311	.02642	.02714	.03055	.03152	.03498	.03625	12
2265	.02317	.02649	.02721	.03063	.03159	.03506	.03633	13
2271	.02323	.02655	.02728	.03070	.03167	.03514	.03642	14
2277	.02330	.02662	.02735	.03077	.03175	.03521	.03650	15
2283	.02336	.02669	.02742	.03084	.03182	.03529	.03658	16
2289	.02343	.02675	.02749	.03091	.03190	.03537	.03666	17
2295	.02349	.02682	.02756	.03098	.03198	.03544	.03674	18
2302	.02356	.02689	.02763	.03106	.03205	.03552	.03683	19
2308	.02362	.02696	.02770	.03113	.03213	.03560	.03691	20
2314	.02369	.02702	.02777	.03120	.03221	.03567	.03699	21
2320	.02375	.02709	.02784	.03127	.03228	.03575	.03708	22
2327	.02382	.02716	.02791	.03134	.03236	.03583	.03716	23
2333	.02388	.02722	.02799	.03142	.03244	.03590	.03724	24
2339	.02395	.02729	.02806	.03149	.03251	.03598	.03732	25
2345	.02402	.02736	.02813	.03156	.03259	.03606	.03741	26
2352	.02408	.02743	.02820	.03163	.03267	.03614	.03749	27
2358	.02415	.02749	.02827	.03171	.03275	.03621	.03758	28
2364	.02421	.02756	.02834	.03178	.03282	.03629	.03766	29
2370	.02428	.02763	.02842	.03185	.03290	.03637	.03774	30
2377	.02435	.02770	.02849	.03193	.03298	.03645	.03783	31
2383	.02441	.02777	.02856	.03200	.03306	.03653	.03791	32
2389	.02448	.02783	.02863	.03207	.03313	.03660	.03799	33
2396	.02454	.02790	.02870	.03214	.03321	.03668	.03808	34
2402	.02461	.02797	.02878	.03222	.03329	.03676	.03816	35
2408	.02468	.02804	.02885	.03229	.03337	.03684	.03825	36
2415	.02474	.02811	.02892	.03236	.03345	.03692	.03833	37
2421	.02481	.02818	.02899	.03244	.03353	.03699	.03842	38
2427	.02488	.02824	.02907	.03251	.03360	.03707	.03850	39
2434	.02494	.02831	.02914	.03258	.03368	.03715	.03858	40
2440	.02501	.02838	.02921	.03266	.03376	.03723	.03867	41
2447	.02508	.02845	.02928	.03273	.03384	.03731	.03875	42
2453	.02515	.02852	.02936	.03281	.03392	.03739	.03884	43
2459	.02521	.02859	.02943	.03288	.03400	.03747	.03892	44
2466	.02528	.02866	.02950	.03295	.03408	.03754	.03901	45
2472	.02535	.02873	.02958	.03303	.03416	.03762	.03909	46
2479	.02542	.02880	.02965	.03310	.03424	.03770	.03918	47
2485	.02548	.02887	.02972	.03318	.03432	.03778	.03927	48
2492	.02555	.02894	.02980	.03325	.03439	.03786	.03935	49
2498	.02562	.02900	.02987	.03333	.03447	.03794	.03944	50
2504	.02569	.02907	.02994	.03340	.03455	.03802	.03952	51
2511	.02576	.02914	.03002	.03347	.03463	.03810	.03961	52
2517	.02582	.02921	.03009	.03355	.03471	.03818	.03969	53
2524	.02589	.02928	.03017	.03362	.03479	.03826	.03978	54
2530	.02596	.02935	.03024	.03370	.03487	.03834	.03987	55
2537	.02603	.02942	.03032	.03377	.03495	.03842	.03995	56
2543	.02610	.02949	.03039	.03385	.03503	.03850	.04004	57
2550	.02617	.02956	.03046	.03392	.03512	.03858	.04013	58
2556	.02624	.02963	.03054	.03400	.03520	.03866	.04021	59
2563	.02630	.02970	.03061	.03407	.03528	.03874	.04030	60

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

	24°		25°		26°		27°		
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
0	08645	.09464	.09389	.10338	10121	.11260	.10899	.12233	0
1	.08657	.09478	.09382	.10353	.10133	.11276	.10913	.1224s	1
2	.08669	.09492	.09394	.10368	.10146	.11292	.10926	.12266	2
3	.08681	.09506	.09406	.10383	.10159	.11308	.10939	.12283	3
4	.08693	.09520	.09418	.10398	.10172	.11323	.10952	.12299	4
5	.08705	.09535	.09431	.10413	.10184	.11339	.10965	.12316	5
6	.08717	.09549	.09443	.10428	.10197	.11355	.10979	.12333	6
7	.08728	.09563	.09455	.10443	.10210	.11371	.10992	.12349	7
8	.08740	.09577	.09468	.10458	.10223	.11387	.11005	.12366	8
9	.08752	.09592	.09480	.10473	.10236	.11403	.11019	.12383	9
10	.08764	.09606	.09493	.10488	.10248	.11419	.11032	.12400	10
11	.08776	.09620	.09505	.10503	.10261	.11435	.11045	.12416	11
12	.08788	.09635	.09517	.10518	.10274	.11451	.11058	.12433	12
13	.08800	.09649	.09530	.10533	.10287	.11467	.11072	.12450	13
14	.08812	.09663	.09542	.10549	.10300	.11483	.11085	.12467	14
15	.08824	.09678	.09554	.10564	.10313	.11499	.11098	.12484	15
16	.08836	.09692	.09567	.10579	.10326	.11515	.11112	.12501	16
17	.08848	.09707	.09579	.10594	.10338	.11531	.11125	.12518	17
18	.08860	.09721	.09592	.10609	.10351	.11547	.11138	.12534	18
19	.08872	.09735	.09604	.10625	.10364	.11563	.11152	.12551	19
20	.08884	.09750	.09617	.10640	.10377	.11579	.11165	.12568	20
21	.08896	.09764	.09629	.10655	.10390	.11595	.11178	.12585	21
22	.08908	.09779	.09642	.10670	.10403	.11611	.11192	.12602	22
23	.08920	.09793	.09654	.10686	.10416	.11627	.11205	.12619	23
24	.08932	.09808	.09666	.10701	.10429	.11643	.11218	.12636	24
25	.08944	.09822	.09679	.10716	.10442	.11659	.11232	.12653	25
26	.08956	.09837	.09691	.10731	.10455	.11675	.11245	.12670	26
27	.08968	.09851	.09704	.10747	.10468	.11691	.11259	.12687	27
28	.08980	.09866	.09716	.10762	.10481	.11708	.11272	.12704	28
29	.08992	.09880	.09729	.10777	.10494	.11724	.11285	.12721	29
30	.09004	.09895	.09741	.10793	.10507	.11740	.11299	.12738	30
31	.09016	.09909	.09754	.10808	.10520	.11756	.11312	.12755	31
32	.09028	.09924	.09767	.10824	.10533	.11772	.11326	.12772	32
33	.09040	.09939	.09779	.10839	.10546	.11789	.11339	.12789	33
34	.09052	.09953	.09792	.10854	.10559	.11805	.11353	.12807	34
35	.09064	.09968	.09804	.10870	.10572	.11821	.11366	.12824	35
36	.09076	.09982	.09817	.10885	.10585	.11838	.11380	.12841	36
37	.09089	.09997	.09829	.10901	.10598	.11854	.11393	.12858	37
38	.09101	.10012	.09842	.10916	.10611	.11870	.11407	.12875	38
39	.09113	.10026	.09854	.10932	.10624	.11886	.11420	.12892	39
40	.09125	.10041	.09867	.10947	.10637	.11903	.11434	.12910	40
41	.09137	.10055	.09880	.10963	.10650	.11919	.11447	.12927	41
42	.09149	.10071	.09892	.10978	.10663	.11936	.11461	.12944	42
43	.09161	.10085	.09905	.10994	.10676	.11952	.11474	.12961	43
44	.09174	.10100	.09918	.11009	.10689	.11968	.11488	.12979	44
45	.09186	.10115	.09930	.11025	.10702	.11985	.11501	.12996	45
46	.09198	.10130	.09943	.11041	.10715	.12001	.11515	.13013	46
47	.09210	.10144	.09955	.11056	.10728	.12018	.11528	.13031	47
48	.09222	.10159	.09968	.11072	.10741	.12034	.11542	.13048	48
49	.09234	.10174	.09981	.11087	.10755	.12051	.11555	.13065	49
50	.09247	.10189	.09993	.11103	.10768	.12067	.11569	.13083	50
51	.09259	.10204	.10006	.11119	.10781	.12084	.11583	.13100	51
52	.09271	.10218	.10019	.11134	.10794	.12100	.11596	.13117	52
53	.09283	.10233	.10032	.11150	.10807	.12117	.11610	.13135	53
54	.09296	.10248	.10044	.11166	.10820	.12133	.11623	.13152	54
55	.09308	.10263	.10057	.11181	.10833	.12150	.11637	.13170	55
56	.09320	.10278	.10070	.11197	.10847	.12166	.11651	.13187	56
57	.09332	.10293	.10082	.11212	.10860	.12183	.11664	.13205	57
58	.09345	.10308	.10095	.11229	.10873	.12199	.11678	.13222	58
59	.09357	.10323	.10108	.11244	.10886	.12216	.11692	.13240	59
60	.09369	.10338	.10121	.11260	.10899	.12233	.11705	.13257	60

20°		21°		22°		23°		
Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
.06031	.06418	.06642	.07115	.07282	.07853	.07950	.08638	0
.06041	.06429	.06652	.07126	.07293	.07866	.07961	.08649	1
.06051	.06440	.06663	.07138	.07303	.07879	.07972	.08663	2
.06061	.06452	.06673	.07150	.07314	.07892	.07984	.08676	3
.06071	.06463	.06684	.07162	.07325	.07904	.07995	.08690	4
.06081	.06474	.06694	.07174	.07336	.07917	.08006	.08703	5
.06091	.06486	.06705	.07186	.07347	.07930	.08018	.08717	6
.06101	.06497	.06715	.07199	.07358	.07943	.08029	.08730	7
.06111	.06508	.06726	.07211	.07369	.07955	.08041	.08744	8
.06121	.06520	.06736	.07223	.07380	.07968	.08052	.08757	9
.06131	.06531	.06747	.07235	.07391	.07981	.08064	.08771	10
.06141	.06542	.06757	.07247	.07402	.07994	.08075	.08784	11
.06151	.06554	.06768	.07259	.07413	.08006	.08086	.08798	12
.06161	.06565	.06778	.07271	.07424	.08019	.08098	.08811	13
.06171	.06577	.06789	.07283	.07435	.08032	.08109	.08825	14
.06181	.06588	.06799	.07295	.07446	.08045	.08121	.08839	15
.06191	.06600	.06810	.07307	.07457	.08058	.08132	.08852	16
.06201	.06611	.06820	.07320	.07468	.08071	.08144	.08866	17
.06211	.06622	.06831	.07332	.07479	.08084	.08155	.08880	18
.06221	.06634	.06841	.07344	.07490	.08097	.08167	.08893	19
.06231	.06645	.06852	.07356	.07501	.08109	.08178	.08907	20
.06241	.06657	.06863	.07368	.07512	.08122	.08190	.08921	21
.06252	.06668	.06873	.07380	.07523	.08135	.08201	.08934	22
.06262	.06680	.06884	.07393	.07534	.08148	.08213	.08948	23
.06272	.06691	.06894	.07405	.07545	.08161	.08225	.08962	24
.06282	.06703	.06905	.07417	.07556	.08174	.08236	.08975	25
.06292	.06715	.06916	.07429	.07568	.08087	.08248	.08989	26
.06302	.06726	.06926	.07442	.07579	.08200	.08259	.09003	27
.06312	.06738	.06937	.07454	.07590	.08213	.08271	.09017	28
.06323	.06749	.06948	.07466	.07601	.08226	.08282	.09030	29
.06333	.06761	.06958	.07479	.07612	.08239	.08294	.09044	30
.06343	.06773	.06969	.07491	.07623	.08252	.08306	.09058	31
.06353	.06784	.06980	.07503	.07634	.08265	.08317	.09072	32
.06363	.06796	.06990	.07516	.07645	.08278	.08329	.09086	33
.06374	.06807	.07001	.07528	.07657	.08291	.08340	.09099	34
.06384	.06819	.07012	.07540	.07668	.08305	.08352	.09113	35
.06394	.06831	.07022	.07553	.07679	.08318	.08364	.09127	36
.06404	.06843	.07033	.07565	.07690	.08331	.08375	.09141	37
.06415	.06854	.07044	.07578	.07701	.08344	.08387	.09155	38
.06425	.06866	.07055	.07590	.07713	.08357	.08399	.09169	39
.06435	.06878	.07065	.07602	.07724	.08370	.08410	.09183	40
.06445	.06889	.07076	.07615	.07735	.08383	.08422	.09197	41
.06456	.06901	.07087	.07627	.07746	.08397	.08434	.09211	42
.06466	.06913	.07098	.07640	.07757	.08410	.08445	.09224	43
.06476	.06925	.07108	.07652	.07769	.08423	.08457	.09238	44
.06486	.06936	.07119	.07665	.07780	.08436	.08469	.09252	45
.06497	.06948	.07130	.07677	.07791	.08449	.08481	.09266	46
.06507	.06960	.07141	.07690	.07802	.08463	.08492	.09280	47
.06517	.06972	.07151	.07702	.07814	.08476	.08504	.09294	48
.06528	.06984	.07162	.07715	.07825	.08489	.08516	.09308	49
.06538	.06995	.07173	.07727	.07836	.08503	.08528	.09323	50
.06548	.07007	.07184	.07740	.07848	.08516	.08539	.09337	51
.06559	.07019	.07195	.07752	.07859	.08529	.08551	.09351	52
.06569	.07031	.07206	.07765	.07870	.08542	.08563	.09365	53
.06580	.07043	.07216	.07778	.07881	.08556	.08575	.09379	54
.06590	.07055	.07227	.07790	.07893	.08569	.08586	.09393	55
.06600	.07067	.07238	.07803	.07904	.08582	.08598	.09407	56
.06611	.07079	.07249	.07816	.07915	.08596	.08610	.09421	57
.06621	.07091	.07260	.07828	.07927	.08609	.08622	.09435	58
.06632	.07103	.07271	.07841	.07938	.08623	.08634	.09449	59
.06642	.07115	.07282	.07853	.07950	.08636	.08645	.09464	60

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	32°		33°		34°		35°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	.15195	.17918	.16133	.19236	.17096	.20622	.18085	.22077	0
1	.15211	.17939	.16149	.19259	.17113	.20645	.18101	.22102	1
2	.15226	.17961	.16165	.19281	.17129	.20669	.18118	.22127	2
3	.15241	.17982	.16181	.19304	.17145	.20693	.18135	.22152	3
4	.15257	.18004	.16196	.19327	.17161	.20717	.18152	.22177	4
5	.15272	.18025	.16212	.19349	.17178	.20740	.18168	.22202	5
6	.15288	.18047	.16228	.19372	.17194	.20764	.18185	.22227	6
7	.15303	.18068	.16244	.19394	.17210	.20788	.18202	.22252	7
8	.15319	.18090	.16260	.19417	.17227	.20812	.18218	.22277	8
9	.15334	.18111	.16276	.19440	.17243	.20836	.18235	.22302	9
10	.15350	.18133	.16292	.19463	.17259	.20859	.18252	.22327	10
11	.15365	.18155	.16308	.19485	.17278	.20883	.18269	.22352	11
12	.15381	.18176	.16324	.19508	.17292	.20907	.18286	.22377	12
13	.15396	.18198	.16340	.19531	.17308	.20931	.18302	.22402	13
14	.15412	.18220	.16355	.19554	.17325	.20955	.18319	.22428	14
15	.15427	.18241	.16371	.19576	.17341	.20979	.18336	.22453	15
16	.15443	.18263	.16387	.19599	.17357	.21003	.18353	.22478	16
17	.15458	.18285	.16403	.19622	.17374	.21027	.18369	.22503	17
18	.15474	.18307	.16419	.19645	.17390	.21051	.18386	.22528	18
19	.15489	.18328	.16435	.19668	.17407	.21075	.18403	.22554	19
20	.15505	.18350	.16451	.19691	.17423	.21099	.18420	.22579	20
21	.15520	.18372	.16467	.19713	.17439	.21123	.18437	.22604	21
22	.15536	.18394	.16483	.19736	.17456	.21147	.18454	.22629	22
23	.15552	.18416	.16499	.19759	.17472	.21171	.18470	.22655	23
24	.15567	.18437	.16515	.19782	.17489	.21195	.18487	.22680	24
25	.15583	.18459	.16531	.19805	.17505	.21220	.18504	.22706	25
26	.15598	.18481	.16547	.19828	.17522	.21244	.18521	.22731	26
27	.15614	.18503	.16563	.19851	.17538	.21268	.18538	.22756	27
28	.15630	.18525	.16579	.19874	.17554	.21292	.18555	.22782	28
29	.15645	.18547	.16595	.19897	.17571	.21316	.18572	.22807	29
30	.15661	.18569	.16611	.19920	.17587	.21341	.18588	.22833	30
31	.15676	.18591	.16627	.19944	.17604	.21365	.18605	.22858	31
32	.15692	.18613	.16644	.19967	.17620	.21389	.18622	.22884	32
33	.15708	.18635	.16660	.19990	.17637	.21414	.18639	.22909	33
34	.15723	.18657	.16676	.20013	.17653	.21438	.18656	.22935	34
35	.15739	.18679	.16692	.20036	.17670	.21462	.18673	.22960	35
36	.15755	.18701	.16708	.20059	.17686	.21487	.18690	.22986	36
37	.15770	.18723	.16724	.20083	.17703	.21511	.18707	.23012	37
38	.15786	.18745	.16740	.20106	.17719	.21535	.18724	.23037	38
39	.15802	.18767	.16756	.20129	.17736	.21560	.18741	.23063	39
40	.15818	.18790	.16772	.20152	.17752	.21584	.18758	.23089	40
41	.15833	.18812	.16788	.20176	.17769	.21609	.18775	.23114	41
42	.15849	.18834	.16805	.20199	.17786	.21633	.18792	.23140	42
43	.15865	.18856	.16821	.20222	.17802	.21658	.18809	.23166	43
44	.15880	.18878	.16837	.20246	.17819	.21682	.18826	.23192	44
45	.15896	.18901	.16853	.20269	.17835	.21707	.18843	.23217	45
46	.15912	.18923	.16869	.20292	.17852	.21731	.18860	.23243	46
47	.15928	.18945	.16885	.20316	.17868	.21756	.18877	.23269	47
48	.15943	.18967	.16902	.20339	.17885	.21781	.18894	.23295	48
49	.15959	.18990	.16918	.20363	.17902	.21805	.18911	.23321	49
50	.15975	.19012	.16934	.20386	.17918	.21830	.18928	.23347	50
51	.15991	.19034	.16950	.20410	.17935	.21855	.18945	.23373	51
52	.16006	.19057	.16966	.20433	.17952	.21879	.18962	.23399	52
53	.16022	.19079	.16983	.20457	.17968	.21904	.18979	.23424	53
54	.16038	.19102	.16999	.20480	.17985	.21929	.18996	.23450	54
55	.16054	.19124	.17015	.20504	.18001	.21953	.19013	.23476	55
56	.16070	.19146	.17031	.20527	.18018	.21978	.19030	.23502	56
57	.16085	.19169	.17047	.20551	.18035	.22003	.19047	.23529	57
58	.16101	.19191	.17064	.20575	.18051	.22028	.19064	.23555	58
59	.16117	.19214	.17080	.20598	.18068	.22053	.19081	.23581	59
60	.16133	.19236	.17096	.20622	.18085	.22077	.19098	.23607	60

	28°		29°		30°		31°	
′s.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
05	.13257	.12538	.14335	.13397	.15470	.14283	.16663	0
19	.13275	.12552	.14354	.13412	.15489	.14298	.16684	1
33	.13292	.12566	.14372	.13427	.15509	.14313	.16704	2
46	.13310	.12580	.14391	.13441	.15528	.14328	.16725	3
60	.13327	.12595	.14409	.13456	.15548	.14343	.16745	4
74	.13345	.12609	.14428	.13470	.15567	.14358	.16766	5
87	.13362	.12623	.14446	.13485	.15587	.14373	.16786	6
01	.13380	.12637	.14465	.13499	.15606	.14388	.16806	7
15	.13398	.12651	.14483	.13514	.15626	.14403	.16827	8
28	.13415	.12665	.14502	.13529	.15645	.14418	.16848	9
42	.13433	.12679	.14521	.13543	.15665	.14433	.16868	10
56	.13451	.12694	.14539	.13558	.15684	.14449	.16889	11
70	.13468	.12708	.14558	.13573	.15704	.14464	.16909	12
83	.13486	.12722	.14576	.13587	.15724	.14479	.16930	13
97	.13504	.12736	.14595	.13602	.15743	.14494	.16950	14
11	.13521	.12750	.14614	.13616	.15763	.14509	.16971	15
25	.13539	.12765	.14632	.13631	.15782	.14524	.16992	16
38	.13557	.12779	.14651	.13646	.15802	.14539	.17012	17
52	.13575	.12793	.14670	.13660	.15822	.14554	.17033	18
66	.13593	.12807	.14689	.13675	.15841	.14569	.17054	19
80	.13610	.12822	.14707	.13690	.15861	.14584	.17075	20
94	.13628	.12836	.14726	.13705	.15881	.14599	.17095	21
07	.13646	.12850	.14745	.13719	.15901	.14615	.17116	22
21	.13664	.12864	.14764	.13734	.15920	.14630	.17137	23
35	.13682	.12879	.14782	.13749	.15940	.14645	.17158	24
49	.13700	.12893	.14801	.13763	.15960	.14660	.17178	25
63	.13718	.12907	.14820	.13778	.15980	.14675	.17199	26
77	.13735	.12921	.14839	.13793	.16000	.14690	.17220	27
91	.13753	.12936	.14858	.13808	.16019	.14706	.17241	28
04	.13771	.12950	.14877	.13822	.16039	.14721	.17262	29
18	.13789	.12964	.14896	.13837	.16059	.14736	.17283	30
32	.13807	.12979	.14914	.13852	.16079	.14751	.17304	31
46	.13825	.12993	.14933	.13867	.16099	.14766	.17325	32
60	.13843	.13007	.14952	.13881	.16119	.14782	.17346	33
74	.13861	.13022	.14971	.13896	.16139	.14797	.17367	34
88	.13879	.13036	.14990	.13911	.16159	.14812	.17388	35
02	.13897	.13051	.15009	.13926	.16179	.14827	.17409	36
16	.13916	.13065	.15028	.13941	.16199	.14843	.17430	37
30	.13934	.13079	.15047	.13955	.16219	.14858	.17451	38
44	.13952	.13094	.15066	.13970	.16239	.14873	.17472	39
57	.13970	.13108	.15085	.13985	.16259	.14888	.17493	40
71	.13988	.13122	.15105	.14000	.16279	.14904	.17514	41
85	.14006	.13137	.15124	.14015	.16299	.14919	.17535	42
99	.14024	.13151	.15143	.14030	.16319	.14934	.17556	43
13	.14042	.13166	.15162	.14044	.16339	.14949	.17577	44
27	.14061	.13180	.15181	.14059	.16359	.14965	.17598	45
41	.14079	.13195	.15200	.14074	.16380	.14980	.17620	46
55	.14097	.13209	.15219	.14089	.16400	.14995	.17641	47
69	.14115	.13223	.15239	.14104	.16420	.15011	.17662	48
83	.14134	.13238	.15258	.14119	.16440	.15026	.17683	49
97	.14152	.13252	.15277	.14134	.16460	.15041	.17704	50
11	.14170	.13267	.15296	.14149	.16481	.15057	.17726	51
25	.14188	.13281	.15315	.14164	.16501	.15072	.17747	52
39	.14207	.13296	.15335	.14179	.16521	.15087	.17768	53
54	.14225	.13310	.15354	.14194	.16541	.15103	.17790	54
68	.14243	.13325	.15373	.14208	.16562	.15118	.17811	55
82	.14262	.13339	.15393	.14223	.16582	.15134	.17832	56
96	.14280	.13354	.15412	.14238	.16602	.15149	.17854	57
10	.14299	.13368	.15431	.14253	.16623	.15164	.17875	58
24	.14317	.13383	.15451	.14268	.16643	.15180	.17896	59
38	.14335	.13397	.15470	.14283	.16663	.15195	.17918	60

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

	40°		41°		42°		43°		
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
0	.23396	.30541	.24529	.32501	.25686	.34563	.26865	.36733	0
1	.23414	.30573	.24548	.32535	.25705	.34599	.26884	.36770	1
2	.23433	.30605	.24567	.32568	.25724	.34634	.26904	.36807	2
3	.23452	.30636	.24586	.32602	.25744	.34669	.26924	.36844	3
4	.23470	.30668	.24605	.32636	.25763	.34704	.26944	.36881	4
5	.23489	.30700	.24625	.32669	.25783	.34740	.26964	.36919	5
6	.23508	.30732	.24644	.32703	.25802	.34775	.26984	.36956	6
7	.23527	.30764	.24663	.32737	.25822	.34811	.27004	.36993	7
8	.23545	.30796	.24682	.32770	.25841	.34846	.27024	.37030	8
9	.23564	.30829	.24701	.32804	.25861	.34882	.27043	.37068	9
10	.23583	.30861	.24720	.32838	.25880	.34917	.27063	.37105	10
11	.23602	.30893	.24739	.32872	.25900	.34953	.27083	.37143	11
12	.23620	.30925	.24759	.32905	.25920	.34988	.27103	.37180	12
13	.23639	.30957	.24778	.32939	.25939	.35024	.27123	.37218	13
14	.23658	.30989	.24797	.32973	.25959	.35060	.27143	.37255	14
15	.23677	.31022	.24816	.33007	.25978	.35095	.27163	.37293	15
16	.23696	.31054	.24835	.33041	.25998	.35131	.27183	.37330	16
17	.23714	.31086	.24854	.33075	.26017	.35167	.27203	.37368	17
18	.23733	.31119	.24874	.33109	.26037	.35203	.27223	.37406	18
19	.23752	.31151	.24893	.33143	.26056	.35238	.27243	.37443	19
20	.23771	.31183	.24912	.33177	.26076	.35274	.27263	.37481	20
21	.23790	.31216	.24931	.33211	.26096	.35310	.27283	.37519	21
22	.23808	.31248	.24950	.33245	.26115	.35346	.27303	.37556	22
23	.23827	.31281	.24970	.33279	.26135	.35382	.27323	.37594	23
24	.23846	.31313	.24989	.33314	.26154	.35418	.27343	.37632	24
25	.23865	.31346	.25008	.33348	.26174	.35454	.27363	.37670	25
26	.23884	.31378	.25027	.33382	.26194	.35490	.27383	.37708	26
27	.23903	.31411	.25047	.33416	.26213	.35526	.27403	.37746	27
28	.23922	.31443	.25066	.33451	.26233	.35562	.27423	.37784	28
29	.23941	.31476	.25085	.33485	.26253	.35598	.27443	.37822	29
30	.23959	.31509	.25104	.33519	.26272	.35634	.27463	.37860	30
31	.23978	.31541	.25124	.33554	.26292	.35670	.27483	.37898	31
32	.23997	.31574	.25143	.33588	.26312	.35707	.27503	.37936	32
33	.24016	.31607	.25162	.33622	.26331	.35743	.27523	.37974	33
34	.24035	.31640	.25182	.33657	.26351	.35779	.27543	.38012	34
35	.24054	.31672	.25201	.33691	.26371	.35815	.27563	.38051	35
36	.24073	.31705	.25220	.33726	.26390	.35852	.27583	.38089	36
37	.24092	.31738	.25240	.33760	.26410	.35888	.27603	.38127	37
38	.24111	.31771	.25259	.33795	.26430	.35924	.27623	.38165	38
39	.24130	.31804	.25278	.33830	.26449	.35961	.27643	.38204	39
40	.24149	.31837	.25297	.33864	.26469	.35997	.27663	.38242	40
41	.24168	.31870	.25317	.33899	.26489	.36034	.27683	.38280	41
42	.24187	.31903	.25336	.33934	.26509	.36070	.27703	.38319	42
43	.24206	.31936	.25356	.33968	.26528	.36107	.27723	.38357	43
44	.24225	.31969	.25375	.34003	.26548	.36143	.27743	.38396	44
45	.24244	.32002	.25394	.34038	.26568	.36180	.27764	.38434	45
46	.24262	.32035	.25414	.34073	.26588	.36217	.27784	.38473	46
47	.24281	.32068	.25433	.34108	.26607	.36253	.27804	.38512	47
48	.24300	.32101	.25452	.34142	.26627	.36290	.27824	.38550	48
49	.24320	.32134	.25472	.34177	.26647	.36327	.27844	.38589	49
50	.24339	.32168	.25491	.34212	.26667	.36363	.27864	.38628	50
51	.24358	.32201	.25511	.34247	.26686	.36400	.27884	.38666	51
52	.24377	.32234	.25530	.34282	.26706	.36437	.27905	.38705	52
53	.24398	.32267	.25549	.34317	.26726	.36474	.27925	.38744	53
54	.24415	.32301	.25569	.34352	.26746	.36511	.27945	.38783	54
55	.24434	.32334	.25589	.34387	.26766	.36548	.27965	.38822	55
56	.24453	.32368	.25608	.34423	.26785	.36585	.27985	.38860	56
57	.24472	.32401	.25627	.34458	.26805	.36622	.28005	.38899	57
58	.24491	.32434	.25647	.34493	.26825	.36659	.28026	.38938	58
59	.24510	.32468	.25666	.34528	.26845	.36696	.28046	.38977	59
60	.24529	.32501	.25686	.34563	.26865	.36733	.28066	.39016	60

X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

	44°		45°		46°		47°	
s.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
36	.39016	.29289	.41421	.30534	.43956	.31800	.46628	0
36	.39055	.29310	.41463	.30555	.43999	.31821	.46674	1
36	.39095	.29330	.41504	.30576	.44042	.31843	.46719	2
37	.39134	.29351	.41545	.30597	.44086	.31864	.46765	3
37	.39173	.29372	.41586	.30618	.44129	.31885	.46811	4
37	.39212	.29392	.41627	.30639	.44173	.31907	.46857	5
37	.39251	.29413	.41669	.30660	.44217	.31928	.46903	6
38	.39291	.29433	.41710	.30681	.44260	.31949	.46949	7
38	.39330	.29454	.41752	.30702	.44304	.31971	.46995	8
38	.39369	.29475	.41793	.30723	.44347	.31992	.47041	9
38	.39409	.29495	.41835	.30744	.44391	.32013	.47087	10
39	.39448	.29516	.41876	.30765	.44435	.32035	.47134	11
39	.39487	.29537	.41918	.30786	.44479	.32056	.47180	12
39	.39527	.29557	.41959	.30807	.44523	.32077	.47226	13
40	.39566	.29578	.42001	.30828	.44567	.32099	.47272	14
40	.39606	.29599	.42042	.30849	.44610	.32120	.47319	15
40	.39646	.29619	.42084	.30870	.44654	.32141	.47365	16
40	.39685	.29640	.42126	.30891	.44698	.32163	.47411	17
41	.39725	.29661	.42168	.30912	.44742	.32184	.47458	18
41	.39764	.29681	.42210	.30933	.44787	.32205	.47504	19
41	.39804	.29702	.42251	.30954	.44831	.32227	.47551	20
42	.39844	.29723	.42293	.30975	.44875	.32248	.47598	21
42	.39884	.29743	.42335	.30996	.44919	.32270	.47644	22
42	.39924	.29764	.42377	.31017	.44963	.32291	.47691	23
43	.39963	.29785	.42419	.31038	.45007	.32312	.47738	24
43	.40003	.29805	.42461	.31059	.45052	.32334	.47784	25
43	.40043	.29826	.42503	.31080	.45096	.32355	.47831	26
44	.40083	.29847	.42545	.31101	.45141	.32377	.47878	27
44	.40123	.29868	.42587	.31122	.45185	.32398	.47925	28
45	.40163	.29888	.42630	.31143	.45229	.32420	.47972	29
45	.40203	.29909	.42672	.31165	.45274	.32441	.48019	30
45	.40243	.29930	.42714	.31186	.45319	.32462	.48066	31
46	.40283	.29951	.42756	.31207	.45363	.32484	.48113	32
46	.40324	.29971	.42799	.31228	.45408	.32505	.48160	33
47	.40364	.29992	.42841	.31249	.45452	.32527	.48207	34
47	.40404	.30013	.42883	.31270	.45497	.32548	.48254	35
47	.40444	.30034	.42926	.31291	.45542	.32570	.48301	36
48	.40485	.30054	.42968	.31312	.45587	.32591	.48349	37
48	.40525	.30075	.43011	.31334	.45631	.32613	.48396	38
49	.40565	.30096	.43053	.31355	.45676	.32634	.48443	39
49	.40606	.30117	.43096	.31376	.45721	.32656	.48491	40
50	.40646	.30138	.43139	.31397	.45766	.32677	.48538	41
50	.40687	.30158	.43181	.31418	.45811	.32699	.48586	42
51	.40727	.30179	.43224	.31439	.45856	.32720	.48633	43
51	.40768	.30200	.43267	.31461	.45901	.32742	.48681	44
51	.40808	.30221	.43310	.31482	.45946	.32763	.48728	45
52	.40849	.30242	.43352	.31503	.45992	.32785	.48776	46
52	.40890	.30263	.43395	.31524	.46037	.32806	.48824	47
53	.40930	.30283	.43438	.31545	.46082	.32828	.48871	48
53	.40971	.30304	.43481	.31567	.46127	.32849	.48919	49
54	.41012	.30325	.43524	.31588	.46173	.32871	.48967	50
54	.41053	.30346	.43567	.31609	.46218	.32893	.49015	51
55	.41093	.30367	.43610	.31630	.46263	.32914	.49063	52
55	.41134	.30388	.43653	.31651	.46309	.32936	.49111	53
56	.41175	.30409	.43696	.31673	.46354	.32957	.49159	54
57	.41216	.30430	.43739	.31694	.46400	.32979	.49207	55
57	.41257	.30451	.43783	.31715	.46445	.33001	.49255	56
58	.41298	.30471	.43826	.31736	.46491	.33022	.49303	57
58	.41339	.30492	.43869	.31758	.46537	.33044	.49351	58
59	.41380	.30513	.43912	.31779	.46582	.33065	.49399	59
59	.41421	.30534	.43956	.31800	.46628	.33087	.49448	60

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS

′	48° Vers.	48° Ex. sec.	49° Vers.	49° Ex. sec.	50° Vers.	50° Ex. sec.	51° Vers.	51° Ex. sec.	′
0	·33087	·49448	·34394	·52425	·35721	·55572	·37068	·58902	60
1	·33109	·49496	·34416	·52476	·35744	·55626	·37091	·58959	59
2	·33130	·49544	·34438	·52527	·35766	·55680	·37113	·59016	58
3	·33152	·49593	·34460	·52579	·35788	·55734	·37136	·59073	57
4	·33173	·49641	·34482	·52630	·35810	·55789	·37158	·59130	56
5	·33195	·49690	·34504	·52681	·35833	·55843	·37181	·59188	55
6	·33217	·49738	·34526	·52732	·35855	·55897	·37204	·59245	54
7	·33238	·49787	·34548	·52784	·35877	·55951	·37226	·59302	53
8	·33260	·49835	·34570	·52835	·35900	·56005	·37249	·59360	52
9	·33282	·49884	·34592	·52886	·35922	·56060	·37272	·59418	51
10	·33303	·49933	·34614	·52938	·35944	·56114	·37294	·59475	50
11	·33325	·49981	·34636	·52989	·35967	·56169	·37317	·59533	49
12	·33347	·50030	·34658	·53041	·35989	·56223	·37340	·59590	48
13	·33368	·50079	·34680	·53092	·36011	·56278	·37362	·59648	47
14	·33390	·50128	·34702	·53144	·36034	·56332	·37385	·59706	46
15	·33412	·50177	·34724	·53196	·36056	·56387	·37408	·59764	45
16	·33434	·50226	·34746	·53247	·36078	·56442	·37430	·59822	44
17	·33455	·50275	·34768	·53299	·36101	·56497	·37453	·59880	43
18	·33477	·50324	·34790	·53351	·36123	·56551	·37476	·59938	42
19	·33499	·50373	·34812	·53403	·36146	·56606	·37498	·59996	41
20	·33520	·50422	·34834	·53455	·36168	·56661	·37521	·60054	40
21	·33542	·50471	·34856	·53507	·36190	·56716	·37544	·60112	39
22	·33564	·50521	·34878	·53559	·36213	·56771	·37567	·60171	38
23	·33586	·50570	·34900	·53611	·36235	·56826	·37589	·60229	37
24	·33607	·50619	·34923	·53663	·36258	·56881	·37612	·60287	36
25	·33629	·50669	·34945	·53715	·36280	·56937	·37635	·60346	35
26	·33651	·50718	·34967	·53768	·36302	·56992	·37658	·60404	34
27	·33673	·50767	·34989	·53820	·36325	·57047	·37680	·60463	33
28	·33694	·50817	·35011	·53872	·36347	·57103	·37703	·60521	32
29	·33716	·50866	·35033	·53924	·36370	·57158	·37726	·60580	31
30	·33738	·50916	·35055	·53977	·36392	·57213	·37749	·60639	30
31	·33760	·50966	·35077	·54029	·36415	·57269	·37771	·60698	29
32	·33782	·51015	·35099	·54082	·36437	·57324	·37794	·60756	28
33	·33803	·51065	·35122	·54134	·36460	·57380	·37817	·60815	27
34	·33825	·51115	·35144	·54187	·36482	·57436	·37840	·60874	26
35	·33847	·51165	·35166	·54240	·36504	·57491	·37862	·60933	25
36	·33869	·51215	·35188	·54292	·36527	·57547	·37885	·60992	24
37	·33891	·51265	·35210	·54345	·36549	·57603	·37908	·61051	23
38	·33912	·51314	·35232	·54398	·36572	·57659	·37931	·61111	22
39	·33934	·51364	·35254	·54451	·36594	·57715	·37954	·61170	21
40	·33956	·51415	·35277	·54504	·36617	·57771	·37976	·61229	20
41	·33978	·51465	·35299	·54557	·36639	·57827	·37999	·61288	19
42	·34000	·51515	·35321	·54610	·36662	·57883	·38022	·61348	18
43	·34022	·51565	·35343	·54663	·36684	·57939	·38045	·61407	17
44	·34044	·51615	·35365	·54716	·36707	·57995	·38068	·61467	16
45	·34065	·51665	·35388	·54769	·36729	·58051	·38091	·61526	15
46	·34087	·51716	·35410	·54822	·36752	·58108	·38113	·61586	14
47	·34109	·51766	·35432	·54876	·36775	·58164	·38136	·61646	13
48	·34131	·51817	·35454	·54929	·36797	·58221	·38159	·61705	12
49	·34153	·51867	·35476	·54982	·36820	·58277	·38182	·61765	11
50	·34175	·51918	·35499	·55036	·36842	·58333	·38205	·61825	10
51	·34197	·51968	·35521	·55089	·36865	·58390	·38228	·61885	9
52	·34219	·52019	·35543	·55143	·36887	·58447	·38251	·61945	8
53	·34241	·52069	·35565	·55196	·36910	·58503	·38274	·62005	7
54	·34262	·52120	·35588	·55250	·36932	·58560	·38296	·62065	6
55	·34284	·52171	·35610	·55303	·36955	·58617	·38319	·62125	5
56	·34306	·52222	·35632	·55357	·36978	·58674	·38342	·62185	4
57	·34328	·52273	·35654	·55411	·37000	·58731	·38365	·62248	3
58	·34350	·52323	·35677	·55465	·37023	·58788	·38388	·62308	2
59	·34372	·52374	·35699	·55518	·37045	·58845	·38411	·62368	1
60	·34394	·52425	·35721	·55572	·37068	·58902	·38434	·62427	0

BLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

	52°		53°		54°		55°		
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
0	.38434	.62427	.39819	.66164	.41221	.70130	.42642	.74345	0
1	.38457	.62487	.39842	.66228	.41245	.70198	.42666	.74417	1
2	.38480	.62548	.39865	.66292	.41269	.70267	.42690	.74490	2
3	.38503	.62609	.39888	.66357	.41292	.70335	.42714	.74562	3
4	.38526	.62669	.39911	.66421	.41316	.70403	.42738	.74635	4
5	.38549	.62730	.39935	.66486	.41339	.70472	.42762	.74708	5
6	.38571	.62791	.39958	.66550	.41363	.70540	.42785	.74781	6
7	.38594	.62852	.39981	.66615	.41386	.70609	.42809	.74854	7
8	.38617	.62913	.40005	.66679	.41410	.70677	.42833	.74927	8
9	.38640	.62974	.40028	.66744	.41433	.70746	.42857	.75000	9
0	.38663	.63035	.40051	.66809	.41457	.70815	.42881	.75073	10
1	.38686	.63096	.40074	.66873	.41481	.70884	.42905	.75146	11
2	.38709	.63157	.40098	.66938	.41504	.70953	.42929	.75219	12
3	.38732	.63218	.40121	.67003	.41528	.71022	.42953	.75293	13
4	.38755	.63279	.40144	.67068	.41551	.71091	.42976	.75366	14
5	.38778	.63341	.40168	.67133	.41575	.71160	.43000	.75440	15
6	.38801	.63402	.40191	.67199	.41599	.71229	.43024	.75513	16
7	.38824	.63464	.40214	.67264	.41622	.71298	.43048	.75587	17
8	.38847	.63525	.40237	.67329	.41646	.71368	.43072	.75661	18
9	.38870	.63587	.40261	.67394	.41670	.71437	.43096	.75734	19
0	.38893	.63648	.40284	.67460	.41693	.71506	.43120	.75808	20
1	.38916	.63710	.40307	.67525	.41717	.71576	.43144	.75882	21
2	.38939	.63772	.40331	.67591	.41740	.71646	.43168	.75956	22
3	.38962	.63834	.40354	.67656	.41764	.71715	.43192	.76031	23
4	.38985	.63895	.40378	.67722	.41788	.71785	.43216	.76105	24
5	.39009	.63957	.40401	.67788	.41811	.71855	.43240	.76179	25
6	.39032	.64019	.40424	.67853	.41835	.71925	.43264	.76253	26
7	.39055	.64081	.40448	.67919	.41859	.71995	.43287	.76328	27
8	.39078	.64144	.40471	.67985	.41882	.72065	.43311	.76402	28
9	.39101	.64206	.40494	.68051	.41906	.72135	.43335	.76477	29
0	.39124	.64268	.40518	.68117	.41930	.72205	.43359	.76552	30
1	.39147	.64330	.40541	.68183	.41953	.72275	.43383	.76626	31
2	.39170	.64393	.40565	.68250	.41977	.72346	.43407	.76701	32
3	.39193	.64455	.40588	.68316	.42001	.72416	.43431	.76776	33
4	.39216	.64518	.40611	.68382	.42024	.72487	.43455	.76851	34
5	.39239	.64580	.40635	.68449	.42048	.72557	.43479	.76926	35
6	.39262	.64643	.40658	.68515	.42072	.72628	.43503	.77001	36
7	.39286	.64705	.40682	.68582	.42096	.72698	.43527	.77077	37
8	.39309	.64768	.40705	.68648	.42119	.72769	.43551	.77152	38
9	.39332	.64831	.40728	.68715	.42143	.72840	.43575	.77227	39
0	.39355	.64894	.40752	.68782	.42167	.72911	.43599	.77303	40
1	.39378	.64957	.40775	.68848	.42191	.72982	.43623	.77378	41
2	.39401	.65020	.40799	.68915	.42214	.73053	.43647	.77454	42
3	.39424	.65083	.40822	.68982	.42238	.73124	.43671	.77530	43
4	.39447	.65146	.40846	.69049	.42262	.73195	.43695	.77606	44
5	.39471	.65209	.40869	.69116	.42285	.73267	.43720	.77681	45
6	.39494	.65272	.40893	.69183	.42309	.73338	.43744	.77757	46
7	.39517	.65336	.40916	.69250	.42333	.73409	.43768	.77833	47
8	.39540	.65399	.40939	.69318	.42357	.73481	.43792	.77910	48
9	.39563	.65462	.40963	.69385	.42381	.73552	.43816	.77986	49
0	.39586	.65526	.40986	.69452	.42404	.73624	.43840	.78062	50
1	.39610	.65589	.41010	.69520	.42428	.73696	.43864	.78138	51
2	.39633	.65653	.41033	.69587	.42452	.73768	.43888	.78215	52
3	.39656	.65717	.41057	.69655	.42476	.73840	.43912	.78291	53
4	.39679	.65780	.41080	.69723	.42499	.73911	.43936	.78368	54
5	.39702	.65844	.41104	.69790	.42523	.73983	.43960	.78445	55
6	.39726	.65908	.41127	.69858	.42547	.74056	.43984	.78521	56
7	.39749	.65972	.41151	.69926	.42571	.74128	.44008	.78598	57
8	.39772	.66036	.41174	.69994	.42595	.74200	.44032	.78675	58
9	.39795	.66100	.41198	.70062	.42619	.74272	.44057	.78752	59
0	.39819	.66164	.41221	.70130	.42642	.74345	.44081	.78829	60

TABLE X.—NATURAL VERSED SINES AND EXTERNAL

	56°		57°		58°		5
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.
0	.44081	.78829	.45536	.83608	.47008	.88708	.48496
1	.44105	.78906	.45560	.83690	.47033	.88796	.48521
2	.44129	.78984	.45585	.83773	.47057	.88884	.48546
3	.44153	.79061	.45609	.83855	.47082	.88972	.48571
4	.44177	.79138	.45634	.83938	.47107	.89060	.48596
5	.44201	.79216	.45658	.84020	.47131	.89148	.48621
6	.44225	.79293	.45683	.84103	.47156	.89237	.48646
7	.44250	.79371	.45707	.84186	.47181	.89325	.48671
8	.44274	.79449	.45731	.84269	.47206	.89414	.48696
9	.44298	.79527	.45756	.84352	.47230	.89503	.48721
10	.44322	.79604	.45780	.84435	.47255	.89591	.48746
11	.44346	.79682	.45805	.84518	.47280	.89680	.48771
12	.44370	.79761	.45829	.84601	.47304	.89769	.48796
13	.44395	.79839	.45854	.84685	.47329	.89858	.48821
14	.44419	.79917	.45878	.84768	.47354	.89948	.48846
15	.44443	.79995	.45903	.84852	.47379	.90037	.48871
16	.44467	.80074	.45927	.84935	.47403	.90126	.48896
17	.44491	.80152	.45951	.85019	.47428	.90216	.48921
18	.44516	.80231	.45976	.85103	.47453	.90305	.48946
19	.44540	.80309	.46000	.85187	.47478	.90395	.48971
20	.44564	.80388	.46025	.85271	.47502	.90485	.48996
21	.44588	.80467	.46049	.85355	.47527	.90575	.49021
22	.44612	.80546	.46074	.85439	.47552	.90665	.49046
23	.44637	.80625	.46098	.85523	.47577	.90755	.49071
24	.44661	.80704	.46123	.85608	.47601	.90845	.49096
25	.44685	.80783	.46147	.85692	.47626	.90935	.49121
26	.44709	.80862	.46172	.85777	.47651	.91026	.49146
27	.44734	.80942	.46196	.85861	.47676	.91116	.49171
28	.44758	.81021	.46221	.85946	.47701	.91207	.49196
29	.44782	.81101	.46246	.86031	.47725	.91297	.49221
30	.44806	.81180	.46270	.86116	.47750	.91388	.49246
31	.44831	.81260	.46295	.86201	.47775	.91479	.49271
32	.44855	.81340	.46319	.86286	.47800	.91570	.49296
33	.44879	.81419	.46344	.86371	.47825	.91661	.49321
34	.44903	.81499	.46368	.86457	.47849	.91752	.49346
35	.44928	.81579	.46393	.86542	.47874	.91844	.49372
36	.44952	.81659	.46417	.86627	.47899	.91935	.49397
37	.44976	.81740	.46442	.86713	.47924	.92027	.49422
38	.45001	.81820	.46466	.86799	.47949	.92118	.49447
39	.45025	.81900	.46491	.86885	.47974	.92210	.49472
40	.45049	.81981	.46516	.86970	.47998	.92302	.49497
41	.45073	.82061	.46540	.87056	.48023	.92394	.49522
42	.45098	.82142	.46565	.87142	.48048	.92486	.49547
43	.45122	.82222	.46589	.87229	.48073	.92578	.49572
44	.45146	.82303	.46614	.87315	.48098	.92670	.49597
45	.45171	.82384	.46639	.87401	.48123	.92762	.49623
46	.45195	.82465	.46663	.87488	.48148	.92855	.49648
47	.45219	.82546	.46688	.87574	.48172	.92947	.49673
48	.45244	.82627	.46712	.87661	.48197	.93040	.49698
49	.45268	.82709	.46737	.87748	.48222	.93133	.49723
50	.45292	.82790	.46762	.87834	.48247	.93226	.49748
51	.45317	.82871	.46786	.87921	.48272	.93319	.49773
52	.45341	.82953	.46811	.88008	.48297	.93412	.49799
53	.45365	.83034	.46836	.88095	.48322	.93505	.49824
54	.45390	.83116	.46860	.88183	.48347	.93598	.49849
55	.45414	.83198	.46885	.88270	.48372	.93692	.49874
56	.45439	.83280	.46909	.88357	.48396	.93785	.49899
57	.45463	.83362	.46934	.88445	.48421	.93879	.49924
58	.45487	.83444	.46959	.88532	.48446	.93973	.49950
59	.45512	.83526	.46983	.88620	.48471	.94066	.49975
60	.45536	.83608	.47008	.88708	.48496	.94160	.50000

ABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	60° Vers.	60° Ex. sec.	61° Vers.	61° Ex. sec.	62° Vers.	62° Ex. sec.	63° Vers.	63° Ex. sec.	′
0	.50000	1.00000	.51519	1.06267	.53053	1.13005	.54601	1.20269	0
1	.50025	1.00101	.51544	1.06375	.53079	1.13122	.54627	1.20385	1
2	.50050	1.00202	.51570	1.06483	.53104	1.13239	.54653	1.20521	2
3	.50076	1.00303	.51595	1.06592	.53130	1.13356	.54679	1.20647	3
4	.50101	1.00404	.51621	1.06701	.53156	1.13473	.54705	1.20773	4
5	.50126	1.00505	.51646	1.06809	.53181	1.13590	.54731	1.20900	5
6	.50151	1.00607	.51672	1.06918	.53207	1.13707	.54757	1.21026	6
7	.50176	1.00708	.51697	1.07027	.53233	1.13825	.54782	1.21153	7
8	.50202	1.00810	.51723	1.07137	.53258	1.13942	.54808	1.21280	8
9	.50227	1.00912	.51748	1.07246	.53284	1.14060	.54834	1.21407	9
10	.50252	1.01014	.51774	1.07356	.53310	1.14178	.54860	1.21535	10
11	.50277	1.01116	.51799	1.07465	.53336	1.14296	.54886	1.21662	11
12	.50303	1.01218	.51825	1.07575	.53361	1.14414	.54912	1.21790	12
13	.50328	1.01320	.51850	1.07685	.53387	1.14533	.54938	1.21918	13
14	.50353	1.01422	.51876	1.07795	.53413	1.14651	.54964	1.22045	14
15	.50378	1.01525	.51901	1.07905	.53439	1.14770	.54990	1.22174	15
16	.50404	1.01628	.51927	1.08015	.53464	1.14889	.55016	1.22302	16
17	.50429	1.01730	.51952	1.08126	.53490	1.15008	.55042	1.22430	17
18	.50454	1.01833	.51978	1.08236	.53516	1.15127	.55068	1.22559	18
19	.50479	1.01936	.52003	1.08347	.53542	1.15246	.55094	1.22688	19
20	.50505	1.02039	.52029	1.08458	.53567	1.15366	.55120	1.22817	20
21	.50530	1.02143	.52054	1.08569	.53593	1.15485	.55146	1.22946	21
22	.50555	1.02246	.52080	1.08680	.53619	1.15605	.55172	1.23075	22
23	.50581	1.02349	.52105	1.08791	.53645	1.15725	.55198	1.23205	23
24	.50606	1.02453	.52131	1.08903	.53670	1.15845	.55224	1.23334	24
25	.50631	1.02557	.52156	1.09014	.53696	1.15965	.55250	1.23464	25
26	.50656	1.02661	.52182	1.09126	.53722	1.16085	.55276	1.23594	26
27	.50682	1.02765	.52207	1.09238	.53748	1.16206	.55302	1.23724	27
28	.50707	1.02869	.52233	1.09350	.53774	1.16326	.55328	1.23855	28
29	.50732	1.02973	.52259	1.09462	.53799	1.16447	.55354	1.23985	29
30	.50758	1.03077	.52284	1.09574	.53825	1.16568	.55380	1.24116	30
31	.50783	1.03182	.52310	1.09686	.53851	1.16689	.55406	1.24247	31
32	.50808	1.03286	.52335	1.09799	.53877	1.16810	.55432	1.24378	32
33	.50834	1.03391	.52361	1.09911	.53903	1.16932	.55458	1.24509	33
34	.50859	1.03496	.52386	1.10024	.53928	1.17053	.55484	1.24640	34
35	.50884	1.03601	.52412	1.10137	.53954	1.17175	.55510	1.24772	35
36	.50910	1.03706	.52438	1.10250	.53980	1.17297	.55536	1.24903	36
37	.50935	1.03811	.52463	1.10363	.54006	1.17419	.55563	1.25035	37
38	.50960	1.03916	.52489	1.10477	.54032	1.17541	.55589	1.25167	38
39	.50986	1.04022	.52514	1.10590	.54058	1.17663	.55615	1.25300	39
40	.51011	1.04128	.52540	1.10704	.54083	1.17786	.55641	1.25432	40
41	.51036	1.04233	.52566	1.10817	.54109	1.17909	.55667	1.25565	41
42	.51062	1.04339	.52591	1.10931	.54135	1.18031	.55693	1.25697	42
43	.51087	1.04445	.52617	1.11045	.54161	1.18154	.55719	1.25830	43
44	.51113	1.04551	.52642	1.11159	.54187	1.18277	.55745	1.25963	44
45	.51138	1.04658	.52668	1.11274	.54213	1.18401	.55771	1.26097	45
46	.51163	1.04764	.52694	1.11388	.54238	1.18524	.55797	1.26230	46
47	.51189	1.04870	.52719	1.11503	.54264	1.18648	.55823	1.26364	47
48	.51214	1.04977	.52745	1.11617	.54290	1.18772	.55849	1.26498	48
49	.51239	1.05084	.52771	1.11732	.54316	1.18895	.55876	1.26632	49
50	.51265	1.05191	.52796	1.11847	.54342	1.19019	.55902	1.26766	50
51	.51290	1.05298	.52822	1.11963	.54368	1.19144	.55928	1.26900	51
52	.51316	1.05405	.52848	1.12078	.54394	1.19268	.55954	1.27035	52
53	.51341	1.05512	.52873	1.12193	.54420	1.19393	.55980	1.27169	53
54	.51366	1.05619	.52899	1.12309	.54446	1.19517	.56006	1.27304	54
55	.51392	1.05727	.52924	1.12425	.54471	1.19642	.56032	1.27439	55
56	.51417	1.05835	.52950	1.12540	.54497	1.19767	.56058	1.27574	56
57	.51443	1.05942	.52976	1.12657	.54523	1.19892	.56084	1.27710	57
58	.51468	1.06050	.53001	1.12773	.54549	1.20018	.56111	1.27845	58
59	.51494	1.06158	.53027	1.12889	.54575	1.20143	.56137	1.27981	59
60	.51519	1.06267	.53053	1.13005	.54601	1.20269	.56163	1.28117	60

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS

′	64° Vers.	64° Ex. sec.	65° Vers.	65° Ex. sec.	66° Vers.	66° Ex. sec.	67° Vers.	67° Ex. sec.	′
0	.56163	1.28117	.57738	1.36620	.59326	1.45859	.60927	1.55930	0
1	.56189	1.28253	.57765	1.36768	.59353	1.46020	.60954	1.56106	1
2	.56215	1.28390	.57791	1.36916	.59379	1.46181	.60980	1.56282	2
3	.56241	1.28526	.57817	1.37064	.59406	1.46342	.61007	1.56458	3
4	.56267	1.28663	.57844	1.37212	.59433	1.46504	.61034	1.56634	4
5	.56294	1.28800	.57870	1.37361	.59459	1.46665	.61061	1.56811	5
6	.56320	1.28937	.57896	1.37509	.59486	1.46827	.61088	1.56988	6
7	.56346	1.29074	.57923	1.37658	.59512	1.46989	.61114	1.57165	7
8	.56372	1.29211	.57949	1.37808	.59539	1.47152	.61141	1.57342	8
9	.56398	1.29349	.57976	1.37957	.59566	1.47314	.61168	1.57520	9
10	.56425	1.29487	.58002	1.38107	.59592	1.47477	.61195	1.57698	10
11	.56451	1.29625	.58028	1.38256	.59619	1.47640	.61222	1.57876	11
12	.56477	1.29763	.58055	1.38408	.59645	1.47804	.61248	1.58054	12
13	.56503	1.29901	.58081	1.38556	.59672	1.47967	.61275	1.58233	13
14	.56529	1.30040	.58108	1.38707	.59699	1.48131	.61302	1.58412	14
15	.56555	1.30179	.58134	1.38857	.59725	1.48295	.61329	1.58591	15
16	.56582	1.30318	.58160	1.39008	.59752	1.48459	.61356	1.58771	16
17	.56608	1.30457	.58187	1.39159	.59779	1.48624	.61383	1.58950	17
18	.56634	1.30596	.58213	1.39311	.59805	1.48789	.61409	1.59130	18
19	.56660	1.30735	.58240	1.39462	.59832	1.48954	.61436	1.59311	19
20	.56687	1.30875	.58266	1.39614	.59859	1.49119	.61463	1.59491	20
21	.56713	1.31015	.58293	1.39766	.59885	1.49284	.61490	1.59672	21
22	.56739	1.31155	.58319	1.39918	.59912	1.49450	.61517	1.59853	22
23	.56765	1.31295	.58345	1.40070	.59938	1.49616	.61544	1.60035	23
24	.56791	1.31436	.58372	1.40222	.59965	1.49782	.61570	1.60217	24
25	.56818	1.31576	.58398	1.40375	.59992	1.49948	.61597	1.60399	25
26	.56844	1.31717	.58425	1.40528	.60018	1.50115	.61624	1.60581	26
27	.56870	1.31858	.58451	1.40681	.60045	1.50282	.61651	1.60763	27
28	.56896	1.31999	.58478	1.40835	.60072	1.50449	.61678	1.60946	28
29	.56923	1.32140	.58504	1.40988	.60098	1.50617	.61705	1.61129	29
30	.56949	1.32282	.58531	1.41142	.60125	1.50784	.61732	1.61313	30
31	.56975	1.32424	.58557	1.41296	.60152	1.50952	.61759	1.61496	31
32	.57001	1.32566	.58584	1.41450	.60178	1.51120	.61785	1.61680	32
33	.57028	1.32708	.58610	1.41605	.60205	1.51289	.61812	1.61864	33
34	.57054	1.32850	.58637	1.41760	.60232	1.51457	.61839	1.62049	34
35	.57080	1.32993	.58663	1.41914	.60259	1.51626	.61866	1.62234	35
36	.57106	1.33135	.58690	1.42070	.60285	1.51795	.61893	1.62419	36
37	.57133	1.33278	.58716	1.42225	.60312	1.51965	.61920	1.62604	37
38	.57159	1.33422	.58743	1.42380	.60339	1.52134	.61947	1.62790	38
39	.57185	1.33565	.58769	1.42536	.60365	1.52304	.61974	1.62976	39
40	.57212	1.33708	.58796	1.42692	.60392	1.52474	.62001	1.63162	40
41	.57238	1.33852	.58822	1.42848	.60419	1.52645	.62027	1.63348	41
42	.57264	1.33996	.58849	1.43005	.60445	1.52815	.62054	1.63535	42
43	.57291	1.34140	.58875	1.43162	.60472	1.52988	.62081	1.63722	43
44	.57317	1.34284	.58902	1.43318	.60499	1.53157	.62108	1.63908	44
45	.57343	1.34429	.58928	1.43476	.60526	1.53329	.62135	1.64097	45
46	.57369	1.34573	.58955	1.43633	.60552	1.53500	.62162	1.64285	46
47	.57396	1.34718	.58981	1.43790	.60579	1.53672	.62189	1.64473	47
48	.57422	1.34863	.59008	1.43948	.60606	1.53845	.62216	1.64662	48
49	.57448	1.35009	.59034	1.44106	.60633	1.54017	.62243	1.64851	49
50	.57475	1.35154	.59061	1.44264	.60659	1.54190	.62270	1.65040	50
51	.57501	1.35300	.59087	1.44423	.60686	1.54363	.62297	1.65229	51
52	.57527	1.35446	.59114	1.44582	.60713	1.54536	.62324	1.65419	52
53	.57554	1.35592	.59140	1.44741	.60740	1.54709	.62351	1.65609	53
54	.57580	1.35738	.59167	1.44900	.60766	1.54883	.62378	1.65799	54
55	.57606	1.35885	.59194	1.45059	.60793	1.55057	.62405	1.65989	55
56	.57633	1.36031	.59220	1.45219	.60820	1.55231	.62431	1.66180	56
57	.57659	1.36178	.59247	1.45378	.60847	1.55405	.62458	1.66371	57
58	.57685	1.36325	.59273	1.45539	.60873	1.55580	.62485	1.66563	58
59	.57712	1.36473	.59300	1.45699	.60900	1.55755	.62512	1.66755	59
60	.57738	1.36620	.59326	1.45859	.60927	1.55930	.62539	1.66947	60

ABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

	68°		69°		70°		71°		
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
	·62539	1·66947	·64163	1·79043	·65798	1·92380	·67443	2·07155	0
	·62566	1·67139	·64190	1·79254	·65825	1·92614	·67471	2·07415	1
	·62593	1·67332	·64218	1·79466	·65853	1·92849	·67498	2·07675	2
	·62620	1·67525	·64245	1·79679	·65880	1·93083	·67526	2·07936	3
	·62647	1·67718	·64272	1·79891	·65907	1·93318	·67553	2·08197	4
	·62674	1·67911	·64299	1·80104	·65935	1·93554	·67581	2·08459	5
	·62701	1·68105	·64326	1·80318	·65962	1·93790	·67608	2·08721	6
	·62728	1·68299	·64353	1·80531	·65989	1·94026	·67636	2·08983	7
	·62755	1·68494	·64381	1·80746	·66017	1·94263	·67663	2·09246	8
	·62782	1·68689	·64408	1·80960	·66044	1·94500	·67691	2·09510	9
	·62809	1·68884	·64435	1·81175	·66071	1·94737	·67718	2·09774	10
	·62836	1·69079	·64462	1·81390	·66099	1·94975	·67746	2·10038	11
	·62863	1·69275	·64489	1·81605	·66126	1·95213	·67773	2·10303	12
	·62890	1·69471	·64517	1·81821	·66154	1·95452	·67801	2·10568	13
	·62917	1·69667	·64544	1·82037	·66181	1·95691	·67829	2·10834	14
	·62944	1·69864	·64571	1·82254	·66208	1·95931	·67856	2·11101	15
	·62971	1·70061	·64598	1·82471	·66236	1·96171	·67884	2·11367	16
	·62998	1·70258	·64625	1·82688	·66263	1·96411	·67911	2·11635	17
	·63025	1·70455	·64653	1·82906	·66290	1·96652	·67939	2·11903	18
	·63052	1·70653	·64680	1·83124	·66318	1·96893	·67966	2·12171	19
	·63079	1·70851	·64707	1·83342	·66345	1·97135	·67994	2·12440	20
	·63106	1·71050	·64734	1·83561	·66373	1·97377	·68021	2·12709	21
	·63133	1·71249	·64761	1·83780	·66400	1·97619	·68049	2·12979	22
	·63161	1·71448	·64789	1·83999	·66427	1·97862	·68077	2·13249	23
	·63188	1·71647	·64816	1·84219	·66455	1·98106	·68104	2·13520	24
	·63215	1·71847	·64843	1·84439	·66482	1·98349	·68132	2·13791	25
	·63242	1·72047	·64870	1·84659	·66510	1·98594	·68159	2·14063	26
	·63269	1·72247	·64898	1·84880	·66537	1·98838	·68187	2·14335	27
	·63296	1·72448	·64925	1·85102	·66564	1·99083	·68214	2·14608	28
	·63323	1·72649	·64952	1·85323	·66592	1·99329	·68242	2·14881	29
	·63350	1·72850	·64979	1·85545	·66619	1·99574	·68270	2·15155	30
	·63377	1·73052	·65007	1·85767	·66647	1·99821	·68297	2·15429	31
	·63404	1·73254	·65034	1·85990	·66674	2·00067	·68325	2·15704	32
	·63431	1·73456	·65061	1·86213	·66702	2·00315	·68352	2·15979	33
	·63458	1·73659	·65088	1·86437	·66729	2·00562	·68380	2·16255	34
	·63485	1·73862	·65116	1·86661	·66756	2·00810	·68408	2·16531	35
	·63512	1·74065	·65143	1·86885	·66784	2·01059	·68435	2·16808	36
	·63539	1·74269	·65170	1·87109	·66811	2·01308	·68463	2·17085	37
	·63566	1·74473	·65197	1·87334	·66839	2·01557	·68490	2·17363	38
	·63594	1·74677	·65225	1·87560	·66866	2·01807	·68518	2·17641	39
	·63621	1·74881	·65252	1·87785	·66894	2·02057	·68546	2·17920	40
	·63648	1·75086	·65279	1·88011	·66921	2·02308	·68573	2·18199	41
	·63675	1·75292	·65306	1·88238	·66949	2·02559	·68601	2·18479	42
	·63702	1·75497	·65334	1·88465	·66976	2·02810	·68628	2·18759	43
	·63729	1·75703	·65361	1·88692	·67003	2·03062	·68656	2·19040	44
	·63756	1·75909	·65388	1·88920	·67031	2·03315	·68684	2·19322	45
	·63783	1·76116	·65416	1·89148	·67058	2·03568	·68711	2·19604	46
	·63810	1·76323	·65443	1·89376	·67086	2·03821	·68739	2·19886	47
	·63838	1·76530	·65470	1·89605	·67113	2·04075	·68767	2·20169	48
	·63865	1·76737	·65497	1·89834	·67141	2·04329	·68794	2·20453	49
	·63892	1·76945	·65525	1·90063	·67168	2·04584	·68822	2·20737	50
	·63919	1·77154	·65552	1·90293	·67196	2·04839	·68849	2·21021	51
	·63946	1·77362	·65579	1·90524	·67223	2·05094	·68877	2·21306	52
	·63973	1·77571	·65607	1·90754	·67251	2·05350	·68905	2·21592	53
	·64000	1·77780	·65634	1·90986	·67278	2·05607	·68932	2·21878	54
	·64027	1·77990	·65661	1·91217	·67306	2·05864	·68960	2·22165	55
	·64055	1·78200	·65689	1·91449	·67333	2·06121	·68988	2·22452	56
	·64082	1·78410	·65716	1·91681	·67361	2·06379	·69015	2·22740	57
	·64109	1·78621	·65743	1·91914	·67388	2·06637	·69043	2·23028	58
	·64136	1·78832	·65771	1·92147	·67416	2·06896	·69071	2·23317	59
	·64163	1·79043	·65798	1·92380	·67443	2·07155	·69098	2·23607	60

9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.45[illegible]	[illegible]
10	69875	2.38[illegible]	7.[illegible]	2.45[illegible]	[illegible]	[illegible]	.74899	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
21	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
23	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
25	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
26	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
27	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
29	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
31	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
32	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
33	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
34	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
35	70068	2 34092	71738	2.53831	73416	2.78171	.75103	3.01
36	70096	2 34493	71766	2 54181	73444	2.78568	.75131	3.02
37	70.24	2 34713	71794	2 54531	73472	2.76966	.75159	3.02
38	70151	2 35025	71822	2.54883	73500	2.77365	.75187	3.03
39	70.79	2 35336	71850	2.55235	73529	2.77765	.75216	3.03
40	70207	2 35649	71877	2 55587	73557	2.78166	.75244	3.03
41	70235	2 35962	71905	2.55940	73585	2.78568	.75272	3.04
42	70263	2 36276	71933	2.56294	73613	2.78970	.75300	3.04
43	70290	2.36590	71961	2.56649	73641	2.79374	.75328	3.05
44	70318	2.36905	71989	2.57005	73669	2.79778	.75356	3.05
45	70346	2.37221	72017	2.57361	73697	2.80183	.75385	3.06

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

	76°		77°		78°		79°		
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
0	.75808	3.13357	.77505	3.44541	.79209	3.80973	.80919	4.24084	0
1	.75836	3.13839	.77533	3.45102	.79237	3.81633	.80948	4.24870	1
2	.75864	3.14323	.77562	3.45664	.79266	3.82294	.80978	4.25658	2
3	.75892	3.14809	.77590	3.46228	.79294	3.82956	.81005	4.26448	3
4	.75921	3.15295	.77618	3.46793	.79323	3.83621	.81033	4.27241	4
5	.75949	3.15782	.77647	3.47360	.79351	3.84288	.81062	4.28036	5
6	.75977	3.16271	.77675	3.47928	.79380	3.84956	.81090	4.28833	6
7	.76005	3.16761	.77703	3.48498	.79408	3.85627	.81119	4.29634	7
8	.76034	3.17252	.77732	3.49069	.79437	3.86299	.81148	4.30436	8
9	.76062	3.17744	.77760	3.49642	.79465	3.86973	.81176	4.31241	9
10	.76090	3.18238	.77788	3.50216	.79493	3.87649	.81205	4.32049	10
11	.76118	3.18733	.77817	3.50791	.79522	3.88327	.81233	4.32859	11
12	.76147	3.19228	.77845	3.51368	.79550	3.89007	.81262	4.33671	12
13	.76175	3.19725	.77874	3.51947	.79579	3.89689	.81290	4.34486	13
14	.76203	3.20224	.77902	3.52527	.79607	3.90373	.81319	4.35304	14
15	.76231	3.20723	.77930	3.53109	.79636	3.91058	.81348	4.36124	15
16	.76260	3.21224	.77959	3.53692	.79664	3.91746	.81376	4.36947	16
17	.76288	3.21726	.77987	3.54277	.79693	3.92436	.81405	4.37772	17
18	.76316	3.22229	.78015	3.54863	.79721	3.93128	.81433	4.38600	18
19	.76344	3.22734	.78044	3.55451	.79750	3.93821	.81462	4.39430	19
20	.76373	3.23239	.78072	3.56041	.79778	3.94517	.81491	4.40263	20
21	.76401	3.23746	.78101	3.56632	.79807	3.95215	.81519	4.41099	21
22	.76429	3.24255	.78129	3.57224	.79835	3.95914	.81548	4.41937	22
23	.76458	3.24764	.78157	3.57819	.79864	3.96616	.81576	4.42778	23
24	.76486	3.25275	.78186	3.58414	.79892	3.97320	.81605	4.43622	24
25	.76514	3.25787	.78214	3.59012	.79921	3.98025	.81633	4.44468	25
26	.76542	3.26300	.78242	3.59611	.79949	3.98733	.81662	4.45317	26
27	.76571	3.26814	.78271	3.60211	.79978	3.99443	.81691	4.46169	27
28	.76599	3.27330	.78299	3.60813	.80006	4.00155	.81719	4.47023	28
29	.76627	3.27847	.78328	3.61417	.80035	4.00869	.81748	4.47881	29
30	.76655	3.28366	.78356	3.62023	.80063	4.01585	.81776	4.48740	30
31	.76684	3.28885	.78384	3.62630	.80092	4.02303	.81805	4.49603	31
32	.76712	3.29406	.78413	3.63238	.80120	4.03024	.81834	4.50468	32
33	.76740	3.29929	.78441	3.63849	.80149	4.03746	.81862	4.51337	33
34	.76769	3.30452	.78470	3.64461	.80177	4.04471	.81891	4.52208	34
35	.76797	3.30977	.78498	3.65074	.80206	4.05197	.81919	4.53081	35
36	.76825	3.31503	.78526	3.65690	.80234	4.05926	.81948	4.53958	36
37	.76854	3.32031	.78555	3.66307	.80263	4.06657	.81977	4.54837	37
38	.76882	3.32560	.78583	3.66925	.80291	4.07390	.82005	4.55720	38
39	.76910	3.33090	.78612	3.67545	.80320	4.08125	.82034	4.56605	39
40	.76938	3.33622	.78640	3.68167	.80348	4.08863	.82063	4.57493	40
41	.76967	3.34154	.78669	3.68791	.80377	4.09602	.82091	4.58383	41
42	.76995	3.34689	.78697	3.69417	.80405	4.10344	.82120	4.59277	42
43	.77023	3.35224	.78725	3.70044	.80434	4.11088	.82148	4.60174	43
44	.77052	3.35761	.78754	3.70673	.80462	4.11835	.82177	4.61073	44
45	.77080	3.36299	.78782	3.71303	.80491	4.12583	.82206	4.61976	45
46	.77108	3.36839	.78811	3.71935	.80520	4.13334	.82234	4.62881	46
47	.77137	3.37380	.78839	3.72569	.80548	4.14087	.82263	4.63790	47
48	.77165	3.37923	.78868	3.73205	.80577	4.14842	.82292	4.64701	48
49	.77193	3.38466	.78896	3.73843	.80605	4.15599	.82320	4.65616	49
50	.77222	3.39012	.78924	3.74482	.80634	4.16359	.82349	4.66533	50
51	.77250	3.39558	.78953	3.75123	.80662	4.17121	.82377	4.67454	51
52	.77278	3.40106	.78981	3.75766	.80691	4.17886	.82406	4.68377	52
53	.77307	3.40656	.79010	3.76411	.80719	4.18652	.82435	4.69304	53
54	.77335	3.41206	.79038	3.77057	.80748	4.19421	.82463	4.70234	54
55	.77363	3.41759	.79067	3.77705	.80776	4.20193	.82492	4.71166	55
56	.77392	3.42312	.79095	3.78355	.80805	4.20966	.82521	4.72102	56
57	.77420	3.42867	.79123	3.79007	.80833	4.21742	.82549	4.73041	57
58	.77448	3.43424	.79152	3.79661	.80862	4.22521	.82578	4.73983	58
59	.77477	3.43982	.79180	3.80316	.80891	4.23301	.82607	4.74929	59
60	.77505	3.44541	.79209	3.80973	.80919	4.24084	.82635	4.75877	60

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

	80°		81°		82°		83°		
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
0	.82685	4·75877	.84357	5·39245	.86083	6·18530	.87813	7·20551	0
1	.82664	4·76829	.84385	5·40422	.86112	6·20020	.87842	7·22500	1
2	.82692	4·77784	.84414	5·41602	.86140	6·21517	.87871	7·24457	2
3	.82721	4·78742	.84443	5·42787	.86169	6·23019	.87900	7·26425	3
4	.82750	4·79703	.84471	5·43977	.86198	6·24529	.87929	7·28402	4
5	.82778	4·80667	.84500	5·45171	.86227	6·26044	.87957	7·30388	5
6	.82807	4·81635	.84529	5·46369	.86256	6·27563	.87986	7·32384	6
7	.82836	4·82606	.84558	5·47572	.86284	6·29095	.88015	7·34390	7
8	.82864	4·83581	.84586	5·48779	.86313	6·30630	.88044	7·36405	8
9	.82893	4·84558	.84615	5·49991	.86342	6·32171	.88073	7·38431	9
10	.82922	4·85539	.84644	5·51208	.86371	6·33719	.88102	7·40466	10
11	.82950	4·86524	.84673	5·52429	.86400	6·35274	.88131	7·42511	11
12	.82979	4·87511	.84701	5·53655	.86428	6·36835	.88160	7·44566	12
13	.83008	4·88502	.84730	5·54886	.86457	6·38403	.88188	7·46632	13
14	.83036	4·89497	.84759	5·56121	.86486	6·39978	.88217	7·48707	14
15	.83065	4·90495	.84788	5·57361	.86515	6·41560	.88246	7·50793	15
16	.83094	4·91496	.84816	5·58606	.86544	6·43148	.88275	7·52889	16
17	.83122	4·92501	.84845	5·59855	.86573	6·44743	.88304	7·54996	17
18	.83151	4·93509	.84874	5·61110	.86601	6·46346	.88333	7·57113	18
19	.83180	4·94521	.84903	5·62369	.86630	6·47955	.88362	7·59241	19
20	.83208	4·95536	.84931	5·63633	.86659	6·49571	.88391	7·61379	20
21	.83237	4·96555	.84960	5·64902	.86688	6·51194	.88420	7·63528	21
22	.83266	4·97577	.84989	5·66176	.86717	6·52825	.88448	7·65688	22
23	.83294	4·98603	.85018	5·67454	.86746	6·54462	.88477	7·67859	23
24	.83323	4·99633	.85046	5·68738	.86774	6·56107	.88506	7·70041	24
25	.83352	5·00666	.85075	5·70027	.86803	6·57759	.88535	7·72234	25
26	.83380	5·01703	.85104	5·71321	.86832	6·59418	.88564	7·74438	26
27	.83409	5·02743	.85133	5·72620	.86861	6·61085	.88593	7·76653	27
28	.83438	5·03787	.85162	5·73924	.86890	6·62759	.88622	7·78880	28
29	.83467	5·04834	.85190	5·75233	.86919	6·64441	.88651	7·81118	29
30	.83495	5·05886	.85219	5·76547	.86947	6·66130	.88680	7·83367	30
31	.83524	5·06941	.85248	5·77866	.86976	6·67826	.88709	7·85628	31
32	.83553	5·08000	.85277	5·79191	.87005	6·69530	.88737	7·87901	32
33	.83581	5·09062	.85305	5·80521	.87034	6·71242	.88766	7·90186	33
34	.83610	5·10129	.85334	5·81856	.87063	6·72962	.88795	7·92482	34
35	.83639	5·11199	.85363	5·83196	.87092	6·74689	.88824	7·94791	35
36	.83667	5·12273	.85392	5·84542	.87120	6·76424	.88853	7·97111	36
37	.83696	5·13350	.85420	5·85893	.87149	6·78167	.88882	7·99444	37
38	.83725	5·14432	.85449	5·87250	.87178	6·79918	.88911	8·01788	38
39	.83754	5·15517	.85478	5·88612	.87207	6·81677	.88940	8·04146	39
40	.83782	5·16607	.85507	5·89979	.87236	6·83443	.88969	8·06515	40
41	.83811	5·17700	.85536	5·91352	.87265	6·85218	.88998	8·08897	41
42	.83840	5·18797	.85564	5·92731	.87294	6·87001	.89027	8·11292	42
43	.83868	5·19898	.85593	5·94115	.87322	6·88792	.89055	8·13699	43
44	.83897	5·21004	.85622	5·95505	.87351	6·90592	.89084	8·16120	44
45	.83926	5·22113	.85651	5·96900	.87380	6·92400	.89113	8·18553	45
46	.83954	5·23226	.85680	5·98301	.87409	6·94216	.89142	8·20999	46
47	.83983	5·24343	.85708	5·99708	.87438	6·96040	.89171	8·23459	47
48	.84012	5·25464	.85737	6·01120	.87467	6·97873	.89200	8·25931	48
49	.84041	5·26590	.85766	6·02538	.87496	6·99714	.89229	8·28417	49
50	.84069	5·27719	.85795	6·03962	.87524	7·01565	.89258	8·30917	50
51	.84098	5·28853	.85823	6·05392	.87553	7·03423	.89287	8·33430	51
52	.84127	5·29991	.85852	6·06828	.87582	7·05291	.89316	8·35957	52
53	.84155	5·31133	.85881	6·08269	.87611	7·07167	.89345	8·38497	53
54	.84184	5·32279	.85910	6·09717	.87640	7·09052	.89374	8·41052	54
55	.84213	5·33429	.85939	6·11171	.87669	7·10946	.89403	8·43620	55
56	.84242	5·34584	.85967	6·12630	.87698	7·12849	.89431	8·46203	56
57	.84270	5·35743	.85996	6·14096	.87726	7·14760	.89460	8·48800	57
58	.84299	5·36906	.86025	6·15568	.87755	7·16681	.89489	8·51411	58
59	.84328	5·38073	.86054	6·17046	.87784	7·18612	.89518	8·54037	59
60	.84357	5·39245	.86083	6·18530	.87813	7·20551	.89547	8·56677	60

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	84° Vers.	84° Ex. sec.	85° Vers.	85° Ex. sec.	86° Vers.	86° Ex. sec.	′
0	.89547	8.56677	.91284	10.47371	.93024	13.33559	0
1	.89576	8.59332	.91313	10.51199	.93053	13.39547	1
2	.89605	8.62002	.91342	10.55052	.93082	13.45586	2
3	.89634	8.64687	.91371	10.58932	.93111	13.51676	3
4	.89663	8.67387	.91400	10.62837	.93140	13.57817	4
5	.89692	8.70103	.91429	10.66769	.93169	13.64011	5
6	.89721	8.72833	.91458	10.70728	.93198	13.70258	6
7	.89750	8.75579	.91487	10.74714	.93227	13.76558	7
8	.89779	8.78341	.91516	10.78727	.93257	13.82913	8
9	.89808	8.81119	.91545	10.82768	.93286	13.89323	9
10	.89836	8.83912	.91574	10.86837	.93315	13.95788	10
11	.89865	8.86722	.91603	10.90934	.93344	14.02310	11
12	.89894	8.89547	.91632	10.95060	.93373	14.08890	12
13	.89923	8.92389	.91661	10.99214	.93402	14.15527	13
14	.89952	8.95248	.91690	11.03397	.93431	14.22223	14
15	.89981	8.98123	.91719	1.07610	.93460	14.28979	15
16	.90010	9.01015	.91748	11.11852	.93489	14.35795	16
17	.90039	9.03923	.91777	11.16125	.93518	14.42672	17
18	.90068	9.06849	.91806	11.20427	.93547	14.49611	18
19	.90097	9.09792	.91835	11.24761	.93576	14.56614	19
20	.90126	9.12752	.91864	11.29125	.93605	14.63679	20
21	.90155	9.15730	.91893	11.33521	.93634	14.70810	21
22	.90184	9.18725	.91922	11.37948	.93663	14.78005	22
23	.90213	9.21739	.91951	11.42408	.93692	14.85268	23
24	.90242	9.24770	.91980	11.46900	.93721	14.92597	24
25	.90271	9.27819	.92009	11.51424	.93750	14.99995	25
26	.90300	9.30887	.92038	11.55982	.93779	15.07462	26
27	.90329	9.33973	.92067	11.60572	.93808	15.14999	27
28	.90358	9.37077	.92096	11.65197	.93837	15.22607	28
29	.90386	9.40201	.92125	11.69856	.93866	15.30287	29
30	.90415	9.43343	.92154	11.74550	.93895	15.38041	30
31	.90444	9.46505	.92183	11.79278	.93924	15.45869	31
32	.90473	9.49685	.92212	11.84042	.93953	15.53772	32
33	.90502	9.52886	.92241	11.88841	.93982	15.61751	33
34	.90531	9.56106	.92270	11.93677	.94011	15.69808	34
35	.90560	9.59346	.92299	11.98549	.94040	15.77944	35
36	.90589	9.62605	.92328	12.03458	.94069	15.86159	36
37	.90618	9.65885	.92357	12.08404	.94098	15.94456	37
38	.90647	9.69186	.92386	12.13388	.94127	16.02835	38
39	.90676	9.72507	.92415	12.18411	.94156	16.11297	39
40	.90705	9.75849	.92444	12.23472	.94186	16.19843	40
41	.90734	9.79212	.92473	12.28572	.94215	16.28476	41
42	.90763	9.82596	.92502	12.33712	.94244	16.37196	42
43	.90792	9.86001	.92531	12.38891	.94273	16.46005	43
44	.90821	9.89428	.92560	12.44112	.94302	16.54903	44
45	.90850	9.92877	.92589	12.49373	.94331	16.63893	45
46	.90879	9.96348	.92618	12.54676	.94360	16.72975	46
47	.90908	9.99841	.92647	12.60021	.94389	16.82152	47
48	.90937	10.03356	.92676	12.65408	.94418	16.91424	48
49	.90966	10.06894	.92705	12.70838	.94447	17.00794	49
50	.90995	10.10455	.92734	12.76312	.94476	17.10262	50
51	.91024	10.14039	.92763	12.81829	.94505	17.19830	51
52	.91053	10.17646	.92792	12.87391	.94534	17.29501	52
53	.91082	10.21277	.92821	12.92999	.94563	17.39274	53
54	.91111	10.24932	.92850	12.98651	.94592	17.49153	54
55	.91140	10.28610	.92879	13.04350	.94621	17.59139	55
56	.91169	10.32313	.92908	13.10096	.94650	17.69233	56
57	.91197	10.36040	.92937	13.15889	.94679	17.79438	57
58	.91226	10.39792	.92966	13.21730	.94708	17.89755	58
59	.91255	10.43569	.92995	13.27620	.94737	18.00185	59
60	.91284	10.47371	.93024	13.33559	.94766	18.10732	60

TABLE X.—NATURAL VERSED SINES AND EXTERNAL S

	87°		88°		89°	
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. s
0	.94766	18.10732	.96510	27.65371	.98255	56.29
1	.94795	18.21397	.96539	27.89440	.98284	57.26
2	.94825	18.32182	.96568	28.13917	.98313	58.27
3	.94854	18.43088	.96597	28.38812	.98342	59.31
4	.94883	18.54119	.96626	28.64137	.98371	60.39
5	.94912	18.65275	.96655	28.89903	.98400	61.50
6	.94941	18.76560	.96684	29.16120	.98429	62.66
7	.94970	18.87976	.96714	29.42802	.98458	63.86
8	.94999	18.99524	.96743	29.69960	.98487	65.11
9	.95028	19.11208	.96772	29.97607	.98517	66.40
10	.95057	19.23028	.96801	30.25758	.98546	67.75
11	.95086	19.34989	.96830	30.54425	.98575	69.16
12	.95115	19.47093	.96859	30.83623	.98604	70.62
13	.95144	19 59341	.96888	31.13366	.98633	72.14
14	.95173	19.71737	.96917	31.43671	.98662	73.73
15	.95202	19.84283	.96946	31.74554	.98691	75.39
16	.95231	19.96982	.96975	32.06030	.98720	77.13
17	.95260	20.09838	.97004	32.38118	.98749	78.94
18	.95289	20.22852	.97033	32.70835	.98778	80.85
19	.95318	20.36027	.97062	33.04199	.98807	82.84
20	.95347	20.49368	.97092	33.38232	.98836	84.94
21	.95377	20.62876	.97121	33.72952	.98866	87.14
22	.95406	20.76555	.97150	34.08380	.98895	89.46
23	.95435	20.90409	.97179	34.44539	.98924	91.91
24	.95464	21.04440	.97208	34.81452	.98953	94.49
25	.95493	21.18653	.97237	35.19141	.98982	97.22
26	.95522	21.33050	.97266	35.57633	.99011	100.1
27	.95551	21.47635	.97295	35.96953	.99040	103.1
28	.95580	21.62413	.97324	36.37127	.99069	106.4
29	.95609	21.77386	.97353	36.78185	.99098	109.8
30	.95638	21.92559	.97382	37.20155	.99127	113.5
31	.95667	22.07935	.97411	37.63068	.99156	117.5
32	.95696	22.23520	.97440	38.06957	.99186	121.7
33	.95725	22.39316	.97470	38.51855	.99215	126.3
34	.95754	22.55328	.97499	38.97797	.99244	131.2
35	.95783	22.71563	.97528	39.44829	.99278	136.5
36	.95812	22.88022	.97557	39.92983	.99302	142.2
37	.95842	23.04712	.97586	40.42266	.99331	148.4
38	.95871	23.21637	.97615	40.92772	.99360	155.2
39	.95900	23 38802	.97644	41.44525	.99389	162.7
40	.95929	23.56212	.97673	41.97571	.99418	170.8
41	.95958	23.73873	.97702	42.51961	.99447	179.9
42	.95987	23.91790	.97731	43.07746	.99476	189.9
43	.96016	24.09969	.97760	43.64980	.99505	201.2
44	.96045	24.28414	.97789	44.23720	.99535	213.8
45	.96074	24.47134	.97819	44.84026	.99564	228.1
46	.96103	24.66132	.97848	45.45963	.99593	244.5
47	.96132	24.85417	.97877	46.09596	.99622	263.4
48	.96161	25.04994	.97906	46.74997	.99651	285.4
49	.96190	25.24869	.97935	47.42241	.99680	311.5
50	.96219	25.45051	.97964	48.11406	.99709	342.7
51	.96248	25.65546	.97993	48.82576	.99738	380.9
52	.96277	25.86360	.98022	49.55840	.99767	428.7
53	.96307	26.07503	.98051	50.31290	.99796	490.1
54	.96336	26.28981	.98080	51.09027	.99825	571.9
55	.96365	26.50804	.98109	51.89156	.99855	686.5
56	.96394	26.72978	.98138	52.71790	.99884	858.4
57	.96423	26.95513	.98168	53.57046	.99913	1144.9
58	.96452	27.18417	.98197	54.45053	.99942	1717.8
59	.96481	27.41700	.98226	55.35946	.99971	3438.7
60	.96510	27.65371	.98255	56.29869	1.00000	Infini

TABLE XI.—REDUCTION OF BAROMETER READING TO 32° F.

Temp. ° Fahr.	Inches.										
	26·0	26·5	27·0	27·5	28·0	28·5	29·0	29·5	30·0	30·5	31·0
45	−.039	−.039	−.040	−.041	−.042	−.042	−.043	−.044	−.045	−.045	−.046
46	.041	.042	.043	.043	.044	.045	.046	.046	.047	.048	.049
47	.043	.044	.045	.046	.047	.048	.048	.049	.050	.051	.052
48	.046	.047	.047	.048	.049	.050	.051	.052	.053	.053	.054
49	.048	.049	.050	.051	.052	.052	.054	.054	.055	.056	.057
50	.050	.051	.052	.053	.054	.055	.056	.057	.058	.059	.060
51	.053	.054	.055	.056	.057	.058	.059	.060	.061	.062	.063
52	.055	.056	.057	.058	.059	.060	.061	.062	.064	.065	.066
53	.057	.058	.060	.061	.062	.063	.064	.065	.066	.067	.068
54	.060	.061	.062	.063	.064	.065	.067	.068	.069	.070	.071
55	.062	.063	.064	.065	.066	.068	.069	.070	.071	.073	.074
56	.064	.065	.067	.068	.069	.070	.072	.073	.074	.075	.077
57	.067	.068	.069	.070	.072	.073	.075	.076	.077	.078	.080
58	.069	.070	.071	.073	.074	.076	.077	.078	.080	.081	.082
59	.072	.073	.074	.075	.077	.078	.080	.081	.083	.084	.085
60	.074	.076	.077	.078	.079	.081	.082	.084	.085	.086	.088
61	.076	.077	.079	.080	.082	.083	.085	.086	.088	.089	.091
62	.079	.080	.082	.083	.085	.086	.088	.089	.091	.092	.094
63	.081	.082	.084	.085	.087	.088	.090	.091	.093	.095	.096
64	.083	.085	.086	.088	.090	.091	.093	.094	.096	.097	.099
65	.086	.087	.089	.090	.092	.093	.095	.097	.099	.100	.102
66	.088	.089	.091	.093	.095	.096	.098	.099	.101	.103	.105
67	.090	.092	.094	.095	.097	.099	.101	.102	.104	.106	.108
68	.093	.094	.096	.098	.100	.101	.103	.105	.107	.108	.110
69	.095	.097	.099	.100	.102	.104	.106	.107	.110	.111	.113
70	.097	.099	.101	.103	.105	.106	.109	.110	.112	.114	.116
71	.100	.101	.103	.105	.107	.109	.111	.113	.115	.117	.119
72	.102	.104	.106	.108	.110	.112	.114	.116	.118	.120	.122
73	.104	.106	.108	.110	.112	.114	.116	.118	.120	.122	.124
74	.107	.109	.111	.113	.115	.117	.119	.121	.123	.125	.127
75	.109	.111	.113	.115	.117	.119	.122	.124	.126	.128	.130
76	.111	.113	.116	.118	.120	.122	.124	.126	.128	.130	.133
77	.114	.116	.118	.120	.122	.124	.127	.129	.131	.133	.136
78	.116	.118	.120	.122	.125	.127	.129	.131	.134	.136	.138
79	.118	.120	.123	.125	.127	.129	.132	.134	.137	.139	.141
80	.121	.123	.125	.127	.130	.132	.135	.137	.139	.141	.144
81	.123	.125	.128	.130	.132	.134	.137	.139	.142	.144	.147
82	.125	.128	.130	.132	.135	.137	.140	.142	.145	.147	.149
83	.128	.130	.133	.135	.138	.140	.142	.145	.147	.149	.152
84	.130	.132	.135	.138	.140	.142	.145	.147	.150	.152	.155
85	.132	.134	.137	.140	.143	.145	.148	.150	.153	.155	.158
86	.135	.137	.140	.142	.145	.148	.150	.153	.155	.158	.161
87	.137	.139	.142	.144	.148	.150	.153	.155	.158	.161	.163
88	.139	.142	.145	.147	.150	.152	.155	.158	.161	.163	.166
89	.142	.144	.147	.150	.153	.155	.158	.161	.164	.166	.169
90	.144	.147	.150	.153	.155	.158	.161	.164	.166	.169	.172
91	−.146	−.149	−.152	−.155	−.158	−.160	−.163	−.166	−.169	−.172	−.175

TABLE XII.—BAROMETRIC ELEVATIONS.*

B	*A*	Diff. for .01.	*B*	*A*	Diff. for .01.	*B*	*A*
Inches.	Feet.	Feet.	Inches.	Feet.	Feet.	Inches.	Feet.
20.0	11.047		23.7	6.423		27.4	2.470
20.1	10.911	−13.6	23.8	6.308	−11.5	27.5	2.371
20.2	10.776	13.5	23.9	6.194	11.4	27.6	2.272
20.3	10.642	13.4	24.0	6.080	11.4	27.7	2.173
20.4	10.508	13.4	24.1	5.967	11.3	27.8	2.075
20.5	10.375	13.3	24.2	5.854	11.3	27.9	1.977
20.6	10.242	13.3	24.3	5.741	11.3	28.0	1.880
20.7	10.110	13.2	24.4	5.629	11.2	28.1	1.783
20.8	9.979	13.1	24.5	5.518	11.1	28.2	1 686
20.9	9.848	13.1	24.6	5.407	11.1	28.3	1.589
21.0	9.718	13.0	24.7	5.296	11.1	28.4	1.493
21.1	9.589	12.9	24.8	5.186	11.0	28.5	1.397
21.2	9.460	12.9	24.9	5.077	10.9	28.6	1.302
21.3	9.332	12.8	25.0	4.968	10.9	28.7	1.207
21.4	9.204	12.8	25.1	4.859	10.9	28.8	1.112
21.5	9.077	12.7	25.2	4.751	10.8	28.9	1.018
21.6	8 951	12.6	25.3	4.643	10.8	29.0	924
21.7	8.825	12.6	25.4	4.535	10.8	29.1	830
21.8	8.700	12.5	25.5	4.428	10.7	29.2	736
21.9	8.575	12.5	25.6	4.321	10.7	29.3	643
22.0	8.451	12.4	25.7	4.215	10.6	29.4	550
22.1	8.327	12.4	25.8	4.109	10.6	29.5	458
22.2	8.204	12.3	25.9	4.004	10.5	29.6	366
22.3	8.082	12.2	26.0	3 899	10.5	29.7	274
22.4	7.960	12.2	26.1	3.794	10.4	29.8	182
22.5	7.838	12.2	26.2	3 690	10.4	29.9	91
22.6	7.717	12.1	26.3	3.586	10.4	30.0	0
22.7	7.597	12.0	26.4	3.483	10.3	30.1	−91
22.8	7.477	12.0	26.5	3.380	10.3	30.2	181
22.9	7 358	11.9	26.6	3 277	10.3	30.3	271
23.0	7.239	11.9	26.7	3.175	10.2	30.4	361
23.1	7.121	11.8	26.8	3.073	10.2	30.5	451
23.2	7.004	11.7	26.9	2 972	10.1	30.6	540
23.3	6.887	11.7	27.0	2.871	10.1	30.7	629
23.4	6.770	11.7	27.1	2 770	10 1	30.8	717
23.5	6.554	11.6	27.2	2.670	10.0	30.9	805
23.6	6.538	11.6	27.3	2.570	10.0	31 0	−893
23.7	6.423	−11.5	27.4	2.470	−10 0		

* Compiled from Report of U. S. C. & G. Survey for 1881. App. 10 Ta

TABLE XIII.—COEFFICIENTS FOR CORRECTIONS FOR TEMPER AND HUMIDITY.*

$t+t'$	*C*	Diff. for 1°.	$t+t'$	*C*	Diff. for 1°.	$t+t'$	*C*
0°	−.1024	10.9	60°	−.0380	10.7	120°	+.0262
10	.0915	10.9	70	.0273	10.7	130	.0368
20	.0806	10.8	80	.0166	10.8	140	.0472
30	.0698	10.6	90	−.0058	10.7	150	.0575
40	.0592	10.6	100	+.0049	10.7	160	.0677
50	.0486	10.6	110	.0156	10.7	170	.0779
60	−.0380		120	+.0262	10.6	180	+.0879

* Compiled from Report of U. S. C. & G. Survey for 1881, App. 10, Table

$$\sin a = \frac{1}{\operatorname{cosec} a} = \frac{\tan a}{\sqrt{1+\tan^2 a}} = \sqrt{\frac{1-\cos 2a}{2}} = \frac{1}{\sqrt{1+\cot^2 a}}$$

$$= \cos a \tan a = \sqrt{1-\cos^2 a} = 2 \sin \tfrac{1}{2}a \cos \tfrac{1}{2}a$$

$$= \frac{1+\cos a}{\cot \frac{1}{2}a} = \frac{2 \tan \frac{1}{2}a}{1+\tan^2 \frac{1}{2}a} = \operatorname{vers} a \cot \tfrac{1}{2}a.$$

$$\cos a = \frac{1}{\sec a} = \frac{\cot a}{\sqrt{1+\cot^2 a}} = \frac{1}{\sqrt{1+\tan^2 a}}$$

$$= 1 - \operatorname{vers} a = \sin a \cot a = \sqrt{1-\sin^2 a} = 2 \cos^2 \tfrac{1}{2}a - 1$$

$$= \sin a \cot \tfrac{1}{2}a - 1 = \cos^2 \tfrac{1}{2}a - \sin^2 \tfrac{1}{2}a = 1 - 2 \sin^2 \tfrac{1}{2}a.$$

$$\tan a = \frac{1}{\cot a} = \frac{\sin a}{\cos a} = \frac{\sec a}{\operatorname{cosec} a} = \frac{1}{\sqrt{\operatorname{cosec}^2 a - 1}}$$

$$= \operatorname{vers} 2a \operatorname{cosec} 2a = \cot a - 2 \cot 2a = \sin a \sec a$$

$$= \frac{\sin 2a}{1+\cos 2a} = \operatorname{exsec} a \cot \tfrac{1}{2}a = \operatorname{exsec} 2a \cot 2a.$$

$$\cot a = \frac{1}{\tan a} = \frac{\cos a}{\sin a} = \frac{\sin 2a}{1-\cos 2a} = \frac{1+\cos 2a}{\sin 2a}$$

$$= \sqrt{\operatorname{cosec}^2 a - 1} = \cot \tfrac{1}{2}a - \operatorname{cosec} a.$$

$$\operatorname{vers} a = 1 - \cos a = \sin a \tan \tfrac{1}{2}a = 2 \sin^2 \tfrac{1}{2}a = \cos a \operatorname{exsec} a.$$

$$\operatorname{exsec} a = \sec a - 1 = \tan a \tan \tfrac{1}{2}a = \operatorname{vers} a \sec a.$$

$$\sin \tfrac{1}{2}a = \sqrt{\frac{\operatorname{vers} a}{2}} = \frac{\sin a}{2 \cos \frac{1}{2}a} = \frac{\operatorname{vers} a \cos \frac{1}{2}a}{\sin a}.$$

$$\cos \tfrac{1}{2}a = \sqrt{\frac{1+\cos a}{2}} = \frac{\sin a}{2 \sin \frac{1}{2}a} = \frac{\sin a \sin \frac{1}{2}a}{\operatorname{vers} a}.$$

$$\tan \tfrac{1}{2}a = \operatorname{vers} a \operatorname{cosec} a = \operatorname{cosec} a - \cot a = \frac{\tan a}{1+\sec a}.$$

$$\cot \tfrac{1}{2}a = \frac{1+\cos a}{\sin a} = \operatorname{cosec} a + \cot a = \frac{\tan a}{\operatorname{exsec} a} = \frac{1}{\operatorname{cosec} a - \cot a}.$$

$$\operatorname{vers} \tfrac{1}{2}a = 1 - \sqrt{\tfrac{1}{2}(1+\cos a)}.$$

$$\operatorname{exsec} \tfrac{1}{2}a = \frac{1}{\sqrt{\frac{1}{2}(1+\cos a)}} - 1.$$

13 $\sin 2a = 2 \sin a \cos a = \dfrac{2 \tan a}{1 + \tan^2 a}$.

14 $\cos 2a = \cos^2 a - \sin^2 a = 1 - 2 \sin^2 a = 2 \cos^2 a - 1$

$= \dfrac{1 - \tan^2 a}{1 + \tan^2 a}$.

15 $\tan 2a = \dfrac{2 \tan a}{1 - \tan^2 a}$.

16 $\cot 2a = \frac{1}{2} \cot a - \frac{1}{2} \tan a = \dfrac{\cot^2 a - 1}{2 \cot a} = \dfrac{1 - \tan^2 a}{2 \tan a}$.

17 $\text{vers } 2a = 2 \sin^2 a = 1 - \cos 2a = 2 \sin a \cos a \tan a$.

18 $\text{exsec } 2a = \dfrac{\tan 2a}{\cot a} = \dfrac{2 \tan^2 a}{1 - \tan^2 a} = \dfrac{2 \sin^2 a}{1 - 2 \sin^2 a}$.

19 $\sin (a \pm b) = \sin a \cos b \pm \cos a \sin b$.

20 $\cos (a \pm b) = \cos a \cos b \mp \sin a \sin b$.

21 $\sin a + \sin b = 2 \sin \frac{1}{2}(a+b) \cos \frac{1}{2}(a-b)$.

22 $\sin a - \sin b = 2 \sin \frac{1}{2}(a-b) \cos \frac{1}{2}(a+b)$.

23 $\cos a + \cos b = 2 \cos \frac{1}{2}(a+b) \cos \frac{1}{2}(a-b)$.

24 $\cos a - \cos b = -2 \sin \frac{1}{2}(a+b) \sin \frac{1}{2}(a-b)$.

Call the sides of any triangle A, B, C, and the opposite angles a, b, and c. Call $s = \frac{1}{2}(A + B + C)$.

25 $\tan \frac{1}{2}(a-b) = \dfrac{A-B}{A+B} \tan \frac{1}{2}(a+b) = \dfrac{A-B}{A+B} \cot \frac{1}{2}c$.

26 $C = (A+B) \dfrac{\cos \frac{1}{2}(a+b)}{\cos \frac{1}{2}(a-b)} = (A-B) \dfrac{\sin \frac{1}{2}(a+b)}{\sin \frac{1}{2}(a-b)}$.

27 $\sin \frac{1}{2}a = \sqrt{\dfrac{(s-B)(s-C)}{BC}}$.

28 $\cos \frac{1}{2}a = \sqrt{\dfrac{s(s-A)}{BC}}$.

29 $\text{vers } a = \dfrac{2(s-B)(s-C)}{BC}$.

30 $\text{Area} = \sqrt{s(s-A)(s-B)(s-C)} = A^2 \dfrac{\sin b \sin c}{2 \sin a}$.

TABLE XV.—USEFUL FORMULÆ AND CONSTANTS.

		Logarithm.
rence of a circle (radius $= r$)	$= 2\pi r$.	
circle	$= \pi r^2$.	
ector (length of arc $= l$)	$= \frac{1}{2}lr$.	
" (angle of arc $= \alpha°$)	$= \frac{\alpha}{360}\pi r^2$.	
egment (chord $= c$, mid. ord. $= m$)	$= \frac{2}{3}cm$ (approx.).	
of a cone or pyramid	= area of base $\times \frac{1}{3}$ height.	
circle to radius 1 rence of a circle to diameter 1 of a sphere to diameter 1	$= \pi =$ 3.1415927	0.497 1499
of a sphere to radius $1 = 4\pi \div 3$	= 4.1887902	0.622 0886
l to radius expressed in degrees	= 57.2957795	1.758 1226
l to radius expressed in minutes	= 3437.7467708	3.536 2739
l to radius expressed in seconds	= 206264.8062471	5.314 4251
f arc of 1°, radius unity	0.01745329	8.241 8774
ne second	= 0.0000048481	4.685 5749
of one cubic foot of water at maximum density (therm. ., barom. 30″)	62.379	1.795 0384
of one cubic foot of water at ordinary temperature (therm.)	62.321	1.794 6349
ion due to gravity at latitude of New York in feet per second	32.15945	1.507 3086
U. S. standard) $= \frac{3600}{3937}$ meter	= 0.914402 m.	9.961 1371
	= 0.304801 m.	9.484 0158
	= 0.025400 m.	8.404 8346
	= 3.28083 feet	0.515 9842
	= 39.3700 inches	1.595 1654
(avoirdupois)	= 0.453592 kilogr.	9.656 6659
m	= 2.20462 pounds	1.343 3341
(U. S. standard)	= 2150.420 cu. in.	
	= 1.244 cu. ft.	
(U. S. standard)	= 231. cu. in.	
	= 0.1337 cu. ft.	

TABLE XVI.—SQUARES, CUBES, SQUARE ROOTS,

No.	Squares.	Cubes.	Square Roots.	Cube Roots.
1	1	1	1.0000000	1.0000000
2	4	8	1.4142136	1.2599210
3	9	27	1.7320508	1.4422496
4	16	64	2.0000000	1.5874011
5	25	125	2.2360680	1.7099759
6	36	216	2.4494897	1.8171206
7	49	343	2.6457513	1.9129312
8	64	512	2.8284271	2.0000000
9	81	729	3.0000000	2.0800837
10	100	1000	3.1622777	2.1544347
11	121	1331	3.3166248	2.2239801
12	144	1728	3.4641016	2.2894286
13	169	2197	3.6055513	2.3513347
14	196	2744	3.7416574	2.4101422
15	225	3375	3.8729833	2.4662121
16	256	4096	4.0000000	2.5198421
17	289	4913	4.1231056	2.5712816
18	324	5832	4.2426407	2.6207414
19	361	6859	4.3588989	2.6684016
20	400	8000	4.4721360	2.7144177
21	441	9261	4.5825757	2.7589243
22	484	10648	4.6904158	2.8020393
23	529	12167	4.7958315	2.8438670
24	576	13824	4.8989795	2.8844991
25	625	15625	5.0000000	2.9240177
26	676	17576	5.0990195	2.9624960
27	729	19683	5.1961524	3.0000000
28	784	21952	5.2915026	3.0365889
29	841	24389	5.3851648	3.0723168
30	900	27000	5.4772256	3.1072325
31	961	29791	5.5677644	3.1413806
32	1024	32768	5.6568542	3.1748021
33	1089	35937	5.7445626	3.2075343
34	1156	39304	5.8309519	3.2396118
35	1225	42875	5.9160798	3.2710663
36	1296	46656	6.0000000	3.3019272
37	1369	50653	6.0827625	3.3322218
38	1444	54872	6.1644140	3.3619754
39	1521	59319	6.2449980	3.3912114
40	1600	64000	6.3245553	3.4199519
41	1681	68921	6.4031242	3.4482172
42	1764	74088	6.4807407	3.4760266
43	1849	79507	6.5574385	3.5033981
44	1936	85184	6.6332496	3.5303483
45	2025	91125	6.7082039	3.5568933
46	2116	97336	6.7823300	3.5830479
47	2209	103823	6.8556546	3.6088261
48	2304	110592	6.9282032	3.6342411
49	2401	117649	7.0000000	3.6593057
50	2500	125000	7.0710678	3.6840314
51	2601	132651	7.1414284	3.7084298
52	2704	140608	7.2111026	3.7325111
53	2809	148877	7.2801099	3.7562858
54	2916	157464	7.3484692	3.7797631
55	3025	166375	7.4161985	3.8029525
56	3136	175616	7.4833148	3.8258624
57	3249	185193	7.5498344	3.8485011
58	3364	195112	7.6157731	3.8708766
59	3481	205379	7.6811457	3.8929965
60	3600	216000	7.7459667	3.9148676

s.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
	226981	7.8102497	3.9364972	.016393443
	238328	7.8740079	3.9578915	.016129032
	250047	7.9372539	3.9790571	.015873016
	262144	8.0000000	4.0000000	.015625000
	274625	8.0622577	4.0207256	.015384615
	287496	8.1240384	4.0412401	.015151515
	300763	8.1853528	4.0615480	.014925373
	314432	8.2462113	4.0816551	.014705882
	328509	8.3066239	4.1015661	.014492754
	343000	8.3666003	4.1212853	.014285714
	357911	8.4261498	4.1408178	.014084507
	373248	8.4852814	4.1601676	.013888889
	389017	8.5440037	4.1793390	.013698630
	405224	8.6023253	4.1983364	.013513514
	421875	8.6602540	4.2171633	.013333333
	438976	8.7177979	4.2358236	.013157895
	456533	8.7749644	4.2543210	.012987013
	474552	8.8317609	4.2726586	.012820513
	493039	8.8881944	4.2908404	.012658228
	512000	8.9442719	4.3088695	.012500000
	531441	9.0000000	4.3267487	.012345679
	551368	9.0553851	4.3444815	.012195122
	571787	9.1104336	4.3620707	.012048193
	592704	9.1651514	4.3795191	.011904762
	614125	9.2195445	4.3968296	.011764706
	636056	9.2736185	4.4140049	.011627907
	658503	9.3273791	4.4310476	.011494253
	681472	9.3808315	4.4479602	.011363636
	704969	9.4339811	4.4647451	.011235955
	729000	9.4868330	4.4814047	.011111111
	753571	9.5393920	4.4979414	.010989011
	778688	9.5916630	4.5143574	.010869565
	804357	9.6436508	4.5306549	.010752688
	830584	9.6953597	4.5468359	.010638298
	857375	9.7467943	4.5629026	.010526316
	884736	9.7979590	4.5788570	.010416667
	912673	9.8488578	4.5947009	.010309278
	941192	9.8994949	4.6104363	.010204082
	970299	9.9498744	4.6260650	.010101010
	1000000	10.0000000	4.6415888	.010000000
	1030301	10.0498756	4.6570095	.009900990
	1061208	10.0995049	4.6723287	.009803922
	1092727	10.1488916	4.6875482	.009708738
	1124864	10.1980390	4.7026694	.009615385
	1157625	10.2469508	4.7176940	.009523810
	1191016	10.2956301	4.7326235	.009433962
	1225043	10.3440804	4.7474594	.009345794
	1259712	10.3923048	4.7622032	.009259259
	1295029	10.4403065	4.7768562	.009174312
	1331000	10.4880885	4.7914199	.009090909
	1367631	10.5356538	4.8058955	.009009009
	1404928	10.5830052	4.8202845	.008928571
	1442897	10.6301458	4.8345881	.008849558
	1481544	10.6770783	4.8488076	.008771930
	1520875	10.7238053	4.8629442	.008695652
	1560896	10.7703296	4.8769990	.008620690
	1601613	10.8166538	4.8909732	.008547009
	1643032	10.8627805	4.9048681	.008474576
	1685159	10.9087121	4.9186847	.008403361
	1728000	10.9544512	4.9324242	.008333333

TABLE XVI.—SQUARES, CUBES, SQUARE ROOTS,

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
1	1	1	1.0000000	1.0000000	1.000000000
2	4	8	1.4142136	1.2599210	.500000000
3	9	27	1.7320508	1.4422496	.333333333
4	16	64	2.0000000	1.5874011	.250000000
5	25	125	2.2360680	1.7099759	.200000000
6	36	216	2.4494897	1.8171206	.166666667
7	49	343	2.6457513	1.9129312	.142857143
8	64	512	2.8284271	2.0000000	.125000000
9	81	729	3.0000000	2.0800837	.111111111
10	100	1000	3.1622777	2.1544347	.100000000
11	121	1331	3.3166248	2.2239801	.090909091
12	144	1728	3.4641016	2.2894286	.083333333
13	169	2197	3.6055513	2.3513347	.076923077
14	196	2744	3.7416574	2.4101422	.071428571
15	225	3375	3.8729833	2.4662121	.066666667
16	256	4096	4.0000000	2.5198421	.062500000
17	289	4913	4.1231056	2.5712816	.058823529
18	324	5832	4.2426407	2.6207414	.055555556
19	361	6859	4.3588989	2.6684016	.052631579
20	400	8000	4.4721360	2.7144177	.050000000
21	441	9261	4.5825757	2.7589243	.047619048
22	484	10648	4.6904158	2.8020393	.045454545
23	529	12167	4.7958315	2.8438670	.043478261
24	576	13824	4.8989795	2.8844991	.041666667
25	625	15625	5.0000000	2.9240177	.040000000
26	676	17576	5.0990195	2.9624960	.038461538
27	729	19683	5.1961524	3.0000000	.037037037
28	784	21952	5.2915026	3.0365889	.035714286
29	841	24389	5.3851648	3.0723168	.034482759
30	900	27000	5.4772256	3.1072325	.033333333
31	961	29791	5.5677644	3.1413806	.032258065
32	1024	32768	5.6568542	3.1748021	.031250000
33	1089	35937	5.7445626	3.2075343	.030303030
34	1156	39304	5.8309519	3.2396118	.029411765
35	1225	42875	5.9160798	3.2710663	.028571429
36	1296	46656	6.0000000	3.3019272	.027777778
37	1369	50653	6.0827625	3.3322218	.027027027
38	1444	54872	6.1644140	3.3619754	.026315789
39	1521	59319	6.2449980	3.3912114	.025641026
40	1600	64000	6.3245553	3.4199519	.025000000
41	1681	68921	6.4031242	3.4482172	.024390244
42	1764	74088	6.4807407	3.4760266	.023809524
43	1849	79507	6.5574385	3.5033981	.023255814
44	1936	85184	6.6332496	3.5303483	.022727273
45	2025	91125	6.7082039	3.5568933	.022222222
46	2116	97336	6.7823300	3.5830479	.021739130
47	2209	103823	6.8556546	3.6088261	.021276600
48	2304	110592	6.9282032	3.6342411	.020833333
49	2401	117649	7.0000000	3.6593057	.020408163
50	2500	125000	7.0710678	3.6840314	.020000000
51	2601	132651	7.1414284	3.7084298	.019607843
52	2704	140608	7.2111026	3.7325111	.019230769
53	2809	148877	7.2801099	3.7562858	.018867925
54	2916	157464	7.3484692	3.7797631	.018518519
55	3025	166375	7.4161985	3.8029525	.018181818
56	3136	175616	7.4833148	3.8258624	.017857143
57	3249	185193	7.5498344	3.8485011	.017543860
58	3364	195112	7.6157731	3.8708766	.017241379
59	3481	205379	7.6811457	3.8929965	.016949153
60	3600	216000	7.7459667	3.9148676	.016666667

Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
32761	5929741	13.4536240	5.6566528	.005524862
33124	6028568	13.4907376	5.6670511	.005494505
33489	6128487	13.5277493	5.6774114	.005464481
33856	6229504	13.5646600	5.6877340	.005434783
34225	6331625	13.6014705	5.6980192	.005405405
34596	6434856	13.6381817	5.7082675	.005376344
34969	6539203	13.6747943	5.7184791	.005347594
35344	6644672	13.7113092	5.7286543	.005319149
35721	6751269	13.7477271	5.7387936	.005291005
36100	6859000	13.7840488	5.7488971	.005263158
36481	6967871	13.8202750	5.7589652	.005235602
36864	7077888	13.8564065	5.7689982	.005208333
37249	7189057	13.8924440	5.7789966	.005181347
37636	7301384	13.9283883	5.7889604	.005154639
38025	7414875	13.9642400	5.7988900	.005128205
38416	7529536	14.0000000	5.8087857	.005102041
38809	7645373	14.0356688	5.8186479	.005076142
39204	7762392	14.0712473	5.8284767	.005050505
39601	7880599	14.1067360	5.8382725	.005025126
40000	8000000	14.1421356	5.8480355	.005000000
40401	8120601	14.1774469	5.8577660	.004975124
40804	8242408	14.2126704	5.8674643	.004950495
41209	8365427	14.2478068	5.8771307	.004926108
41616	8489664	14.2828569	5.8867653	.004901961
42025	8615125	14.3178211	5.8963685	.004878049
42436	8741816	14.3527001	5.9059406	.004854369
42849	8869743	14.3874946	5.9154817	.004830918
43264	8998912	14.4222051	5.9249921	.004807692
43681	9129329	14.4568323	5.9344721	.004784689
44100	9261000	14.4913767	5.9439220	.004761905
44521	9393931	14.5258390	5.9533418	.004739336
44944	9528128	14.5602198	5.9627320	.004716981
45369	9663597	14.5945195	5.9720926	.004694836
45796	9800344	14.6287388	5.9814240	.004672897
46225	9938375	14.6628783	5.9907264	.004651163
46656	10077696	14.6969385	6.0000000	.004629630
47089	10218313	14.7309199	6.0092450	.004608295
47524	10360232	14.7648231	6.0184617	.004587156
47961	10503459	14.7986486	6.0276502	.004566210
48400	10648000	14.8323970	6.0368107	.004545455
48841	10793861	14.8660687	6.0459435	.004524887
49284	10941048	14.8996644	6.0550489	.004504505
49729	11089567	14.9331845	6.0641270	.004484305
50176	11239424	14.9666295	6.0731779	.004464286
50625	11390625	15.0000000	6.0822020	.004444444
51076	11543176	15.0332964	6.0911994	.004424779
51529	11697083	15.0665192	6.1001702	.004405286
51984	11852352	15.0996689	6.1091147	.004385965
52441	12008989	15.1327460	6.1180332	.004366812
52900	12167000	15.1657509	6.1269257	.004347826
53361	12326391	15.1986842	6.1357924	.004329004
53824	12487168	15.2315462	6.1446337	.004310345
54289	12649337	15.2643375	6.1534495	.004291845
54756	12812904	15.2970585	6.1622401	.004273504
55225	12977875	15.3297097	6.1710058	.004255319
55696	13144256	15.3622915	6.1797466	.004237288
56169	13312053	15.3948043	6.1884628	.004219409
56644	13481272	15.4272486	6.1971544	.004201681
57121	13651919	15.4596248	6.2058218	.004184100
57600	13824000	15.4919334	6.2144650	.004166667

TABLE XVI.—SQUARES, CUBES, SQUARE ROOTS,

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals
241	58081	13997521	15.5241747	6.2230843	
242	58564	14172488	15.5563492	6.2316797	
243	59049	14348907	15.5884573	6.2402515	
244	59536	14526784	15.6204994	6.2487998	
245	60025	14706125	15.6524758	6.2573248	
246	60516	14886936	15.6843871	6.2658266	
247	61009	15069223	15.7162336	6.2743054	
248	61504	15252992	15.7480157	6.2827613	
249	62001	15438249	15.7797338	6.2911946	
250	62500	15625000	15.8113883	6.2996053	
251	63001	15813251	15.8429795	6.3079935	
252	63504	16003008	15.8745079	6.3163596	
253	64009	16194277	15.9059737	6.3247035	
254	64516	16387064	15.9373775	6.3330256	
255	65025	16581375	15.9687194	6.3413257	
256	65536	16777216	16.0000000	6.3496042	
257	66049	16974593	16.0312195	6.3578611	
258	66564	17173512	16.0623784	6.3660968	
259	67081	17373979	16.0934769	6.3743111	
260	67600	17576000	16.1245155	6.3825043	
261	68121	17779581	16.1554944	6.3906765	
262	68644	17984728	16.1864141	6.3988279	
263	69169	18191447	16.2172747	6.4069585	
264	69696	18399744	16.2480768	6.4150687	
265	70225	18609625	16.2788206	6.4231583	
266	70756	18821096	16.3095064	6.4312276	
267	71289	19034163	16.3401346	6.4392767	
268	71824	19248832	16.3707055	6.4473057	
269	72361	19465109	16.4012195	6.4553148	
270	72900	19683000	16.4316767	6.4633041	
271	73441	19902511	16.4620776	6.4712736	
272	73984	20123648	16.4924225	6.4792236	
273	74529	20346417	16.5227116	6.4871541	
274	75076	20570824	16.5529454	6.4950653	
275	75625	20796875	16.5831240	6.5029572	
276	76176	21024576	16.6132477	6.5108300	
277	76729	21253933	16.6433170	6.5186839	
278	77284	21484952	16.6733320	6.5265189	
279	77841	21717639	16.7032931	6.5343351	
280	78400	21952000	16.7332005	6.5421326	
281	78961	22188041	16.7630546	6.5499116	
282	79524	22425768	16.7928556	6.5576722	
283	80089	22665187	16.8226038	6.5654144	
284	80656	22906304	16.8522995	6.5731385	
285	81225	23149125	16.8819430	6.5808443	
286	81796	23393656	16.9115345	6.5885323	
287	82369	23639903	16.9410743	6.5962023	
288	82944	23887872	16.9705627	6.6038545	
289	83521	24137569	17.0000000	6.6114890	
290	84100	24389000	17.0293864	6.6191060	
291	84681	24642171	17.0587221	6.6267054	
292	85264	24897088	17.0880075	6.6342874	
293	85849	25153757	17.1172428	6.6418522	
294	86436	25412184	17.1464282	6.6493998	
295	87025	25672375	17.1755640	6.6569302	
296	87616	25934336	17.2046505	6.6644437	
297	88209	26198073	17.2336879	6.6719403	
298	88804	26463592	17.2626765	6.6794200	
299	89401	26730899	17.2916165	6.6868831	
300	90000	27000000	17.3205081	6.6943295	

es.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
)1	27270901	17.3493516	6.7017593	.003322259
)4	27543608	17.3781472	6.7091729	.003311258
)9	27818127	17.4068952	6.7165700	.003300330
,6	28094464	17.4355958	6.7239508	.003289474
)5	28372625	17.4642492	6.7313155	.003278689
)8	28652616	17.4928557	6.7386641	.003267974
)9	28934443	17.5214155	6.7459967	.003257329
)4	29218112	17.5499288	6.7533134	.003246753
)1	29503629	17.5783958	6.7606143	.003236246
)0	29791000	17.6068169	6.7678995	.003225806
)1	30080231	17.6351921	6.7751690	.003215434
)4	30371328	17.6635217	6.7824229	.003205128
)9	30664297	17.6918060	6.7896613	.003194888
)6	30959144	17.7200451	6.7968844	.003184713
)5	31255875	17.7482393	6.8040921	.003174603
)8	31554496	17.7763888	6.8112847	.003164557
)9	31855013	17.8044938	6.8184620	.003154574
)4	32157432	17.8325545	6.8256242	.003144654
)1	32461759	17.8605711	6.8327714	.003134796
)0	32768000	17.8885438	6.8399037	.003125000
)1	33076161	17.9164729	6.8470213	.003115265
)4	33386248	17.9443584	6.8541240	.003105590
)9	33698267	17.9722008	6.8612120	.003095975
'6	34012224	18.0000000	6.8682855	.003086420
)5	34328125	18.0277564	6.8753443	.003076923
'6	34645976	18.0554701	6.8823888	.003067485
)9	34965783	18.0831413	6.8894188	.003058104
)4	35287552	18.1107703	6.8964345	.003048780
)1	35611289	18.1383571	6.9034359	.003039514
)0	35937000	18.1659021	6.9104232	.003030303
)1	36264691	18.1934054	6.9173964	.003021148
)4	36594368	18.2208672	6.9243556	.003012048
)9	36926037	18.2482876	6.9313008	.003003003
)6	37259704	18.2756669	6.9382321	.002994012
)5	37595375	18.3030052	6.9451496	.002985075
)6	37933056	18.3303028	6.9520533	.002976190
)9	38272753	18.3575598	6.9589434	.002967359
14	38614472	18.3847763	6 9658198	.002958580
21	38958219	18.4119526	6.9726826	.002949853
)0	39304000	18.4390889	6.9795321	.002941176
31	39651821	18.4661853	6.9863681	.002932551
34	40001688	18.4932420	6.9931906	.002923977
49	40353607	18.5202592	7.0000000	.002915452
36	40707534	18.5472370	7.0067962	.002906977
25	41063625	18.5741756	7.0135791	.002898551
16	41421736	18.6010752	7.0203490	.002890173
)9	41781923	18.6279360	7.0271058	.002881844
)4	42144192	18.6547581	7.0338497	.002873563
)1	42508549	18.6815417	7.0405806	.002865330
)0	42875000	18.7082869	7.0472987	.002857143
)1	43243551	18.7349940	7.0540041	.002849003
)4	43614208	18.7616630	7.0606967	.002840909
)9	43986977	18.7882942	7.0673767	.002832861
16	44361864	18.8148877	7.0740440	.002824859
25	44738875	18.8414437	7.0806988	.002816901
36	45118016	18.8679623	7.0873411	.002808989
49	45499293	18.8944436	7.0939709	.002801120
64	45882712	18.9208879	7.1005885	.002793296
81	46268279	18.9472953	7.1071937	.002785515
00	46656000	18.9736660	7.1137866	.002777778

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Re
241	58081	13997521	15.5241747	6.2230843	.0
242	58564	14172488	15.5563492	6.2316797	.0
243	59049	14348907	15.5884573	6.2402515	.0
244	59536	14526784	15.6204994	6.2487998	.0
245	60025	14706125	15.6524758	6.2573248	.0
246	60516	14886936	15.6843871	6.2658266	.0
247	61009	15069223	15.7162336	6.2743054	.0
248	61504	15252992	15.7480157	6.2827613	.0
249	62001	15438249	15.7797338	6.2911946	.0
250	62500	15625000	15.8113883	6.2996053	.0
251	63001	15813251	15.8429795	6.3079935	.0
252	63504	16003008	15.8745079	6.3163596	.0
253	64009	16194277	15.9059737	6.3247035	.0
254	64516	16387064	15.9373775	6.3330256	.0
255	65025	16581375	15.9687194	6.3413257	.0
256	65536	16777216	16.0000000	6.3496042	.0
257	66049	16974593	16.0312195	6.3578611	.0
258	66564	17173512	16.0623784	6.3660968	.0
259	67081	17373979	16.0934769	6.3743111	.0
260	67600	17576000	16.1245155	6.3825043	.0
261	68121	17779581	16.1554944	6.3906765	.0
262	68644	17984728	16.1864141	6.3988279	.0
263	69169	18191447	16.2172747	6.4069585	.0
264	69696	18399744	16.2480768	6.4150687	.0
265	70225	18609625	16.2788206	6.4231583	.0
266	70756	18821096	16.3095064	6.4312276	.0
267	71289	19034163	16.3401346	6.4392767	.0
268	71824	19248832	16.3707055	6.4473057	.0
269	72361	19465109	16.4012195	6.4553148	.0
270	72900	19683000	16.4316767	6.4633041	.0
271	73441	19902511	16.4620776	6.4712736	.0
272	73984	20123648	16.4924225	6.4792236	.0
273	74529	20346417	16.5227116	6.4871541	.0
274	75076	20570824	16.5529454	6.4950653	.0
275	75625	20796875	16.5831240	6.5029572	.0
276	76176	21024576	16.6132477	6.5108300	.0
277	76729	21253933	16.6433170	6.5186839	.0
278	77284	21484952	16.6733320	6.5265189	.0
279	77841	21717639	16.7032931	6.5343351	.0
280	78400	21952000	16.7332005	6.5421326	.0
281	78961	22188041	16.7630546	6.5499116	.0
282	79524	22425768	16.7928556	6.5576722	.0
283	80089	22665187	16.8226038	6.5654144	.0
284	80656	22906304	16.8522995	6.5731385	.0
285	81225	23149125	16.8819430	6.5808443	.0
286	81796	23393656	16.9115345	6.5885323	.0
287	82369	23639903	16.9410743	6.5962023	.0
288	82944	23887872	16.9705627	6.6038545	.0
289	83521	24137569	17.0000000	6.6114890	.0
290	84100	24389000	17.0293864	6.6191060	.0
291	84681	24642171	17.0587221	6.6267054	.0
292	85264	24897088	17.0880075	6.6342874	.0
293	85849	25153757	17.1172428	6.6418522	.0
294	86436	25412184	17.1464282	6.6493998	.0
295	87025	25672375	17.1755640	6.6569302	.0
296	87616	25934336	17.2046505	6.6644437	.0
297	88209	26198073	17.2336879	6.6719403	.0
298	88804	26463592	17.2626765	6.6794200	.0
299	89401	26730899	17.2916165	6.6868831	.0
300	90000	27000000	17.3205081	6.6943295	.0

Cubes.	Square Roots.	Cube Roots.	Reciprocals.
74618461	20.5182845	7.4948113	.002375297
75151448	20.5426386	7.5007406	.002369668
75686967	20.5669638	7.5066607	.002364066
76225024	20.5912603	7.5125715	.002358491
76765625	20.6155281	7.5184730	.002352941
77308776	20.6397674	7.5243652	.002347418
77854483	20.6639783	7.5302482	.002341920
78402752	20.6881609	7.5361221	.002336449
78953589	20.7123152	7.5419867	.002331002
79507000	20.7364414	7.5478423	.002325581
80062991	20.7605395	7.5536888	.002320186
80621568	20.7846097	7.5595263	.002314815
81182737	20.8086520	7.5653548	.002309469
81746504	20.8326667	7.5711743	.002304147
82312875	20.8566536	7.5769849	.002298851
82881856	20.8806130	7.5827865	.002293578
83453453	20.9045450	7.5885793	.002288330
84027672	20.9284495	7.5943633	.002283105
84604519	20.9523268	7.6001385	.002277904
85184000	20.9761770	7.6059049	.002272727
85766121	21.0000000	7.6116626	.002267574
86350888	21.0237960	7.6174116	.002262443
86938307	21.0475652	7.6231519	.002257336
87528384	21.0713075	7.6288837	.002252252
88121125	21.0950231	7.6346067	.002247191
88716536	21.1187121	7.6403213	.002242152
89314623	21.1423745	7.6460272	.002237136
89915392	21.1660105	7.6517247	.002232143
90518849	21.1896201	7.6574138	.002227171
91125000	21.2132034	7.6630943	.002222222
91733851	21.2367606	7.6687665	.002217295
92345408	21.2602916	7.6744303	.002212389
92959677	21.2837967	7.6800857	.002207506
93576664	21.3072758	7.6857328	.002202643
94196375	21.3307290	7.6913717	.002197802
94818816	21.3541565	7.6970023	.002192982
95443993	21.3775583	7.7026246	.002188184
96071912	21.4009346	7.7082388	.002183406
96702579	21.4242853	7.7138448	.002178649
97336000	21.4476106	7.7194426	.002173913
97972181	21.4709106	7.7250325	.002169197
98611128	21.4941853	7.7306141	.002164502
99252847	21.5174348	7.7361877	.002159827
99897344	21.5406592	7.7417532	.002155172
100544625	21.5638587	7.7473109	.002150538
101194696	21.5870331	7.7528606	.002145923
101847563	21.6101828	7.7584023	.002141328
102503232	21.6333077	7.7639361	.002136752
103161709	21.6564078	7.7694620	.002132196
103823000	21.6794834	7.7749801	.002127660
104487111	21.7025344	7.7804904	.002123142
105154048	21.7255610	7.7859928	.002118644
105823817	21.7435632	7.7914875	.002114165
106496424	21.7715411	7.7969745	.002109705
107171875	21.7944947	7.8024538	.002105263
107850176	21.8174242	7.8079254	.002100840
108531333	21.8403297	7.8133892	.002096436
109215352	21.8632111	7.8188456	.002092050
109902239	21.8860686	7.8242942	.002087683
110592000	21.9089023	7.8297353	.002083333

No.	Squares.	Cubes.	Square Roots.	Cube Roots.
481	231361	111284641	21.9317122	7.8351688
482	232324	111980168	21.9544984	7.8405949
483	233289	112678587	21.9772610	7.8460134
484	234256	113379904	22.0000000	7.8514244
485	235225	114084125	22.0227155	7.8568281
486	236196	114791256	22.0454077	7.8622242
487	237169	115501303	22.0680765	7.8676130
488	238144	116214272	22.0907220	7.8729944
489	239121	116930169	22.1133444	7.8783684
490	240100	117649000	22.1359436	7.8837352
491	241081	118370771	22.1585198	7.8890946
492	242064	119095488	22.1810730	7.8944468
493	243049	119823157	22.2036033	7.8997917
494	244036	120553784	22.2261108	7.9051294
495	245025	121287375	22.2485955	7.9104599
496	246016	122023936	22.2710575	7.9157832
497	247009	122763473	22.2934968	7.9210994
498	248004	123505992	22.3159136	7.9264085
499	249001	124251499	22.3383079	7.9317104
500	250000	125000000	22.3606798	7.9370053
501	251001	125751501	22.3830293	7.9422931
502	252004	126506008	22.4053565	7.9475739
503	253009	127263527	22.4276615	7.9528477
504	254016	128024064	22.4499443	7.9581144
505	255025	128787625	22.4722051	7.9633743
506	256036	129554216	22.4944438	7.9686271
507	257049	130323843	22.5166605	7.9738731
508	258064	131096512	22.5388553	7.9791122
509	259081	131872229	22.5610283	7.9843444
510	260100	132651000	22.5831796	7.9895697
511	261121	133432831	22.6053091	7.9947883
512	262144	134217728	22.6274170	8.0000000
513	263169	135005697	22.6495033	8.0052049
514	264196	135796744	22.6715681	8.0104032
515	265225	136590875	22.6936114	8.0155946
516	266256	137388096	22.7156334	8.0207794
517	267289	138188413	22.7376340	8.0259574
518	268324	138991832	22.7596134	8.0311287
519	269361	139798359	22.7815715	8.0362935
520	270400	140608000	22.8035085	8.0414515
521	271441	141420761	22.8254244	8.0466030
522	272484	142236648	22.8473193	8.0517479
523	273529	143055667	22.8691933	8.0568862
524	274576	143877824	22.8910463	8.0620180
525	275625	144703125	22.9128785	8.0671432
526	276678	145531578	22.9346899	8.0722620
527	277729	146363183	22.9564806	8.0773743
528	278784	147197952	22.9782506	8.0824800
529	279841	148035889	23.0000000	8.0875794
530	280900	148877000	23.0217289	8.0926723
531	281961	149721291	23.0434372	8.0977589
532	283024	150568768	23.0651252	8.1028390
533	284089	151419437	23.0867928	8.1079128
534	285156	152273304	23.1084400	8.1129803
535	286225	153130375	23.1300670	8.1180414
536	287296	153990656	23.1516738	8.1230962
537	288369	154854153	23.1732605	8.1281447
538	289444	155720872	23.1948270	8.1331870
539	290521	156590819	23.2163735	8.1382230
540	291600	157464000	23.2379001	8.1432529

Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
292681	158340421	23.2594067	8.1482765	.001848429
293764	159220088	23.2808935	8.1532939	.001845018
294849	160103007	23.3023604	8.1583051	.001841621
295936	160989184	23.3238076	8.1633102	.001838235
297025	161878625	23.3452351	8.1683092	.001834862
298116	162771336	23.3666429	8.1733020	.001831502
299209	163667323	23.3880311	8.1782888	.001828154
300304	164566592	23.4093998	8.1832695	.001824818
301401	165469149	23.4307490	8.1882441	.001821494
302500	166375000	23.4520788	8.1932127	.001818182
303601	167284151	23.4733892	8.1981753	.001814882
304704	168196608	23.4946802	8.2031319	.001811594
305809	169112377	23.5159520	8.2080825	.001808318
306916	170031464	23.5372046	8.2130271	.001805054
308025	170953875	23.5584380	8.2179657	.001801802
309136	171879616	23.5796522	8 2228985	.001798561
310249	172808693	23.6008474	8.2278254	.001795332
311364	173741112	23.6220236	8.2327463	.001792115
312481	174676879	23.6431808	8.2376614	.001788909
313600	175616000	23.6643191	8.2425706	.001785714
314721	176558481	23.6854386	8.2474740	.001782531
315844	177504328	23.7065392	8.2523715	.001779359
316969	178453547	23.7276210	8.2572633	.001776199
318096	179406144	23.7486842	8.2621492	.001773050
319225	180362125	23.7697286	8.2670294	.001769912
320356	181321496	23.7907545	8.2719039	.001766784
321489	182284263	23.8117618	8.2767726	.001763668
322624	183250432	23.8327506	8.2816355	.001760563
323761	184220009	23.8537209	8.2864928	.001757469
324900	185193000	23.8746728	8.2913444	.001754386
326041	186169411	23.8956063	8.2961903	.001751313
327184	187149248	23.9165215	8.3010304	.001748252
328329	188132517	23.9374184	8.3058651	.001745201
329476	189119224	23.9582971	8.3106941	.001742160
330625	190109375	23.9791576	8 3155175	.001739130
331776	191102976	24.0000000	8.3203353	.001736111
332929	192100033	24.0208243	8.3251475	.001733102
334084	193100552	24.0416306	8.3299542	.001730104
335241	194104539	24.0624188	8.3347553	.001727116
336400	195112000	24.0831891	8 3395509	.001724138
337561	196122941	24.1039416	8.3443410	.001721170
338724	197137368	24.1246762	8.3491256	.001718213
339889	198155287	24.1453929	8.3539047	.001715266
341056	199176704	24.1660919	8.3586784	.001712329
342225	200201625	24.1867732	8.3634466	.001709402
343396	201230056	24.2074369	8.3682095	.001706485
344569	202262003	24.2280829	8.3729668	.001703578
345744	203297472	24.2487113	8.3777188	.001700680
346921	204336469	24.2693222	8.3824653	.001697793
348100	205379000	24.2899156	8.3872065	.001694915
349281	206425071	24.3104916	8.3919423	.001692047
350464	207474688	24.3310501	8.3966729	.001689189
351649	208527857	24 3515913	8.4013981	.001686341
352836	209584584	24.3721152	8.4061180	.001683502
354025	210644875	24.3926218	8 4108326	.001680672
355216	211708736	24.4131112	8.4155419	.001677852
356409	212776173	24.4335834	8.4202460	.001675042
357604	213847192	24.4540385	8.4249448	.001672241
358801	214921799	24.4744765	8.4296383	.001669449
360000	216000000	24.4948974	8.4343267	.001666667

No.	Squares.	Cubes.	Square Roots.	Cube Roots.
601	361201	217081801	24.5153013	8.4390098
602	362404	218167208	24.5356883	8.4436877
603	363609	219256227	24.5560583	8.4483605
604	364816	220348864	24.5764115	8.4530281
605	366025	221445125	24.5967478	8.4576906
606	367236	222545016	24.6170673	8.4623479
607	368449	223648543	24.6373700	8.4670001
608	369664	224755712	24.6576560	8.4716471
609	370881	225866529	24.6779254	8.4762892
610	372100	226981000	24.6981781	8.4809261
611	373321	228099131	24.7184142	8.4855579
612	374544	229220928	24.7386338	8.4901848
613	375769	230346397	24.7588368	8.4948065
614	376996	231475544	24.7790234	8.4994233
615	378225	232608375	24.7991935	8.5040350
616	379456	233744896	24.8193473	8.5086417
617	380689	234885113	24.8394847	8.5132435
618	381924	236029032	24.8596058	8.5178403
619	383161	237176659	24.8797106	8.5224321
620	384400	238328000	24.8997992	8.5270189
621	385641	239483061	24.9198716	8.5316009
622	386884	240641848	24.9399278	8.5361780
623	388129	241804367	24.9599679	8.5407501
624	389376	242970624	24.9799920	8.5453173
625	390625	244140625	25.0000000	8.5498797
626	391876	245314376	25.0199920	8.5544372
627	393129	246491883	25.0399681	8.5589899
628	394384	247673152	25.0599282	8.5635377
629	395641	248858189	25.0798724	8.5680807
630	396900	250047000	25.0998008	8.5726189
631	398161	251239591	25.1197134	8.5771523
632	399424	252435968	25.1396102	8.5816809
633	400689	253636137	25.1594913	8.5862047
634	401956	254840104	25.1793566	8.5907238
635	403225	256047875	25.1992063	8.5952380
636	404496	257259456	25.2190404	8.5997476
637	405769	258474853	25.2388589	8.6042525
638	407044	259694072	25.2586619	8.6087526
639	408321	260917119	25.2784493	8.6132480
640	409600	262144000	25.2982213	8.6177388
641	410881	263374721	25.3179778	8.6222248
642	412164	264609288	25.3377189	8.6267063
643	413449	265847707	25.3574447	8.6311830
644	414736	267089984	25.3771551	8.6356551
645	416025	268336125	25.3968502	8.6401226
646	417316	269586136	25.4165301	8.6445855
647	418609	270840023	25.4361947	8.6490437
648	419904	272097792	25.4558441	8.6534974
649	421201	273359449	25.4754784	8.6579465
650	422500	274625000	25.4950976	8.6623911
651	423801	275894451	25.5147016	8.6668310
652	425104	277167808	25.5342907	8.6712665
653	426409	278445077	25.5538647	8.6756974
654	427716	279726264	25.5734237	8.6801237
655	429025	281011375	25.5929678	8.6845456
656	430336	282300416	25.6124969	8.6889630
657	431649	283593393	25.6320112	8.6933759
658	432964	284890312	25.6515107	8.6977843
659	434281	286191179	25.6709953	8.7021882
660	435600	287496000	25.6904652	8.7065877

).	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
	436921	288804781	25.7099203	8.7109827	.001512859
	438244	290117528	25.7293607	8.7153734	.001510574
	439569	291434247	25.7487864	8.7197596	.001508296
	440896	292754944	25.7681975	8.7241414	.001506024
	442225	294079625	25.7875939	8.7285187	.001503759
	443556	295408296	25.8069758	8.7328918	.001501502
	444889	296740963	25.8263431	8.7372604	.001499250
	446224	298077632	25.8456960	8.7416246	.001497006
	447561	299418309	25.8650343	8.7459846	.001494768
0	448900	300763000	25.8843582	8.7503401	.001492537
	450241	302111711	25.9036677	8.7546913	.001490313
	451584	303464448	25.9229628	8.7590383	.001488095
	452929	304821217	25.9422435	8.7633809	.001485884
	454276	306182024	25.9615100	8.7677192	.001483680
	455625	307546875	25.9807621	8.7720532	.001481481
	456976	308915776	26.0000000	8.7763830	.001479290
	458329	310288733	26.0192237	8.7807084	.001477105
	459684	311665752	26.0384331	8.7850296	.001474926
	461041	313046839	26.0576284	8.7893466	.001472754
0	462400	314432000	26.0768096	8.7936593	.001470588
	463761	315821241	26.0959767	8.7979679	.001468429
	465124	317214568	26.1151297	8.8022721	.001466276
	466489	318611987	26.1342687	8.8065722	.001464129
	467856	320013504	26.1533937	8.8108681	.001461988
	469225	321419125	26.1725047	8.8151598	.001459854
	470596	322828856	26.1916017	8.8194474	.001457726
	471969	324242703	26.2106848	8.8237307	.001455604
	473344	325660672	26.2297541	8.8280099	.001453488
	474721	327082769	26.2488095	8.8322850	.001451379
0	476100	328509000	26.2678511	8.8365559	.001449275
	477481	329939371	26.2868789	8.8408227	.001447178
	478864	331373888	26.3058929	8.8450854	.001445087
	480249	332812557	26.3248932	8.8493440	.001443001
	481636	334255384	26.3438797	8.8535985	.001440922
	483025	335702375	26.3628527	8.8578489	.001438849
	484416	337153536	26.3818119	8.8620952	.001436782
	485809	338608873	26.4007576	8.8663375	.001434720
	487204	340068392	26.4196896	8.8705757	.001432665
	488601	341532099	26.4386081	8.8748099	.001430615
0	490000	343000000	26.4575131	8.8790400	.001428571
	491401	344472101	26.4764046	8.8832661	.001426534
	492804	345948408	26.4952826	8.8874882	.001424501
	494209	347428927	26.5141472	8.8917063	.001422475
	495616	348913664	26.5329983	8.8959204	.001420455
	497025	350402625	26.5518361	8.9001304	.001418440
	498436	351895816	26.5706605	8.9043366	.001416431
	499849	353393243	26.5894716	8.9085387	.001414427
	501264	354894912	26.6082694	8.9127369	.001412429
	502681	356400829	26.6270539	8.9169311	.001410437
0	504100	357911000	26.6458252	8.9211214	.001408451
	505521	359425431	26.6645833	8.9253078	.001406470
	506944	360944128	26.6833281	8.9294902	.001404494
	508369	362467097	26.7020598	8.9336687	.001402525
	509796	363994344	26.7207784	8.9378433	.001400560
	511225	365525875	26.7394839	8.9420140	.001399501
	512656	367061696	26.7581763	8.9461809	.001396648
	514089	368601813	26.7768557	8.9503438	.001394700
	515524	370146232	26.7955220	8.9545029	.001392758
	516961	371694959	26.8141754	8.9586581	.001390821
0	518400	373248000	26.8328157	8.9628095	.001388889

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Re
721	519841	374805361	26.8514432	8.9669570	.0
722	521284	376367048	26.8700577	8.9711007	.0
723	522729	377933067	26.8886593	8.9752406	.0
724	524176	379503424	26.9072481	8.9793766	.0
725	525625	381078125	26.9258240	8.9835089	.0
726	527076	382657176	26.9443872	8.9876373	.0
727	528529	384240583	26.9629375	8.9917620	.0
728	529984	385828352	26.9814751	8.9958829	.0
729	531441	387420489	27.0000000	9.0000000	.0
730	532900	389017000	27.0185122	9.0041134	.0
731	534361	390617891	27.0370117	9.0082229	.0
732	535824	392223168	27.0554985	9.0123288	.0
733	537289	393832837	27.0739727	9.0164309	.0
734	538756	395446904	27.0924344	9.0205293	.0
735	540225	397065375	27.1108834	9.0246239	.0
736	541696	398688256	27.1293199	9.0287149	.0
737	543169	400315553	27.1477439	9.0328021	.0
738	544644	401947272	27.1661554	9.0368857	.0
739	546121	403583419	27.1845544	9.0409655	.0
740	547600	405224000	27.2029410	9.0450419	.0
741	549081	406869021	27.2213152	9.0491142	.0
742	550564	408518488	27.2396769	9.0531831	.0
743	552049	410172407	27.2580263	9.0572482	.0
744	553536	411830784	27.2763634	9.0613098	.0
745	555025	413493625	27.2946881	9.0653677	.0
746	556516	415160936	27.3130006	9.0694220	.0
747	558009	416832723	27.3313007	9.0734726	.0
748	559504	418508992	27.3495887	9.0775197	.0
749	561001	420189749	27.3678644	9.0815631	.0
750	562500	421875000	27.3861279	9.0856030	.0
751	564001	423564751	27.4043792	9.0896392	.0
752	565504	425259008	27.4226184	9.0936719	.0
753	567009	426957777	27.4408455	9.0977010	.0
754	568516	428661064	27.4590604	9.1017265	.0
755	570025	430368875	27.4772633	9.1057485	.0
756	571536	432081216	27.4954542	9.1097669	.0
757	573049	433798093	27.5136330	9.1137818	.0
758	574564	435519512	27.5317998	9.1177931	.0
759	576081	437245479	27.5499546	9.1218010	.0
760	577600	438976000	27.5680975	9.1258053	.0
761	579121	440711081	27.5862284	9.1298061	.0
762	580644	442450728	27.6043475	9.1338034	.0
763	582169	444194947	27.6224546	9.1377971	.0
764	583696	445943744	27.6405499	9.1417874	.0
765	585225	447697125	27.6586334	9.1457742	.0
766	586756	449455096	27.6767050	9.1497576	.0
767	588289	451217663	27.6947648	9.1537375	.0
768	589824	452984832	27.7128129	9.1577139	.0
769	591361	454756609	27.7308492	9.1616869	.0
770	592900	456533000	27.7488739	9.1656565	.0
771	594441	458314011	27.7668868	9.1696225	.0
772	595984	460099648	27.7848880	9.1735852	.0
773	597529	461889917	27.8028775	9.1775445	.0
774	599076	463684824	27.8208555	9.1815003	.0
775	600625	465484375	27.8388218	9.1854527	.0
776	602176	467288576	27.8567766	9.1894018	.0
777	603729	469097433	27.8747197	9.1933474	.0
778	605284	470910952	27.8926514	9.1972897	.0
779	606841	472729139	27.9105715	9.2012286	.0
780	608400	474552000	27.9284801	9.2051641	.0

Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
609961	476379541	27.9463772	9.2090962	.001280410
611524	478211768	27.9642629	9.2130250	.001278772
613089	480048687	27.9821372	9.2169505	.001277139
614656	481890304	28.0000000	9.2208726	.001275510
616225	483736625	28.0178515	9.2247914	.001273885
617796	485587656	28.0356915	9.2287068	.001272265
619369	487443403	28.0535203	9.2326189	.001270648
620944	489303872	28.0713377	9.2365277	.001269036
622521	491169069	28.0891438	9.2404333	.001267427
624100	493039000	28.1069386	9.2443355	.001265823
625681	494913671	28.1247222	9.2482344	.001264223
627264	496793088	28.1424946	9.2521300	.001262626
628849	498677257	28.1602557	9.2560224	.001261034
630436	500566184	28.1780056	9.2599114	.001259446
632025	502459875	28.1957444	9.2637973	.001257862
633616	504358336	28.2134720	9.2676798	.001256281
635209	506261573	28.2311884	9.2715592	.001254705
636804	508169592	28.2488938	9.2754352	.001253133
638401	510082399	28.2665881	9.2793081	.001251564
640000	512000000	28.2842712	9.2831777	.001250000
641601	513922401	28.3019434	9.2870440	.001248439
643204	515849608	28.3196045	9.2909072	.001246883
644809	517781627	28.3372546	9.2947671	.001245330
646416	519718464	28.3548938	9.2986239	.001243781
648025	521660125	28.3725219	9.3024775	.001242236
649636	523606616	28.3901391	9.3063278	.001240695
651249	525557943	28.4077454	9.3101750	.001239157
652864	527514112	28.4253408	9.3140190	.001237624
654481	529475129	28.4429253	9.3178599	.001236094
656100	531441000	28.4604989	9.3216975	.001234568
657721	533411731	28.4780617	9.3255320	.001233046
659344	535387328	28.4956137	9.3293634	.001231527
660969	537367797	28.5131549	9.3331916	.001230012
662596	539353144	28.5306852	9.3370167	.001228501
664225	541343375	28.5482048	9.3408386	.001226994
665856	543338496	28.5657137	9.3446575	.001225490
667489	545338513	28.5832119	9.3484731	.001223990
669124	547343432	28.6006993	9.3522857	.001222494
670761	549353259	28.6181760	9.3560952	.001221001
672400	551368000	28.6356421	9.3599016	.001219512
674041	553387661	28.6530976	9.3637049	.001218027
675684	555412248	28.6705424	9.3675051	.001216545
677329	557441767	28.6879766	9.3713022	.001215067
678976	559476224	28.7054002	9.3750963	.001213592
680625	561515625	28.7228132	9.3788873	.001212121
682276	563559976	28.7402157	9.3826752	.001210654
683929	565609283	28.7576077	9.3864600	.001209190
685584	567663552	28.7749891	9.3902419	.001207729
687241	569722789	28.7923601	9.3940206	.001206273
688900	571787000	28.8097206	9.3977964	.001204819
690561	573856191	28.8270706	9.4015691	.001203369
692224	575930368	28.8444102	9.4053387	.001201923
693889	578009537	28.8617394	9.4091054	.001200480
695556	580093704	28.8790582	9.4128690	.001199041
697225	582182875	28.8963666	9.4166297	.001197605
698896	584277056	28.9136646	9.4203873	.001196172
700569	586376253	28.9309523	9.4241420	.001194743
702244	588480472	28.9482297	9.4278936	.001193317
703921	590589719	28.9654967	9.4316423	.001191895
705600	592704000	28.9827535	9.4353880	.001190476

No.	Squares.	Cubes.	Square Roots.	Cube Roots.
841	707281	594823321	29·0000000	9·4391307
842	708964	596947688	29·0172363	9·4428704
843	710649	599077107	29·0344623	9·4466072
844	712336	601211584	29·0516781	9·4503410
845	714025	603351125	29·0688837	9·4540719
846	715716	605495736	29·0860791	9·4577999
847	717409	607645423	29·1032644	9·4615249
848	719104	609800192	29·1204396	9·4652470
849	720801	611960049	29·1376046	9·4689661
850	722500	614125000	29·1547595	9·4726824
851	724201	616295051	29·1719043	9·4763957
852	725904	618470208	29·1890390	9·4801061
853	727609	620650477	29·2061637	9·4838136
854	729316	622835864	29·2232784	9·4875182
855	731025	625026375	29·2403830	9·4912200
856	732736	627222016	29·2574777	9·4949188
857	734449	629422793	29·2745623	9·4986147
858	736164	631628712	29·2916370	9·5023078
859	737881	633839779	29·3087018	9·5059980
860	739600	636056000	29·3257566	9·5096854
861	741321	638277381	29·3428015	9·5133699
862	743044	640503928	29·3598365	9·5170515
863	744769	642735647	29·3768616	9·5207303
864	746496	644972544	29·3938769	9·5244063
865	748225	647214625	29·4108823	9·5280794
866	749956	649461896	29·4278779	9·5317497
867	751689	651714363	9·4448637	9·5354172
868	753424	653972032	29·4618397	9·5390818
869	755161	656234909	29·4788059	9·5427437
870	756900	658503000	29·4957624	9·5464027
871	758641	660776311	29·5127091	9·5500589
872	760384	663054848	29·5296461	9·5537123
873	762129	665338617	29·5465734	9·5573630
874	763876	667627624	29·5634910	9·5610108
875	765625	669921875	29·5803989	9·5646559
876	767376	672221376	29·5972972	9·5682982
877	769129	674526133	29·6141858	9·5719377
878	770884	676836152	29·6310648	9·5755745
879	772641	679151439	29·6479342	9·5792085
880	774400	681472000	29·6647939	9·5828397
881	776161	683797841	29·6816442	9·5864682
882	777924	686128968	29·6984848	9·5900939
883	779689	688465387	29·7153159	9·5937169
884	781456	690807104	29·7321375	9·5973373
885	783225	693154125	29·7489496	9·6009548
886	784996	695506456	29·7657521	9·6045696
887	786769	697864103	29·7825452	9·6081817
888	788544	700227072	29·7993289	9·6117911
889	790321	702595369	29·8161030	9·6153977
890	792100	704969000	29·8328678	9·6190017
891	793881	707347971	29·8496231	9·6226030
892	795664	709732288	29·8663690	9·6262016
893	797449	712121957	29·8831056	9·6297975
894	799236	714516984	29·8998328	9·6333907
895	801025	716917375	29·9165506	9·6369812
896	802816	719323136	29·9332591	9·6405690
897	804609	721734273	29·9499583	9·6441542
898	806404	724150792	29·9666481	9·6477367
899	808201	726572699	29·9833287	9·6513166
900	810000	729000000	30·0000000	9·6548938

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
901	811801	731432701	30·0166620	9·6584684	·001109878
902	813604	733870808	30·0333148	9·6620403	·001108647
903	815409	736314327	30·0499584	9·6656096	·001107420
904	817216	738763264	30·0665928	9·6691762	·001106195
905	819025	741217625	30·0832179	9·6727403	·001104972
906	820836	743677416	30·0998339	9·6763017	·001103753
907	822649	746142643	30·1164407	9·6798604	·001102536
908	824464	748613312	30·1330383	9·6834166	·001101322
909	826281	751089429	30·1496269	9·6869701	·001100110
910	828100	753571000	30·1662063	9·6905211	·001098901
911	829921	756058031	30·1827765	9·6940694	·001097695
912	831744	758550528	30·1993377	9·6976151	·001096491
913	833569	761048497	30·2158899	9·7011583	·001095290
914	835396	763551944	30·2324329	9·7046989	·001094092
915	837225	766060875	30·2489669	9·7082369	·001092896
916	839056	768575296	30·2654919	9·7117723	·001091703
917	840889	771095213	30·2820079	9·7153051	·001090513
918	842724	773620632	30·2985148	9·7188354	·001089325
919	844561	776151559	30·3150128	9·7223631	·001088139
920	846400	778688000	30·3315018	9·7258883	·001086957
921	848241	781229961	30·3479818	9·7294109	·001085776
922	850084	783777448	30·3644529	9·7329309	·001084599
923	851929	786330467	30·3809151	9·7364484	·001083423
924	853776	788889024	30·3973683	9·7399634	·001082251
925	855625	791453125	30·4138127	9·7434758	·001081081
926	857476	794022776	30·4302481	9·7469857	·001079914
927	859329	796597983	30·4466747	9·7504930	·001078749
928	861184	799178752	30·4630924	9·7539979	·001077586
929	863041	801765089	30·4795013	9·7575002	·001076426
930	864900	804357000	30·4959014	9·7610001	·001075269
931	866761	806954491	30·5122926	9·7644974	·001074114
932	868624	809557568	30·5286750	9·7679922	·001072961
933	870489	812166237	30·5450487	9·7714845	·001071811
934	872356	814780504	30·5614136	9·7749743	·001070664
935	874225	817400375	30·5777697	9·7784616	·001069519
936	876096	820025856	30·5941171	9·7819466	·001068376
937	877969	822656953	30·6104557	9·7854288	·001067236
938	879844	825293672	30·6267857	9·7889087	·001066098
939	881721	827936019	30·6431069	9·7923861	·001064963
940	883600	830584000	30·6594194	9·7958611	·001063830
941	885481	833237621	30·6757233	9·7993336	·001062699
942	887364	835896888	30·6920185	9·8028036	·001061571
943	889249	838561807	30·7083051	9·8062711	·001060445
944	891136	841232384	30·7245830	9·8097362	·001059322
945	893025	843908625	30·7408523	9·8131989	·001058201
946	894916	846590536	30·7571130	9·8166591	·001057082
947	896809	849278123	30·7733651	9·8201169	·001055966
948	898704	851971392	30·7896086	9·8235723	·001054852
949	900601	854670349	30·8058436	9·8270252	·001053741
950	902500	857375000	30·8220700	9·8304757	·001052632
951	904401	860085351	30·8382879	9·8339238	·001051525
952	906304	862801408	30·8544972	9·8373695	·001050420
953	908209	865523177	30·8706981	9·8408127	·001049318
954	910116	868250664	30·8868904	9·8442536	·001048218
955	912025	870983875	30·9030743	9·8476920	·001047120
956	913936	873722816	30·9192497	9·8511280	·001046025
957	915849	876467493	30·9354166	9·8545617	·001044932
958	917764	879217912	30·9515751	9·8579929	·001043841
959	919681	881974079	30·9677251	9·8614218	·001042753
960	921600	884736000	30·9838668	9·8648483	·001041667

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Recip
961	923521	887503681	31.0000000	9.8682724	.001
962	925444	890277128	31.0161248	9.8716941	.001
963	927369	893056347	31.0322413	9.8751135	.001
964	929296	895841344	31.0483494	9.8785305	.001
965	931225	898632125	31.0644491	9.8819451	.001
966	933156	901428696	31.0805405	9.8853574	.001
967	935089	904231063	31.0966236	9.8887673	.001
968	937024	907039232	31.1126984	9.8921749	.001
969	938961	909853209	31.1287648	9.8955801	.001
970	940900	912673000	31.1448230	9.8989830	.001
971	942841	915498611	31.1608729	9.9023835	.001
972	944784	918330048	31.1769145	9.9057817	.001
973	946729	921167317	31.1929479	9.9091776	.001
974	948676	924010424	31.2089731	9.9125712	.001
975	950625	926859375	31.2249900	9.9159624	.001
976	952576	929714176	31.2409987	9.9193513	.001
977	954529	932574833	31.2569992	9.9227379	.001
978	956484	935441352	31.2729915	9.9261222	.001
979	958441	938313739	31.2889757	9.9295042	.001
980	960400	941192000	31.3049517	9.9328839	.001
981	962361	944076141	31.3209195	9.9362613	.001
982	964324	946966168	31.3368792	9.9396363	.001
983	966289	949862087	31.3528308	9.9430092	.001
984	968256	952763904	31.3687743	9.9463797	.001
985	970225	955671625	31.3847097	9.9497479	.001
986	972196	958585256	31.4006369	9.9531138	.001
987	974169	961504803	31.4165561	9.9564775	.001
988	976144	964430272	31.4324673	9.9598389	.001
989	978121	967361669	31.4483704	9.9631981	.001
990	980100	970299000	31.4642654	9.9665549	.001
991	982081	973242271	31.4801525	9.9699095	.001
992	984064	976191488	31.4960315	9.9732619	.001
993	986049	979146657	31.5119025	9.9766120	.001
994	988036	982107784	31.5277655	9.9799599	.001
995	990025	985074875	31.5436206	9.9833055	.001
996	992016	988047936	31.5594677	9.9866488	.001
997	994009	991026973	31.5753068	9.9899900	.001
998	996004	994011992	31.5911380	9.9933289	.001
999	998001	997002999	31.6069613	9.9966656	.001
1000	1000000	1000000000	31.6227766	10.0000000	.001
1001	1002001	1003003001	31.6385840	10.0033322	.000
1002	1004004	1006012008	31.6543836	10.0066622	.000
1003	1006009	1009027027	31.6701752	10.0099899	.000
1004	1008016	1012048064	31.6859590	10.0133155	.000
1005	1010025	1015075125	31.7017349	10.0166389	.000
1006	1012036	1018108216	31.7175030	10.0199601	.000
1007	1014049	1021147343	31.7332633	10.0232791	.000
1008	1016064	1024192512	31.7490157	10.0265958	.000
1009	1018081	1027243729	31.7647603	10.0299104	.000
1010	1020100	1030301000	31.7804972	10.0332228	.000
1011	1022121	1033364331	31.7962262	10.0365330	.000
1012	1024144	1036433728	31.8119474	10.0398410	.000
1013	1026169	1039509197	31.8276609	10.0431469	.000
1014	1028196	1042590744	31.8433666	10.0464506	.000
1015	1030225	1045678375	31.8590646	10.0497521	.000
1016	1032256	1048772096	31.8747549	10.0530514	.000
1017	1034289	1051871913	31.8904374	10.0563485	.000
1018	1036324	1054977832	31.9061123	10.0596435	.000
1019	1038361	1058089859	31.9217794	10.0629364	.000
1020	1040400	1061208000	31.9374388	10.0662271	.000

TABLE XVII.—CUBIC YARDS PER 100 FEET OF LEVEL SECTIONS. SLOPE 1 : 1.

Depth, d	Base 12 feet.	Base 14 feet.	Base 16 feet.	Base 18 feet.	Base 20 feet.	Base 28 feet.	Base 30 feet.	Base 32 feet.
1	48	56	63	70	78	107	115	122
2	104	119	133	148	163	222	237	252
3	167	189	211	233	256	344	367	389
4	237	267	296	326	356	474	504	533
5	315	352	389	426	463	611	648	685
6	400	444	489	533	578	756	800	844
7	493	544	596	648	700	907	959	1011
8	593	652	711	770	830	1067	1126	1185
9	700	767	833	900	967	1233	1300	1367
10	815	889	963	1037	1111	1407	1481	1556
11	937	1019	1100	1181	1263	1589	1670	1752
12	1067	1156	1244	1333	1422	1778	1867	1956
13	1204	1300	1396	1493	1589	1974	2070	2167
14	1348	1452	1556	1659	1763	2178	2281	2385
15	1500	1611	1722	1833	1944	2389	2500	2611
16	1659	1778	1896	2015	2133	2607	2726	2844
17	1826	1952	2078	2204	2330	2833	2959	3085
18	2000	2133	2267	2400	2533	3067	3200	3333
19	2181	2322	2463	2604	2744	3307	3448	3589
20	2370	2519	2667	2815	2963	3556	3704	3852
21	2567	2722	2878	3033	3189	3811	3967	4122
22	2770	2933	3096	3259	3422	4074	4237	4400
23	2981	3152	3322	3493	3663	4344	4515	4685
24	3200	3378	3556	3733	3911	4622	4800	4978
25	3426	3611	3796	3981	4167	4907	5093	5278
26	3659	3852	4044	4237	4430	5200	5393	5585
27	3900	4100	4300	4500	4700	5500	5700	5900
28	4148	4356	4563	4770	4978	5807	6015	6222
29	4404	4619	4833	5048	5263	6122	6337	6552
30	4667	4889	5111	5333	5556	6444	6667	6889
31	4937	5167	5396	5626	5856	6774	7004	7233
32	5215	5452	5689	5926	6163	7111	7348	7585
33	5500	5744	5989	6233	6478	7456	7700	7944
34	5793	6044	6296	6548	6800	7807	8059	8311
35	6093	6352	6611	6870	7130	8167	8426	8685
36	6400	6667	6933	7200	7467	8533	8800	9067
37	6715	6989	7263	7537	7811	8907	9181	9456
38	7037	7319	7600	7881	8163	9289	9570	9852
39	7367	7656	7944	8233	8522	9678	9967	10256
40	7704	8000	8296	8593	8889	10074	10370	10667
41	8048	8352	8656	8959	9263	10478	10781	11085
42	8400	8711	9022	9333	9644	10889	11200	11511
43	8759	9078	9396	9715	10033	11307	11626	11944
44	9126	9452	9778	10104	10430	11733	12059	12385
45	9500	9833	10167	10500	10833	12167	12500	12833
46	9881	10222	10563	10904	11244	12607	12948	13289
47	10270	10619	10967	11315	11663	13056	13404	13752
48	10667	11022	11378	11733	12089	13511	13867	14222
49	11070	11433	11796	12159	12522	13974	14337	14700
50	11481	11852	12222	12593	12963	14444	14815	15185
51	11900	12278	12656	13033	13411	14922	15300	15678
52	12326	12711	13096	13481	13867	15407	15793	16178
53	12759	13152	13544	13937	14330	15900	16293	16685
54	13200	13600	14000	14400	14800	16400	16800	17200
55	13648	14056	14463	14870	15278	16907	17315	17722
56	14104	14519	14933	15348	15763	17422	17837	18252
57	14567	14989	15411	15833	16256	17944	18367	18789
58	15037	15467	15896	16326	16756	18474	18904	19333
59	15515	15952	16389	16826	17263	19011	19448	19885
60	16000	16444	16889	17333	17778	19556	20000	20444

TABLE XVII.—CUBIC YARDS PER 100 FEET OF LEVEL

SECTIONS. SLOPE 1.5 : 1.

Depth d	Base 12 feet.	Base 14 feet.	Base 16 feet.	Base 18 feet.	Base 20 feet.	Base 28 feet.	Base 30 feet.	Base 32 feet.
1	50	57	65	72	80	109	117	124
2	111	126	141	156	170	230	244	259
3	183	206	228	250	272	361	383	406
4	267	296	326	356	385	504	533	563
5	361	398	435	472	509	657	694	731
6	467	511	556	600	644	822	867	911
7	583	635	687	739	791	998	1050	1102
8	711	770	830	889	948	1185	1244	1304
9	850	917	983	1050	1117	1383	1450	1517
10	1000	1074	1148	1222	1296	1593	1667	1741
11	1161	1243	1324	1406	1487	1813	1894	1976
12	1333	1422	1511	1600	1689	2044	2133	2222
13	1517	1613	1709	1806	1902	2287	2383	2480
14	1711	1815	1919	2022	2126	2541	2644	2748
15	1917	2028	2139	2250	2361	2806	2917	3028
16	2133	2252	2370	2489	2607	3081	3200	3319
17	2361	2487	2613	2739	2865	3369	3494	3620
18	2600	2733	2867	3000	3133	3667	3800	3933
19	2850	2991	3131	3272	3413	3976	4117	4257
20	3111	3259	3407	3556	3704	4296	4444	4593
21	3383	3539	3694	3850	4006	4628	4783	4939
22	3667	3830	3993	4156	4319	4970	5133	5296
23	3961	4131	4302	4472	4642	5324	5494	5665
24	4267	4444	4622	4800	4978	5689	5867	6044
25	4583	4769	4954	5139	5324	6065	6250	6435
26	4911	5104	5296	5489	5681	6452	6644	6837
27	5250	5450	5650	5850	6050	6850	7050	7250
28	5600	5807	6015	6222	6430	7259	7467	7674
29	5961	6176	6391	6606	6820	7680	7894	8109
30	6333	6556	6778	7000	7222	8111	8333	8556
31	6717	6946	7176	7406	7635	8554	8783	9013
32	7111	7348	7585	7822	8059	9007	9244	9481
33	7517	7761	8006	8250	8494	9472	9717	9961
34	7933	8185	8437	8689	8941	9948	10200	10452
35	8361	8620	8880	9139	9398	10435	10694	10954
36	8800	9067	9333	9600	9867	10933	11200	11467
37	9250	9524	9798	10072	10346	11443	11717	11991
38	9711	9993	10274	10556	10837	11963	12244	12526
39	10183	10472	10761	11050	11339	12494	12783	13072
40	10667	10963	11259	11556	11852	13037	13333	13630
41	11161	11465	11769	12072	12376	13591	13894	14198
42	11667	11978	12289	12600	12911	14156	14467	14778
43	12183	12502	12820	13139	13457	14731	15050	15369
44	12711	13037	13363	13689	14015	15319	15644	15970
45	13250	13585	13917	14250	14583	15917	16250	16583
46	13800	14141	14481	14822	15163	16526	16867	17207
47	14361	14709	15057	15406	15754	17146	17494	17843
48	14933	15289	15644	16000	16356	17778	18133	18489
49	15517	15880	16243	16606	16969	18420	18783	19146
50	16111	16481	16852	17222	17593	19074	19444	19815
51	16717	17094	17472	17850	18228	19739	20117	20494
52	17333	17719	18104	18489	18874	20415	20800	21185
53	17961	18354	18746	19139	19531	21102	21494	21887
54	18600	19000	19400	19800	20200	21800	22200	22600
55	19250	19657	20065	20472	20880	22509	22917	23324
56	19911	20326	20741	21156	21570	23230	23644	24059
57	20583	21006	21428	21850	22272	23961	24383	24805
	21267	21696	22126	22556	22985	24704	25133	25563
	21961	22398	22835	23272	23709	25457	25894	26331
	37	23111	23556	24000	24444	26222	26667	27111

.E XVII.—CUBIC YARDS PER 100 FEET OF LEVEL SECTIONS.
CORRECTIVE *PERCENTAGE* FACTORS.

To be applied when cross-sections are *not* level. See § 95.

Side slope = 1.5:1 or $\beta = 33°41'$.

s-e ce e.	b=12 feet and d=			b=20 feet and d=			b=30 feet and d=		
Per-ent	10 feet.	20 feet.	50 feet.	10 feet.	20 feet.	50 feet.	10 feet.	20 feet.	50 feet.
	%	%	%	%	%	%	%	%	%
9	1.9	1.8	1.8	2.1	1.8	1.8	2.3	2.0	1.8
18	8.2	7.7	7.5	9.0	8.0	7.6	10.0	8.4	7.7
27	21	20	19	23	21	20	26	22	20
36	46	44	43	51	45	44	57	48	44
57	327	324	317	358	336	321	400	354	326

Side slope = 1:1 or $\beta = 45°$.

s-e ce e.	b=12 feet and d=			b=20 feet and d=			b=30 feet and d=		
Per-ent	10 feet.	20 feet.	50 feet.	10 feet.	20 feet.	50 feet.	10 feet.	20 feet.	50 feet.
	%	%	%	%	%	%	%	%	%
9	0.9	0.8	0.8	1.0	0.9	0.8	1.2	0.9	0.8
18	3.7	3.4	3.2	4.3	3.6	3.3	5.0	4.0	3.4
27	9.0	8.2	7.8	10.3	8.7	8.0	12.1	9.5	8.2
36	18	16	15	20	17	16	24	19	16
57	58	53	50	67	56	51	78	61	53

TABLE XVIII.—ANNUAL CHARGE AGAINST A TIE, BASED ON THE ORIGINAL COST AND ASSUMED LIFE OF THE TIE; INTEREST COMPOUNDED AT 5%. (See § 217.)

Original cost of tie in cents.	Life of tie in years.																	
	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
20	7.34	5.64	4.62	3.94	3.46	3.09	2.81	2.59	2.41	2.26	2.13	2.02	1.93	1.85	1.77	1.71	1.65	1.60
25	9.18	7.05	5.77	4.92	4.32	3.87	3.52	3.24	3.01	2.82	2.66	2.53	2.41	2.31	2.22	2.14	2.07	2.01
30	11.02	8.46	6.93	5.91	5.18	4.64	4.22	3.89	3.61	3.38	3.19	3.03	2.89	2.77	2.66	2.57	2.48	2.41
35	12.85	9.87	8.08	6.90	6.05	5.42	4.92	4.53	4.21	3.95	3.73	3.54	3.37	3.23	3.10	2.99	2.90	2.81
40	14.69	11.28	9.24	7.88	6.91	6.19	5.63	5.18	4.81	4.51	4.26	4.04	3.85	3.79	3.55	3.42	3.31	3.21
45	16.52	12.69	10.39	8.87	7.78	6.96	6.33	5.83	5.42	5.08	4.79	4.55	4.34	4.15	3.99	3.85	3.72	3.61
50	18.36	14.10	11.55	9.85	8.64	7.74	7.03	6.48	6.02	5.64	5.32	5.05	4.82	4.61	4.43	4.28	4.14	4.01
55	20.20	15.51	12.70	10.84	9.51	8.51	7.74	7.12	6.62	6.21	5.86	5.56	5.30	5.07	4.88	4.71	4.55	4.41
60	22.03	16.92	13.86	11.82	10.37	9.28	8.44	7.77	7.22	6.77	6.39	6.06	5.78	5.54	5.32	5.13	4.96	4.81
65	23.87	18.33	15.01	12.81	11.23	10.06	9.14	8.42	7.83	7.33	6.92	6.57	6.26	6.00	5.77	5.56	5.38	5.22
70	25.70	19.74	16.17	13.79	12.10	10.83	9.85	9.07	8.43	7.90	7.45	7.07	6.74	6.46	6.21	5.99	5.79	5.62
75	27.54	21.15	17.32	14.78	12.96	11.60	10.55	9.72	9.03	8.46	7.98	7.58	7.22	6.92	6.65	6.42	6.20	6.02
80	29.38	22.56	18.48	15.76	13.83	12.38	11.25	10.38	9.63	9.03	8.52	8.08	7.71	7.38	7.10	6.84	6.62	6.42
85	31.21	23.97	19.63	16.75	14.69	13.15	11.96	11.01	10.23	9.59	9.05	8.59	8.19	7.84	7.54	7.27	7.03	6.82
90	33.05	25.38	20.79	17.73	15.55	13.92	12.66	11.66	10.84	10.15	9.58	9.09	8.67	8.30	7.98	7.70	7.45	7.22
95	34.88	26.79	21.94	18.71	16.42	14.70	13.37	12.30	11.44	10.72	10.12	9.60	9.15	8.76	8.42	8.12	7.88	7.62
100	36.72	28.20	23.10	19.70	17.28	15.47	14.07	12.95	12.04	11.28	10.65	10.10	9.63	9.23	8.87	8.55	8.27	8.02
For each 5	1.836	1.410	1.155	.985	.864	.774	.703	.648	.602	.564	.532	.505	.482	.461	.443	.428	.414	.401

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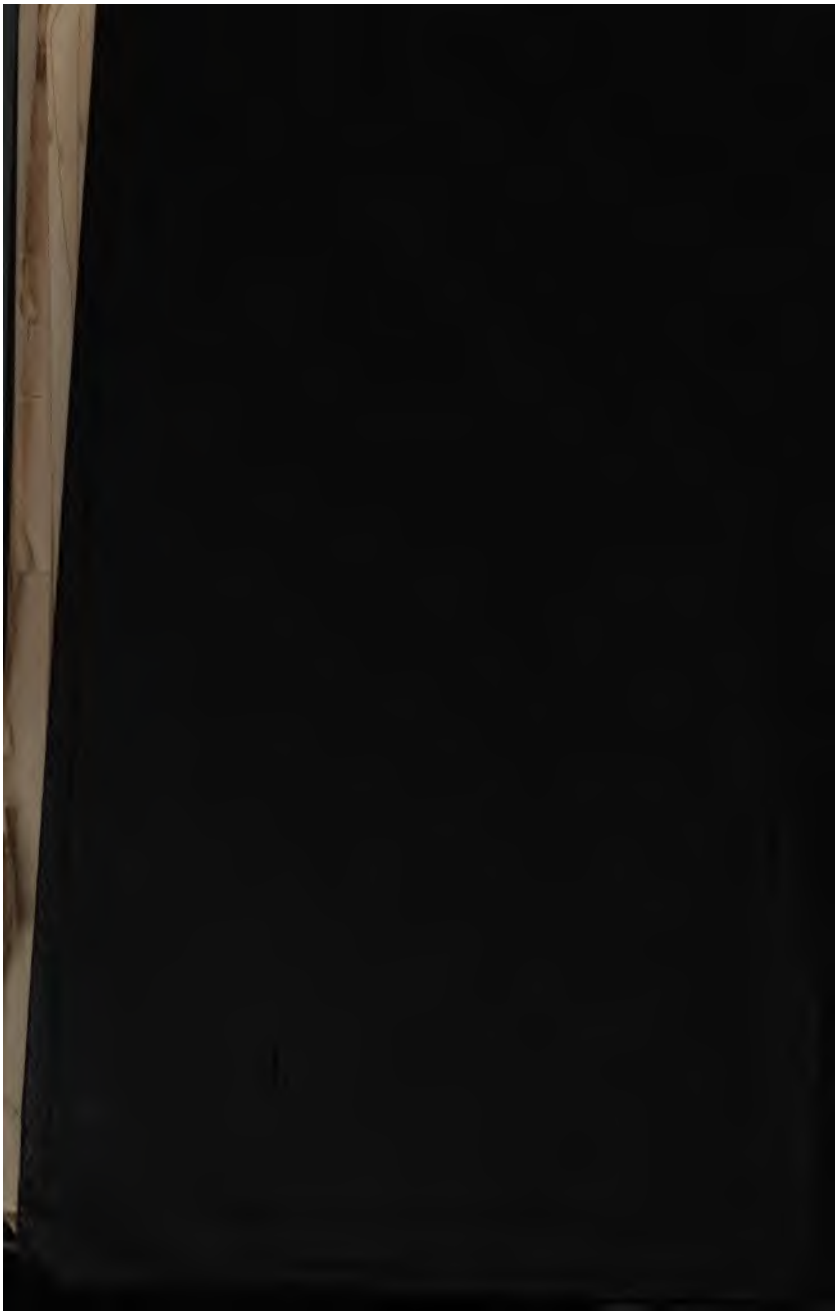
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